

News of $B \rightarrow \gamma$ form factors from $LCSR$

CP³ Origins
Cosmology & Particle Physics



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Particle Phenomenology From the Early Universe to High Energy Colliders

structure

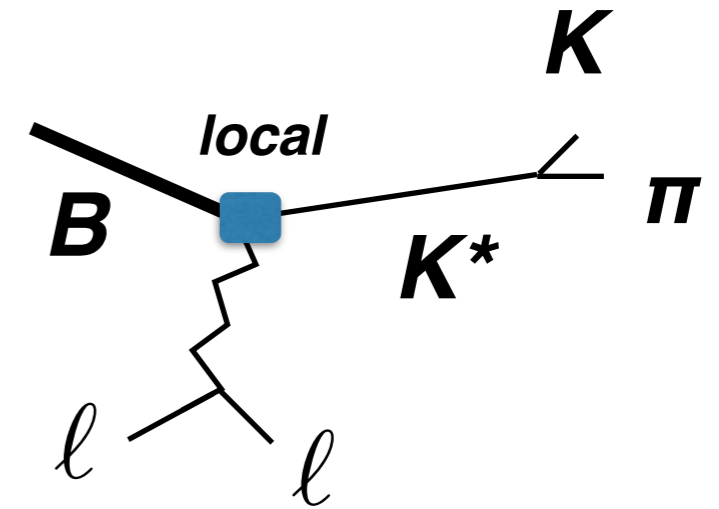
I. form factors

- ***error correlations***
- ***use of equation of motion***
- ***background effects***

II. phenomenological discussion

- ***$B_s \rightarrow \phi$ vs $B \rightarrow K^*$ tension***
- ***$|V_{ub}|$ from $B \rightarrow (\rho, \omega)lv$***
- ***short comment charm resonances in $B \rightarrow K^{(*)}ll$***

Definition of form factors



- **tensor & vector form factors**

$$\langle K^*(p, \eta) | \bar{s} i q_\nu \sigma^{\mu\nu} (1 \pm \gamma_5) b | \bar{B}(p_B) \rangle = P_1^\mu T_1(q^2) \pm P_2^\mu T_2(q^2) \pm P_3^\mu T_3(q^2)$$

$$\langle K^*(p, \eta) | \bar{s} \gamma^\mu (1 \mp \gamma_5) b | \bar{B}(p_B) \rangle = P_1^\mu \mathcal{V}_1(q^2) \pm P_2^\mu \mathcal{V}_2(q^2) \pm P_3^\mu \mathcal{V}_3(q^2) \pm P_P^\mu \mathcal{V}_P(q^2)$$

- **4 directions:**

$$P_P^\mu = i(\eta^* \cdot q) q^\mu ,$$

$$P_1^\mu = 2\epsilon^\mu_{\alpha\beta\gamma} \eta^{*\alpha} p^\beta q^\gamma ,$$

$$P_2^\mu = i\{(m_B^2 - m_{K^*}^2)\eta^{*\mu} - (\eta^* \cdot q)(p + p_B)^\mu\} ,$$

$$P_3^\mu = i(\eta^* \cdot q) \left\{ q^\mu - \frac{q^2}{m_B^2 - m_{K^*}^2} (p + p_B)^\mu \right\}$$

- **in terms of traditional notation:**

$$\mathcal{V}_P(q^2) = \frac{-2m_{K^*}}{q^2} A_0(q^2) , \quad \mathcal{V}_1(q^2) = \frac{-V(q^2)}{m_B + m_{K^*}} , \quad \mathcal{V}_2(q^2) = \frac{-A_1(q^2)}{m_B - m_{K^*}} ,$$

$$\mathcal{V}_3(q^2) = \left(\frac{m_B + m_{K^*}}{q^2} A_1(q^2) - \frac{m_B - m_{K^*}}{q^2} A_2(q^2) \right) \equiv \frac{2m_{K^*}}{q^2} A_3(q^2) .$$

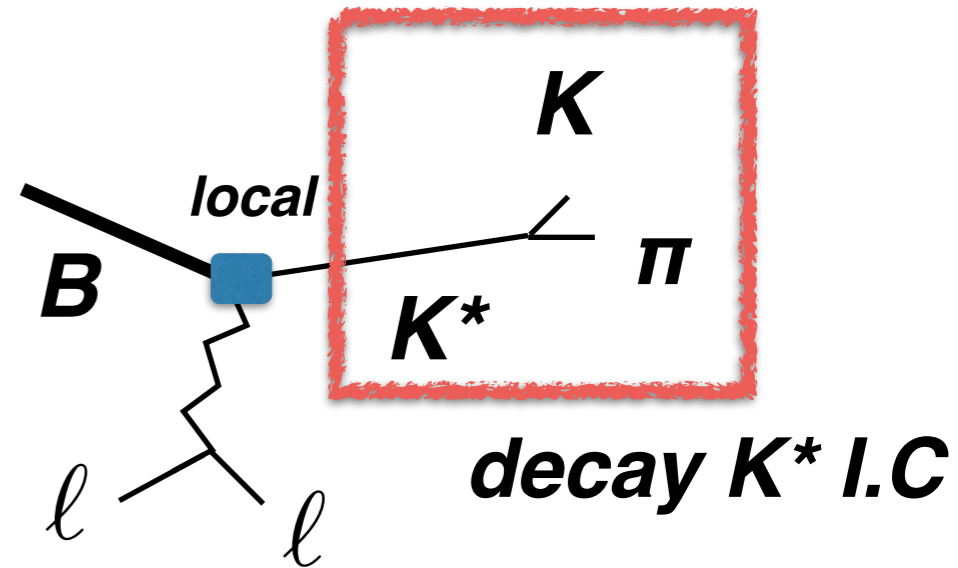
algebraically:

$$T_1(0) = T_2(0)$$

regularity:

$$A_0(0) = A_3(0)$$

Form factors & LCSR use appropriate correlation function Γ



• **sum rule on one line:**

$$\frac{V(q^2)}{p_B^2 - m_B^2} + \int_{\text{threshold}} \frac{ds}{\pi} \frac{\text{Im}\Gamma^V(s, q^2)}{(s - p_B^2 - i0)} = \Gamma^V(p_B^2, q^2)|_{\text{LCOPE}}$$

exact equation

want
 $\langle K^* | V_\mu | B \rangle$

estimate
 $\langle K^* | V_\mu | B\pi\pi \rangle + \dots$

compute
twist & α_s -expansion

$$V[\{m_b, \alpha_s, f^\parallel, f^\perp, \dots\} | \{s_0, M_{\text{Borel}}\}](q^2)$$

input \Rightarrow correlation between form factors I.A

sum rule parameters some help equation of motion I.B

I.A results & error correlations

computation based on [Ball & RZ'04](#) + O(ms)-tree + updated hadronic input

[Bharucha, Straub, RZ 1503.05534](#)

Error correlation of form factors

- idea: use input-uncertainty matrix to generate pseudo-data $O(100\text{pts})$ for all 7 form factors
 \Rightarrow fit-ansatz with $(\alpha_0, \alpha_1, \dots)$ -parameters
provide full **correlation-matrix** “easy-to-implement”

- we use:***

$$F_i(q^2) = \frac{1}{1 - q^2/m_{R,i}^2} \sum_k \alpha_k^i [z(q^2) - z(0)]^k,$$

*z-expansion
around single pole*

k=0..2

LCSR: $0 < q^2 < 14\text{GeV}^2$

k=0..2

“entire range” combined with lattice

from [Horgan, Liu, Meinel, Wingate'13](#)

