Daniel F Litim
University of Sussex

from asymptotic freedom to asymptotic safety

based on: DFL, F Sannino, 1406.2337
DFL, M Mojaza, F Sannino, 1501.03061
and work in prep.

Portorož 2015
Particle Phenomenology From the Early Universe to High Energy Colliders
April 7 - 10 2015, Portorož, Slovenia

standard model & beyond

University of Sussex

- local QFT for fundamental interactions
  - strong nuclear force
  - weak force
  - electromagnetic force
  - perturbatively renormalisable & predictive

standard model & beyond

University of Sussex

local QFT for fundamental interactions

- **strong** nuclear force
- **weak** force
- **electromagnetic** force
- perturbatively renormalisable & **predictive**

key feature

asymptotic freedom

- couplings achieve **non-interacting** Gaussian fixed point in the UV

standard model & beyond

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local QFT for fundamental interactions

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key feature

asymptotic freedom

couplings achieve non-interacting Gaussian fixed point in the UV

fundamental definition of QFT ⟷ UV fixed point

Wilson '71

standard model & beyond

University of Sussex

local QFT for fundamental interactions

- strong nuclear force
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~~asymptotic freedom~~

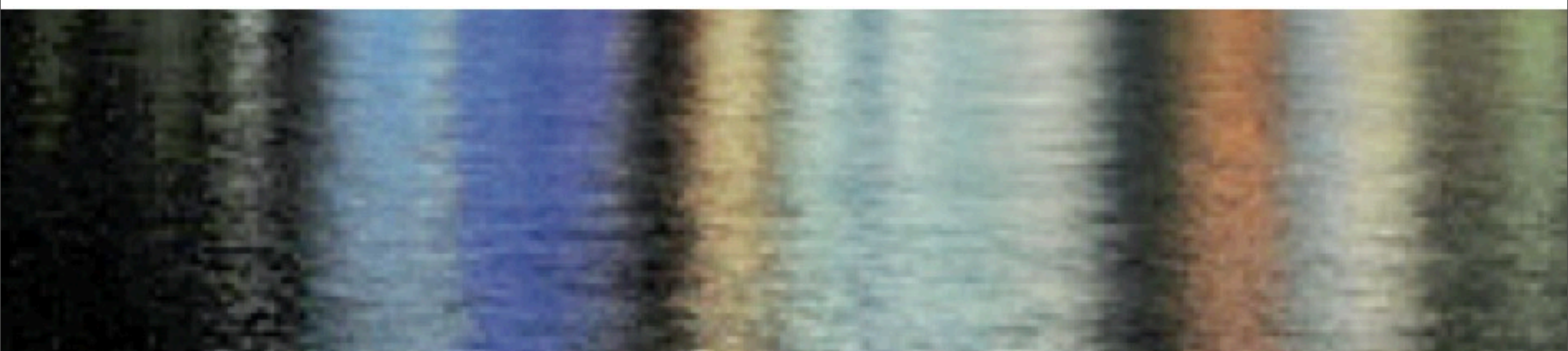
~~couplings achieve non-interacting Gaussian fixed point in the UV~~

today:

asymptotic safety

couplings achieve interacting non-Gaussian fixed point in the UV

asymptotic safety from perturbation theory



gauge theory with fermions

SU(NC) YM with NF fermions:

$$\alpha_g = \frac{g^2\, N_c}{(4\pi)^2}$$

$t = \ln \mu/\Lambda$

$$\partial_t \alpha_g = -B\, \alpha_g^2$$

$\alpha_* \ll 1$

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$B > 0$ **: asymptotic freedom**

UV fixed point $\qquad \alpha_* = 0$

$B < 0$ **: no asymptotic freedom**

UV fixed point? $\qquad \alpha_* \neq 0$

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↑

2-loop

gauge theory with fermions

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$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\alpha_g^3$$ $\alpha_* \ll 1$

$$\alpha_* = 0 \qquad \alpha_g^* = B/C$$

perturbation theory reliable provided that $\alpha_g^* \ll 1$

gauge theory with fermions

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$\alpha_* \ll 1$

$$\alpha_g^* = B/C$$

large-NF,NC (Veneziano) limit:
ϵ continuous

$$\epsilon = \frac{N_f}{N_c} - \frac{11}{2}$$

Veneziano '79

we consider $$0 < -B \equiv -B\,(\epsilon) \ll 1$$

gauge theory with fermions

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however:

no perturbative UV fixed point

in gauge theories with fermionic matter $(C > 0)$ Caswell '74

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→ **scalar fields & Yukawa couplings required**

gauge-Yukawa theory

$$\alpha_y = \frac{y^2 N_c}{(4\pi)^2} \qquad \alpha_g = \frac{g^2\, N_c}{(4\pi)^2}$$

$t = \ln \mu/\Lambda$

$\alpha_* \ll 1$

$$\partial_t \alpha_g = -B\,\alpha_g^2 + C\,\alpha_g^3 - D\,\alpha_g^2\,\alpha_y$$

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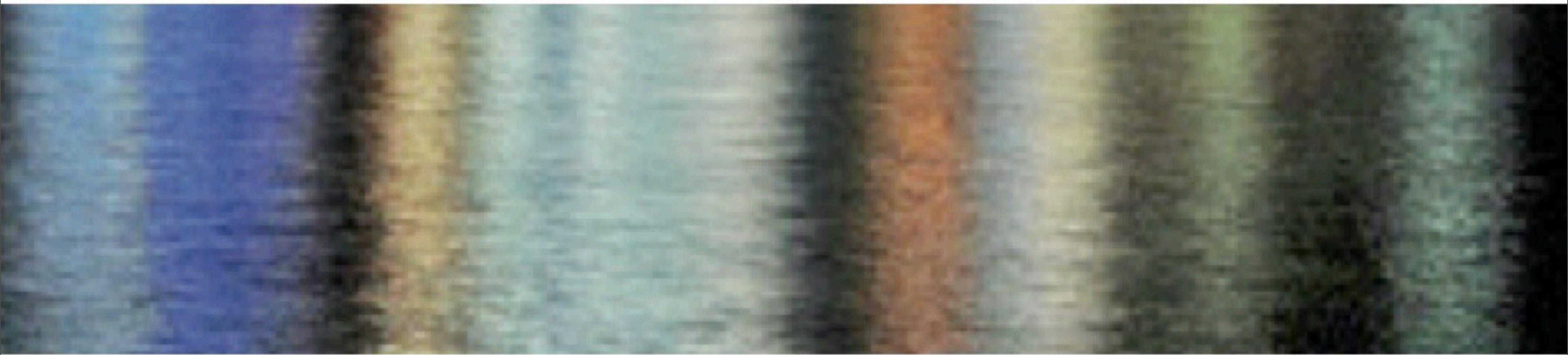
sensible interacting UV fixed point

