# PDF combinations and $\alpha_S(M_Z^2)$

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**Introduction**.

Much more combination of different PDF set in producing results than a few years ago.

Overwhelming (external) desire to have some agreement/consistency between different groups in order to produce combined predictions using e.g. PDF4LHC recommendations.

Three questions.

1. Would like a common value of $\alpha_S(M_Z^2)$. What value (and should it be dependent on perturbative order)?

2. What should the uncertainty on $\alpha_S(M_Z^2)$ be?

3. How should PDF and $\alpha_S(M_Z^2)$ uncertainties be combined? Desire (again external) for decoupling of the uncertainties.

I will discuss the issues and make some suggestions (some more concrete than others).

## Best value(s) of $\alpha_S(m_Z^2)$?

World average of $\alpha_S(m_Z^2) = 0.1186 \pm 0.0006$ (rather small uncertainty).

## MMHT2014 – $\alpha_S(m_Z^2)$ as a data point.

$\alpha_S(m_Z^2)$ coming out similar to 2008 fit. Still a NLO/NNLO difference. Both fairly compatible with global average. Try inputting this as data point.

Try world average (minus DIS data) of $\alpha_S(m_Z^2) = 0.1187 \pm 0.0007$.

At NLO best fit gives $\alpha_S(m_Z^2) = 0.1120 \rightarrow 0.1195$ with $\Delta\chi^2 < 2$.

At NNLO best fit gives $\alpha_S(m_Z^2) = 0.1172 \rightarrow 0.1177$, i.e. very close to $0.118$. $\Delta\chi^2 < 2$

Also force $\alpha_S(m_Z^2) = 0.118$. At NNLO basically no further change. At NLO $\Delta\chi^2 \sim 16$, but no single set deteriorates very significantly.

Usually general agreement in extractions from PDF fits that the NNLO values of $\alpha_S(M_Z^2)$ are $0.0002$ smaller than the NLO values of $\alpha_S(M_Z^2)$?

MMHT2014 – $\alpha_S(M_Z^2) = 0.1195 \rightarrow 0.1178$

ABM14(11) – $\alpha_S(M_Z^2) = 0.1180 \rightarrow 0.1135$

JR14 – $\alpha_S(M_Z^2) = 0.1158(0.1191) \rightarrow 0.1136(0.1162)$

NNPDF2.1 – $\alpha_S(M_Z^2) = 0.1191 \rightarrow 0.1173$

CT10 – $\alpha_S(M_Z^2) = 0.118 \rightarrow 0.1159$

HERAPDF1.6 – $\alpha_S(M_Z^2) = 0.1202$ at NLO and general preference for $\sim 0.118$ at NNLO.

Central values differ far more than NLO $\rightarrow$ NNLO trend, but agree quite well in fits using a GM-VFNS heavy flavour scheme, and in these cases agree quite well with world average at NLO (bit high?) and NNLO (bit low?).

## Proposal

For common value settle on round value of $\alpha_S(M_Z^2) = 0.118$ at NNLO.

Very close to world average and also close to many (most) of determinations at NNLO in PDF fits.

More precise choice (e.g 0.118x) would be more time dependent.

Would personally suggest a higher value, $\alpha_S(M_Z^2) = 0.120$ at NLO. Closer to preferred value of many PDF sets, and higher value at NLO consistent with PDF determinations and would lead to more perturbative stability in predictions.

However, MMHT2014 set up to run with $\alpha_S(M_Z^2) = 0.118$ at NLO and NNLO if necessary.

**Uncertainty?**

World average of $\delta\alpha_S(m_Z^2) = \pm 0.0006$ (rather small uncertainty).

**Proposal (sort of).**

Previously used $\delta\alpha_S(m_Z^2) = \pm 0.0012$ in PDF4LHC recommendation.

However, this also had an effective additional uncertainty of $\delta\alpha_S(m_Z^2) = \pm 0.001$ from spread of CTEQ/CT10, MSTW and NNPDF values, i.e. $\alpha_S(m_Z^2) = 0.1171 - 0.119$ at NNLO and $\alpha_S(m_Z^2) = 0.118 - 0.1202$ at NLO. Added linearly in uncertainty.

Hence, with common central value I suggest something more like $\delta\alpha_S(m_Z^2) = \pm 0.0012$ as moderately conservative choice.

