

(The dark side of) Monte Carlo tools for LHC physics

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Outline

- MC Status with SM examples/backgrounds
- DM searches at colliders
- Accurate simulations for simplified models
 - s-channel
 - t-channel
- Outlook





Accurate predictions for observables in hadronic collisions depend on the knowledge of both parton distribution functions and partonic cross sections.

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Perturbative expansion

- Leading order (LO) calculations typically give only the order of magnitude of cross sections and distributions
 - the scale of αs is not defined
 - jets partons: jet structure starts to appear only beyond LO
 - Born topology might not be leading at the LHC
- To obtain reliable predictions at least NLO is needed
- NNLO allows to quantify uncertainties

Furthermore:

- Resummation of the large logarithmic terms at phase space boundaries
- NLO ElectroWeak corrections ($\alpha_s^2 = \alpha_W$)
- Fully exclusive predictions available in terms of event simulation that can be used in experimental analysis





Predictive (NLO) Monte Carlo Generators

DEFINITION: A Monte Carlo generator is a code that can produce fully exclusive events (up to particle level) as distributed in Nature.

$$\sigma_X = \sum_{a,b} \int_0^1 dx_1 dx_2 f_a(x_1, \mu_F^2) f_b(x_2, \mu_F^2) \times \hat{\sigma}_{ab \to X}(x_1, x_2, \alpha_S(\mu_R^2), \frac{Q^2}{\mu_F^2}, \frac{Q^2}{\mu_R^2})$$



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A **predictive** MC associates a history to short-distance events obtained from a parton-level (at least) next-to-leading order calculation avoid double counting and keeping the formal fixed-order accuracy.



Predictions in QCD: before the LHC





Predictive MC: progress

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Predictive MC: progress





Predictive MC: progress







NLO calculations have 3+1 parts



Virtual part













- Loops have been for long the bottleneck of NLO computations
- Loop integration and integration of the reals over the m+1 phase space leads to Universal divergences that cancel when two contributions are added. A combination scheme is needed (Dipoles, FKS, Antennas)
- * A lot of work was necessary for each computation (see the MCFM project)



NLO+PS matching

Parton Shower Monte Carlo provide a simulation of all the stages of the hadronic collision: merge QCD matrix element + shower in the soft collinear approximation +hadronization model



The MC@NLO method has been extended to deal with samples of different jet multiplicities (merging) keeping NLO accuracy (FxFx in MG5aMC, ME@NLOPS in SHERPA).

The POWHEG method has been extended via the MINLO technique to obtain inclusive samples without merging scales.

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NLO+PS is widely available

- MadGraph5_aMC@NLO: Fully automatic framework, where all the elements of a NLO+PS computation in the SM and (BSM) are automatically generated. FxFx is available for merging at NLO. Loop-induced available.
- **POWHEG-BOX** Framework which allows to promote a standard NLO calculation into a MC at NLO generator. Very popular choice. Interfaces to automatic codes available. Tens of SM and several BSM processes implemented.
- SHERPA+OpenLoops Flexible framework MC@NLO CS dipoles, MEPS@NLO, Fully automatic except for the virtuals which are mostly currently provided by OpenLoops....
- New Entries: HERWIG7, WHIZARD (e+e-)



If the only tool you have is a hammer, it is tempting to treat everything as if it were a nail.

Abraham Maslow (1966)

Predictions in SM for the LHC: status 2016



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Representative SM results

Process	Relevance for DM
Z + jets : data vs MEPS@NLO (FxFx)	Mono and multi-jets + missing Et (X _{DM} X _{DM} + jets)
Z + ttbar : giant K-factors	ttbar + missing Et
Z + H : loop induced contribution (and giant K-factors)	mono Higgs
Processes with intermediate resonances at NLO	t-channel mediators at NLO

Z+jets comparison with ATLAS data



Exclusive jet rates for Z+jets. The NLO merged samples (Sherpa and aMC@NLO FxFx) have up to 2 jets at NLO and 3 (5 for Sherpa) jets at LO. The inclusive Z production at NLO+PS (red curve, left plot) falls short. Similar results hold for W+ jets.