$$D\,F - C\,E > 0$$

asymptotic safety from template gauge-Yukawa theory



gauge-Yukawa theory

Lagrangean

$$L_{\rm YM} = -\frac{1}{2}{\rm Tr}\, F^{\mu\nu}F_{\mu\nu}$$

gauge — Nc colours

$$L_F = {\rm Tr}\left(\overline{Q}\, i\not{D}\, Q\right)$$
$$L_Y = y\,{\rm Tr}\left(\overline{Q}\, H\, Q\right)$$

Yukawa — Nf flavours

$$L_H = {\rm Tr}\left(\partial_\mu H^\dagger\, \partial^\mu H\right)$$
$$L_U = -u\,{\rm Tr}\left(H^\dagger H\right)^2$$
$$L_V = -v\left({\rm Tr}\, H^\dagger H\right)^2 .$$

Higgs — Nf times Nf

gauge-Yukawa theory

Lagrangean

$$L_{\rm YM} = -\frac{1}{2}{\rm Tr}\, F^{\mu\nu} F_{\mu\nu}$$

$$L_F = {\rm Tr}\left(\overline{Q}\, i \not{D}\, Q\right)$$

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$$L_H = {\rm Tr}\left(\partial_\mu H^\dagger\, \partial^\mu H\right)$$

$$L_U = -u\, {\rm Tr}\,(H^\dagger H)^2$$

$$L_V = -v\, ({\rm Tr}\, H^\dagger H)^2\,.$$

couplings

$$\alpha_g = \frac{g^2\, N_C}{(4\pi)^2}\,, \qquad \alpha_y = \frac{y^2\, N_C}{(4\pi)^2}$$

$$\alpha_h = \frac{u\, N_F}{(4\pi)^2}\,, \qquad \alpha_v = \frac{v\, N_F^2}{(4\pi)^2}\,.$$

small parameter:

$$0 < \epsilon \ll 1 \qquad \epsilon = \frac{N_f}{N_c} - \frac{11}{2}$$

no asymptotic freedom

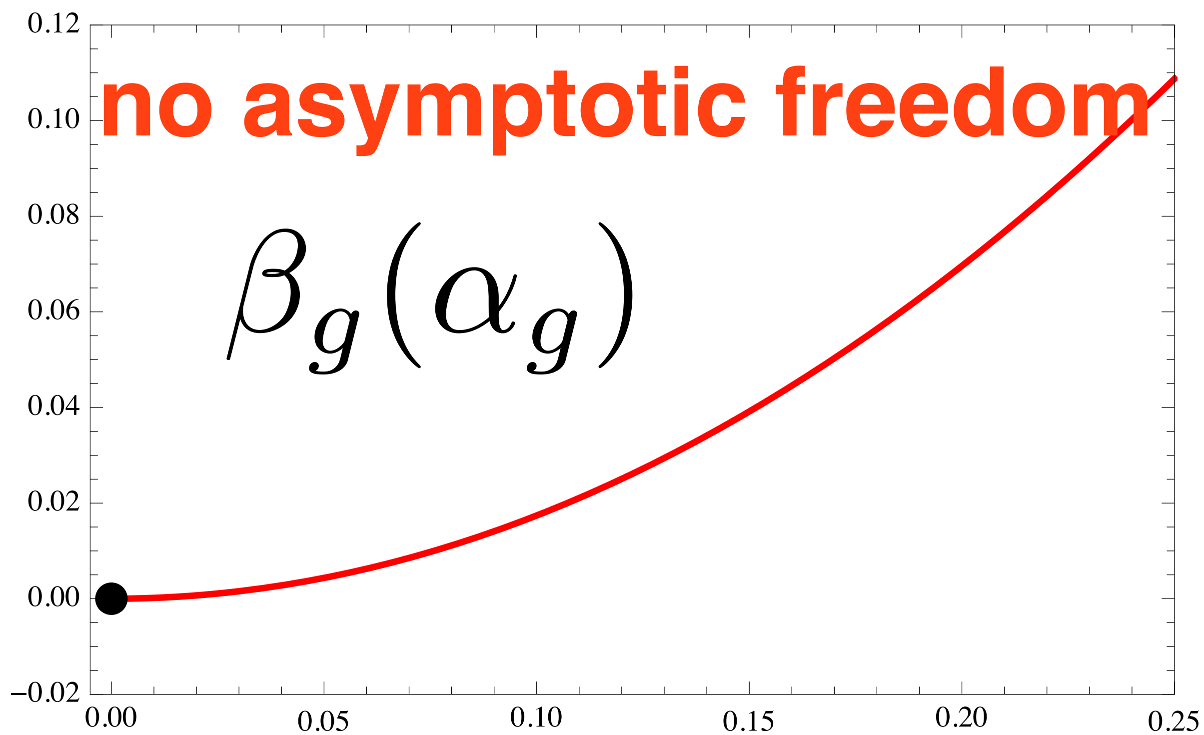
gauge-Yukawa theory

$$\beta_g = \alpha_g^2 \left\{ \frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{2} + \epsilon\right)^2 \alpha_y \right\}$$ gauge

$$\beta_y = \alpha_y \left\{ (13 + 2\epsilon)\,\alpha_y - 6\,\alpha_g \right\}.$$ Yukawa

$$\beta_h = -(11 + 2\epsilon)\,\alpha_y^2 + 4\alpha_h(\alpha_y + 2\alpha_h)$$

$$\beta_v = 12\alpha_h^2 + 4\alpha_v\,(\alpha_v + 4\alpha_h + \alpha_y)\ .$$ Higgs