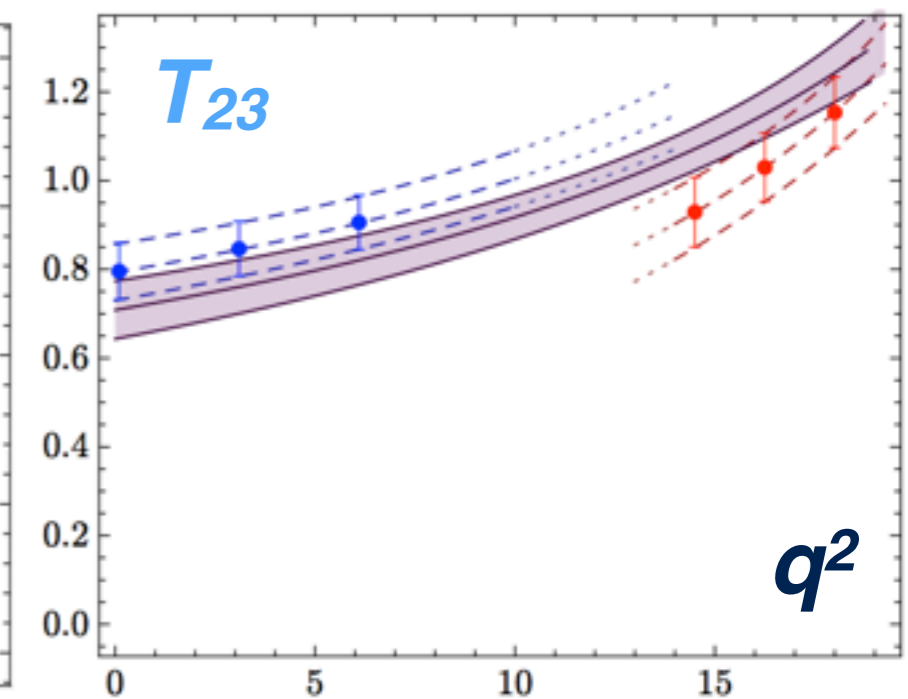
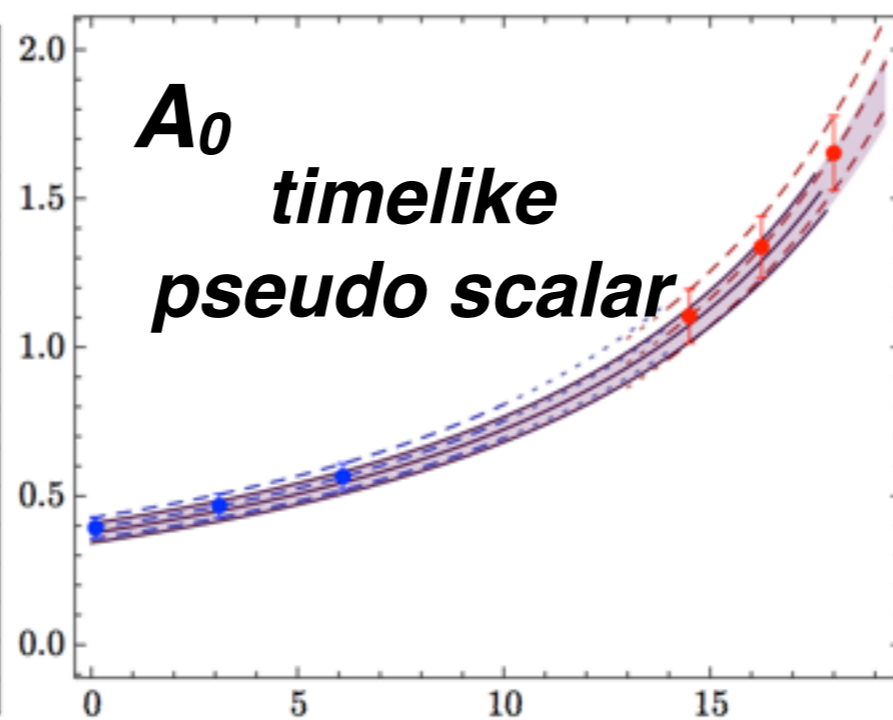
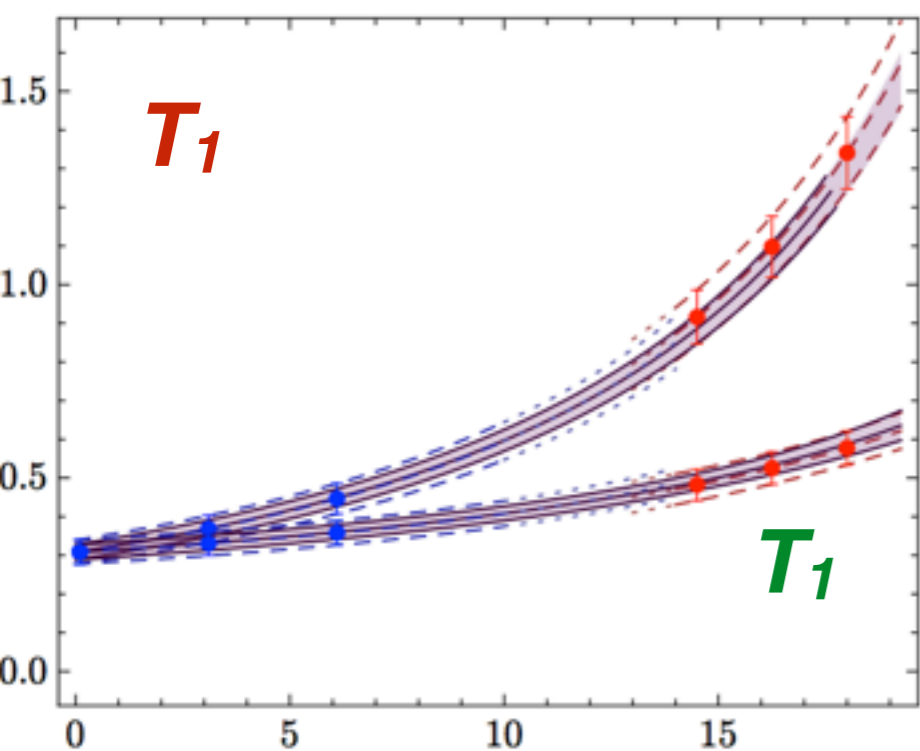
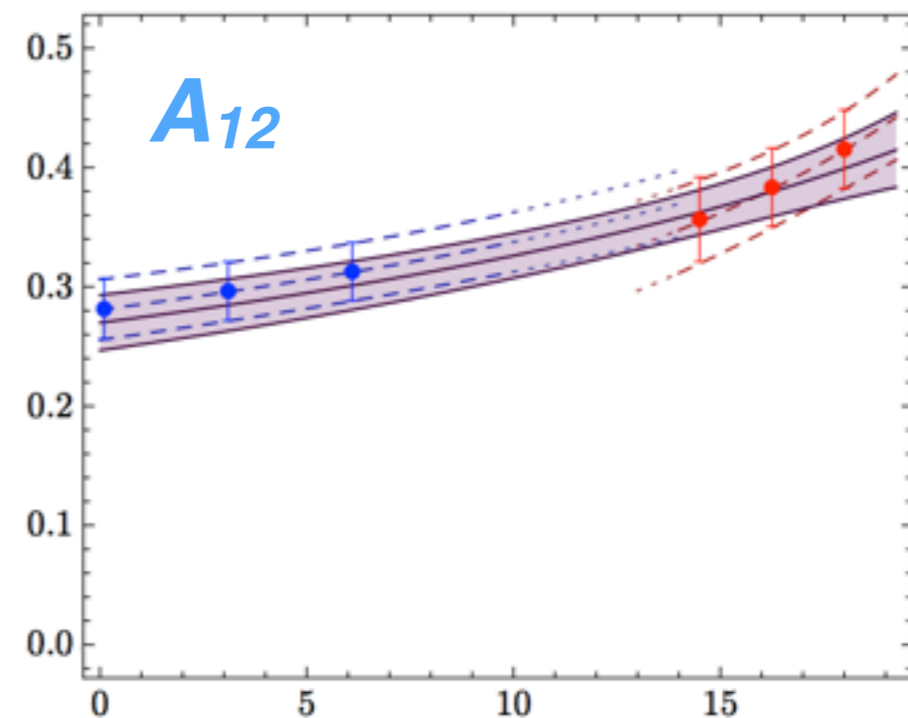
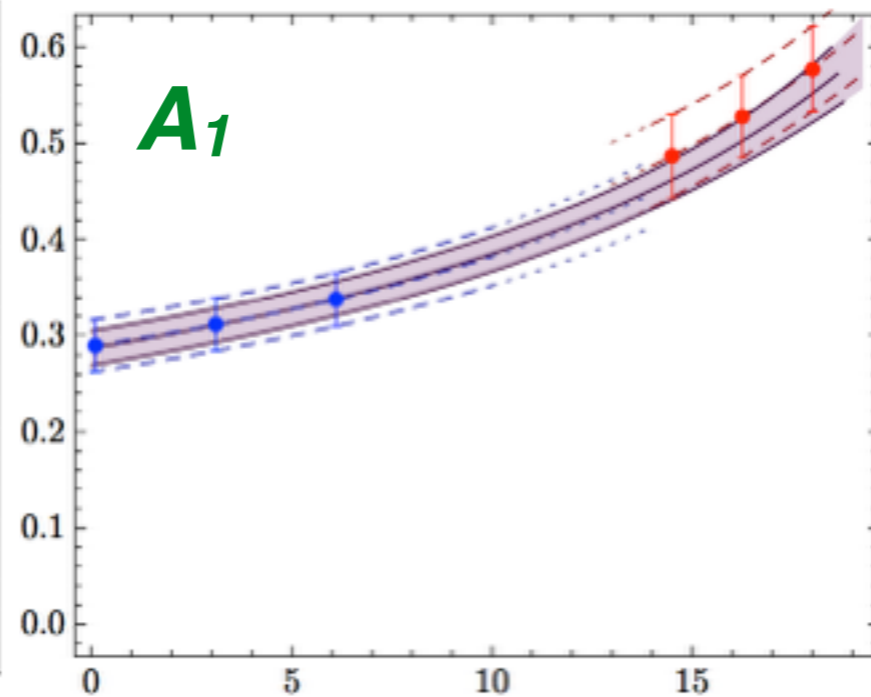
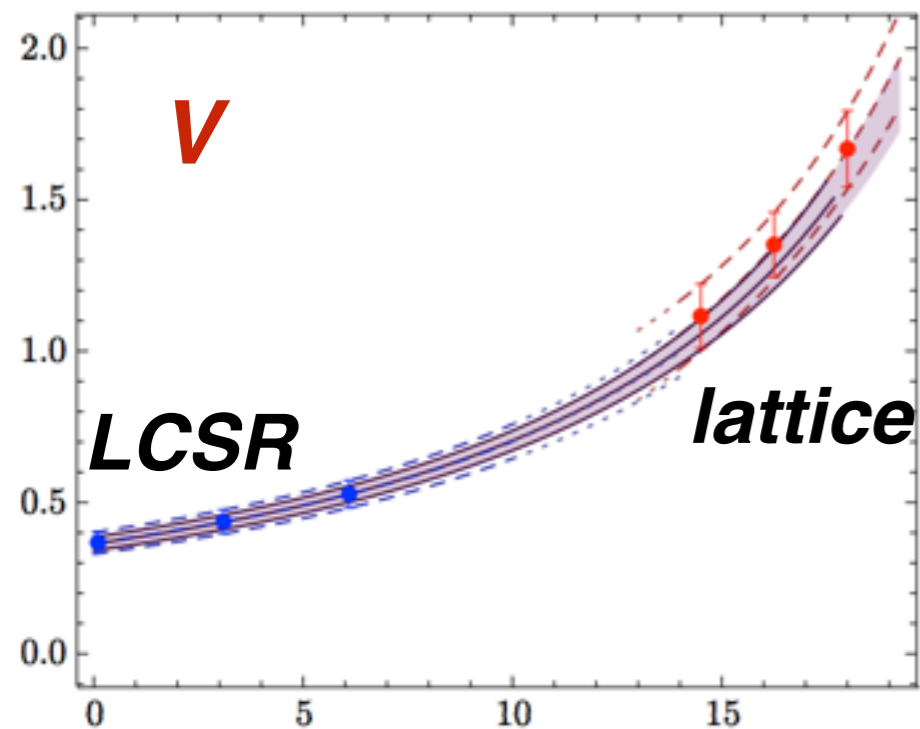
note: lattice with correlated errors as well

