Lepton dipole moments in a low-scale seesaw mSUGRA model

Amon Ilakovac

Department of Physics, University of Zagreb, 10000 Zagreb pp332, Croatia

Based on : A. Pilaftsis, L. Popov, A.I., PRD 89 (2014) 015001

- Motivation; Model
- Amplitudes : diagrams, form factor structure
- Numerical results for a_μ, electron EDM: only from SB phases of A and B.

Motivation

Experiment

Observable	Upper Limit	Future sensitivity
d_e	1.05×10^{-27} e cm $[1, 2, 3]$	$10^{-29} - 10^{-31}$ e cm $[3]$
a_μ^{exp}	Present sensitivity $(\delta a_\mu / a_\mu)$	Future sensitivity
$(116592089) \times 10^{-11}$	0.54×10^{-6} $[1]$	0.14×10^{-6} $[4, 5]$

Table 1: Current upper limits and future sensitivities of CLFV observables, electron EDM and muon MDM.

[1] J. Beringer, (PDG) PRD 86 (2012) 010001

[2] J. Hudson, Nature 473 (2011) 493

[3] M. Jung, JHEP 1305 (2013) 168; Kumar, (2013), 1312.5416 [YbF, ThO]

[4] B.L. Roberts, NPPS 218 (2011) 237

[5] G. Venanzoni, arXiv:1203.1501; J.Phys.Conf.Ser. 349 (2012) 012008

Theory

- $d_e > 10^{-33}$ e cm, $(a_\mu^{th} - a_\mu^{exp})/a_\mu^{exp} > \delta a_\mu / a_\mu$: would be independent sign for a physics BSM

– ν-osc & a_μ, d_e (& CLFV) would give the information on a scale of new physics

(ν_RMSSM) model

- Seesaw mechanism: Neutrino mass matrix (m_e diagonal basis; at scale m_N)

$$M_\nu = \begin{pmatrix} 0 & m_D^T \\ m_D & M_M \end{pmatrix}, \qquad \begin{aligned} M_\nu B^{\nu\dagger} &= 0, \quad m_{n_i} \approx m_{n_j}, \quad i,j > 3, \\ m_D &= \sqrt{2} M_W s_\beta g^{-1} h_\nu^\dagger \end{aligned}$$

$$h_\nu = \begin{pmatrix} 0 & 0 & 0 \\ a e^{-\frac{i\pi}{4}} & b e^{-\frac{i\pi}{4}} & c e^{-\frac{i\pi}{4}} \\ a e^{\frac{i\pi}{4}} & b e^{\frac{i\pi}{4}} & c e^{\frac{i\pi}{4}} \end{pmatrix} \qquad h_\nu = \begin{pmatrix} a^* & b^* & c^* \\ a^* e^{-\frac{2\pi i}{3}} & b^* e^{-\frac{2\pi i}{3}} & c^* e^{-\frac{2\pi i}{3}} \\ a^* e^{\frac{2\pi i}{3}} & b^* e^{\frac{2\pi i}{3}} & c^* e^{\frac{2\pi i}{3}} \end{pmatrix}$$

· $\nu_\ell^{SM} = (Bn)_\ell = (B^\nu \nu)_\ell + (B^N N)_\ell$: B diagonalizes M_ν

· ν masses : sym. breaking; radiatively induced

- Sneutrino mass matrix

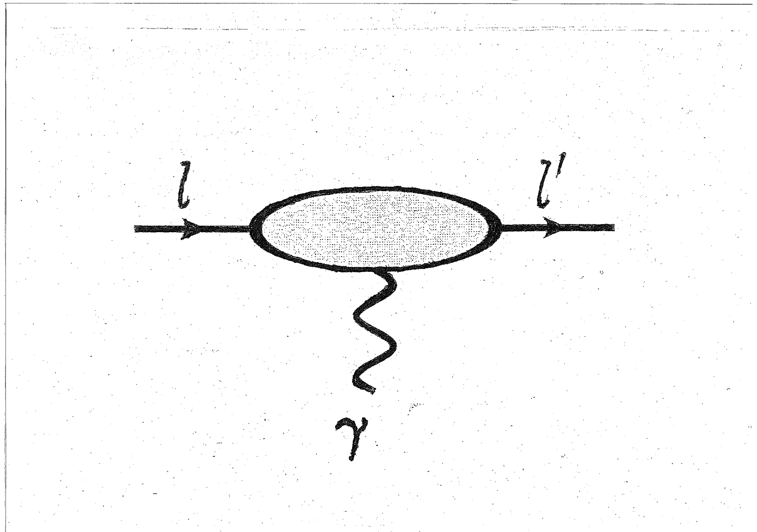
$$M_{\tilde{\nu}}^2 \;=\; \begin{pmatrix} H_1 & N & 0 & M \\ N^\dagger & H_2^T & M^T & M_B \\ 0 & M^* & H_1^T & N^* \\ M^\dagger & M_B^\dagger & N^T & H_2 \end{pmatrix}, \qquad M_{\tilde{\nu}}^2 \stackrel{SUSY}{\longrightarrow} \begin{pmatrix} M_\nu M_\nu^\dagger & 0_{6\times 6} \\ 0_{6\times 6} & M_\nu^\dagger M_\nu \end{pmatrix}$$

$$\begin{aligned} H_1 &= m_{\tilde{L}}^2 + (\frac{1}{2} M_Z^2 c_{2\beta} \mathbf{1}) + (m_D m_D^\dagger) \\ H_2 &= m_{\tilde{\nu}}^2 + (m_D^\dagger m_D) + (M_M^\dagger M_M) \\ M &= m_D (A_\nu - \mu\, ct_\beta) \\ N &= m_D M_M^\dagger, \qquad M_B \equiv \frac{1}{2} B_{IJ} (M_\nu)_{IJ} \;\to\; 0; b_\nu \end{aligned}$$

- N-$\tilde{N}$ sector nearly supersymmetric if $m_N > m_{SUSY}$ and $h_\nu \leq 0.2$

Amplitudes

Amplitudes : diagrams



We took $\tan\beta < 40$.

Amplitudes : structure

$$\mathcal{T}_\mu^{\gamma l' l} = \frac{e\,\alpha_w}{8\pi M_W^2}\,\bar{l}'[(F_\gamma^L)_{l'l}\,(q^2\gamma_\mu - \not{q}q_\mu)P_L + (F_\gamma^R)_{l'l}\,(q^2\gamma_\mu - \not{q}q_\mu)P_R$$
$$+(G_\gamma^L)_{l'l}\,i\sigma_{\mu\nu}q^\nu P_L + (G_\gamma^R)_{l'l}\,i\sigma_{\mu\nu}q^\nu P_R]\,l,$$

Dipole moments

Lagrangian and dipole moments

$$\mathcal{L} = \bar{l}[\gamma(i\partial^\mu + eA^\mu) - m_l - \frac{e}{2m_l}\sigma^{\mu\nu}(F_l + iG_l\gamma_5)\partial_\nu A_\mu]l$$

$$a_l = F_l \qquad d_l = eG_l/m_l$$

Amplitude and dipole moments

$$i\mathcal{T}^{\gamma ll} = \frac{ie\alpha_w}{8\pi M_W^2}[(G_\gamma^L)_{ll}i\sigma_{\mu\nu}q^\nu P_L + G_\gamma^R)_{ll}i\sigma_{\mu\nu}q^\nu P_R]$$

$$a_l = \frac{\alpha_w m_l}{8\pi M_W^2}[(G_\gamma^L)_{ll} + (G_\gamma^R)_{ll}] \qquad d_l = \frac{e\alpha_w}{8\pi M_W^2}i[(G_\gamma^L)_{ll} - (G_\gamma^R)_{ll}]$$

Possible sources of lepton EDM (CPV)

$$A_\nu = h_\nu A_0 e^{i\phi} \qquad -(A_\nu)^{ij}\tilde{\nu}_R^c(h_{uL}^+\tilde{e}_{jL} - h_{uL}^0\tilde{\nu}_{jL})$$

$$b_\nu = B_0 e^{i\theta} m_N \mathbf{1}_3 \qquad (b_\nu)_{ii}\tilde{\nu}_{Ri}\tilde{\nu}_{Ri}$$

$$\Delta_{\mathrm{CP}}^{LR} = \tilde{B}_{lkA}^L \tilde{B}_{lkA}^{R*} \qquad \Delta_{\mathrm{CP}}^{RL} = \tilde{B}_{lkA}^R B_{lkA}^{L*}$$

Scaling behaviour of MSSM contribution dipole moments

$$a_l^{MSSM} \quad \propto \quad \frac{m_l^2}{M_{SUSY}^2} \tan\beta \, \mathsf{sign}(\mu M_{1,2}) \qquad (\text{checked})$$

$$d_l \quad \propto \quad \frac{e\, m_l}{M_{SUSY}^2} \tan\beta \sin(\phi_{CP}) \qquad (\text{expected,checked})$$

$$d_l \quad \propto \quad \frac{e\, m_l\, f(m_0)}{M_N^x} \tan\beta, \quad 2/3 < x < 1 \qquad (\text{found})$$

mSUGRA Framework

Boundary conditions and RGEs:

1. SM parameters at M_Z scale (Fusaoka and Koide PRD57 (1998) 3986).
2. Neutrino Yukawa and heavy neutrino masses at heavy neutrino scale m_N, (Pilaftsis PRL95 (081602) 2005, PRD72 (2005) 113001, PRD83 (2011) 076007; J. Kersten, A.Y. Smirnov, PRD76 (2007) 073005)

$$
\begin{aligned}
m_{N_i} &= m_N, \\
h_\nu &= \begin{pmatrix} 0 & 0 & 0 \\ ae^{-\frac{i\pi}{4}} & be^{-\frac{i\pi}{4}} & ce^{-\frac{i\pi}{4}} \\ ae^{\frac{i\pi}{4}} & be^{\frac{i\pi}{4}} & ce^{\frac{i\pi}{4}} \end{pmatrix} \qquad h_\nu = \begin{pmatrix} a^* & b^* & c^* \\ a^*e^{-\frac{2\pi i}{3}} & b^*e^{-\frac{2\pi i}{3}} & c^*e^{-\frac{2\pi i}{3}} \\ a^*e^{\frac{2\pi i}{3}} & b^*e^{\frac{2\pi i}{3}} & c^*e^{\frac{2\pi i}{3}} \end{pmatrix}
\end{aligned}
$$

3. mSUGRA conditions at gauge unification scale $g_1 = g_2 = g_3$,

$$
\begin{aligned}
m^2_{H_1,H_2} &= m_0^2, \qquad m^2_{\tilde{u},\tilde{d},\tilde{e},\tilde{n}} = m_0^2\,\mathbf{1} \\
M_{1,2,3} &= M_{1/2}, \qquad A_{u,d,e,n} = A_0\, h_{u,d,e,n}\ .
\end{aligned}
$$