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[Frederix et al., 2015]

Z+jets comparison with CMS data

ata vs HERWIG+ 5 data vs HERWIG++ CMS data vs PYTHIA8 10^{0} ਵੀਵੇ 10⁻¹ · 응 왕 10⁻¹ 1 do 0 db 10 10^{-2} 2.5 1.0 1.5 2.0 3.0 0.5 2.5 2.0 3.0 1.0 2.0 2.5 1.5 $\Delta \phi(Z,J1)$ [rad] $\Delta \phi(\text{ZJ1})$ [rad] $\Delta\phi(J1,J2)$ [rad]

Correlations among final state objects provide sensitive observables to check the ability of the NLO merged sample to describe the data compared to the NLO+PS inclusive sample.



ttZ at NLO(+PS)

Giant K-factors at high pt(tt) are due to a soft-collinear double log (same situation as in Vjj) especially for quark initiated production. ttVj is instead stable.

[Pagani et al, 2015]

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One-loop processes at the Born level

Many important processes at the LHC, mostly related to Higgs physics, are mediated by loops. Sometimes they are dominant (such as gg>h) more often they are part of NNLO corrections, but enhanced by gluon pdfs (gg>ZH, gg>VV).

One loop processes are Born level and therefore can be treated exactly as tree-level processes when merging to the shower. Sherpa+OpenLoops and MG5_aMC (and now also Herwig7) have the out-of-the box capability to have MC generators with merging implemented (see eg. [Hirschi and Mattelaer 2015]).

Going NLO+PS is straightforward (the IR structure is NLO) apart from the fact that very difficult two-loop amplitudes need to be calculated, which are presently mostly unknown.







Loop Induced : ZH

ZH production goes through Drell-Yan but also gg fusion. Results for LI processes merged at LO show the importance of $2\rightarrow 3$ for high-pT.







NLO with intermediate top resonances









[Cascioli et al., 2014]

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NLO+PS with intermediate resonances

[Jezo and Nason, 2015] [Frederix et al, 2016]

eee k

decay

production

Amplitudes squared are expressed in terms of sum of contributions from the BWs. Modified FKS methods have been proposed to deal correctly with double logs of Γ in POWHEG and MG5_aMC.



Shift in the reconstructed top mass due to differences in the final state radiation treatment of the b quark. Impact on the top mass reconstruction to be investigated.

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DM Models

Most natural approach to connect with DD.EFT's assume the only low-lying state is the DM.

Powerful

approach once the symmetries of the BSM are established.



C o m p l e t e models, can be most easily constrained u s i n g a plethora of information.

Ambitious

approach to BSM. The most famous example is SUSY.

Built around one particle (DM) and one interaction (mediator) possibly accessible at the LHC.

Essential

approach that encompasses both EFT and UV aspects



Search for DM at the LHC: s-channel mediators



Direct mediator search



 $Y_{SM/DM}$ = mediators mixing with SM Z or H



Search for DM at the LHC: s-channel mediators





Search for DM : s-channel mediators

Rules of the thumb for s-channel-mediators phenomenology at the LHC:

- Spin-0 couples to massive states (MFV) and to gluons/ photons via loops
- Spin-1 mediator couples to fermions (currents)
- Spin-2 couples to everything (energy-momentum)



Higgs-like pheno : loopinduced ggF and tt associated production.



Z'-like pheno, qqbar fusion, W/Z/gamma associated prod.