Combined LCSR & lattice plots

\perp -helicity

\parallel -helicity

0-helicity



q^2

I.B the use of the equation of motion (EOM)

Grinstein Pirjol'04 study correction to Isgur-Wise relation

Hambrock, Hiller, Schacht, RZ '13 first application LCSR

Bharucha, Straub, RZ '15 more systematic exploitation

- ***constrains vector-to-tensor form factor for fixed helicity***
- ***importance for $B \rightarrow K^* l l$ since zero of helicity amplitude largely determined by form factors***

$$H_{\perp}^{B \rightarrow V l l} \sim ..C_7^{\text{eff}} T_1(q^2) + ..C_9^{\text{eff}} V(q^2) + \text{long distance}$$

In particular $P_5' \sim \text{Re}[H_0 H_{\perp}]$ for instance

EOM in QFT \Leftrightarrow relations between correlation functions

- the following equation valid on $\langle K^* | \dots | B \rangle$:

$$i\partial^\nu (\bar{s} i \sigma_{\mu\nu} (\gamma_5) b) = - (m_s \pm m_b) \bar{s} \gamma_\mu (\gamma_5) b + i\partial_\mu (\bar{s} (\gamma_5) b) - 2\bar{s} i \overleftarrow{D}_\mu (\gamma_5) b,$$

- leads to 4 **equation of motion**

$$T_1(q^2) + (m_b + m_s) \mathcal{V}_1(q^2) + \mathcal{D}_1(q^2) = 0,$$

$$T_2(q^2) + (m_b - m_s) \mathcal{V}_2(q^2) + \mathcal{D}_2(q^2) = 0,$$

$$T_3(q^2) + (m_b - m_s) \mathcal{V}_3(q^2) + \mathcal{D}_3(q^2) = 0,$$

$$(m_b - m_s) \mathcal{V}_P(q^2) + \left(\mathcal{D}_P(q^2) - \frac{q^2}{m_b + m_s} \mathcal{V}_P(q^2) \right) = 0.$$

where \mathcal{D}_i 's are form factors of derivative operator:

$$\langle K^*(p, \eta) | \bar{s} (2i \overleftarrow{D})^\mu (1 \pm \gamma_5) b | \bar{B}(p_B) \rangle = P_1^\mu \mathcal{D}_1(q^2) \pm P_2^\mu \mathcal{D}_2(q^2) \pm P_3^\mu \mathcal{D}_3(q^2) \pm P_P^\mu \mathcal{D}_P(q^2)$$

Use of EOM

$$T_1(q^2) + (m_b + m_s)\mathcal{V}_1(q^2) + \mathcal{D}_1(q^2) = 0$$

- Any form factor determination has to **obey EOM** \Rightarrow **consistency check**
 - LCSR checked EOM at tree-level including $O(m_s)$ -corrections works upon use of EOM of vector meson distribution amplitudes
 - lattice (future computations)
- Recall $F_i = F_i\{m_b, \alpha_s, f^\parallel, f^\perp, \dots\} | \{s_0, M_{\text{Borel}}\} (q^2)$
 One way to obey EOM set: $s_0[T_1] = s_0[V_1] = s_0[D_1]$
 - eliminates the major source of uncertainty T_1/V -ratio [rest $O(1\%)$]
 - of course this has to be questioned

• ... yet: $T_1(q^2) + (m_b + m_s)\mathcal{V}_1(q^2) + \mathcal{D}_1(q^2) = 0$

$$0.294 \quad -0.272 \quad -0.022$$

$$s_0^{T_1} \simeq 35 \text{ GeV}^2 \quad s_0^V = s_0^{T_1} \pm 1 \text{ GeV}^2 \quad s_0^{\mathcal{D}_1} = s_0^{T_1} \begin{pmatrix} +15 \\ -6.5 \end{pmatrix} \text{ GeV}^2$$