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gauge-Yukawa theory

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Higgs

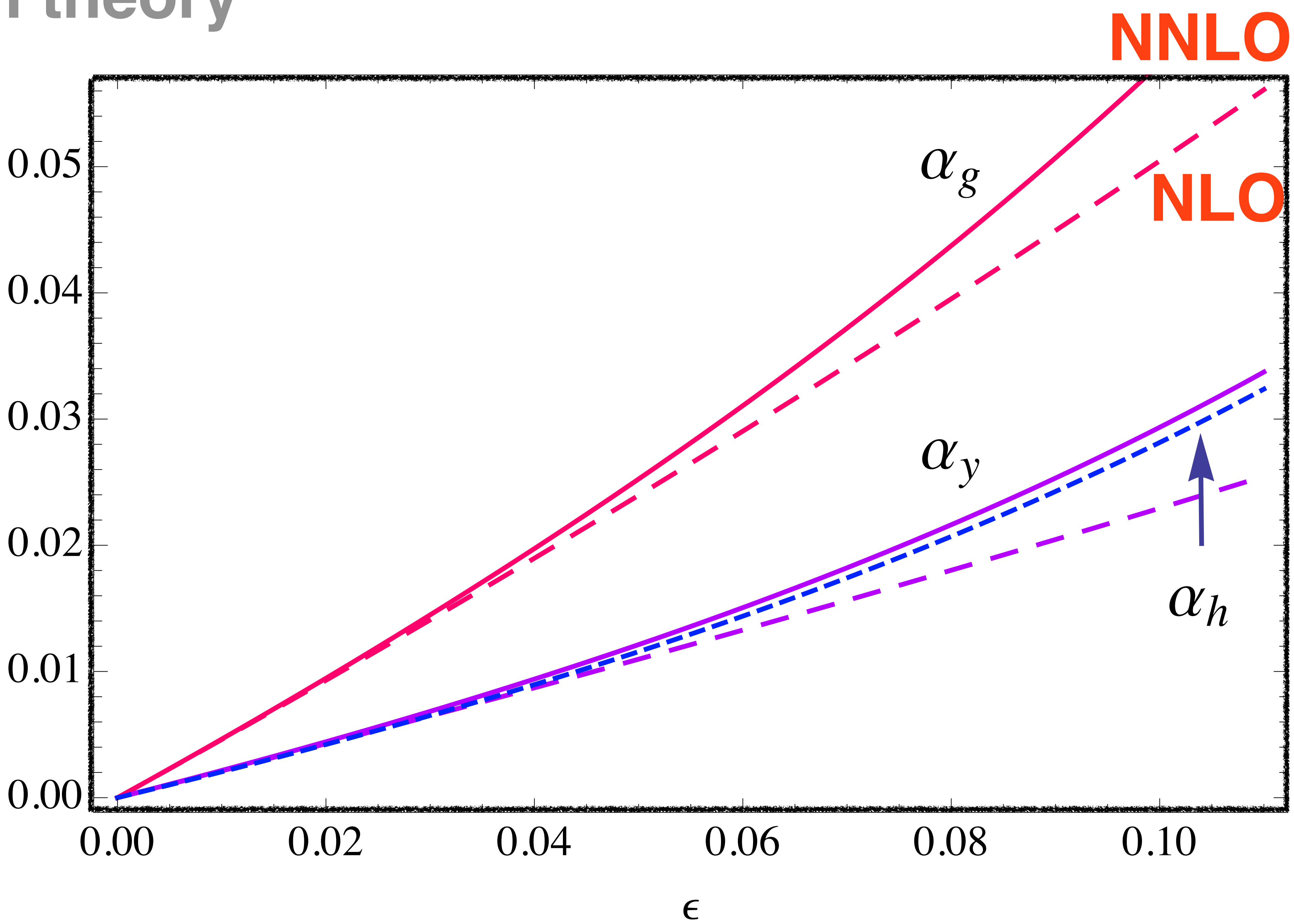
$$\beta_v = 12\alpha_h^2 + 4\alpha_v\,(\alpha_v + 4\alpha_h + \alpha_y) \ .$$