Graviton pheno.



s-channel spin-0 mediators

spin-0 (A/P)	MCFM	POWHEG-BOX	MG5aMC
X _{DM} X _{DM} +jets	I -jet at NLO ''HEFT''	I-jet at NLO+PS	NLO+PS and FxFx in the HEFT. MEPS@LO loop induced [Backovic et al., 2015]
$X_{\text{DM}} X_{\text{DM}}$ +γ	NLO (light quarks)		NLO+PS and FxFx in the HEFT. MEPS@LO loop induced
$X_{DM} X_{DM} + Z \text{ or } W$			NLO+PS and FxFx in the HEFT. MEPS@LO loop induced [M. Neubert, J. Wang, C. Zhang, 2015]
X _{DM} X _{DM} +heavy quarks			NLO+PS [Backovic et al., 2015]



s-channel spin-1 mediators

spin-1 (A/P)	MCFM	POWHEG-BOX	MG5aMC
X _{DM} X _{DM} +jets	I-jet at NLO	I-jet at NLO+PS	NLO+PS and FxFx [Backovic et al., 2015]
X _{DM} X _{DM} +γ (+jets)	NLO		NLO+PS (and FxFx)
X _{DM} X _{DM} +Z or W (+jets)			NLO+PS (and FxFx) [M. Neubert, J. Wang, C. Zhang, 2015]
X _{DM} X _{DM} +heavy quarks			NLO+PS [Backovic et al., 2015]



s-channel spin-2 mediator

[Das, Degrande, Hirschi, Shao, in progress]

spin-2	MG5aMC
X _{DM} X _{DM} + 0, I, (2) jets	NLO+PS (and FxFx)
$X_{\text{DM}} X_{\text{DM}}$ +γ	NLO+PS
X _{DM} X _{DM} +Z	NLO+PS
X _{DM} X _{DM} +W ⁺⁻	NLO+PS
X _{DM} X _{DM} +t tbar	NLO+PS



Xdm

SM

Xdm

Search for DM at the LHC: t-channel mediators







DM

- Search for DM and mediators are normally entangled.
- SUSY-like searches.
- jets+mET golden channel



Search for DM at the LHC: t-channel mediators

[Alwall et al., 2008]



1. $pp \to \tilde{q}\tilde{q}^{\dagger} + (0, 1, 2)j;$

- 2. $pp \to \chi \tilde{q}^{\dagger}, \bar{\chi} \tilde{q} + (0, 1, 2)j$ with no mass-shell integration for internal squark lines;
- 3. $pp \rightarrow \chi \bar{\chi} + (0, 1, 2)j$ with no mass-shell integration for internal squark lines.

[De Aquino et al., 2012] [Martini et al., 2015]





DM @ 1-loop : NLO+PS and Loop-Induced

Including loops for BSM brings additional complications:

- Renormalization of the \mathscr{L}_{NP} needs to be performed and in case of use of numerical loop techniques the full set of counteterms is needed.
- At NLO, processes mix with LO resonant contributions and in order to keep NLO accuracy in inclusive samples a MC friendly procedure to subtract real resonant diagrams must be in place.
- If non-renormalizable operators appear, higher-dimensional ranks appear in the integrals as well as extra UV divergences and full mixing RGE are needed.

Not to mention that given the plethora of models that are conceivable and/or the complexity of BSM constructions, at least a minimal level of automation is needed.



Dark Matter at NLO+PS (& Loop-Induced)

First work in this direction by [Haisch et al., 2013] [Haisch et al., 2013] [Haisch et al., 2014] [Crivellin et al., 2015] [Haisch and Re, 2015] where they used the known analytic results collected in MCFM and also implemented some of them in the POWHEG-Box.





Dark Matter production at NLO+PS and LI

A Simplified Model Lagrangian has been implemented in FeynRules and counter terms obtained automatically with NLOCT. Full automation of Simplified Models possible for Loop-Induced and NLO+PS.

- s-channel scalar and vector mediators coupling [Backovic et al., 2015] to quarks, NLO+PS
- s-channel models with coupling to the top, loop-induced processes. [Mattelaer and Vryonidou, 2015]
- s-channel scalar and vector mediators coupling [M. Neubert, J. Wang, C. Zhang, 2015] to quarks and vector bosons

http://feynrules.irmp.ucl.ac.be/wiki/DMsimp



Dark Matter production at NLO+PS

[Backovic et al., 2015]



- tt+X_{DM}X_{DM} with (pseudo-)scalar mediator is calculated at NLO+PS. Different shapes.
- X_{DM}X_{DM} + (0,)1,2 jets is merged at NLO (FxFx)
 : no significant changes with respect to the NLO
 +PS sample for 1 jet at NLO.