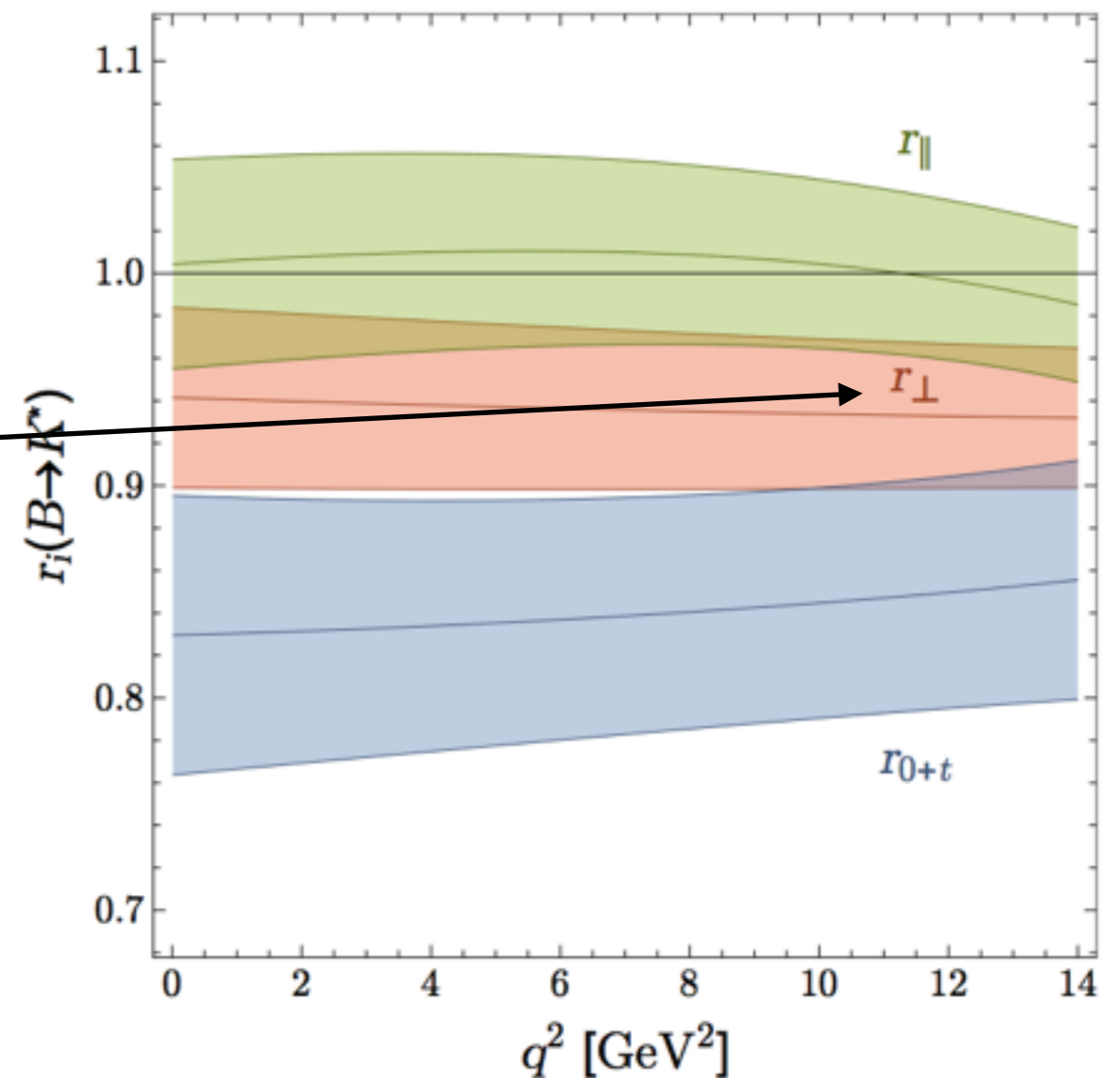
+55%
-63% -shift in \mathcal{D}_1

- Hence **if** D_1 is considered form factor then $|s_0^{T_1} - s_0^V| < 1 \text{ GeV}^2$

checked that **twist** and α_s -expansion is controlled
 (\Rightarrow more than a numerical accident)

- Vector-tensor form factor ratios determined up to 4-6%

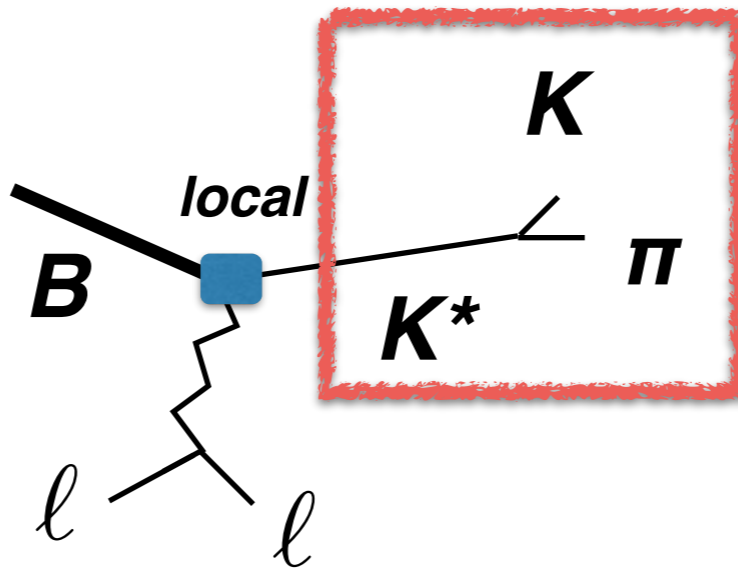
$$r_{\perp}(q^2) = \frac{m_b + m_s}{m_B + m_{K^*}} \frac{V(q^2)}{T_1(q^2)}$$



note added

- similar to large energy [Charles et al '98](#) limit and SCET investigations [Beneke Feldmann '00](#), [Bauer et al'01](#)
- similarity: both use equation of motion
- difference: LCSR EOM in QCD — SCET EOM effective theory $1/m_b$
- \Rightarrow ratios equal up to $1/m_b$ to “SCET-ratios” in [Beneke Feldmann '00](#)

I.C background effects (decaying vector meson)



background effects

question background is present in theory
and experiment (important consistent treatment)

- $B \rightarrow \rho(\rightarrow \pi\pi)lv = \text{signal} \dots \pi\pi$ in P-wave
 - 1) subtract S-wave experiment (no extra error for theory)
 - 2) what about resonant versus non-resonant $\pi\pi$ in P-wave?
- hard to disentangle in theory (in practice) and experiment
main point: argue it might not be necessary

treat $\tau \rightarrow (\pi\pi)_{P-w} lv$ same way in extraction of f_ρ as in $B \rightarrow \rho(\rightarrow \pi\pi)lv$

ρ vs $\pi\pi$ -distribution amplitude

skip no time

- using 2-pion DA (def e.g. Polyakov'98) to describe $B(\rightarrow\pi\pi)|v$ requires determination of the 2-pion DA
- for 0th Gegenbauer moment of vector 2-pion DA = pion form factor

$$F_i^{B\rightarrow\pi\pi}(q^2) = \begin{cases} \rho\text{-DA} : \frac{\langle\pi\pi|\rho\rangle}{m_{\pi\pi}^2 - m_\rho^2 + im_\rho\Gamma_\rho} \underbrace{\langle\rho|V_\mu|0\rangle}_{\sim f_\rho^\parallel} f_B^\mu(q^2) + \dots \\ \pi\pi\text{-DA} : \underbrace{\langle\pi\pi|V_\mu|0\rangle}_{\sim F^{\pi\rightarrow\pi}(m_{\pi\pi}^2)} f_B^\mu(q^2) + \dots \end{cases} \rightarrow \frac{f_\rho^\parallel m_\rho g_{\rho\pi\pi}}{m_{\pi\pi}^2 - m_\rho^2 - im_\rho\Gamma_\rho}$$

