Hadronic Effects and Observables in $B \to \pi \ell^+ \ell^-$ decay

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Particle Phenomenology Workshop, Portoroz, Slovenia (April 8, 2015)

⊛ The first observation of $b \to d\ell^+\ell^-$ transition

- LHCb measurement:
  $BR(B^+ \to \pi^+\mu^+\mu^-) = (2.3 \pm 0.6 \pm 0.1) \times 10^{-8}$
  [LHCb Collab.,1210.2645 (2012)]
  no q^2 distribution, $B \to \pi J/\psi_{\to\mu^+\mu^-}$, $B \to \pi\psi(2S)_{\to\mu^+\mu^-}$ excluded?
- more detailed LHCb measurements ahead
- $b \to d$ FCNC sector may hide something new
  there is still some deviation of $B_d \to \mu^+\mu^-$ from SM $\oplus$ lattice.
- effective Hamiltonian

$$H_{\rm eff}^{b\to d} = \frac{4G_F}{\sqrt{2}}\Big[-\lambda_t(C_9\mathcal{O}_9 + C_{10}\mathcal{O}_{10} + C_7\mathcal{O}_{7\gamma})$$

$$-\lambda_t\sum_{i=3,4,5,6,8g} C_i\,\mathcal{O}_i + \lambda_c\sum_{i=1,2} C_i\,\mathcal{O}_i^c + \lambda_u\sum_{i=1,2} C_i\,\mathcal{O}_i^u\Big] + h.c.\,,$$

  $\lambda_p = V_{pb}V^*_{pd}$, $(p = u, c, t)$, $\lambda_u \lesssim \lambda_c \sim \lambda_t$,

  hereafter $\lambda_t = -(\lambda_u + \lambda_c)$, $\sim \lambda_u$ and $\sim \lambda_c$ separation

- direct CP violation should be noticeable in SM

⊛ Effective operators

- "direct" $b \to d\ell\ell$, $b \to d\gamma$ operators:

$$O_{9(10)} = \frac{\alpha_{em}}{4\pi}[\bar{d}_L\gamma_\mu b_L]\ell\gamma^\mu(\gamma_5)\ell, \quad C_9(m_b) \simeq 4.4, \quad C_{10}(m_b) \simeq= -4.7,$$

$$O_{7\gamma} = -\frac{em_b}{8\pi^2}[\bar{d}\sigma_{\mu\nu}(1+\gamma_5)b]F^{\mu\nu}\,, \quad C_7(m_b) \simeq -0.3$$

- quark-gluon operators, combined with quark e.m. current:

$$O_1^{(c)} = [\bar{d}_L\gamma_\rho c_L][\bar{c}_L\gamma^\rho b_L]\,, \qquad C_1(m_b) \simeq 1.1$$
$$O_2^{(c)} = [\bar{c}_L\gamma_\rho c_L][\bar{d}_L\gamma^\rho b_L]\,, \qquad C_2(m_b) \simeq -0.25$$

$$O_1^{(u)} = [\bar{d}_L\gamma_\rho u_L][\bar{u}_L\gamma^\rho b_L]\,,$$
$$O_2^{(u)} = [\bar{u}_L\gamma_\rho u_L][\bar{d}_L\gamma^\rho b_L]\,,$$

$$O_{8g} = -\frac{m_b}{8\pi^2}\bar{d}\sigma_{\mu\nu}(1+\gamma_5)bG^{\mu\nu}\,, \quad C_8(m_b) \simeq 0.2$$

O_{3-6} - quark-penguin operators , $\quad C_{3,4,5,6} < 0.03$

- difference with respect to $b \to s\ell\ell$: enhancement of $O_{1,2}^{(u)}$, $SU(3)_f$ violation