<u>exact</u>
UV fixed point

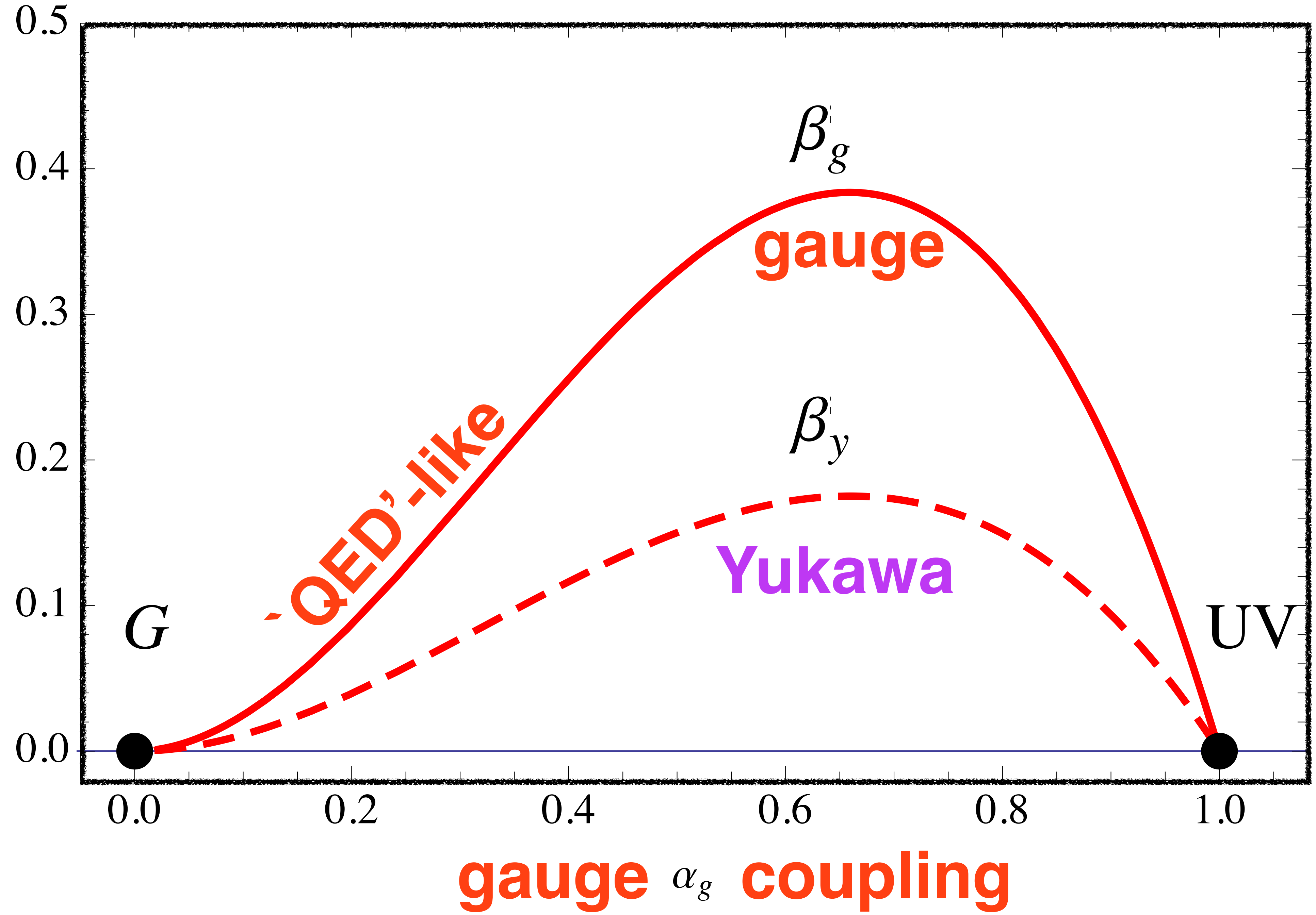
$$\begin{aligned} \alpha_g^* &= 0.4561\,\epsilon + 0.7808\,\epsilon^2 + \mathcal{O}(\epsilon^3) \\ \alpha_y^* &= 0.2105\,\epsilon + 0.5082\,\epsilon^2 + \mathcal{O}(\epsilon^3) \\ \alpha_h^* &= 0.1998\,\epsilon + 0.5042\,\epsilon^2 + \mathcal{O}(\epsilon^3) \\ \alpha_v^* &= -0.1373\,\epsilon + \mathcal{O}(\epsilon^2) \end{aligned}$$

results

UV fixed point from perturbation theory



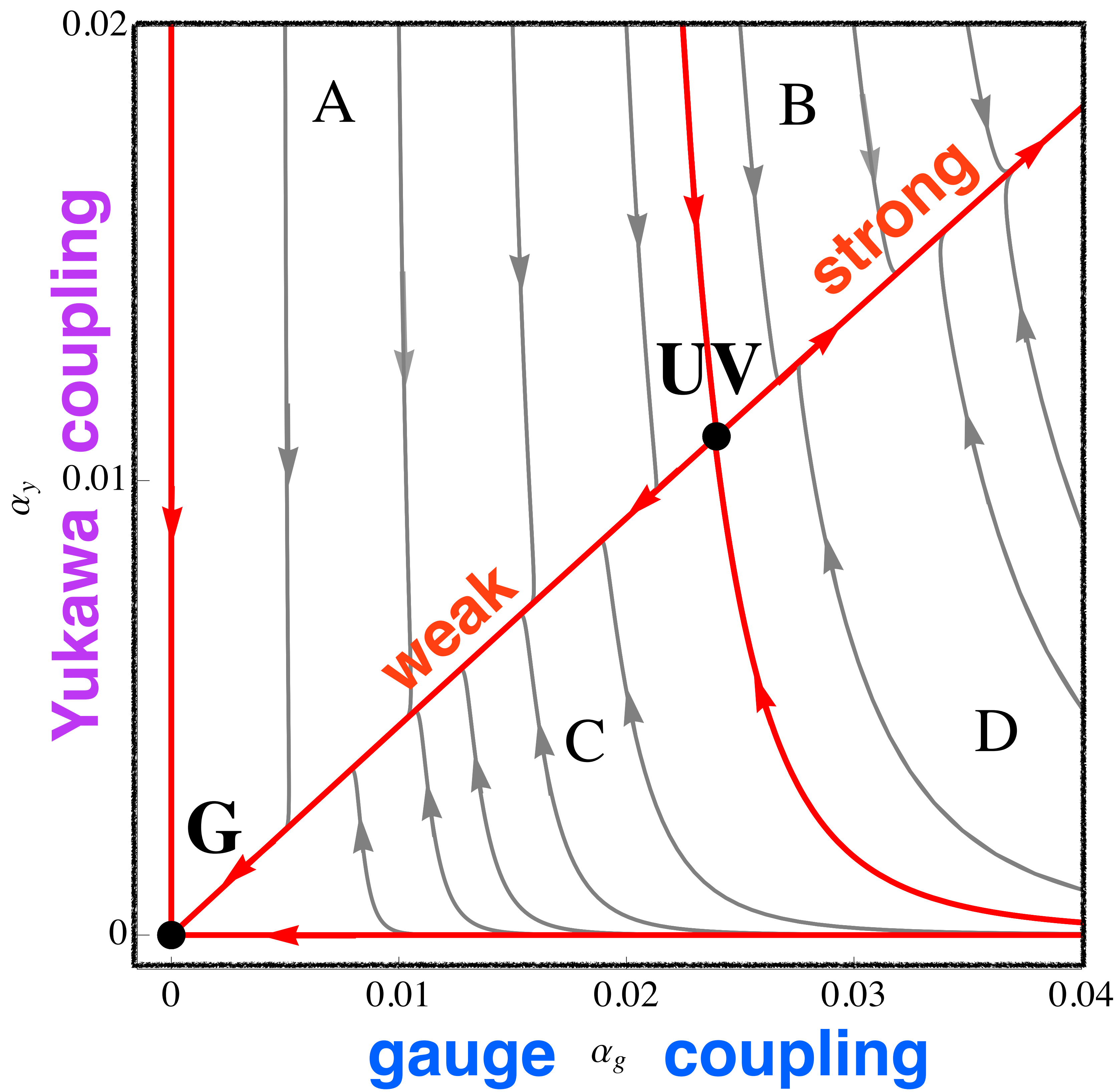
results



interacting UV fixed point entirely due to `fluctuations'