repeats other moments and current

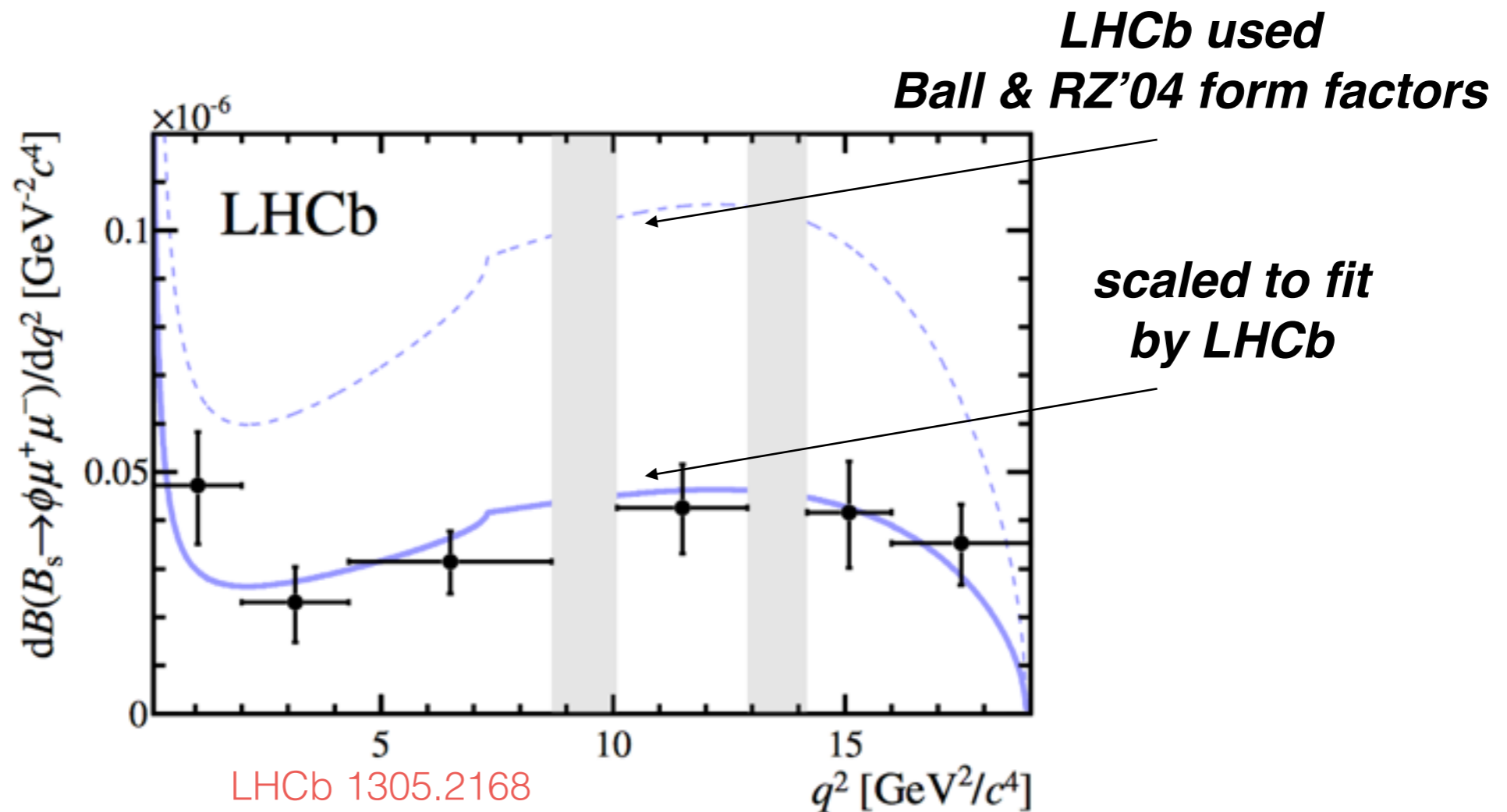
- yet higher moments or tensor 2-pion DA no experimental info available
- ρ -DA uncertainties in (other) parameters take care of background effects in error budget

around ρ -meson peak do not see pragmatic advantage in near future of using 2-pion DA

II. phenomenological discussion

II.A $B_s \rightarrow \phi$ vs $B \rightarrow K^*$ tension

II.B $|V_{ub}|$ from $B \rightarrow (\rho, \omega) l \nu$



- new predictions picture same: “we’re off by factor of 2”
shape ok — is there a **problem** with **form factor normalisation**?
look at ratio $B_s \rightarrow \phi / B \rightarrow K^*$ where normalisation effects cancel ...

B_s → φ vs B → K* tension

- at q²=0 to photons

$$R_{K^*\phi}^{(\gamma)} \equiv \frac{\text{BR}(B^0 \rightarrow K^{*0}\gamma)}{\text{BR}(B_s \rightarrow \phi\gamma)}$$

Lyon, RZ '13	LHCb '12 1202.6267
0.78(18)	1.23(32)

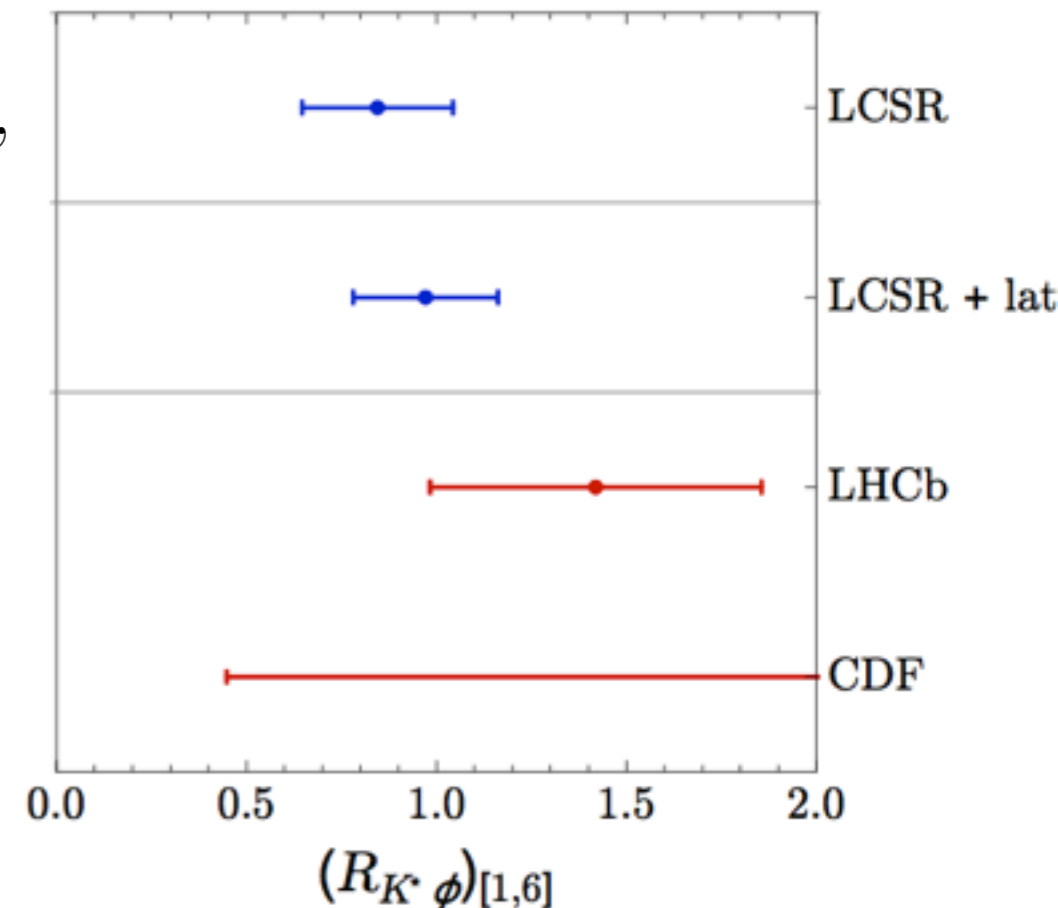
- statistically not significant but persists at higher q²

$$R_{K^*\phi}[q_1, q_2] \equiv \frac{d\text{BR}(B^0 \rightarrow K^{*0}\ell^+\ell^-)/dq^2|_{[q_1, q_2]}}{d\text{BR}(B_s \rightarrow \phi\ell^+\ell^-)/dq^2|_{[q_1, q_2]}}$$

origin of differences?

- lifetimes (effect small)
- weak annihilation taken from Lyon, RZ '13
- form factors determined mainly determined by decay constants ...