4. MSSM$+3N$ RGE equations (P. Chankowski and S. Pokorski, IJMP A17 (2002) 575, S. Petcov et al. NPB676 (2004) 453).

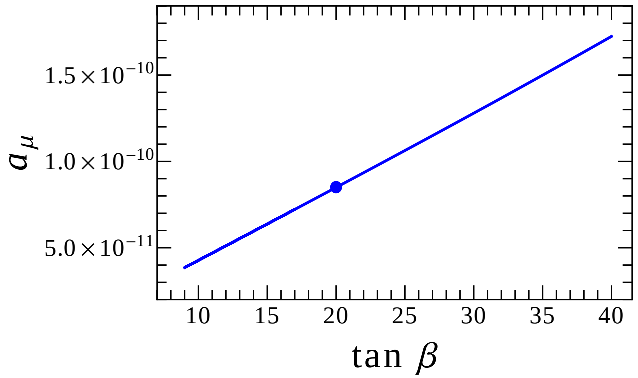
Numerical results for for lepton dipole moments

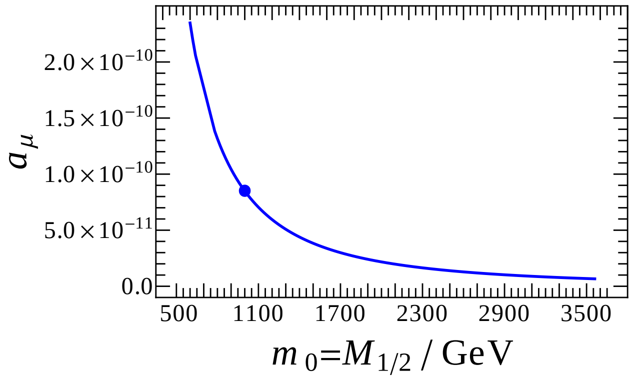
Choice of parameters

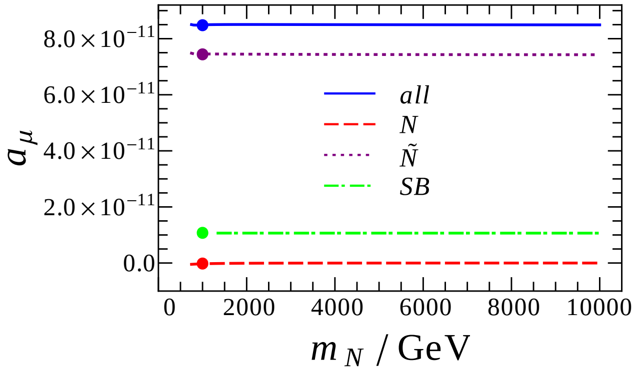
1. $m_0 = 1000$ GeV, $A_0 = -4000$ GeV, $M_{1/2} = 1000$ GeV
   consistent with $m_h \approx 126$ GeV
   consistent with $m_{\tilde{g}},\ m_{\tilde{q}} > 1$ GeV
   in agreement with lightest neutralino as a dark matter candidate
2. $sign(\mu) > 0$
3. $\tan\beta = 20$ in most of calculations
4. Yukawa parameters:
   model 1: $a = b$, $c = 0$; $a = c$, $b = 0$; $b = c$, $a = 0$
   model 2: $a = b = c$
   Pertubativity condition $Trh_\nu^\dagger h_\nu < 4\pi$:
   model 1: $a < 0.34$
   model 2: $a < 0.23$
5. $m_N < 10$ TeV: consistency with resonant leptogenesis

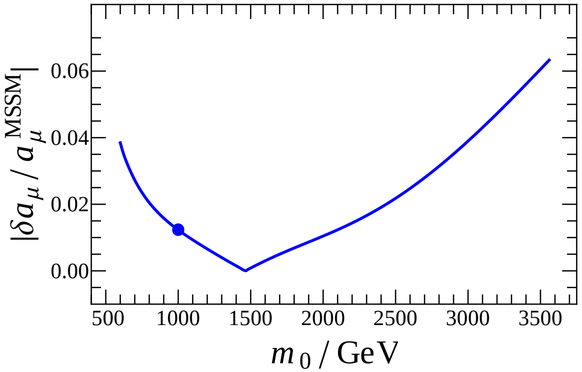
Choice of baseline parameters

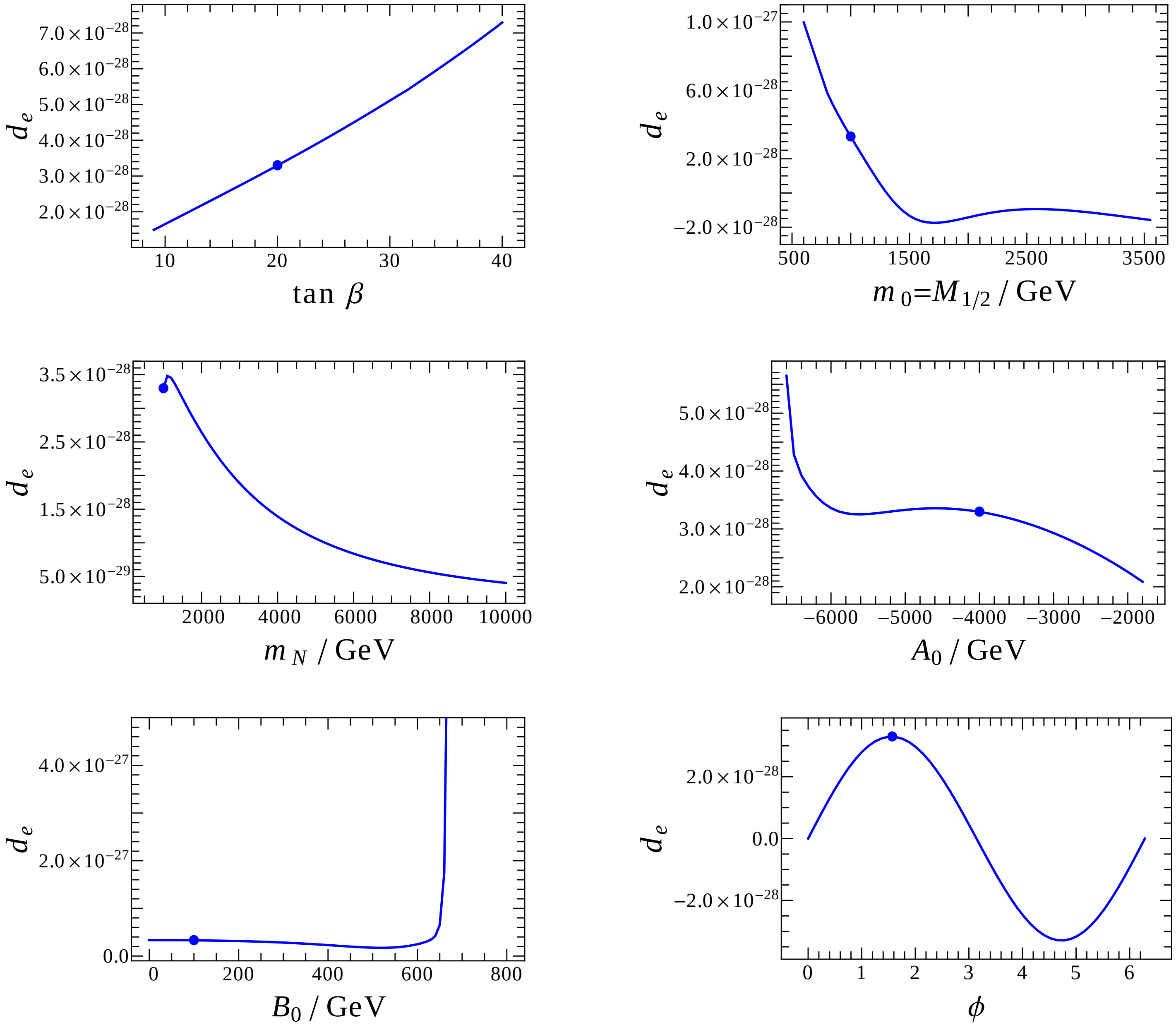
$$m_0 = 1\ \text{TeV}\,, \quad M_{1/2} = 1\ \text{TeV}\,, \quad A_0 = -4\ \text{TeV}\,, \quad \tan\beta = 20\,,$$
$$m_N = 1\ \text{TeV}\,, \quad B_0 = 0.1\ \text{TeV}\,, \quad a = b = c = 0.05\,,$$



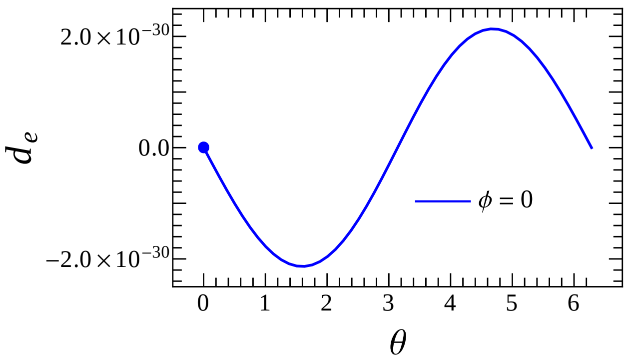


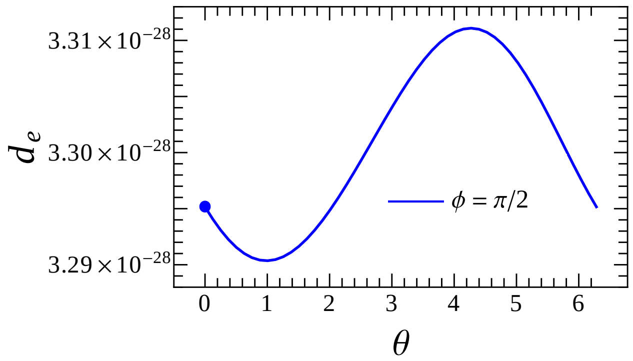






d_e
7.0×10^{-28}
6.0×10^{-28}
5.0×10^{-28}
4.0×10^{-28}
3.0×10^{-28}
2.0×10^{-28}
10 20 30 40
$\tan\beta$
d_e
1.0×10^{-27}
6.0×10^{-28}
2.0×10^{-28}
-2.0×10^{-28}
500 1500 2500 3500
$m_0 = M_{1/2}$ / GeV
d_e
3.5×10^{-28}
2.5×10^{-28}
1.5×10^{-28}
5.0×10^{-29}
2000 4000 6000 8000 10000
m_N / GeV
d_e
5.0×10^{-28}
4.0×10^{-28}
3.0×10^{-28}
2.0×10^{-28}
−6000 −5000 −4000 −3000 −2000
A_0 / GeV
d_e
4.0×10^{-27}
2.0×10^{-27}
0.0
0 200 400 600 800
B_0 / GeV
d_e
2.0×10^{-28}
0.0
-2.0×10^{-28}
0 1 2 3 4 5 6
ϕ