((*Not attached to this special value - something similar seems fine.*)

## Combining PDF and $\alpha_S(M_Z^2)$ uncertainty?

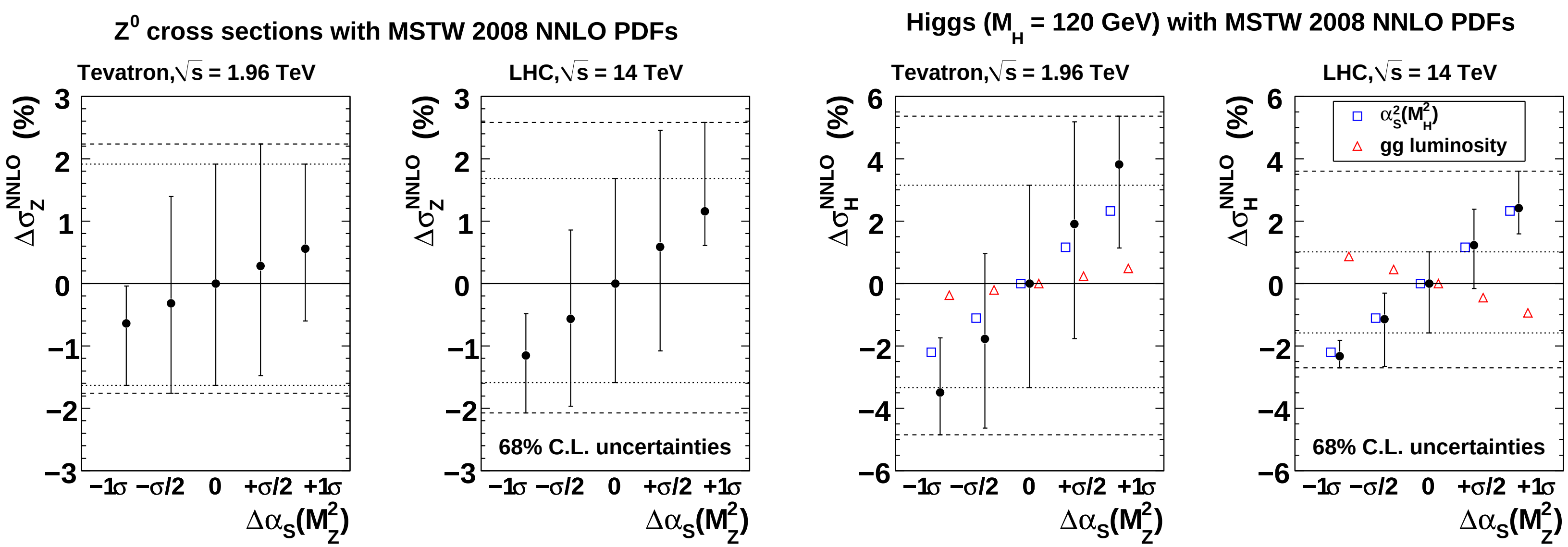
Different PDF sets have had a variety of different procedures for this, and each were applied individually in literal PDF4LHC prescription.

In practice some simplification was allowed if necessary.

## MSTW procedure

Produced PDF sets (with reduced PDF uncertainties) for one and half sigma values of $\alpha_S(M_Z^2)$ uncertainty.

Total uncertainty given by envelope of all PDF uncertainties at each $\alpha_S(M_Z^2)$ value.




Complicated, and $\alpha_S(M_Z^2)$ uncertainty not simply decoupled from PDFs. Conservative. Some way between adding uncertainty linearly and quadratically in practice.

## NNPDF procedure

The combined PDF+$\alpha_S$ uncertainty can be found by combining in a new ensemble replicas that have been extracted with different $\alpha_S(M_Z)$.

The number of replicas corresponding to a given value $\alpha_s = \alpha_s^{(j)}$ is

$$N_{\rm rep}^{\alpha_s^{(j)}} \propto \exp\left(-\frac{\left(\alpha_s^{(j)}-\alpha_s^{(0)}\right)^2}{2(\Delta\alpha_s)^2}\right).$$

The fraction of replicas is $(0.02, 0.08, 0.24, 0.32, 0.24, 0.08, 0.02)$ with $\alpha_s(M_Z) = 0.116, 0.117, 0.118, 0.119, 0.120, 0.121, 0.122$ respectively.