results

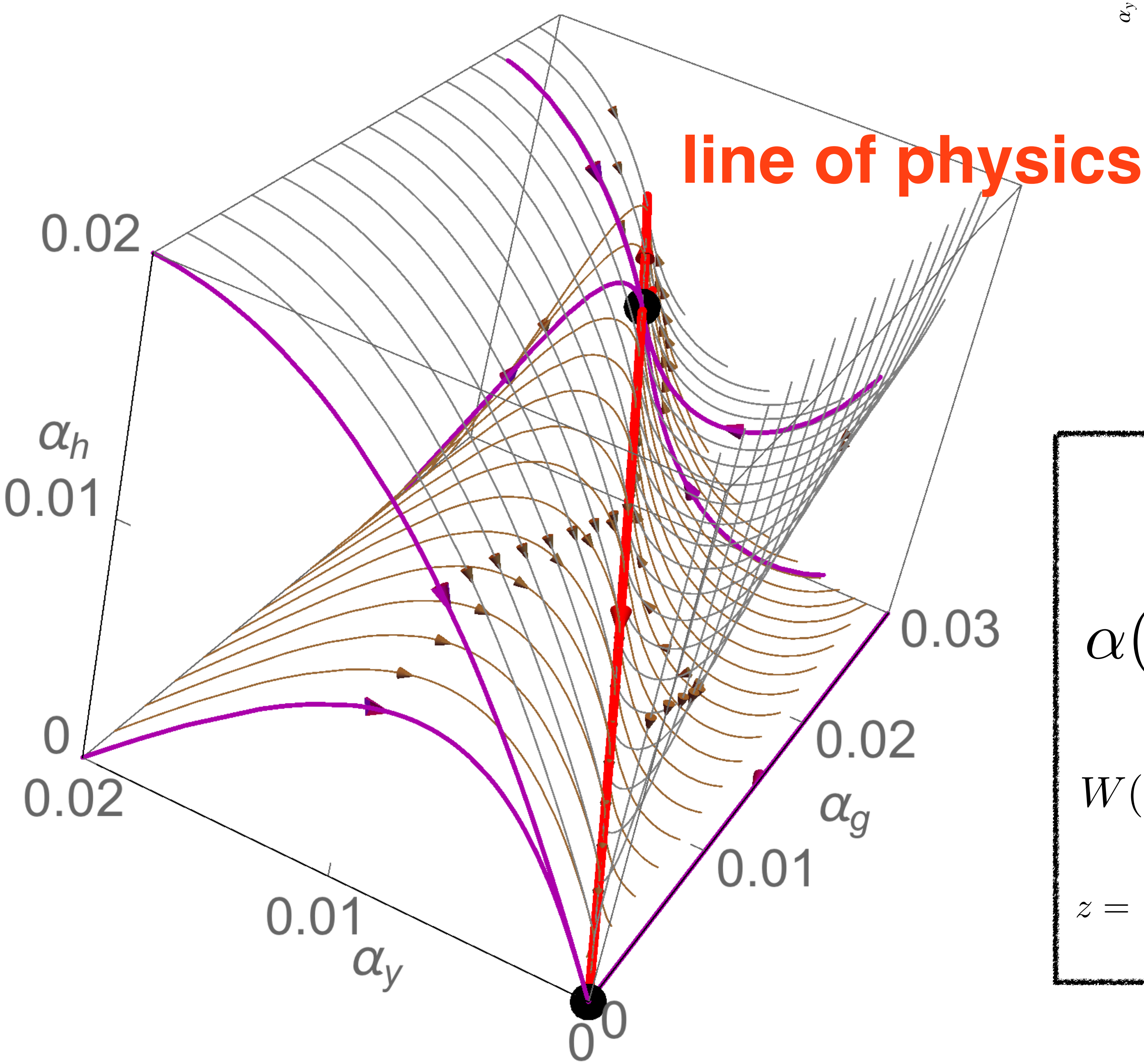
phase diagram

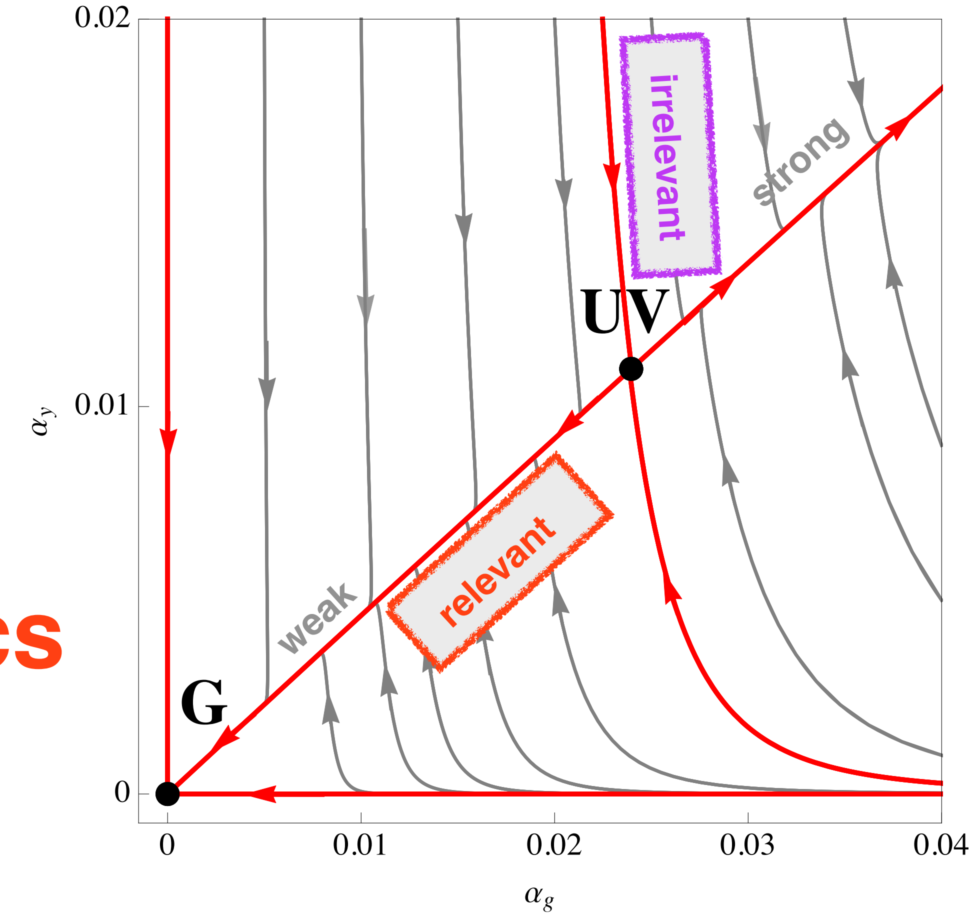


exact UV FP

strict perturbative control

phase diagram





leading order

$$\alpha(\mu) = \frac{\alpha_*}{1 + W(\mu)}$$

$$W(\mu) = W_{\rm Lambert}[z(\mu)]$$

$$z = \left(\frac{\mu_0}{\mu}\right)^{-B\cdot\alpha_*}\left(\frac{\alpha_*}{\alpha_0} - 1\right)\exp\left(\frac{\alpha_*}{\alpha_0} - 1\right).$$