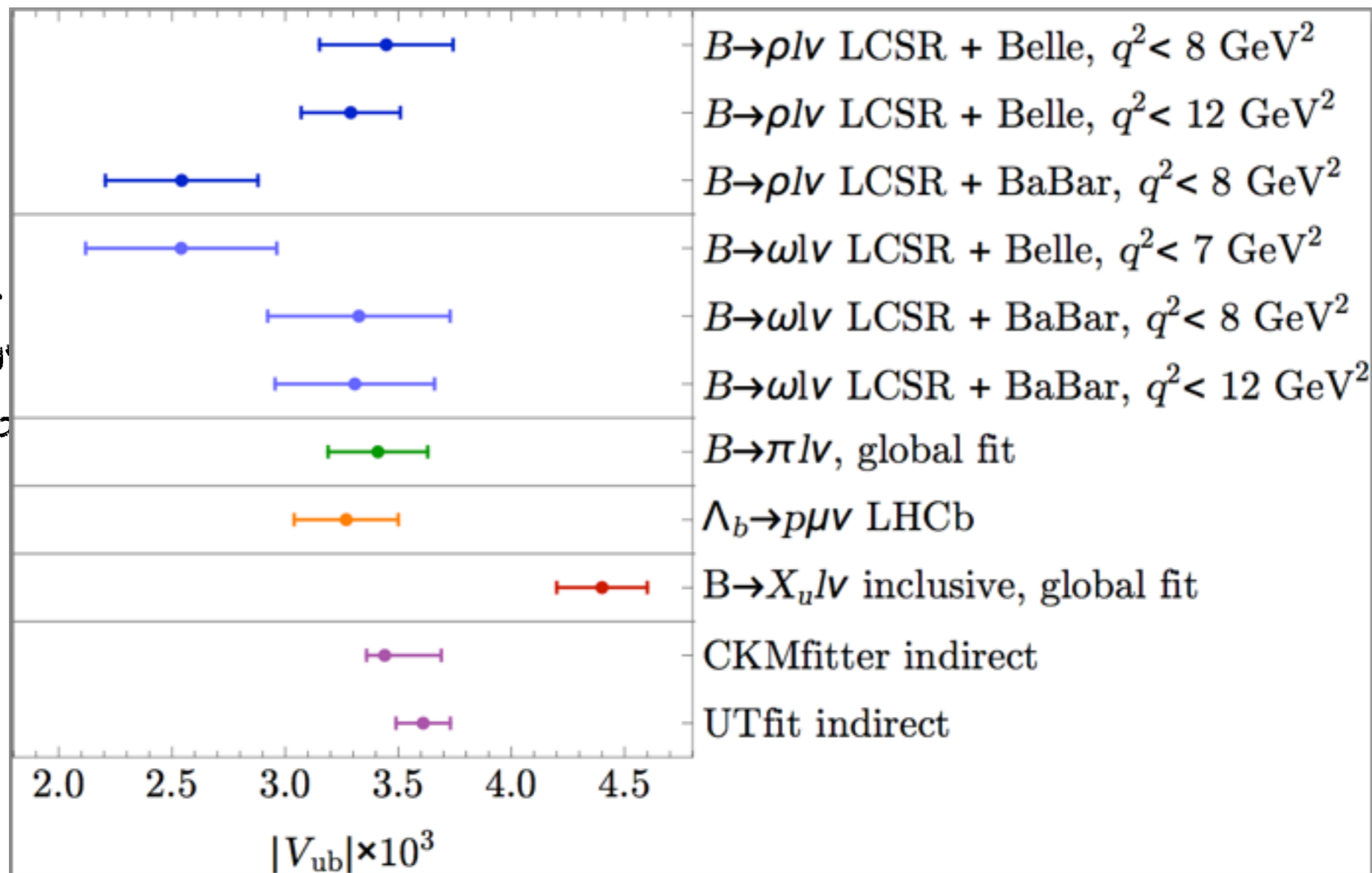
calls for test of form factors?



$|V_{ub}|$ from $B \rightarrow (\rho, \omega) l \nu$

involves vector form factors

consistent with other exclusive determinations



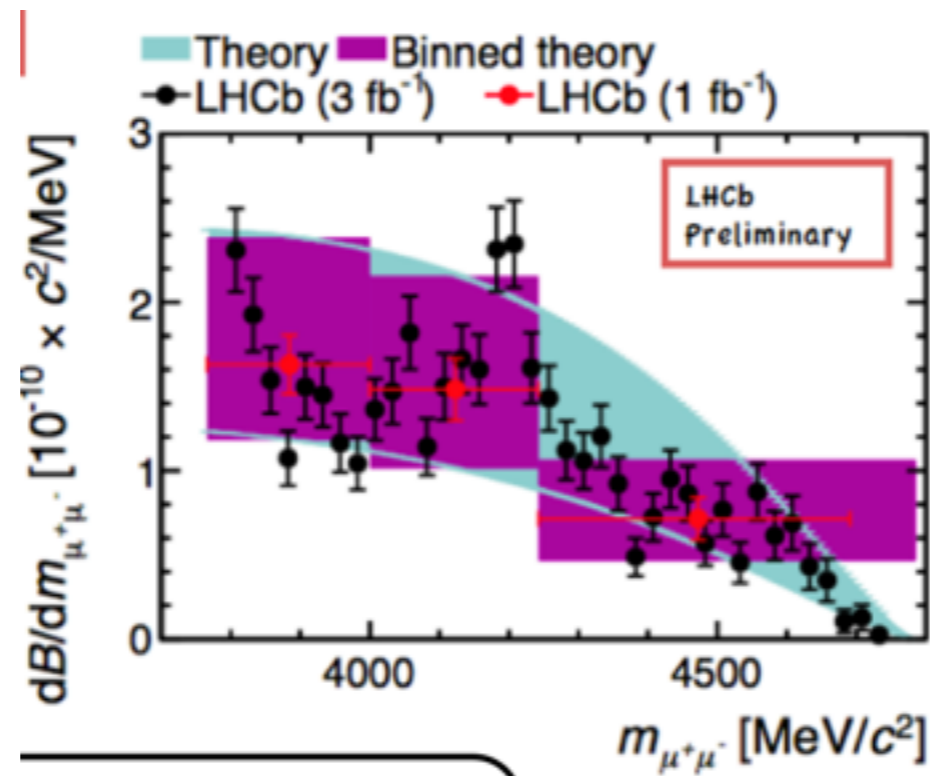
\Rightarrow no sign of (serious) normalisation problems

as questioned by $B_s \rightarrow \phi \mu \mu$

note: B-factory $|V_{ub}|$ -values (could raise) if S-wave subtracted using ang-analysis