Dark Matter production at NLO+PS : Z+mET

[M. Neubert, J. Wang, C. Zhang, 2015]





- Significant effects due to NLO corrections
- Comparison with SM backgrounds also calculated at NLO+PS



Spin-2 s-channel mediator at NLO

[Das, Degrande, Hirschi, Shao, in progress]

- Pauli-Fierz Lagrangian implemented in FeynRules and upgraded to NLO in QCD with NLOCT. Validated, to be made public.
- K-factors large and distributions affected
- Unitarity violating behaviour for nonuniversal couplings can be carefully studied.
- Interesting phenomenology also in view of the 750 GeV hints.





Spin-2 s-channel mediator at NLO



Mono-jet and ttbar associated production of a universal spin-2 mediator.



Loop-induced $X_{DM} X_{DM} + (H, Z, Y)$

[Mattelaer and Vryonidou, 2015]



Scalar mediator coupled to the top leads to $X_{DM} X_{DM} + H$, Z or \mathcal{V} . Selection rules are found. Very different shape of Etmiss depending on the associated production. The effect of extra gluon radiation on the shape Etmiss can also be studied.

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Loop-Induced X_{DM} X_{DM} + jets merged

[Mattelaer and Vryonidou, 2015]



Scalar mediator coupled to the top leads to $X_{DM} X_{DM}$ + jets via loops. Merged samples built automatically and compared to SM at NLO+PS.



Loop-induced $X_{DM} X_{DM}$ + jet (t-channel top/stop)

[Fuks and Vryonidou, in progress]



$$\mathcal{L}_{3} = D_{\mu}\sigma_{3}^{\dagger}D^{\mu}\sigma_{3} - m_{3}^{2}\sigma_{3}^{\dagger}\sigma_{3} + \frac{i}{2}\bar{\chi}\partial\!\!\!/\chi - \frac{1}{2}m_{\chi}\bar{\chi}\chi + \left[\sigma_{3}\bar{t}(\tilde{g}_{L}P_{L} + \tilde{g}_{R}P_{R})\chi + \text{h.c.}\right],$$

SUSY-like production of "neutralino's" via a top-stop loop.

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Summary

- Rapid and impressive progress in techniques in the last years has lead to:
 - Full automation of the computation of NLO QCD corrections in the SM.
 - New techniques and their general (process-independent) implementation of matching/ merging with parton shower programs.
 - A new generation of MC generators that are NLO accurate.
- New results are being obtained for:
 - SM and BSM loop induced processes with jet multiplicities merged
 - NLO in QCD + PS for EFT
 - NLO in QCD + PS for SUSY
 - NLO in QCD + PS for Dark Matter Simplified models



SUSY at NLO+PS

First work in this direction by [Gavin, Hangst, Kraemer, Muhlleinter, Pellen, Popenda, Spira, 2013] where they used the known analytic results collected in PROSPINO and implement them by hand in the POWHEG-Box. Only issue to solve the separation of resonant contributions appearing at NLO (similar situation as in the SM for tW at NLO which is interfering with tt at NLO).