2.0×10^{-30}
0.0
-2.0×10^{-30}
d_e
0 1 2 3 4 5 6
θ
$\phi = 0$


3.31×10^{-28}
3.30×10^{-28}
3.29×10^{-28}
d_e
0 1 2 3 4 5 6
θ
$\phi = \pi/2$

Summary

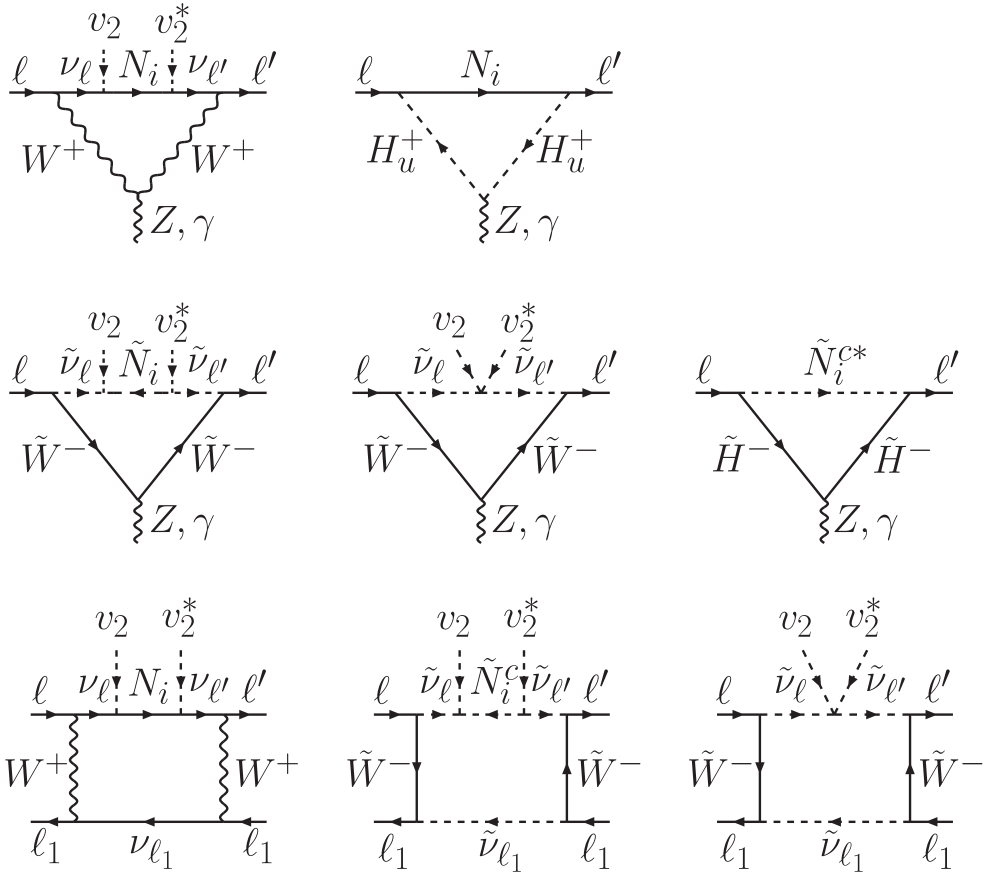
- We made an analysis of the lepton dipole moments, with particular regard to muon magnetic dipole moment a_μ and electron electric dipole moment d_e. Up to our knowledge such analysis has been done for the first time in a model with a low scale seesaw mechanism. We showed that a_μ satisfies scaling behaviour as in MSSM, and the heavy neutrino and sneutrino contributions do not numerically change the MSSM prediction for a_μ. For d_e we found a scaling behaviour which almost agrees with the naive scaling prediction. That is new result. Further, at one loop level, only the additional phases of the soft SUSY breaking bilinear and trilinear couplings induce d_e, while the potential source of CPV from $\nu_R MSSM$ vertices which are not complex conjugate to each other give numerically zero contribution to d_e. That is in accord with the result obtained for the one loop result for d_e in models with high scale seesaw mechanism.

Thank you

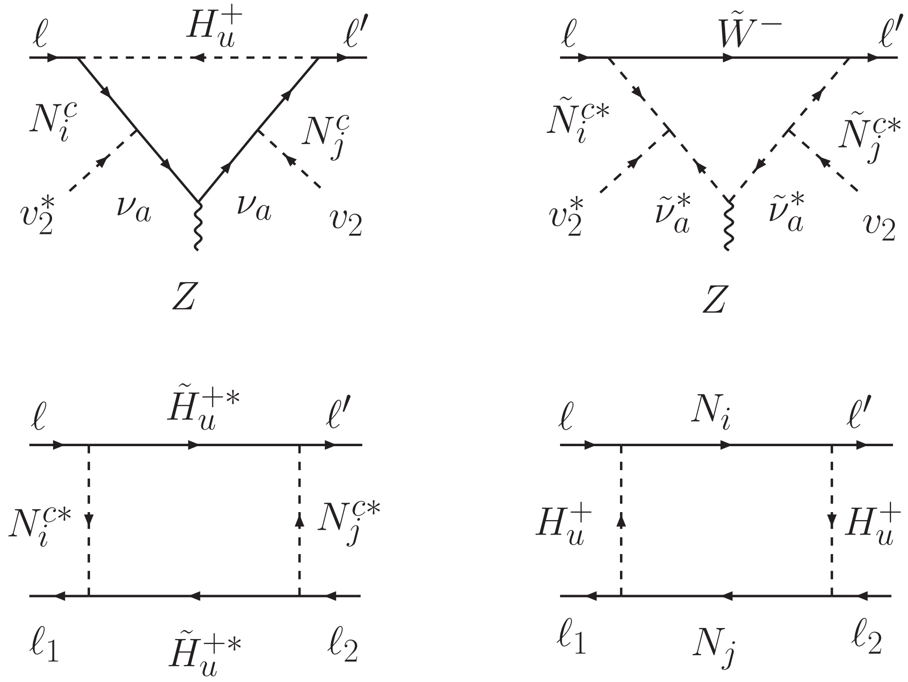
Amplitudes : Dominant contributions

- dominant terms in lowest order in g_W and v_u (Y_ν)

Two Yukawas



Four Yukawas



Form factors

$$
\begin{aligned}
(F_\gamma^{\ell\ell'})^N &= \frac{\Omega_{\ell\ell'}}{6s_\beta^2} \underbrace{\ln\frac{m_N^2}{M_W^2}}_{=\lambda_N}, \\
(F_\gamma^{\ell\ell'})^{\tilde N} &= \frac{\Omega_{\ell\ell'}}{3s_\beta^2}\sum_{k=1}^{2}\mathcal{V}_{k1}^2 \ln\frac{m_N^2}{\tilde m_{\tilde\chi_k^2}}, \\
(G_\gamma^{\ell\ell'})^N &= -\Omega_{\ell\ell'}\left(\frac{1}{6s_\beta^2}+\frac{5}{6}\right) \\
(G_\gamma^{\ell\ell'})^{\tilde N} &= \Omega_{\ell\ell'}\left(\frac{1}{6s_\beta^2}+g_\gamma\right) \\
g_\gamma &= -\sum_{k=1}^{2}\left[\mathcal{V}_{k1}^2\frac{2M_W^2}{m_{\tilde\chi_i}^2} g_{\gamma,1}\left(\frac{m_{\tilde\nu}^2}{m_{\tilde\chi_i}^2}\right)+\mathcal{V}_{k1}\mathcal{U}_{k1}\frac{\sqrt{2}}{c_\beta}\frac{M_W^2}{m_{\tilde\chi_i}^2} g_{\gamma,2}\left(\frac{m_{\tilde\nu}^2}{m_{\tilde\chi_i}^2}\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
(F_Z^{\ell\ell'})^N &= -\frac{3\Omega_{\ell\ell'}}{2}\ln\frac{m_N^2}{M_W^2} - \frac{\Omega_{\ell\ell'}^2}{2s_\beta^2}\frac{m_N^2}{M_W^2},\\
(F_Z^{\ell\ell'})^{\tilde N} &= \frac{\Omega_{\ell\ell'}}{2}\ln\frac{m_N^2}{\tilde m_1^2}\Big(-\frac{1}{2}+2s_W^2+\frac{1}{s_\beta^2}f_Z\Big)\\
f_Z &= \sum_{k,l=1}^{2}\frac{m_{\tilde\chi_k}m_{\tilde\chi_l}}{M_W^2}\big(\mathcal{V}_{k2}\mathcal{U}_{k1}\mathcal{U}_{l1}\mathcal{V}_{l2}+\frac{1}{2}\mathcal{V}_{k2}\mathcal{U}_{k2}\mathcal{U}_{l2}\mathcal{V}_{l2}-s_W^2\delta_{kl}\mathcal{V}_{k2}\mathcal{V}_{l2}\big)
\end{aligned}
$$

$$
\begin{aligned}
(F_{box}^{\ell\ell'\ell_1\ell_2})^N &= -(\Omega_{\ell\ell'}\delta_{\ell_2\ell_1}+\Omega_{\ell\ell_1}\delta_{\ell_2\ell'})+\frac{1}{4s_\beta^4}(\Omega_{\ell\ell'}\Omega_{\ell_2\ell_1}+\Omega_{\ell\ell_1}\Omega_{\ell_2\ell'})\frac{m_N^2}{M_W^2}\\
(F_{box}^{\ell\ell'\ell_1\ell_2})^{\tilde N} &= (\Omega_{\ell\ell'}\delta_{\ell_2\ell_1}+\Omega_{\ell\ell_1}\delta_{\ell_2\ell'})\ f_{box}^{\ell}+\frac{1}{4s_\beta^4}(\Omega_{\ell\ell'}\Omega_{\ell_2\ell_1}+\Omega_{\ell\ell_1}\Omega_{\ell_2\ell'})\frac{m_N^2}{M_W^2}\\
f_{box}^{\ell} &= \sum_{k,l=1}^{2}\mathcal{V}_{k1}^2\mathcal{V}_{l1}^2 f_{box,1}^{\ell}(\lambda_{\tilde\chi_k},\lambda_{\tilde\chi_l},\lambda_{\tilde\nu},\lambda_N)+\mathcal{V}_{k2}\mathcal{V}_{k1}\mathcal{V}_{l2}\mathcal{V}_{l1}f_{box,2}^{\ell}()
\end{aligned}
$$

$$(F^{\ell\ell' uu}_{box})^N = -4(F^{\ell\ell' dd}_{box})^N = 4\Omega_{e\mu}$$

$$(F^{\ell\ell' uu}_{box})^{\tilde{N}} = \sum_{k,l=1}^{2} \mathcal{V}^2_{k1}\mathcal{V}^2_{l1} f^u_{box}(\lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_l}, \lambda_{\tilde{d}}, \lambda_N)$$