II.C comment charm resonances in $B \rightarrow K^{(*)} \ell \ell$

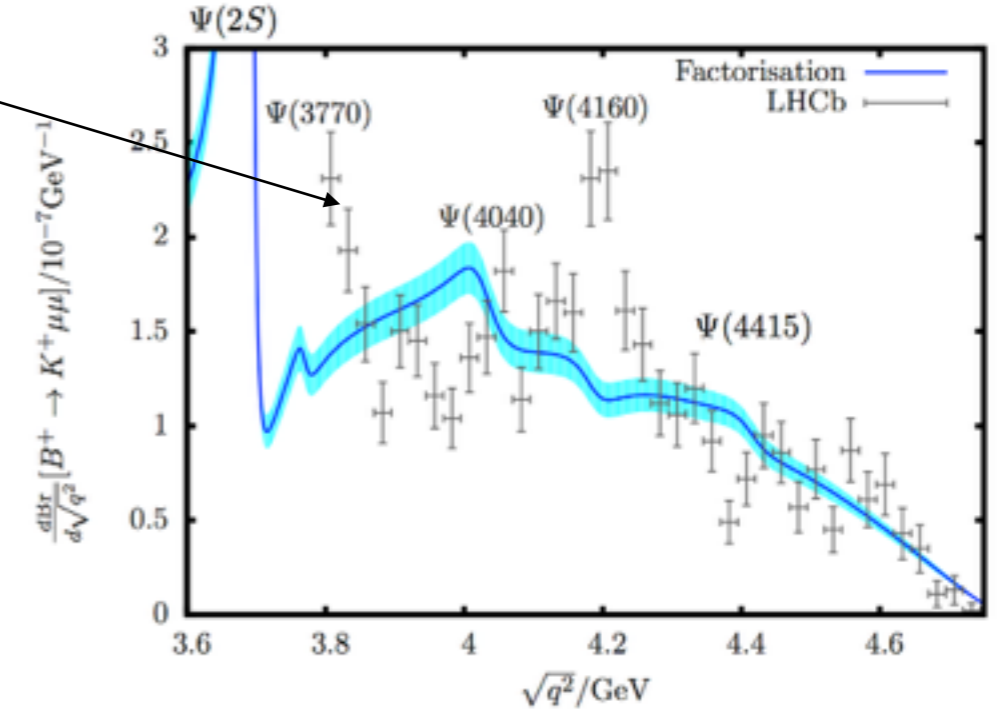
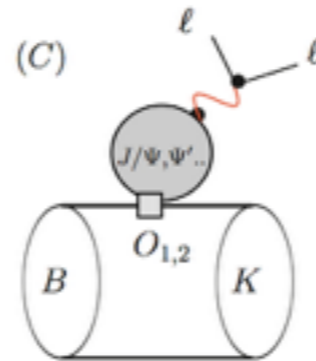
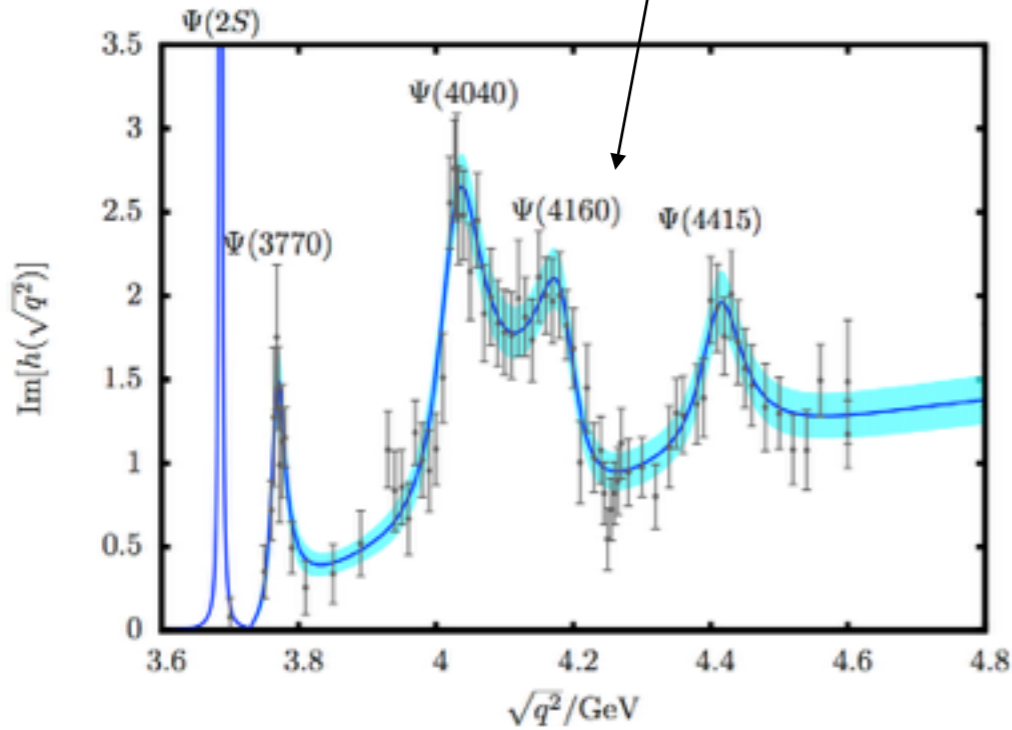
$$BF(B \rightarrow K \ell \ell)$$



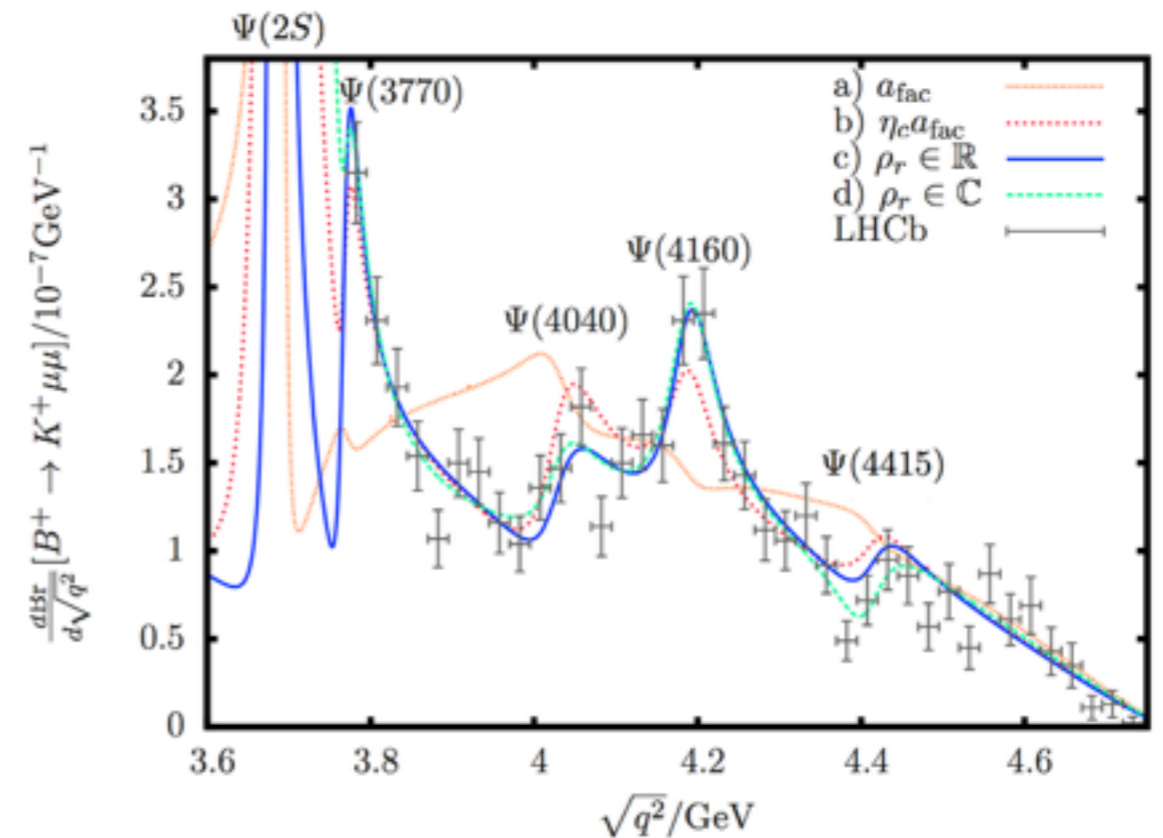
LHCb PRL 111 (2013)