⊛ Decay amplitude

- compact form: valid for $(\ell = e, \mu)$

$$A(B \to \pi \ell^+ \ell^-) = \frac{G_F}{\sqrt{2}} \lambda_t \frac{\alpha_{\rm em}}{\pi} f^+_{B\pi}(q^2) \Bigg[(\bar{\ell}\gamma^\mu \ell)\, p_\mu \Bigg(C_9 + \Delta C_9^{(B\pi)}(q^2) + \frac{2m_b}{m_B + m_\pi} C_7^{\rm eff} \frac{f^T_{B\pi}(q^2)}{f^+_{B\pi}(q^2)} \Bigg) + (\bar{\ell}\gamma^\mu \gamma_5 \ell)\, p_\mu C_{10} \Bigg],$$

- nonlocal effects of $O_{i=1,2,3,...6,8g} \otimes j_{em}$ accumulated in (process- and q^2-dependent)

$$\Delta C_9^{(B\pi)}(q^2) = -\left(\eta_u \Delta C_9^{(u)}(q^2) + \eta_c \Delta C_9^{(c)}(q^2) \right), \qquad \eta_p \equiv \frac{\lambda_p}{\lambda_t},$$

$$\Delta C_9^{(p)}(q^2) \equiv 16\pi^2 \frac{\mathcal{H}^{(p)}(q^2)}{f^+_{B\pi}(q^2)}, \quad (p = u, c),$$

- nonlocal effects defined via two correlation functions

$$\mathcal{H}^{(p)}(q^2) \left[(p\cdot q) q_\mu - q^2 p_\mu \right] = i \int d^4x e^{iqx} \langle \pi(p) | \mathrm{T} \Big\{ j^{\rm em}_\mu(x), \Big[C_1 \mathcal{O}_1^p(0) + C_2 \mathcal{O}_2^p(0) + \sum_{k=3-6,8g} C_k \mathcal{O}_k(0) \Big] \Big\} | B(p+q) \rangle, \quad (p = u, c),$$

⊛ input: $B \to \pi$ form factors

- form factors :

$$\langle \pi(p)|\bar{d}\gamma^\mu b|B(p+q)\rangle = 2p^\mu f^+_{B\pi}(q^2) + \mathcal{O}(q^\mu),$$

$$\langle \pi(p)|\bar{d}\sigma^{\mu\nu}q_\nu b|B(p+q)\rangle = 2p^\mu q^2 \frac{if^T_{B\pi}(q^2)}{m_B + m_\pi} + O(q^\mu)$$

- the update of the vector form factor from LCSR:

I. S.Imsong, A.K., T. Mannel and D. van Dyk, JHEP **1502** (2015) 126 [arXiv:1409.7816 [hep-ph]]

$$f^+_{B\pi}(q^2) = \frac{f^+_{B\pi}(0)}{1 - q^2/m^2_{B*}} \left\{ 1 + b^+_1 \left[z(q^2, t_0) - z(0, t_0) - \frac{1}{3}\left(z(q^2, t_0)^3 - z(0, t_0)^3 \right) \right] \right.$$
$$\left. + b^+_2 \left[z(q^2, t_0)^2 - z(0, t_0)^2 + \frac{2}{3}\left(z(q^2, t_0)^3 - z(0, t_0)^3 \right) \right] \right\},$$

$f^+_{B\pi}(0) = 0.307 \pm 0.020$, $b^+_1 = -1.31 \pm 0.42$, $b^+_2 = -0.904 \pm 0.444$

- the ratio $f^T_{B\pi}(q^2)/f^+_{B\pi}(q^2)$ from LCSR

G. Duplancic, A.K., T. Mannel, B. Melic and N. Offen, JHEP **0804** (2008) 014 [arXiv:0801.1796 [hep-ph]].

⊛ The method to calculate $\Delta C_9^{(B\pi)(q^2)}$

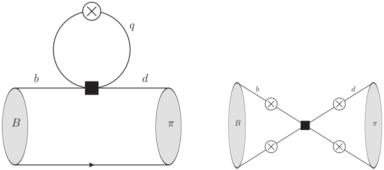
originally used for $B \to K\ell\ell$ in