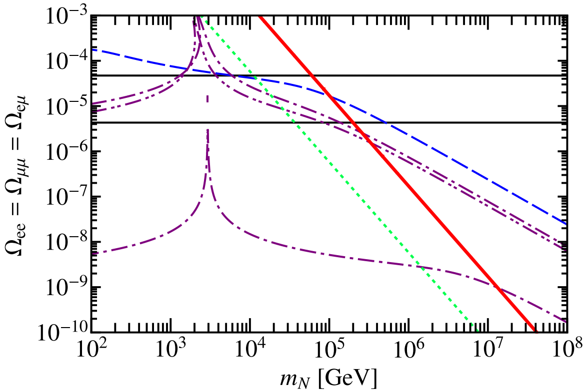
$$(F^{\ell\ell' dd}_{box})^{\tilde{N}} = \sum_{k,l=1}^{2} \mathcal{V}^2_{k1}\mathcal{V}^2_{l1} f^d_{box}(\lambda_{\tilde{\chi}_k}, \lambda_{\tilde{\chi}_l}, \lambda_{\tilde{u}}, \lambda_N)$$

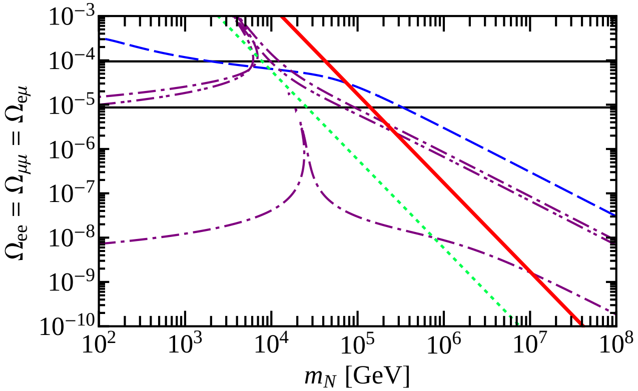
SUSY limit; cancelations, enhancements:

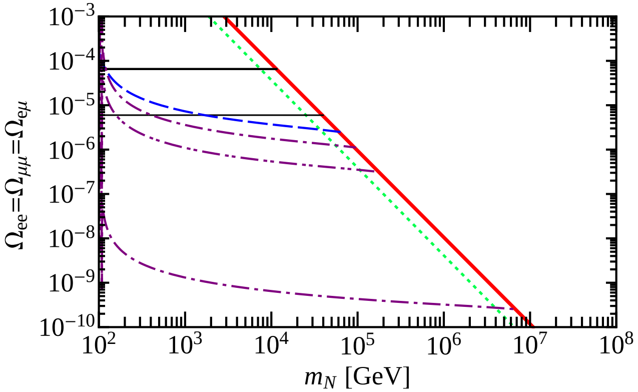
- $\tilde{m}^2_{\tilde{\chi}_{1,2}} \stackrel{SL}{\to} M^2_W, \quad t_\beta \stackrel{SL}{\to} 1, \quad \mu \stackrel{SL}{\to} 0$ (Barbieri, Giudice PLB309)
- $(G^{\ell\ell'}_\gamma)^N + (G^{\ell\ell'}_\gamma)^{\tilde{N}} \stackrel{SL}{=} 0$: Ferrara, Remiddi PLB53 (1974) 347
- box form factors : positive interference
- Y^4_ν terms : become important when $Y_\nu/g_W \sim 1$ $\quad (\Omega_{\ell\ell'}\frac{m^2_N}{M^2_W} = 2(Y^\dagger Y)_{\ell\ell'}/g^2_W)$

  (A. Pilaftsis, A.I, NPB437 (1995) 491)

Numerical estimates







$\tan\beta = 3$

$m_0 = 100$ GeV, $M_{1/2} = 250$ GeV

$A_0 = 100$ GeV

$\Omega_{\mu e} = \Omega_{ee} = \Omega_{\mu\mu}$, other $\Omega_{\ell\ell'} = 0$

Upper bounds

$B(\mu^- \to e^-\gamma)$	1.2×10^{-11}	[1]
	1×10^{-13}	[2]
$B(\mu^- \to e^-e^-e^+)$	1×10^{-12}	[1]
$R^{Ti}_{\mu e}$	4.3×10^{-12}	[3]
	1×10^{-18}	[4]
$R^{Au}_{\mu e}$	7×10^{-13}	[5]

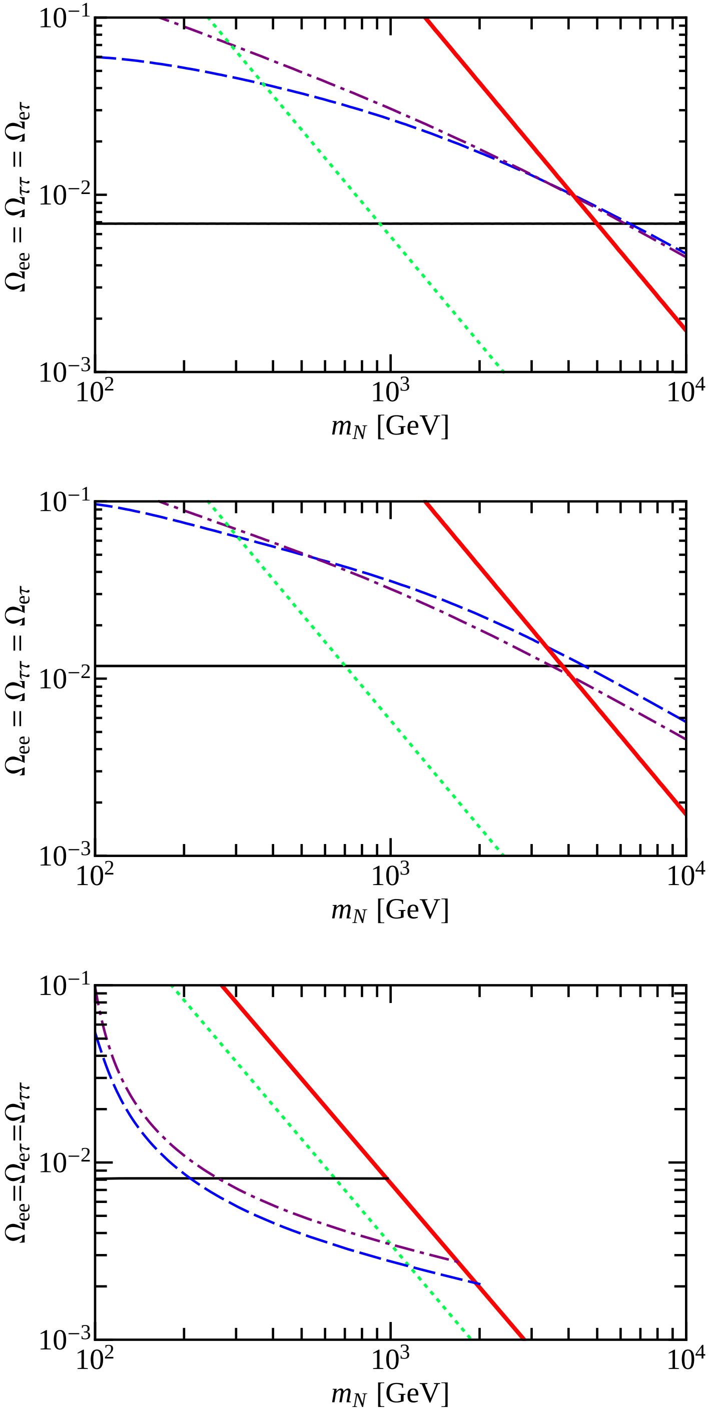
[1] Amsler, PLB 667 (2008) 1

[2] Ritt, NPBPS 162 (2006) 279

[3] Dohmen, PLB 317 (1993) 631

[4] Kuno, NPBPS 149 (2005) 376

[5] Bertl, EPJC 47 (2006) 337



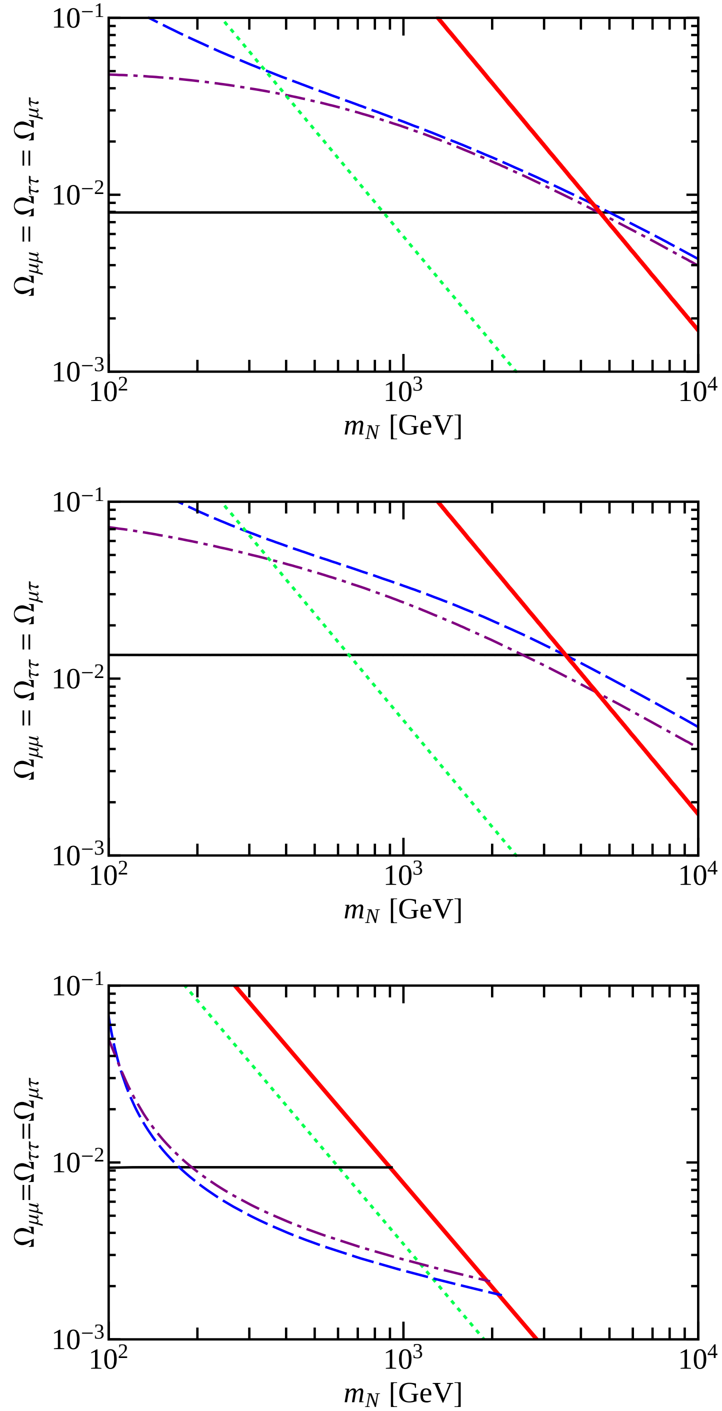
$\Omega_{\tau e} = \Omega_{ee} = \Omega_{\tau\tau}$, other $\Omega_{\ell\ell'} = 0$