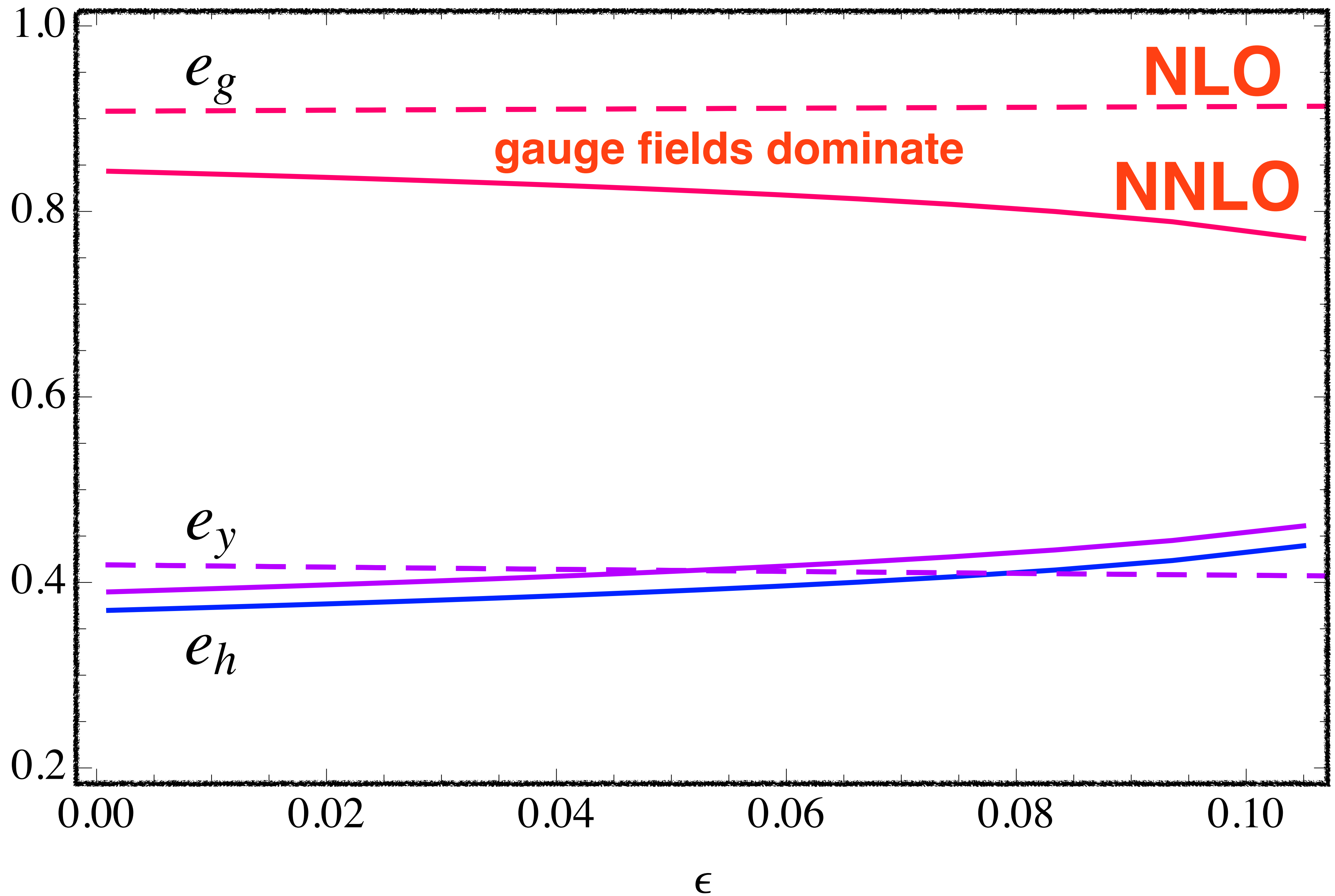
results

UV-relevant eigendirection

gauge

Yukawa

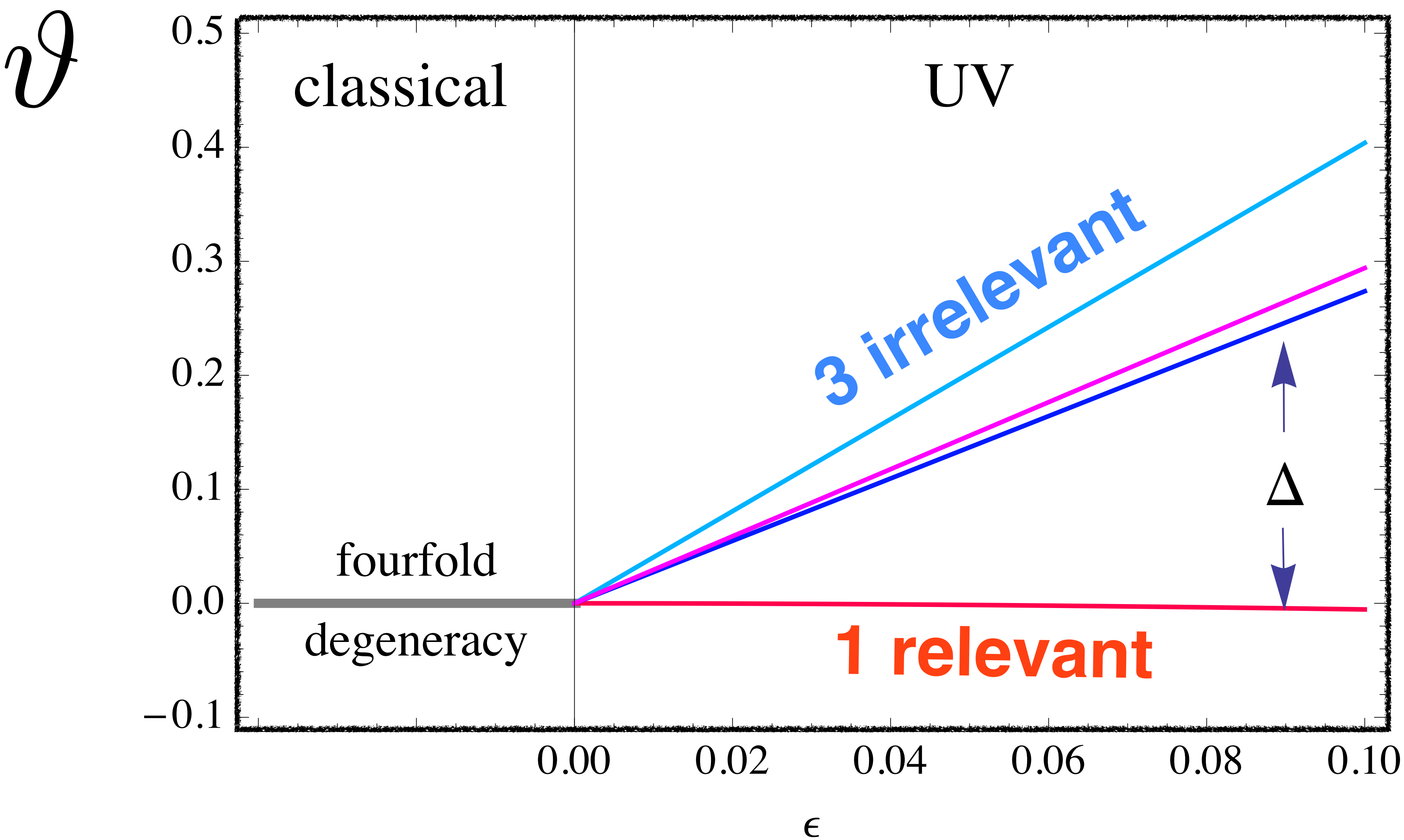
Higgs



results

UV scaling exponents