Only squark pair production implemented



SUSY colored scalar production

[Degrande et al., 2015]

The full chain from the SUSY Lagrangian to the computation of the counter terms done automatically and UFO model available. Full automation of SUSY at NLO+PS achieved.



$$\mathcal{L}_{3} = D_{\mu}\sigma_{3}^{\dagger}D^{\mu}\sigma_{3} - m_{3}^{2}\sigma_{3}^{\dagger}\sigma_{3} + \frac{i}{2}\bar{\chi}\partial\!\!\!/\chi - \frac{1}{2}m_{\chi}\bar{\chi}\chi + \left[\sigma_{3}\bar{t}(\tilde{g}_{L}P_{L} + \tilde{g}_{R}P_{R})\chi + \text{h.c.}\right],$$

10.10	8 TeV	
m_3 [GeV]	$\sigma^{\rm LO}$ [pb]	$\sigma^{\rm NLO}$ [pb]
100	$389.3^{+34.2\%}_{-23.9\%}$	$554.8^{+14.9\%}_{-13.5\%}{}^{+1.6\%}_{-1.6\%}$
250	$4.118^{+40.4\%}_{-27.2\%}$	$5.503^{+13.1\%}_{-13.7\%}{}^{+3.7\%}_{-3.7\%}$
500	$(6.594 \times 10^{-2})^{+45.5\%}_{-29.1\%}$	$(7.764 \times 10^{-2})^{+12.1\%}_{-14.1\%}{}^{+6.7\%}_{-6.7\%}$
750	$(3.504 \times 10^{-3})^{+48.8\%}_{-30.5\%}$	$\bigl(3.699\times10^{-3}\bigr)^{+12.3\%}_{-14.6\%}{}^{+10.2\%}_{-10.2\%}$
1000	$(2.875 \times 10^{-4})^{+51.5\%}_{-31.5\%}$	$\left(2.775\times10^{-4}\right)^{+13.1\%}_{-15.2\%}{}^{+15.5\%}_{-15.5\%}$
	8 TeV	
m. [CoV]		8 TeV
$m_8 ~[{ m GeV}]$	$\sigma^{\rm LO}$ [pb]	8 TeV $\sigma^{\rm NLO}$ [pb]
$m_8 \; [\text{GeV}]$ 100	$\frac{\sigma^{\rm LO}~[\rm pb]}{3854^{+34.4\%}_{-24.1\%}}$	8 TeV $\sigma^{\rm NLO} \text{ [pb]}$ $5573^{+14.9\%}_{-13.6\%}{}^{+1.6\%}_{-1.6\%}$
$m_8 \; [GeV]$ 100 250	$\begin{array}{c} \sigma^{\rm LO} \ [\rm pb] \\ 3854^{+34.4\%}_{-24.1\%} \\ 38.89^{+41.3\%}_{-27.7\%} \end{array}$	8 TeV $\sigma^{\text{NLO}} \text{ [pb]}$ 5573 ^{+14.9% +1.6%} -13.6% -1.6% 54.32 ^{+14.5% +3.9%} -14.6% -3.9%
$m_8 [GeV]$ 100 250 500	$\begin{array}{r} \sigma^{\rm LO} \ [\rm pb] \\ 3854^{+34.4\%}_{-24.1\%} \\ 38.89^{+41.3\%}_{-27.7\%} \\ 0.5878^{+47.6\%}_{-30.0\%} \end{array}$	$\begin{array}{r} 8 \ {\rm TeV} \\ & \sigma^{\rm NLO} \ [\rm pb] \\ \\ 5573^{+14.9\%}_{-13.6\%} {}^{+1.6\%}_{-1.6\%} \\ \\ 54.32^{+14.5\%}_{-14.6\%} {}^{+3.9\%}_{-3.9\%} \\ \\ 0.7431^{+15.8\%}_{-16.2\%} {}^{+7.6\%}_{-7.6\%} \end{array}$
$m_8 [GeV]$ 100 250 500 750	$\begin{array}{c} \sigma^{\rm LO} \ [\rm pb] \\ 3854^{+34.4\%}_{-24.1\%} \\ 38.89^{+41.3\%}_{-27.7\%} \\ 0.5878^{+47.6\%}_{-30.0\%} \\ (2.977 \times 10^{-2})^{+52.0\%}_{-31.9\%} \end{array}$	8 TeV $\sigma^{\text{NLO}} \text{ [pb]}$ 5573 ^{+14.9% +1.6%} -13.6% -1.6% 54.32 ^{+14.5% +3.9%} 54.32 ^{+14.5% +3.9%} 0.7431 ^{+15.8% +7.6%} (3.353 × 10 ⁻²) ^{+17.2% +12.1%} -17.3% -12.1%