A. K., T. Mannel and Y. M. Wang, JHEP **1302** (2013) 010 [arXiv:1211.0234 [hep-ph]],

- calculate the correlation functions $\mathcal{H}^{(u,c)}(q^2 < 0)$ at $|q^2| \gg \Lambda^2_{QCD}$
  OPE and convolution of hard-scattering amplitudes and meson distribution amplitudes (DA's) and/or soft form factors
- include soft-gluon nonfactorizable contributions
  A.K., T. Mannel, A. A. Pivovarov and Y.-M. Wang, JHEP **1009** (2010) 089 [arXiv:1006.4945 [hep-ph]].
- for NLO (hard-gluon) contributions we use
  QCD factorization (QCDF) (at $q^2 < 0$!)
  M. Beneke, T. Feldmann and D. Seidel, Nucl. Phys. B **612** (2001) 25; [hep-ph/0106067];
  Eur. Phys. J. C **41**, 173 (2005) [hep-ph/0412400].
- effect of penguin operators $O_{3,4,5,6,8g}$ at the level of $\leq 10\%$ in the observables
- $\mathcal{H}^{(u,c)}(q^2 > 0)$ obtained via hadronic dispersion relation in q^2
- valid at large recoil $q^2 \ll m_b^2$, below open charm threshold;

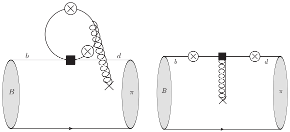
⊛ Calculating nonlocal amplitude at $q^2 < 0$

- LO diagrams: factorizable loop and weak annihilation (QCDF)



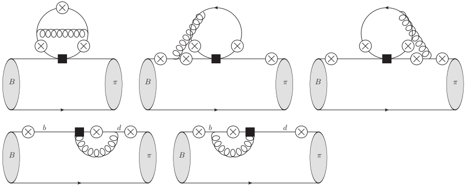
- effect of weak annihilation enhanced by $C_{1,2}$
  dominant γ^* emission from spectator *u* quark in *B*
  (NLO corrections not available, it is timely to calculate them)

- soft-gluon nonfactorizable contraibutions
  (LCSR with *B*-meson DA):



⊛ NLO contributions

- generate Im- part at $q^2 < 0$ dual to FSI
- NLO factorizable effects

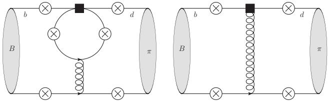


two-loop diagrams with c and massless u loops

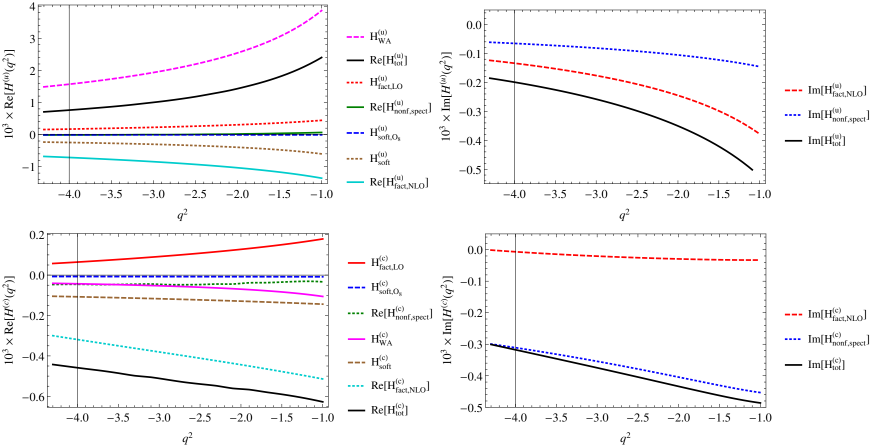
H. H. Asatryan, H. M. Asatrian, C. Greub and M. Walker, Phys. Rev. D **65** (2002) 074004 [hep-ph/0109140];