$$\vartheta_1 < 0 < \vartheta_2 < \vartheta_4 < \vartheta_3$$



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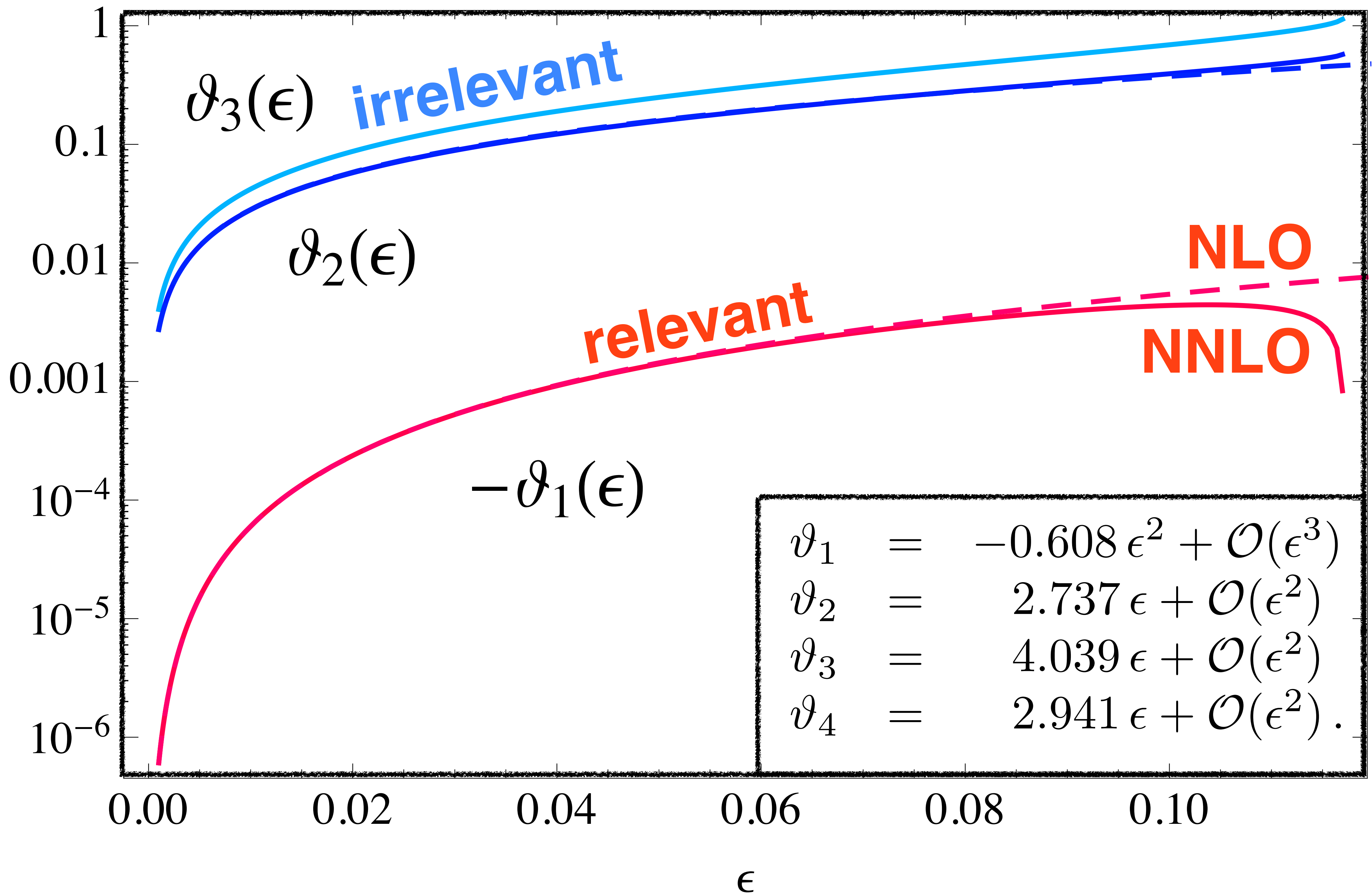