Upper bounds

$B(\tau^- \to e^-\gamma)$	3.3×10^{-8}	[1]
$B(\tau^- \to e^-e^-e^+)$	2.7×10^{-8}	[2]
$B(\tau^- \to e^-\mu^-\mu^+)$	2.7×10^{-8}	[2]

[1] Aubert, PRL 104 (2010) 021802

[2] Hayasaka, PRL 687 (2010) 139



$\Omega_{\tau\mu} = \Omega_{\mu\mu} = \Omega_{\tau\tau}$, other $\Omega_{\ell\ell'} = 0$

Upper bounds

$B(\tau^- \to \mu^- \gamma)$	4.4×10^{-8}	[1]
$B(\tau^- \to \mu^- \mu^- \mu^+)$	2.1×10^{-8}	[2]
$B(\tau^- \to \mu^- e^- e^+)$	1.8×10^{-8}	[2]

[1] Aubert, PRL 104 (2010) 021802

[2] Hayasaka, PRL 687 (2010) 139

Summary

- We have shown that in the low-scaled supersymmetric seesaw models sneutrinos might give large effects indenpenent of SUSY breaking mechanism.
- Due to SUSY the $\ell \to \ell' \gamma$ are suppressed.
- That makes $\mu \to e$ conversion especially interesting candidate for finding LFV. $\mu \to 3e$ and $\tau \to 3e$ give complementary information on LFV.
- Inclusion of the mSUGRA boundary conditions strongly influences te final results of the model. Particularly it leads to lo larger theoretical prediction for LFV observables $R_{\mu e}$, $\mu \to 3e$ and $\tau \to 3$ leptons by up to factor of 25. The branching fractions for $\ell \to \ell' \gamma$ variation show smaller variation – they are slightly larger than those obtained in in MSSM+3N without mSUGRA boundary condition, but larger than results obtained in non-supersymmetric version of the model.

Charged LFV in a low-scale seesaw mSUGRA model

Amon Ilakovac

Department of Physics, University of Zagreb, 10000 Zagreb pp332, Croatia

Based on : A. Pilaftsis, L. Popov, A.I., PRD 87 (2013) 053014

- The model: two sources of LFV: soft-SUSY breaking sector; neutrino Yukawa sector: supersymmetric.
- Amplitudes : diagrams, form factor structure
- Numerical results for $\mu \to e$ conversion, $\mu \to 3e$, $\tau \to 3e/e+2\mu \ldots a_\mu$, electron EDM: dominance of Z-boson and box amplitudes, EDM - only from SB phases of A and B.

Motivation for LFV

Experiment

Observable	Upper Limit	Future sensitivity
$B(\mu \to e\gamma)$	2.4×10^{-12} [1]	$1 - 2 \times 10^{-13}$ [6], 10^{-14} [6]
$B(\mu \to eee)$	10^{-12} [2]	10^{-16} [8], 10^{-17} [7]
$R_{\mu e}^{\mathrm{Ti}}$	4.3×10^{-12} [3],	$3 - 7 \times 10^{-17}$ [10, 9], 10^{-18} [11, 7]
$R_{\mu e}^{\mathrm{Au}}$	7×10^{-13} [4]	$3 - 7 \times 10^{-17}$ [10, 9], 10^{-18} [11, 7]
$B(\tau \to e\gamma)$	3.3×10^{-8} [5]	$1 - 2 \times 10^{-9}$ [13, 12]
$B(\tau \to \mu\gamma)$	4.4×10^{-8} [5]	2×10^{-9} [13, 12]
$B(\tau \to eee)$	2.7×10^{-8} [5]	2×10^{-10} [13, 12]
$B(\tau \to e\mu\mu)$	2.7×10^{-8} [5]	10^{-10} [12]
$B(\tau \to \mu\mu\mu)$	2.1×10^{-8} [5]	2×10^{-10} [13, 12]
$B(\tau \to \mu ee)$	1.8×10^{-8} [5]	10^{-10} [12]

Table 2: Current upper limits and future sensitivities of CLFV observables.

[1] J. Adam, PRL (MEG) 107 (2011) 171801
[2] U. Bellgart, (SINDRUM) NPB 299 (1988) 1
[3] C. Dohmen, (SINDRUM II) PLB 317 (1993) 631
[4] W. Bertl, EPJ C47 (2006) 337
[5] See A.I., arXiv:1212.5939, Ref. [11]

[6] B.A. Golden (MEG) PhD 2012, J. Adam (MEG) PhD 2012

[7] J.L. Hewett, arXiv:1205.2671

[8] N. Berger, (μ3e) JPCS 408, 122070 (2013)

[9] A. Kurup (COMET) NPPS 218, 38 (2011)

[10] R.J. Abrams (Mu2e) arXiv:1211.7019; E.C. Dukes NPPS 218 (2011) 44

[11] Y. Kuno (PRISM) NPPS 149 (2005) 376; R.J. Barow (PRISM) , NPPS 218 (2011) 44

[12] K. Hayasaka, JPCS 171 (2009) 012079

[13] M. Bona (SuperB), arXiv:0709.0451

Theory

- LFV: found in neutrino oscillations only: sign for a physics BSM: scale not determined;
– CLFV and neutrino oscillations – information on a scale of new physics

Standard MSSM+3N LFV

Leptonic part of the superpotential

$$W = h_e^{ij} E_{iR}^c H_{dL} \cdot L_{jL} + h_\nu^{ij} N_{iR}^c H_{uL} \cdot L_{jL} + \frac{1}{2} M_M^{ij} N_{iR}^c N_{jR}^c$$

LFV : Borzumati, Masiero PRL (1986) 961;

$$\mathcal{M}_{\tilde{e}}^2 = \begin{pmatrix} M_{\tilde{L}}^2 + (m_e m_e^\dagger) + D_1 \mathbf{1} & m_e (A_e^* - \mu t_\beta \mathbf{1}) \\ (A_e^T - \mu^* t_\beta \mathbf{1}) m_e^\dagger & M_{\tilde{e}}^2 + (m_e^\dagger m_e) + D_2 \mathbf{1} \end{pmatrix}$$

$$(\Delta M_{\tilde{L}}^2)_{ij} \approx -\frac{1}{8\pi^2} (3m_0^2 + A_0^2) h_\nu^\dagger h_\nu \log \frac{M_X}{M_N},$$

$$(A_e)_{ij} \approx -\frac{3}{8\pi^2} A_0 h_e h_\nu^\dagger h_\nu \log \frac{M_X}{M_N},$$

Since recently : in SUSY LFV studies LFV induced by soft-SUSY breaking terms only

LFV in low-scale seesaw models (ν_RMSSM)

- **New supersymmetric LFV mechanism: $\mathbf{m_N \gtrsim 1}$ TeV**

- LFV parameters in N sector:

$$\Omega_{\ell\ell'} = \frac{v_u^2}{2m_N^2}(h_\nu^\dagger h_\nu)_{\ell\ell'} = B^*_{\ell N_i} B_{\ell' N_i}$$

- Neutrino mass matrix (m_e diagonal basis; at scale m_N)

$$M_\nu = \begin{pmatrix} 0 & m_D^T \\ m_D & M_M \end{pmatrix}, \qquad \begin{aligned} M_\nu B^{\nu\dagger} &= 0, \quad m_{n_i} \approx m_{n_j}, \quad i,j > 3, \\ m_D &= \sqrt{2} M_W s_\beta g^{-1} h_\nu^\dagger \end{aligned}$$

$$h_\nu = \begin{pmatrix} 0 & 0 & 0 \\ a e^{-\frac{i\pi}{4}} & b e^{-\frac{i\pi}{4}} & c e^{-\frac{i\pi}{4}} \\ a e^{\frac{i\pi}{4}} & b e^{\frac{i\pi}{4}} & c e^{\frac{i\pi}{4}} \end{pmatrix} \qquad h_\nu = \begin{pmatrix} a^* & b^* & c^* \\ a^* e^{-\frac{2\pi i}{3}} & b^* e^{-\frac{2\pi i}{3}} & c^* e^{-\frac{2\pi i}{3}} \\ a^* e^{\frac{2\pi i}{3}} & b^* e^{\frac{2\pi i}{3}} & c^* e^{\frac{2\pi i}{3}} \end{pmatrix}$$

· $\nu_\ell^{SM} = (Bn)_\ell = (B^\nu \nu)_\ell + (B^N N)_\ell$: B diagonalizes M_ν