H. M. Asatrian, K. Bieri, C. Greub and M. Walker, Phys. Rev. D **69** (2004) 074007 [hep-ph/0312063].

- Nonfactorizable hard spectator scattering



⊛ Numerical results: $\mathcal{H}^{u,c}(q^2 < 0)$ for $B^\pm \to \pi^\pm \ell^+ \ell^-$



uncertainties $\leq \pm 30\%$ mainly from λ_B, f_B, μ, less from $f^+_{B\pi}$

⊛ Accessing $\Delta C_9(q^2 > 0)$

- hadronic dispersion relation

$$\mathcal{H}^{(p)}(q^2) = \mathcal{H}^{(p)}(q_0^2) + (q^2 - q_0^2)\Bigg[\sum_{V=\rho,\omega,\phi,J/\psi,\psi(2S)} \frac{f_V A^p_{BV\pi}}{(m_V^2 - q_0^2)(m_V^2 - q^2 - i m_V \Gamma_V^{\rm tot})} + \int\limits_{s_h^p}^{\infty} ds \frac{\rho_h^{(p)}(s)}{(s - q_0^2)(s - q^2 - i\epsilon)}\Bigg], \quad (p = u, c)\,.$$

- one subtraction at $q_0^2 = -1.0\ \mathrm{GeV}^2$ for a better convergence
- f_V -decay constants of vector mesons,
  $A^{u,c}_{BV\pi}$ are the $\sim \lambda_{u,c}$ parts of $B \to V\pi$ nonleptonic amplitudes ,
  their moduli fixed using experimental information and QCDF
- integral over continuum/excited states replaced with semi-local duality approximation (LO contributions)) up to $s = 4m_D^2$, the tail $4m_D^2 < s < \infty$ fitted in polynomial form
- relative phases emerging, to be fitted by matching to l.h.s.
- finally obtain $\Delta C_9^{(B\pi)}(q^2 > 0)$, adding up the calculated $\mathcal{H}^{(u,c)}(q^2 > 0)$ weighted by $\lambda_{u.c}$, divided by $f^+_{B\pi}(q^2)$

⊛ Input: Nonleptonic decays $B \to V\pi$

- $B \to \rho(\omega)\pi$ decays, separate $\sim \lambda_u$ and $\sim \lambda_c$ parts of the amplitudes calculated from QCDF;
  cross-check with experiment:

Channel	Observable	Experiment	QCDF*)	QCDF, this work
$B^\pm \to \rho^0\pi^\pm$	$\mathrm{BR} \times 10^{-6}$	8.3 ± 1.2	$11.9^{+7.8}_{-6.1}$	$9.4^{+2.9}_{-1.9}$
	$\mathcal{A}_{CP}$	$0.18^{+0.09}_{-0.17}$	0.04 ± 0.19	0.08 ± 0.14
$B^\pm \to \omega^0\pi^\pm$	$\mathrm{BR} \times 10^{-6}$	6.9 ± 0.5	$8.8^{+5.4}_{-4.3}$	$8.8^{+2.8}_{-1.7}$
	$\mathcal{A}_{CP}$	-0.04 ± 0.06	-0.02 ± 0.04	-0.06 ± 0.06

*) M. Beneke and M. Neubert, Nucl. Phys. B **675** (2003) 333 [hep-ph/0308039].