The central value and PDF+$\alpha_S$ uncertainty are obtained by computing the mean value,

$$\mathcal{O}_0 \; = \; \langle \mathcal{O} \rangle_{rep} = \frac{1}{N_{rep}} \sum_{j=1}^{N_\alpha} \sum_{k_j=1}^{N_{\rm rep}^{\alpha_s^{(j)}}} \mathcal{O}\left({\rm PDF}^{(k_j,j)}, \alpha_s^{(j)}\right),$$

and the standard deviation

$$\sigma^{NNPDF}(\alpha_s{+}PDF) = \left[\frac{1}{N_{rep}-1} \sum_{j=1}^{N_\alpha} \sum_{k_j=1}^{N_{\rm rep}^{\alpha_s^{(j)}}} \left(\mathcal{O}\left({\rm PDF}^{(k_j,j)}, \alpha_s^{(j)}\right) - \mathcal{O}_0\right)^2\right]^{1/2}$$

## CTEQ/CT10 procedure

Procedure based on effectively including $\alpha_S(M_Z^2)$ uncertainty on same footing as PDF parameter uncertainty, and hence including it as one of the parameters contributing to the uncertainty, (which is what done by ABM and JR).

However, in arXiv:1004.4624 it was shown that when one has a set of orthogonal eigenvectors based on parameters $a_0 \cdots a_n$ the total uncertainty obtained from all $n+1$ orthogonal eigenvectors is equivalent to

$$(\Delta\sigma_X^{n+1})^2 = \sqrt{(\Delta\sigma_X^{0,a_i\mathrm{free}})^2 + (\Delta\sigma_X^{n})^2}$$

i.e. if $a_0 = \alpha_S(M_Z^2)$ the uncertainty of the quantity including $\alpha_S(M_Z^2)$ variation is that for PDF variation alone added in quadrature with the uncertainty due to $a_0 = \alpha_S(M_Z^2)$ while PDF parameters are allowed to vary.

Hence, PDFs for best fit at $\alpha_S(M_Z^2) \pm \delta\alpha_S(M_Z^2)$ act simply as an additional pair of eigenvectors.

Assumes central PDFs are at best fit when $\alpha_S(M_Z^2)$ is left free and $\delta\alpha_S(M_Z^2)$ is uncertainty obtained from the fit.

Table compares CTEQ results including $\alpha_S(M_Z^2)$ as an eigenvector (CTEQFAS) and adding the uncertainty in quadrature (CTEQ+CTEQAS).

Generally good, if not perfect agreement.

| Process | CTEQ6.6+CTEQ6.6AS | | | | CTEQ6.6FAS |
|---|---|---|---|---|---|
| $t\bar{t}$ (171 GeV) | $\sigma_0$ | $\Delta\sigma_{PDF}$ | $\Delta\sigma_{\alpha_s}$ | $\Delta\sigma$ | $\sigma_0 \pm \Delta\sigma$ |
| LHC, 7 TeV | 157.41 | 10.97 | 7.54 | 13.31 | $160.10 \pm 13.93$ |
| LHC, 10 TeV | 396.50 | 18.75 | 16.10 | 24.71 | $400.48 \pm 25.74$ |
| LHC, 14 TeV | 877.19 | 28.79 | 30.78 | 42.15 | $881.62 \pm 44.27$ |
| $gg \to H$ (120 GeV) | $\sigma_0$ | $\Delta\sigma_{PDF}$ | $\Delta\sigma_{\alpha_s}$ | $\Delta\sigma$ | $\sigma_0 \pm \Delta\sigma$ |
| Tevatron, 1.96 TeV | 0.63 | 0.042 | 0.032 | 0.053 | $0.64 \pm 0.055$ |
| LHC, 7 TeV | 10.70 | 0.31 | 0.32 | 0.45 | $10.70 \pm 0.48$ |
| LHC, 10 TeV | 20.33 | 0.66 | 0.56 | 0.87 | $20.28 \pm 0.93$ |
| LHC, 14 TeV | 35.75 | 1.31 | 0.94 | 1.61 | $35.63 \pm 1.70$ |
| $gg \to H$ (160 GeV) | $\sigma_0$ | $\Delta\sigma_{PDF}$ | $\Delta\sigma_{\alpha_s}$ | $\Delta\sigma$ | $\sigma_0 \pm \Delta\sigma$ |
| Tevatron, 1.96 TeV | 0.26 | 0.026 | 0.015 | 0.030 | $0.26 \pm 0.031$ |
| LHC, 7 TeV | 5.86 | 0.16 | 0.18 | 0.24 | $5.88 \pm 0.26$ |
| LHC, 10 TeV | 11.73 | 0.33 | 0.33 | 0.47 | $11.72 \pm 0.50$ |
| LHC, 14 TeV | 21.48 | 0.68 | 0.56 | 0.88 | $21.43 \pm 0.94$ |
| $gg \to H$ (250 GeV) | $\sigma_0$ | $\Delta\sigma_{PDF}$ | $\Delta\sigma_{\alpha_s}$ | $\Delta\sigma$ | $\sigma_0 \pm \Delta\sigma$ |
| Tevatron, 1.96 TeV | 0.055 | 0.0099 | 0.0044 | 0.011 | $0.058 \pm 0.012$ |
| LHC, 7 TeV | 2.30 | 0.085 | 0.081 | 0.12 | $2.32 \pm 0.12$ |
| LHC, 10 TeV | 5.08 | 0.14 | 0.15 | 0.21 | $5.10 \pm 0.22$ |
| LHC, 14 TeV | 10.03 | 0.26 | 0.27 | 0.37 | $10.04 \pm 0.41$ |