- Sneutrino mass matrix

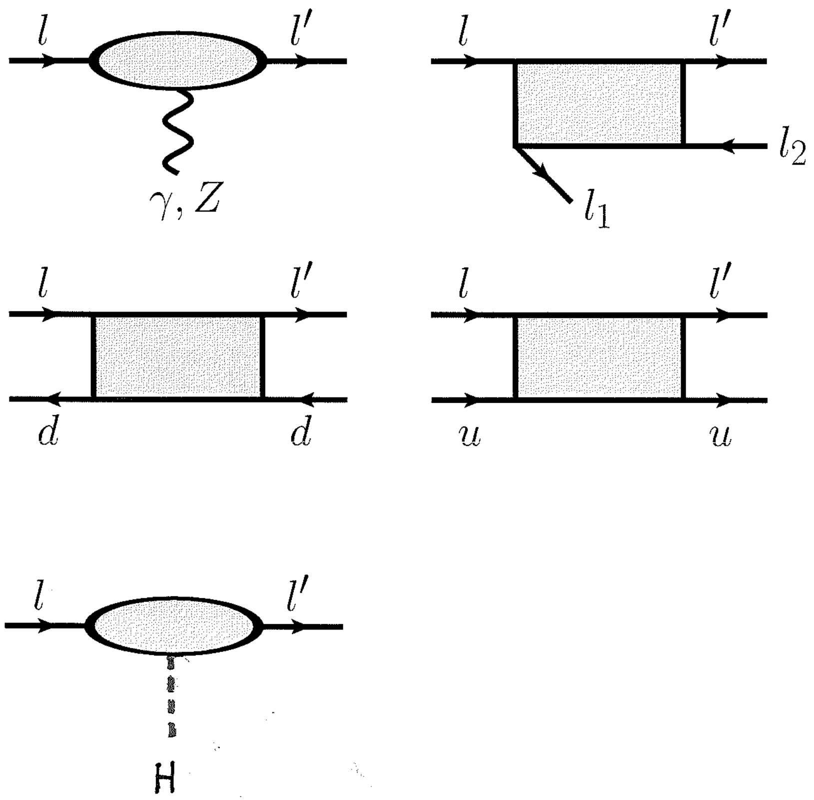
$$M_{\tilde{\nu}}^2 = \begin{pmatrix} H_1 & N & 0 & M \\ N^\dagger & H_2^T & M^T & M_B \\ 0 & M^* & H_1^T & N^* \\ M^\dagger & M_B^\dagger & N^T & H_2 \end{pmatrix}, \qquad M_{\tilde{\nu}}^2 \xrightarrow{SUSY} \begin{pmatrix} M_\nu M_\nu^\dagger & 0_{6\times 6} \\ 0_{6\times 6} & M_\nu^\dagger M_\nu \end{pmatrix}$$

$$\begin{aligned} H_1 &= m_{\tilde{L}}^2 + (\frac{1}{2}M_Z^2 c_{2\beta}\mathbf{1}) + (m_D m_D^\dagger) \\ H_2 &= m_{\tilde{\nu}}^2 + (m_D^\dagger m_D) + (M_M^\dagger M_M) \\ M &= m_D(A_\nu - \mu\, ct_\beta) \\ N &= m_D M_M^\dagger, \qquad M_B \equiv \frac{1}{2}B_{IJ}(M_\nu)_{IJ} \to 0; b_\nu \end{aligned}$$

- N-$\tilde{N}$ sector nearly supersymmetric if $m_N > m_{SUSY}$ and $h_\nu \leq 0.2$

Amplitudes

Amplitudes : diagrams



We took $\tan\beta < 20$. Neutral Higgs (h, H, A) contr. not taken into account

Amplitudes : structure

$$
\begin{aligned}
\mathcal{T}_\mu^{\gamma l' l} &= \frac{e\,\alpha_w}{8\pi M_W^2}\,\bar{l}'[(F_\gamma^L)_{l'l}\,(q^2\gamma_\mu - \not{q}q_\mu)P_L + (F_\gamma^R)_{l'l}\,(q^2\gamma_\mu - \not{q}q_\mu)P_R \\
&\quad + (G_\gamma^L)_{l'l}\,i\sigma_{\mu\nu}q^\nu P_L + (G_\gamma^R)_{l'l}\,i\sigma_{\mu\nu}q^\nu P_R]\,l, \\
\mathcal{T}_\mu^{Z l' l} &= \frac{g_w\,\alpha_w}{8\pi\cos\theta_w}\,\bar{l}'[(F_Z^L)_{l'l}\,\gamma_\mu P_L + (F_Z^R)_{l'l}\,\gamma_\mu P_R]\,l, \\
\mathcal{T}_\gamma^{l l' l_1 l_2} &= \frac{\alpha_w^2 s_w^2}{2M_W^2}\{\delta_{l_1 l_2}\,\bar{l}'\,[(F_\gamma^L)_{l'l}\,\gamma_\mu P_L + (F_\gamma^R)_{l'l}\,\gamma_\mu P_R \\
&\quad + \frac{(\not{p}-\not{p}')}{(p-p')^2}((G_\gamma^L)_{l'l}\,\gamma_\mu P_L + (G_\gamma^R)_{l'l}\,\gamma_\mu P_R)]\,l\ \ \bar{l}_1\gamma^\mu l_2^C\ -\ [l' \leftrightarrow l_1]\}\ , \\
\mathcal{T}_Z^{l l' l_1 l_2} &= \frac{\alpha_w^2}{2M_W^2}[\delta_{l_1 l_2}\,\bar{l}'((F_Z^L)_{l'l}\,\gamma_\mu P_L + (F_Z^R)_{l'l}\,\gamma_\mu P_R)l \\
&\quad \times\ \bar{l}_1(g_L^l\,\gamma^\mu P_L + g_R^l\,\gamma^\mu P_R)l_2^C - (l' \leftrightarrow l_1)]\ ,
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{T}_{\rm box}^{ll'l_1l_2} &= -\frac{\alpha_w^2}{4M_W^2}\Big(B_{\ell V}^{LL}\,\bar{l}'\gamma_\mu P_L l\;\bar{l}_1\gamma^\mu P_L l_2^C + B_{\ell V}^{RR}\,\bar{l}'\gamma_\mu P_R l\;\bar{l}_1\gamma^\mu P_R l_2^C \\
&\quad + B_{\ell V}^{LR}\,\bar{l}'\gamma_\mu P_L l\;\bar{l}_1\gamma^\mu P_R l_2^C + B_{\ell V}^{RL}\,\bar{l}'\gamma_\mu P_R l\;\bar{l}_1\gamma^\mu P_L l_2^C \\
&\quad + B_{\ell S}^{LL}\,\bar{l}' P_L l\;\bar{l}_1 P_L l_2^C + B_{\ell S}^{RR}\,\bar{l}' P_R l\;\bar{l}_1 P_R l_2^C \\
&\quad + B_{\ell S}^{LR}\,\bar{l}' P_L l\;\bar{l}_1 P_R l_2^C + B_{\ell S}^{RL}\,\bar{l}' P_R l\;\bar{l}_1 P_L l_2^C \\
&\quad + B_{\ell T}^{LL}\,\bar{l}'\sigma_{\mu\nu} P_L l\;\bar{l}_1\sigma^{\mu\nu} P_L l_2^C + B_{\ell T}^{RR}\,\bar{l}'\sigma_{\mu\nu} P_R l\;\bar{l}_1\sigma^{\mu\nu} P_R l_2^C\Big) \\
&\equiv -\frac{\alpha_w^2}{4M_W^2}\sum_{X,Y=L,R}\;\sum_{A=V,S,T} B_{\ell A}^{XY}\,\bar{l}'\Gamma_A^X l\;\bar{l}_1\Gamma_A^Y l_2^C\;,
\end{aligned}
$$

$\mathcal{T}_{\rm box}^{ll'dd}$ and $\mathcal{T}_{\rm box}^{ll'uu}$ have the same structure as $\mathcal{T}_{\rm box}^{ll'l_1l_2}$

- form factors
- new form factors

Form factors

Contributions

1. γ, Z, l-box, sl-box; h, H, A not included
2. Each form factor in principle has heavy neutrino (N), sneutrino ($\tilde{N}$) and soft SUSY breaking SB contributions, for instance

$$(F_\gamma^L)_{l'l} \quad = \quad F_{l'l\gamma}^{N} + F_{l'l\gamma}^{L,\tilde{N}} + F_{l'l\gamma}^{L,\mathrm{SB}}$$

A.I., A. Pilaftsis, PRD80 (2009) 091902 : N, $\tilde{N}$; γ, Z, l-box, sl-box; $\nu_R MSSM$

M. Hirsch, F. Staub, A. Vicente, Phys.Rev. D85 (2012) 113013, A. Abada, D. Das, A. Vicente, C. Weiland: N, $\tilde{N}$, SB; γ, Z, higgs, l-box, sl-box, but no N-box, MSISM

A.I., A. Pilaftsis, L. Popov, PRD 87 (2013) 5, 053014: N, $\tilde{N}$, SB; γ, Z, l-box, sl-box but no higgs; $\nu_R MSSM$

M. E. Krauss, W. Porod, F. Staub, A. Abada, A. Vicente, C. Weiland, PRD90 (2014) 013008, MSISM: SUSY-Z not dominant; JHEP 11 (2014) 048 detailed and complete calculation

SUSY limit; cancelations:

- $\tilde{m}^2_{\tilde{\chi}_{1,2}} \stackrel{SL}{\to} M_W^2$, $\quad t_\beta \stackrel{SL}{\to} 1$, $\quad \mu \stackrel{SL}{\to} 0$ (Barbieri, Giudice PLB309)
- $(G_\gamma^{\ell\ell'})^N + (G_\gamma^{\ell\ell'})^{\tilde{N}} \stackrel{SL}{=} 0$: Ferrara, Remiddi PLB53 (1974) 347

mSUGRA Framework

Boundary conditions and RGEs:

1. SM parameters at M_Z scale (Fusaoka and Koide PRD57 (1998) 3986).

2. Neutrino Yukawa and heavy neutrino masses at heavy neutrino scale m_N,
 (Pilaftsis PRL95 (081602) 2005, PRD72 (2005) 113001, PRD83 (2011) 076007;
 J. Kersten, A.Y. Smirnov, PRD76 (2007) 073005)

$$\begin{aligned} m_{N_i} &= m_N, \\ h_\nu &= \begin{pmatrix} 0 & 0 & 0 \\ ae^{-\frac{i\pi}{4}} & be^{-\frac{i\pi}{4}} & ce^{-\frac{i\pi}{4}} \\ ae^{\frac{i\pi}{4}} & be^{\frac{i\pi}{4}} & ce^{\frac{i\pi}{4}} \end{pmatrix} \qquad h_\nu = \begin{pmatrix} a^* & b^* & c^* \\ a^*e^{-\frac{2\pi i}{3}} & b^*e^{-\frac{2\pi i}{3}} & c^*e^{-\frac{2\pi i}{3}} \\ a^*e^{\frac{2\pi i}{3}} & b^*e^{\frac{2\pi i}{3}} & c^*e^{\frac{2\pi i}{3}} \end{pmatrix} \end{aligned}$$