pronounced $J^{PC} = 1^-$ charm resonance structure

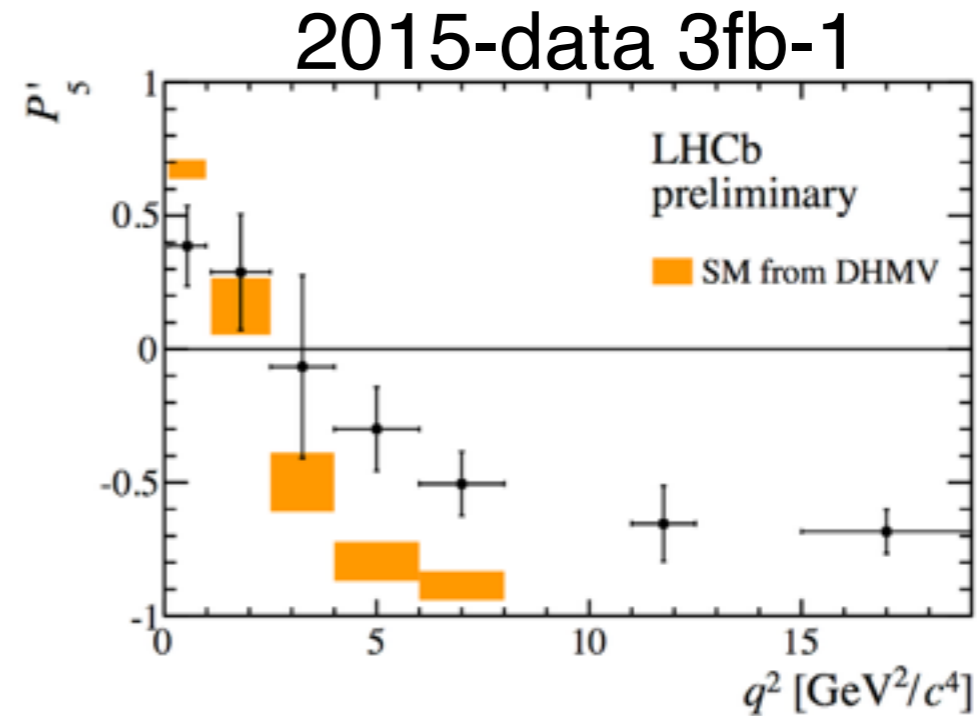
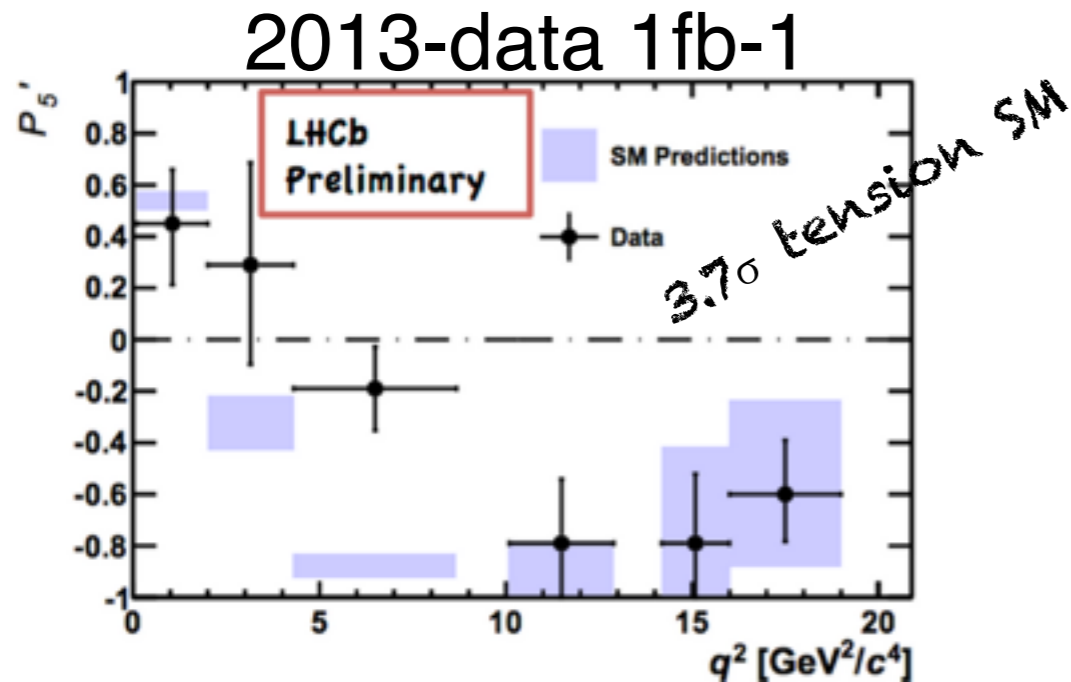
- Using a fit to BES-II data $e^+e^- \rightarrow \text{hadrons}$ able to check status of “naive” factorisation at high q^2 in $B \rightarrow K\pi\pi$



height of resonances in naive fac. by factor $\sim (-2.5)$ fits the data well

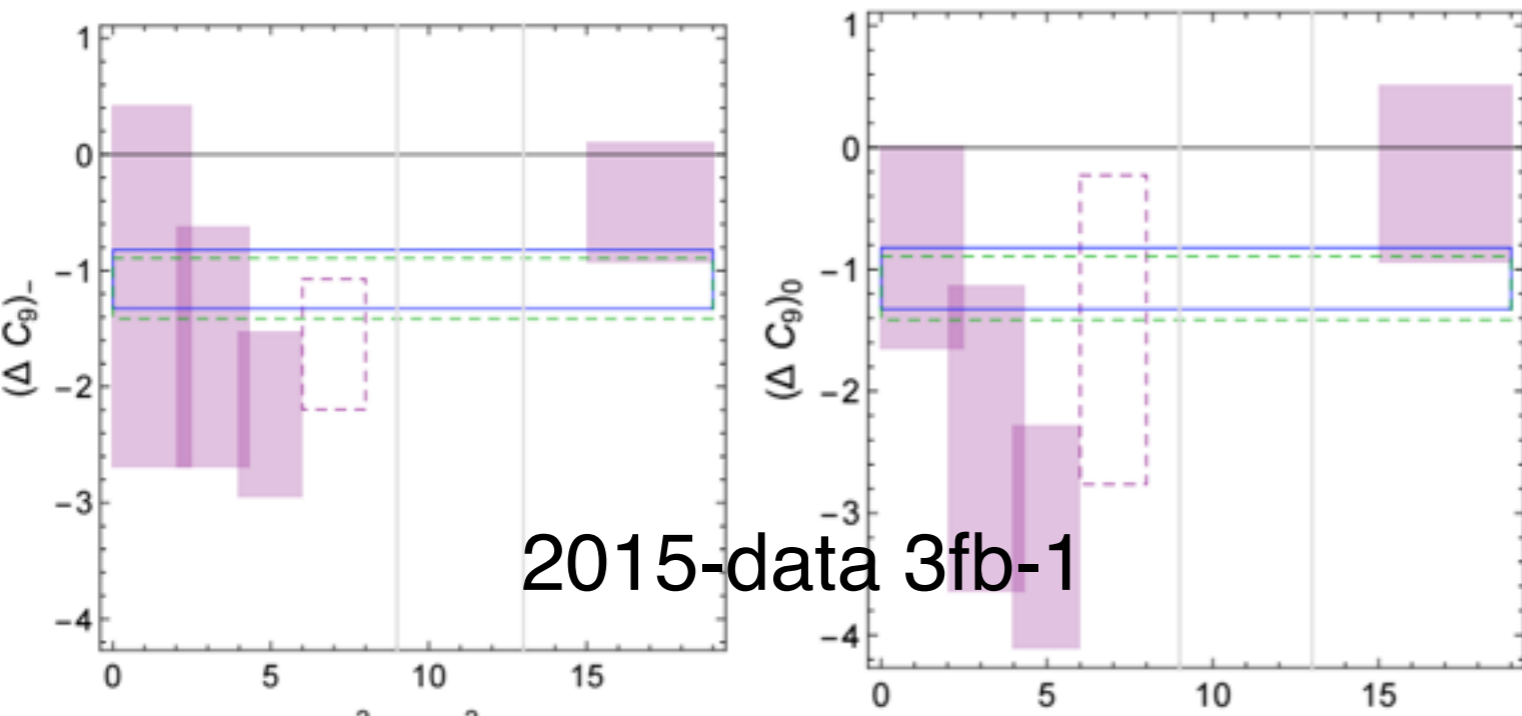


- Led us to speculate P_5' -anomaly in $B \rightarrow K^{(*)} \ell \ell$ might be related to charm (since charm pronounced)



- 1) pronounced to J/ψ 2) accommodated by photon penguin C_{10} not nec.

Straub's talk Moriond'15 (proceedings & Wolfgang's talk)



- effect same sign as in naive fac. in “-” versus “0” helicity
- my comment: that's what $B \rightarrow J/\psi K^*$ experimental angular analysis predicts for $J/\psi, \psi(2S)$ -contributions

conclusions and summary

- whether global $\Delta C_9^{\text{short-distance}} \simeq -1$ remains is tricky question — needs more data
- then **R_K -anomaly** (2.6σ) came along and there **charm** should play **no role** and this points towards true short-distance new physics talks by [Crivellin, Hiller, Altmannshofer, Nardecchia, Vicente](#)
- **equation of motion & correlated errors** for form factors help to predict angular observables like P_5' with higher precision

thanks for your attention