$$\mathcal{L}_8 = \frac{1}{2} D_\mu \sigma_8 D^\mu \sigma_8 - \frac{1}{2} m_8^2 \sigma_8 \sigma_8 + \frac{\hat{g}_g}{\Lambda} \sigma_8 G_{\mu\nu} G^{\mu\nu} + \sum_{q=u,d} \left[\sigma_8 \bar{q} (\hat{g}_q^L P_L + \hat{g}_q^R P_R) q + \text{h.c.} \right] ,$$

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SUSY gluino pair production

[Degrande et al., 2015]

..including gluino pair production which technically very challenging (Majorana nature of the gluinos).

$m_{\tilde{g}}$ [GeV]	$\sigma^{\rm LO}$ [pb]	$\sigma^{ m NLO}$ [pb]
200	$2104^{+30.3\%}_{-21.9\%}{}^{+14.0\%}_{-14.0\%}$	$3183^{+10.8\%}_{-11.6\%}{}^{+1.8\%}_{-1.8\%}$
500	$15.46^{+34.7\%}_{-24.1\%}{}^{+19.5\%}_{-19.5\%}$	$24.90^{+12.5\%}_{-13.4\%}{}^{+3.7\%}_{-3.7\%}$
750	$1.206^{+35.9\%}_{-24.6\%}{}^{+23.5\%}_{-23.5\%}$	$2.009^{+13.5\%}_{-14.1\%}{}^{+5.5\%}_{-5.5\%}$
1000	$1.608 \cdot 10^{-1+36.3\%+26.4\%}_{-24.8\%-26.4\%}$	$2.743 \cdot 10^{-1+14.4\%}_{-14.8\%}{}^{+7.3\%}_{-7.3\%}$
1500	$6.264 \cdot 10^{-3+36.2\%+29.4\%}_{-24.7\%-29.4\%}$	$1.056\cdot 10^{-2+16.1\%}_{15.8\%}{}^{+11.3\%}_{11.3\%}$
2000	$4.217 \cdot 10^{-4+35.6\%+29.8\%}_{-24.5\%-29.8\%}$	$6.327 \cdot 10^{-4+17.7\%+17.8\%}_{-17.8\%}$

$$\begin{split} \mathcal{L}_{\text{SQCD}} &= D_{\mu} \tilde{q}_{L}^{\dagger} D^{\mu} \tilde{q}_{L} + D_{\mu} \tilde{q}_{R}^{\dagger} D^{\mu} \tilde{q}_{R} + \frac{i}{2} \bar{\tilde{g}} D^{\mu} \tilde{g} \\ &- m_{\tilde{q}_{L}}^{2} \tilde{q}_{L}^{\dagger} \tilde{q}_{L} - m_{\tilde{q}_{R}}^{2} \tilde{q}_{R}^{\dagger} \tilde{q}_{R} - \frac{1}{2} m_{\tilde{g}} \bar{\tilde{g}} \tilde{g} \\ &+ \sqrt{2} g_{s} \Big[- \tilde{q}_{L}^{\dagger} T (\bar{\tilde{g}} P_{L} q) + (\bar{q} P_{L} \tilde{g}) T \tilde{q}_{R} + \text{h.c.} \Big] \\ &- \frac{g_{s}^{2}}{2} \Big[\tilde{q}_{R}^{\dagger} T \tilde{q}_{R} - \tilde{q}_{L}^{\dagger} T \tilde{q}_{L} \Big] \Big[\tilde{q}_{R}^{\dagger} T \tilde{q}_{R} - \tilde{q}_{L}^{\dagger} T \tilde{q}_{L} \Big] , \end{split}$$



lσ[pb per bin

dσ[pb per bin] ≍