3. mSUGRA conditions at gauge unification scale $g_1 = g_2 = g_3$,

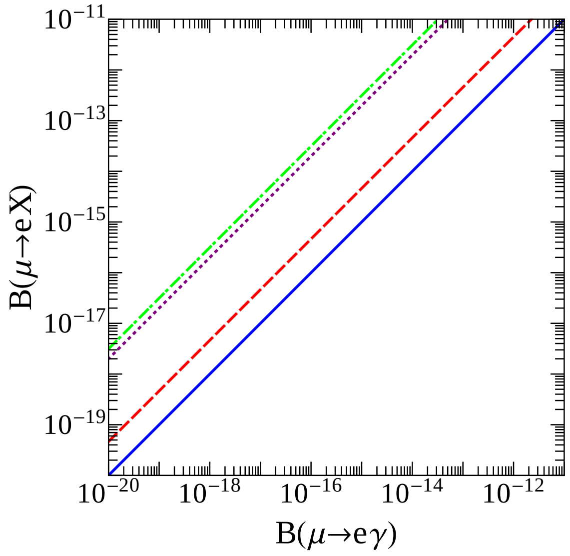
$$\begin{aligned} m^2_{H_1,H_2} &= m_0^2, \qquad m^2_{\tilde{u},\tilde{d},\tilde{e},\tilde{n}} = m_0^2\,\mathbf{1} \\ M_{1,2,3} &= M_{1/2}, \qquad A_{u,d,e,n} = A_0\, h_{u,d,e,n}\ . \end{aligned}$$

4. MSSM+$3N$ RGE equations (P. Chankowski and S. Pokorski, IJMP A17 (2002) 575,
 S. Petcov et al. NPB676 (2004) 453).

Numerical results for CLFV

Choice of parameters

1. $m_0 = 1000$ GeV, $A_0 = -4000$ GeV, $M_{1/2} = 1000$ GeV
   consistent with $m_h \approx 126$ GeV
   consistent with $m_{\tilde{g}},\ m_{\tilde{q}} > 1$ GeV
   in agreement with lightest neutralino as a dark matter candidate
2. $sign(\mu) > 0$
3. $\tan\beta = 20$ in most of calculations
4. Yukawa parameters:
   model 1: $a = b$, $c = 0$; $a = c$, $b = 0$; $b = c$, $a = 0$
   model 2: $a = b = c$
   Pertubativity condition $Tr h_\nu^\dagger h_\nu < 4\pi$:
   model 1: $a < 0.34$
   model 2: $a < 0.23$
5. $m_N < 10$ TeV: consistency with resonant leptogenesis



$B(\mu \to e\gamma)$ $B(\mu \to eee)$ $R_{\mu e}^{Ti}$ $R_{\mu e}^{Au}$

$m_0 = M_{1/2} = 1$ TeV, $A_0 = -3$ TeV

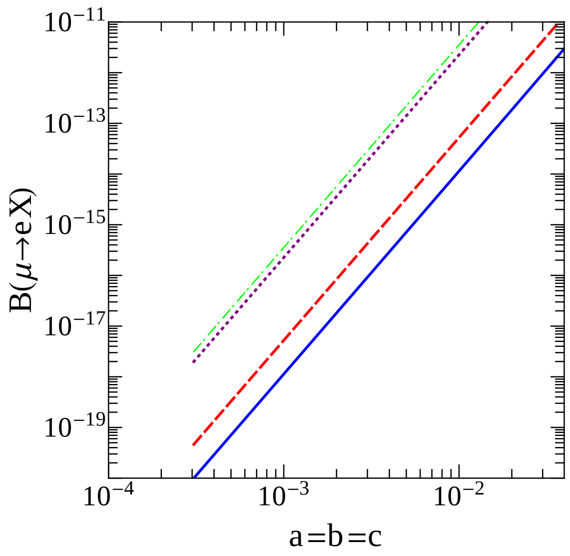
$m_N = 1$ TeV, $\tan\beta = 10$

model 2: $a = b = c$

pertubativity condition $Tr h_\nu^\dagger h_\nu < 4\pi$

quadratic Yukawa dependence

$R_{\mu e}^{Au}$, $R_{\mu e}^{Ti}, B(\mu \to eee) > B(\mu \to e\gamma)$



$B(\mu \to e\gamma)$ $B(\mu \to eee)$ $R_{\mu e}^{Ti}$ $R_{\mu e}^{Au}$

$m_0 = M_{1/2} = 1$ TeV, $A_0 = -3$ TeV

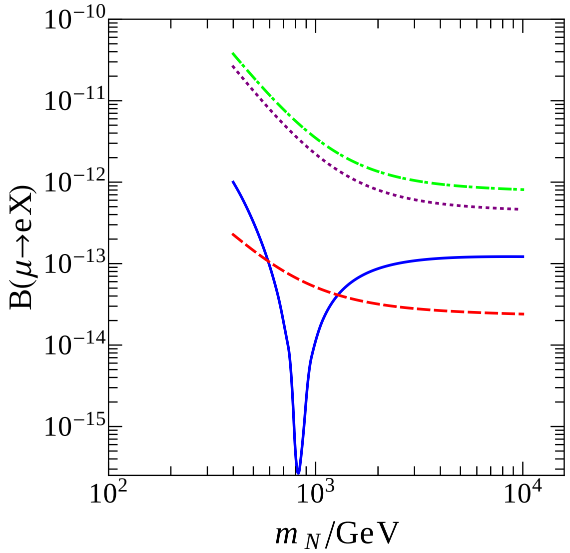
$m_N = 1$ TeV, $\tan\beta = 10$

model 2: $a = b = c$

pertubativity condition $Tr h_\nu^\dagger h_\nu < 4\pi$

quadratic Yukawa dependence

$R_{\mu e}^{Au}$, $R_{\mu e}^{Ti}, B(\mu \to eee) > B(\mu \to e\gamma)$



$B(\mu \to e\gamma)$ $\quad$ $B(\mu \to eee)$ $\quad$ $R^{Ti}_{\mu e}$ $\quad$ $R^{Au}_{\mu e}$

$m_0 = M_{1/2} = 1$ TeV, $A_0 = -3$ TeV

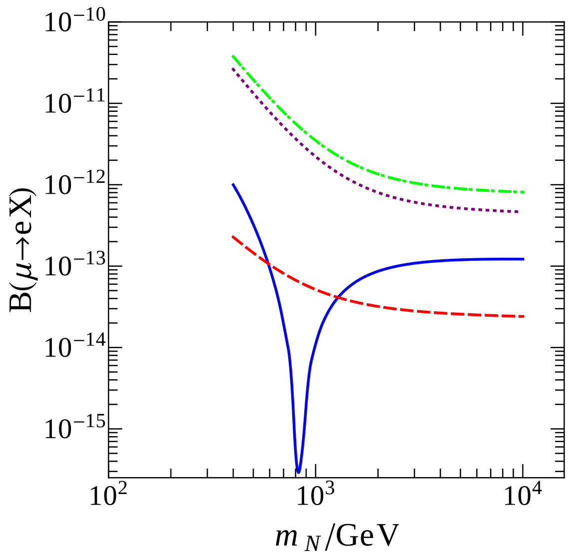
a: $B(\mu \to eee) = 10^{-12}$ for $m_N = 400$ GeV

$\tan\beta = 10$

model 2: $a = b = c$

$B(\mu \to e\gamma)$: cancelation of N, $\tilde{N}$ and SB contributions

$R^{Au}_{\mu e}$, $R^{Ti}_{\mu e} > B(\mu \to e\gamma)$



$B(\mu \to e\gamma)$ $\quad$ $B(\mu \to eee)$ $\quad$ $R^{Ti}_{\mu e}$ $\quad$ $R^{Au}_{\mu e}$

$m_0 = M_{1/2} = 1$ TeV, $A_0 = -3$ TeV

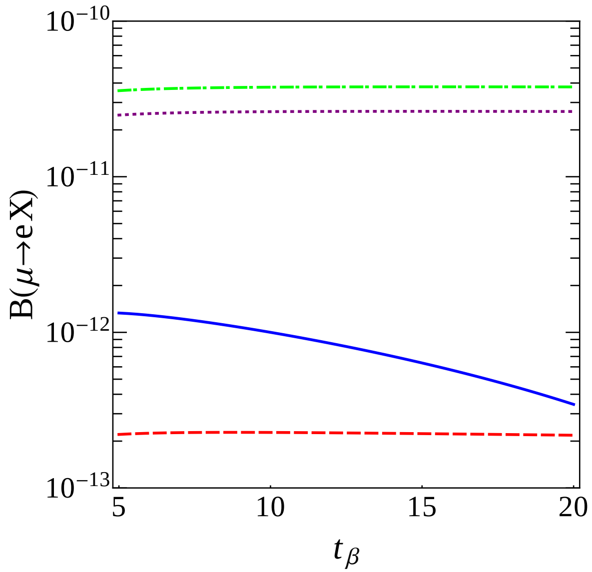
a: $B(\mu \to eee) = 10^{-12}$ for $m_N = 400$ GeV

$\tan\beta = 10$

model 1: $a = b$, $c = 0$

$B(\mu \to e\gamma)$: cancelation of N, $\tilde{N}$ and SB contributions

$R^{Au}_{\mu e}$, $R^{Ti}_{\mu e} > B(\mu \to e\gamma)$



$B(\mu \to e\gamma)$ $B(\mu \to eee)$ $R_{\mu e}^{Ti}$ $R_{\mu e}^{Au}$

$m_0 = M_{1/2} = 1$ TeV, $A_0 = -3$ TeV

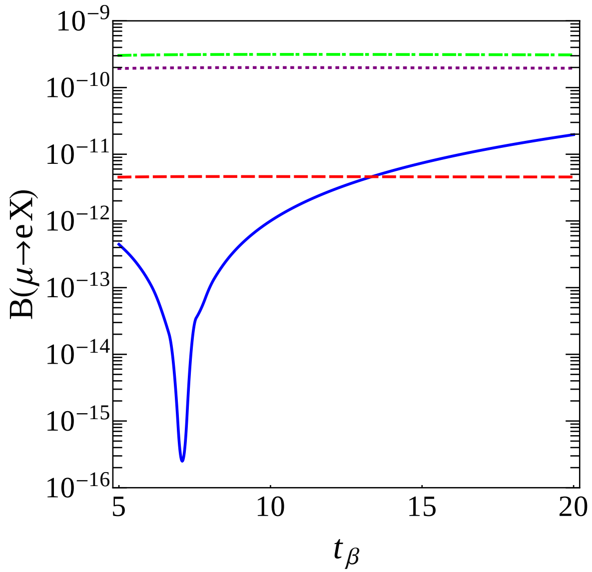
$m_N = 400$ GeV

a: $B(\mu \to eee) = 10^{-12}$ for $\tan\beta = 10$

model 2: $a = b = c$

weak dependence on $\tan\beta$

$R_{\mu e}^{Au},\ R_{\mu e}^{Ti} > B(\mu \to e\gamma)$



$B(\mu \to e\gamma)$ $B(\mu \to eee)$ $R_{\mu e}^{Ti}$ $R_{\mu e}^{Au}$

$m_0 = M_{1/2} = 1$ TeV, $A_0 = -3$ TeV

$m_N = 1$ TeV

a: $B(\mu \to eee) = 10^{-12}$ for $m_N = 1$ TeV

model 2: $a = b = c$

$B(\mu \to e\gamma)$: cancelation of N, $\tilde{N}$ and SB contributions

$R_{\mu e}^{Au},\ R_{\mu e}^{Ti} > B(\mu \to e\gamma)$

$$R_1 \equiv \frac{B(l \to l' l_1 l_1^c)}{B(l \to l'\gamma)} \to \frac{\alpha}{3\pi}\left(\ln \frac{m_l^2}{m_{l'}^2} - 3\right)$$

$$R_2 \equiv \frac{B(l \to l' l' l'^c)}{B(l \to l'\gamma)} \to \frac{\alpha}{3\pi}\left(\ln \frac{m_l^2}{m_{l'}^2} - \frac{11}{4}\right)$$

$$R_3 \equiv \frac{R^J_{\mu e}}{B(\mu \to e\gamma)}$$

$$\to 16\alpha^4 \frac{\Gamma_\mu}{\Gamma_{\rm capture}} Z Z^4_{eff} |F(-\mu^2)|^2$$