**Simple idea for proposal**.

Adding PDF uncertainty in quadrature with the uncertainty entirely due to $\alpha_S(M_Z^2)$ (with PDFs left free) always gives approximately the same answer as more sophisticated methods, and in some procedures exactly the same answer.

Therefore for ease of calculation (and presentation), and with no very obvious lack of accuracy, suggest:

Find overall best value and PDF uncertainty of quantity $\sigma^0 \pm \delta\sigma^{\rm PDF}$ from combination of PDFs (in some manner, e.g. replicas generated from each group, Meta PDFs, ...) at an agreed central value of $\alpha_S^0(M_Z^2)$.

At $\alpha_S^0(M_Z^2) \pm \delta\alpha_S(M_Z^2)$ calculate best prediction for quantity based on combination of the same PDFs. Difference between this and $\sigma^0$ is $\pm\delta\sigma^{\alpha_S}$

Define total uncertainty $\delta\sigma = \sqrt{(\delta\sigma^{\rm PDF})^2 + (\delta\sigma^{\alpha_S})^2}$.

Independent of means of combining PDFs at a particular $\alpha_S(M_Z^2)$.

If $\delta\alpha_S(M_Z^2)$ is not automatically the separation between PDFs at $\alpha_S^0(M_Z^2)$ and available PDF sets, e.g $\delta\alpha_S(M_Z^2) = 0.0012$ while PDF sets are available with separations of 0.001, then rescale $\pm\delta\sigma^{\alpha_S}$ by the desired change in $\alpha_S(M_Z^2)$ divided by actual change in $\alpha_S(M_Z^2)$, i.e. $0.0012/0.001 = 1.2$ in our example.

Likely only to be a small correction, and in practice cross section dependence on $\alpha_S^0(M_Z^2)$ is usually quite linear in vicinity of best fit.

Alternatively. Once $\delta\alpha_S(M_Z^2)$ is decided each group makes a central set with $\alpha_S^0(M_Z^2) \pm \delta\alpha_S(M_Z^2)$ available.

Caveat Does strictly rely on central set having ($\alpha_S(M_Z^2)$) corresponding to best fit and having uncertainty as determined from a fit (potentially with, or without $\alpha_S(M_Z^2)$) as a data point.

Part of reason for suggesting higher NLO value of $\alpha_S(M_Z^2)$ than NNLO value - however, MMHT2014 NLO eigenvectors for $\alpha_S(M_Z^2) = 0.118$ and $\alpha_S(M_Z^2) = 0.120$ map rather well onto each other (NLO eigenvectors do not map very well to NNLO eigenvectors), so perhaps wouldn't be too bad.

Not clear that use of this procedure would lead to greater inaccuracies than the intrinsic "error on the error".

## Alternatives

1. Do the same as before in essence, i.e. calculated PDF+ $\alpha_S(M_Z^2)$ uncertainty for each group and take envelope. Simpler if each have common central value, but still a lot of work and complicated.

Not easy to separate out simply into PDF uncertainty and $\alpha_S(M_Z^2)$ uncertainty.

2. Use the NNPDF approach of replicas distributed in terms of $\alpha_S(M_Z^2)$.

Only applicable if combination to be made explicitly in terms of replicas from each group.

Larger amount of work and requires replicas to be made available by each group at appropriate values of $\alpha_S(M_Z^2)$.

Again, not easy to separate out simply into PDF uncertainty and $\alpha_S(M_Z^2)$ uncertainty.

3. As in simple proposal but add $\alpha_S(M_Z^2)$ uncertainty linearly to PDF uncertainty. Just as simple but more conservative uncertainty.

Not sure there is much real justification for this other than being conservative. Could simply increase $\delta\alpha_S(M_Z^2)$ and achieve same end.

Possible to think of other approaches, but would probably include some element of those mentioned.

Would almost certainty be more complicated/time consuming, with no particularly obvious gain.