- $B \to J/\psi\pi$ and $B \to \psi(2S)\pi$ not well described by QCDF, but $\sim \lambda_u$ part is small (originating from quark-penguins), we use BR's and upper limits for A_{CP};

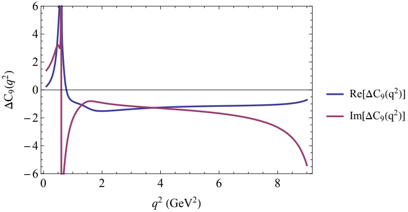
- moduli of nonleptonic amplitudes used in the dispersion relation

Mode	$\lvert A^u_{BV\pi}\rvert$	$\lvert A^c_{BV\pi}\rvert$	Mode	$\lvert A^u_{BV\pi}\rvert$	$\lvert A^c_{BV\pi}\rvert$
$B^\pm \to \rho\pi^\pm$	$20.7^{+2.6}_{-2.1}$	$1.2^{+0.9}_{-0.4}$	$B^0 \to \rho\pi^0$	$3.6^{+0.4}_{-0.5}$	0
$B^\pm \to \omega\pi^\pm$	$19.1^{+2.7}_{-2.0}$	$0.3^{+0.3}_{-0.1}$	$B^0 \to \omega\pi^0$	0	0
$B^\pm \to J/\psi\pi^\pm$	$0.1^{+1.3}_{-0.1}$	29.2 ± 1.7	$B^0 \to J/\psi\pi^0$	$0.0^{+0.9}_{-0.0}$	19.8 ± 1.1
$B^\pm \to \psi(2S)\pi^\pm$	$2.8^{+6.7}_{-2.8}$	32.9 ± 2.2	$B^0 \to \psi(2S)\pi^0$	$2.0^{+4.7}_{-2.0}$	23.3 ± 1.6
$B^\pm \to \varphi^0\pi^\pm$	0	0	$B^0 \to \varphi^0\pi^0$	0	0

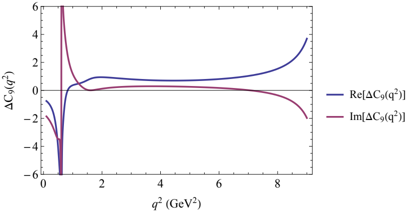
⊛ Result for C_9 at large recoil: *PRELIMINARY*

- the real and imaginary part of $\Delta C_9(q^2 > 0)$ (upper panel) and (lower panel)

$B^- \to \pi^- \ell^+ \ell^-$



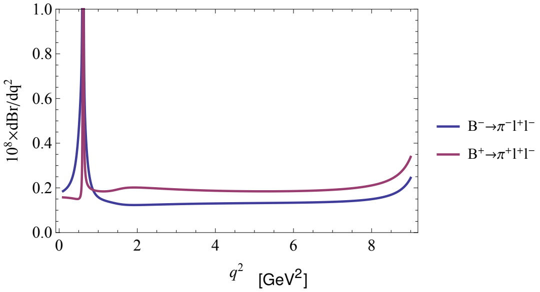
$B^+ \to \pi^+ \ell^+ \ell^-$



⊛ Observables: Differential width *PRELIMINARY*

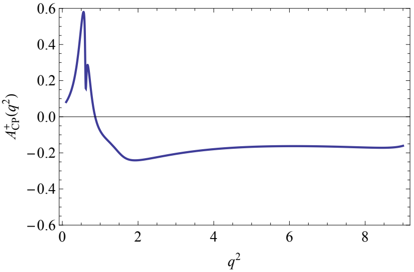
$$\frac{d\mathrm{Br}(B \to \pi \ell^+ \ell^-)}{dq^2} = \frac{G_F^2 \alpha_{\rm em}^2 |\lambda_t|^2}{1536\pi^5 m_B^3} |f^+_{B\pi}(q^2)|^2 \lambda^{3/2}(m_B^2, q^2, m_\pi^2)$$

$$\times \left\{ \left| C_9 + \Delta C_9^{B\pi}(q^2) + \frac{2m_b}{m_B + m_\pi} C_7 \frac{f^T_{B\pi}(q^2)}{f^+_{B\pi}(q^2)} \right|^2 + |C_{10}|^2 \right\} \tau_B \,.$$



⊛ Observables: *PRELIMINARY*

- differential *CP*-asymmetry



- Bins of partially integrated width (in 10^{-8}) and CP-asymmetry (error analysis on the way, smaller errors for A_{CP} expected)