$(G^L_\gamma)_{l'l}$, $(G^R_\gamma)_{l'l}$ only:

$R_1(\tau \to e\mu\mu) = 1/90$,

$R_1(\tau \to e\mu\mu) = 1/419$

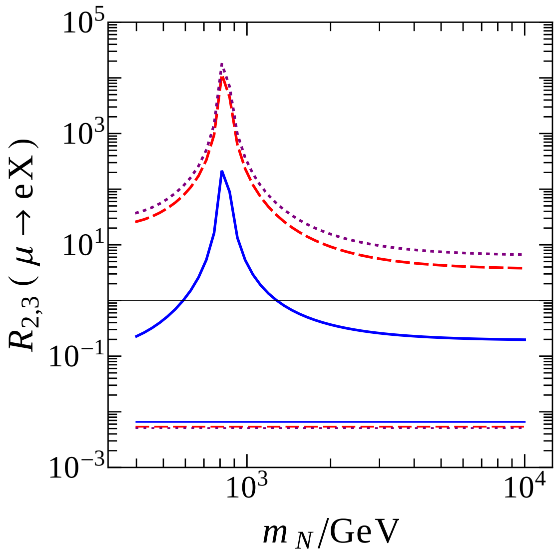
$R_2(\mu \to eee) = 1/159$,

$R_2(\tau \to eee) = 1/91$,

$R_2(\tau \to \mu\mu\mu) = 1/460$

$R_3^{Ti} = 1/198$,

$R_3^{Au} = 1/188$



$R_2(\mu \to eee)$, R_3^{Ti}, R_3^{Au}

$m_0 = M_{1/2} = 1$ TeV, $A_0 = -3$ TeV

$m_N = 400$ GeV, $\tan\beta = 10$

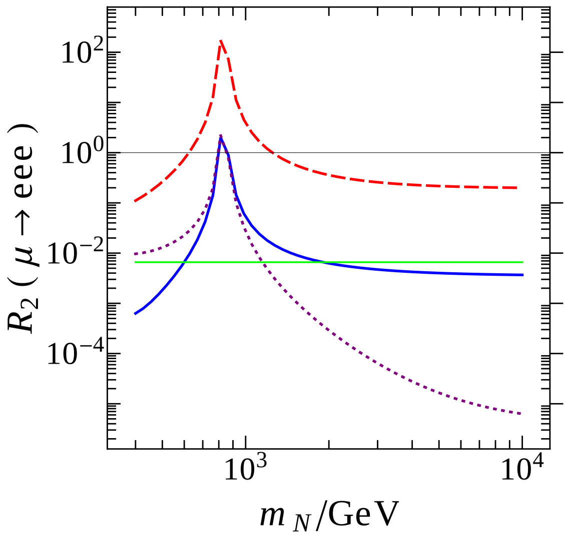
model 1 : $a = b$, $c = 0$

$R_2(\mu \to eee)$: full: $0.2 - 10^2$, $(G^{L,R}_\gamma)_{l'l}$ only: $1/159$

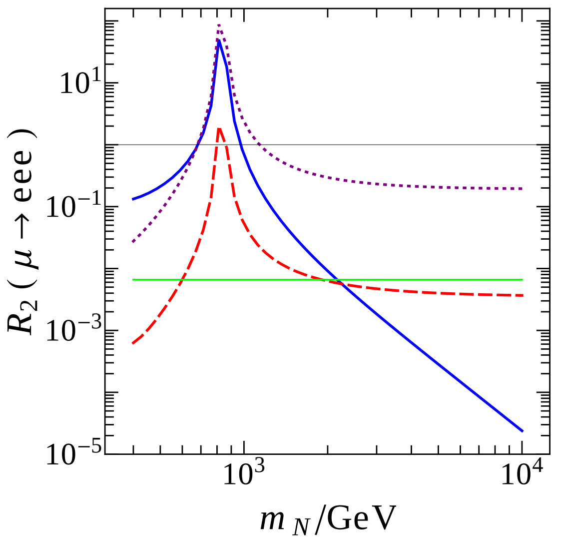
R_3^{Ti}: full: $3 - 10^4$, $(G^{L,R}_\gamma)_{l'l}$ only: $1/198$

R_3^{Au}: full: $6 - 2 \times 10^2$, $(G^{L,R}_\gamma)_{l'l}$ only: $1/188$

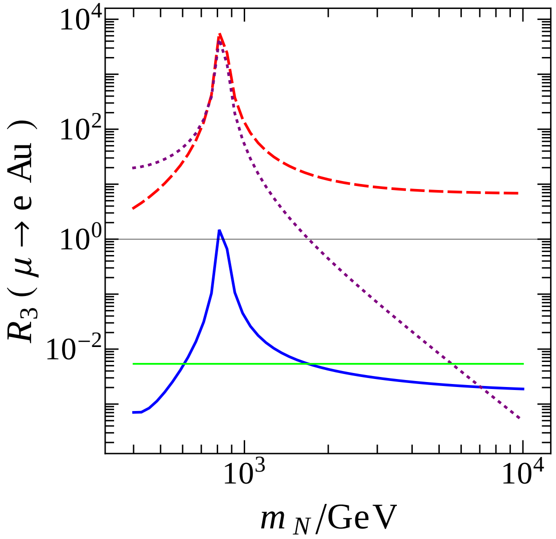
- source of strong enhancement?



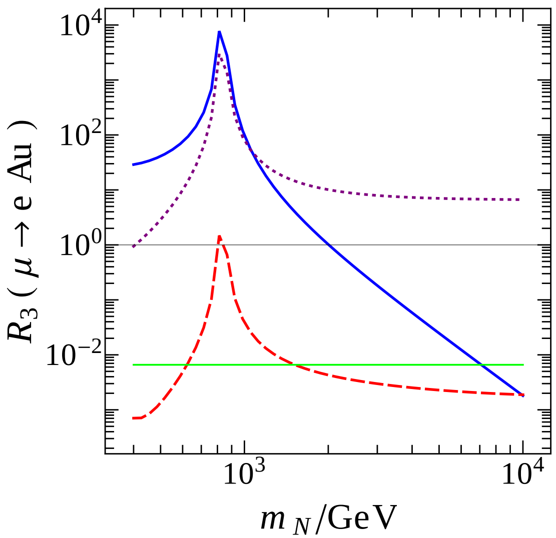
$R_2(\mu \to eee)$: form factor contributions
G_γ and F_γ, F_Z, box, $G_\gamma^{L,R}$ only
$\tan\beta = 10$
a : $B(\mu \to e\gamma) = 10^{12}$ for $m_N = 400$ GeV
- dominance of the F_Z contribution



$R_2(\mu \to eee)$: N, $\tilde{N}$, SB, $G_\gamma^{L,R}$ only
$\tan\beta = 10$
a : $B(\mu \to e\gamma) = 10^{12}$ for $m_N = 400$ GeV
- dominance of N for $m_N < 1$ TeV,
- dominance of SB for $m_N > 1$ TeV



R_3^{Au} : form factor contributions
G_γ and F_γ, F_Z, box, $G_\gamma^{L,R}$ only
$\tan\beta = 10$
$a:\ B(\mu \to e\gamma) = 10^{12}$ for $m_N = 400$ GeV
- dominance of F_Z cont. for $m_N > 1$ TeV
- dominance of box cont. for $m_N < 1$ TeV



R_3^{Au}: N, $\tilde{N}$, SB, $G_\gamma^{L,R}$ only
$\tan\beta = 10$
$a:\ B(\mu \to e\gamma) = 10^{12}$ for $m_N = 400$ GeV
- dominance of N cont. for $m_N < 1$ TeV,
- dominance of SB cont. for $m_N > 1$ TeV

Summary

- We have carefully studied the N, $\tilde{N}$ and soft SB contributions to LFV. For the first time complete set of box diagrams is included. Complete set of chiral amplitudes is included ($B^{LR}_{\ell S}$, $B^{RL}_{\ell S}$) - this decomposition is valid for any model.
- We have shown that in $\mu \to eee$ N Z-boson-mediated graphs dominate for $m_N < 1$ TeV and soft SB Z-boson-mediated graphs dominate for $m_N > 1$ TeV. In $\mu \to e$ conversion in nuclei N box graphs dominate for $m_N < 1$ TeV and soft SB Z-boson-mediated graphs dominate for $m_N > 1$ TeV. It is interesting that the low-scale seesaw model setup strongly influences soft SB part of the amplitude.
- Due to partial cancelation of N and $\tilde{N}$ contributions in magnetic dipole amplitudes the $l \to l'\gamma$ amplitudes are suppressed relative to other CLFV amplitudes.
- Due to pertubativity condition on Yukawa couplings, the CLFV amplitudes are dominated by quadratic Yukawa contributions, while quartic contributions are small.

- The dependence of LFV amplitudes on $\tan\beta$ for $5 \leq \tan\beta \leq 20$ is weak, except for $l \to l'\gamma$ processes. ($B_s \to \mu\mu$)

- Relative to the MSSM with ordinary seesaw mechanism, $l \to l' l_1 l_2$ and $\mu \to e$ conversion branching ratios are enhanced $2-3$ orders of magnitude in the region of parametric space where are no accidental cancelations of amplitudes. Opposed to the high-scale seesaw MSSM models, in the low-scale seesaw MSSM models $l \to l' l_1 l_2$ and $\mu \to e$ may give stronger constraint to the model parameters than $l \to l'\gamma$ processes.

Thank you