vacuum stability

vacuum must be stable classically
and quantum-mechanically

$$V \propto \alpha_v \mathrm{Tr}(H^\dagger H)^2 + \alpha_h (\mathrm{Tr} H^\dagger H)^2$$

stability

$$\alpha_h > 0 \quad \text{and} \quad \alpha_h + \alpha_v \geq 0 \qquad H_c \propto \delta_{ij}$$

$$\alpha_h < 0 \quad \text{and} \quad \alpha_h + \alpha_v / N_F \geq 0 \qquad H_c \propto \delta_{i1}$$

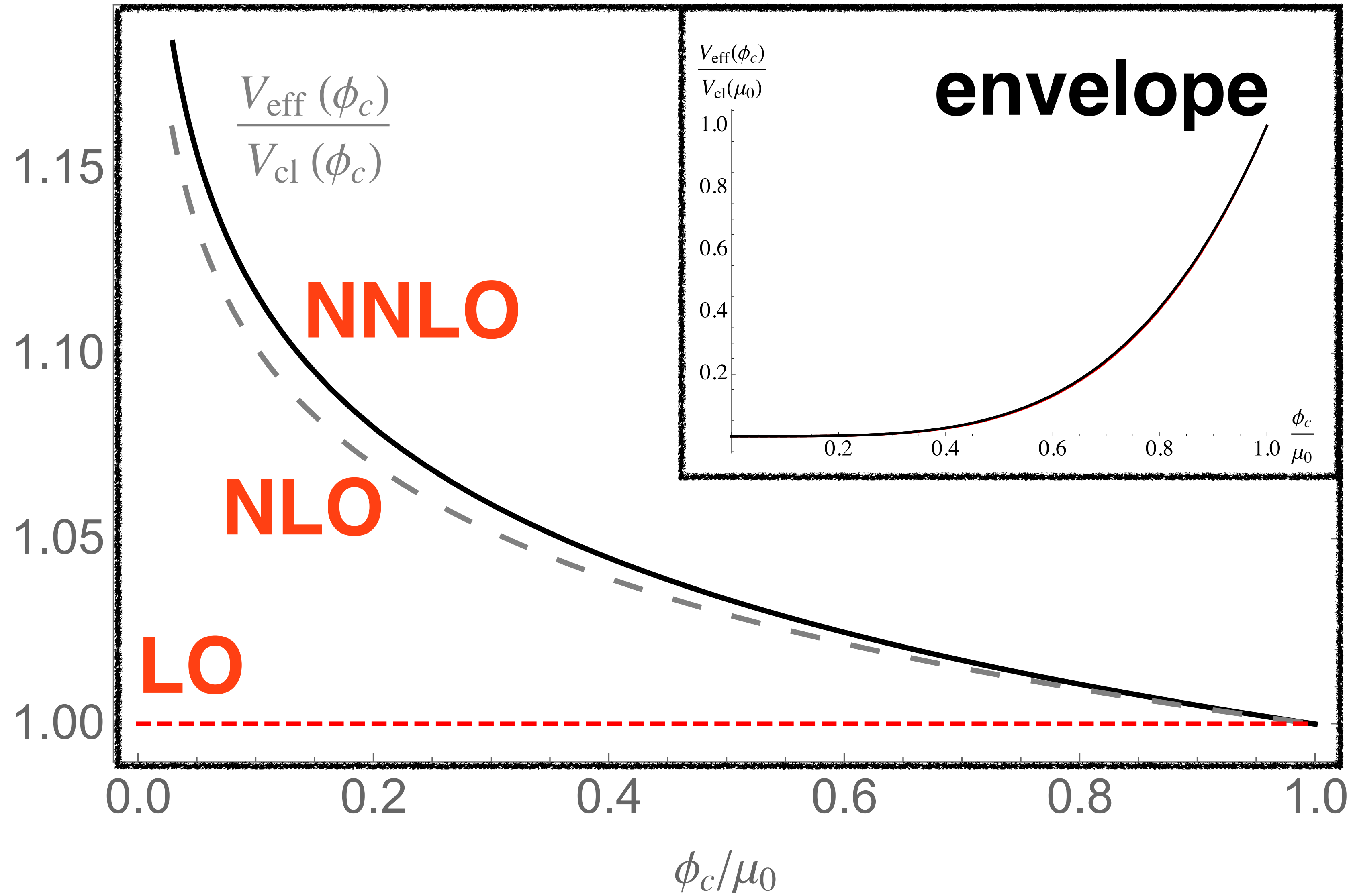
UV FP:

$$0 < \alpha_h^* + \alpha_v^*$$ **ok**

vacuum stability

quantum stability: Coleman-Weinberg type resummation of logs

$$\left(\mu_0 \frac{\partial}{\partial \mu_0} - \gamma(\alpha_j)\,\phi_c \frac{\partial}{\partial \phi_c} + \sum_i \beta_i(\alpha_j) \frac{\partial}{\partial \alpha_i}\right) V_{\rm eff}(\phi_c, \mu_0, \alpha_j) = 0$$



effective potential well-defined for all scales

conclusions

fundamental QFTs beyond asymptotic freedom

4D matter-gauge theories

- **exact proof** of asymptotic safety
- sensible **UV finite theory**
- **all types of fields** required
- no additional (super-)symmetry

conclusions

fundamental QFTs beyond asymptotic freedom

4D matter-gauge theories

exact proof of asymptotic safety
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what's next?

BSM, naturalness?
asymptotic safety at strong coupling?
4D quantum gravity?