$[q_1^2, q_2^2]$	$\mathrm{Br}^-(q_1^2 \le q^2 \le q_2^2)$	$\mathrm{Br}^+(q_1^2 \le q^2 \le q_2^2)$	$A_{CP}(q_1^2 \le q^2 \le q_2^2)$
[0.05, 2.0]	0.511	0.426	0.091
[2.0, 6.0]	0.519	0.759	-0.188
[6.0, 9.0]	0.457	0.639	-0.166
[0.05, 8.0]	1.308	1.573	-0.092

- $B^0 \to \pi^0 \ell^+ \ell^-$, and isospin asymmetry in progress

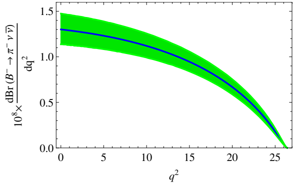
⊛ pure FCNC decay $B \to \pi\nu\bar{\nu}$

- only one effective operator in SM, nonlocal effects absent

($B \to K\nu\bar{\nu}$, M. Bartsch, M. Beylich, G. Buchalla and D.-N. Gao, JHEP **0911** (2009) [arXiv:0909.1512])

$$\mathcal{H}_{\rm eff}^{b\to d\nu\bar{\nu}} = -\frac{4G_F}{\sqrt{2}}\lambda_t C^\nu \mathcal{O}^\nu, \quad \mathcal{O}^\nu = \frac{\alpha_{\rm em}}{4\pi}(\bar{d}_L\gamma_\mu b_L)(\bar{\nu}\gamma^\mu(1-\gamma_5)\nu)$$

$$\sum_{\ell=e,\mu,\tau}\frac{d{\rm Br}(B\to\pi\nu_\ell\bar{\nu}_\ell)}{dq^2} = \tau_B\frac{G_F^2\alpha_{\rm em}^2}{256\pi^5 m_B^3}|V_{td}V_{tb}^*|^2\lambda^{3/2}(q^2)|C^\nu|^2|f_+^{(B\pi)}(q^2)|^2$$



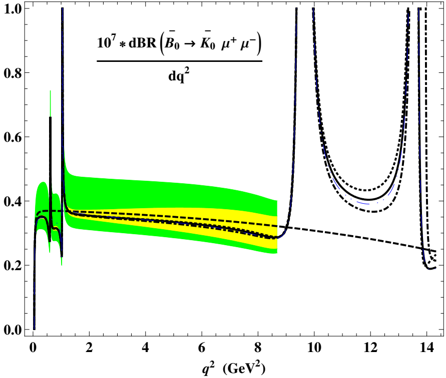
$$q^2 = m_B^2 + m_\pi^2 - 2m_B E_\pi$$

Conclusions

- hadronic input and observables in $B \to \pi\ell\ell$ in the whole large recoil region, $0 < q^2 \lesssim m^2_{J/\psi}$
- the price: dependence on the hadronic ansatz in the dispersion relation
- accuracy improvable, common input with $B \to K\ell\ell$
- most interesting - direct CP -asymmetry less dependent on the input
- in the region of intermediate $q^2 = 2 - 6$ GeV^2, the results are in the ballpark of "direct" QCDF predictions

  W. S. Hou, M. Kohda and F. Xu, Phys. Rev. D **90**, no. 1, 013002 (2014) [arXiv:1403.7410 [hep-ph]]
- in progress:
  - isospin asymmetry ($B^0 \to \pi^0\ell\ell$ could be difficult for LHCb)
  - error analysis for observables
  - generic new physics operators, to estimate the deviations

BACKUP



$\frac{10^7 * dBR(\bar{B}_0 \to \bar{K}_0\ \mu^+ \mu^-)}{dq^2}$
1.0
0.8
0.6
0.4
0.2
0.0
0
2
4
6
8
10
12
14
q^2 (GeV^2)