# Factorization properties of three-body non-leptonic $B$ decays

Javier Virto

Universität Siegen

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Theoretishe Physik 1

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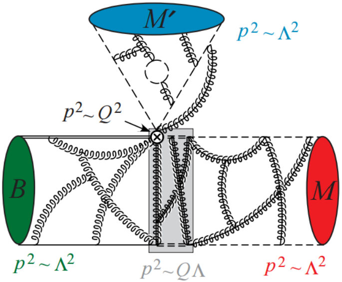
# Levels of Theoretical Complexity in $B$ decays

| | | | |
|---|---|---|---|
| 1. | **Inclusive semileptonic:** | $B \to X\ell\nu$ | HQE |
| 2. | **Exclusive leptonic:** | $B \to \ell\ell$ | $f_B$ |
| 3. | **Inclusive rad./dileptonic:** | $B \to X\gamma, X\ell\ell$ | HQE + factorization |
| 4. | **Exclusive semileptonic:** | $B \to M\ell\nu$ | $F^{B\to M}$ |
| 5. | **Exclusive rad./dileptonic:** | $B \to V\gamma, M\ell\ell$ | $F^{B\to M}$ + factorization |
| 6. | **Exclusive non-leptonic:** | $B \to M_1 M_2$ | factorization + $F^{B\to M}$ + $\Phi_M$ |
| | | $B \to M_1 M_2 M_3$ | factorization + ??? |

- All interesting for SM parameters, New Physics, CP-violation, ...
- Hadronic contributions remain a challenge $\to$ especially in non-leptonic

# Factorization in two-body non-leptonics



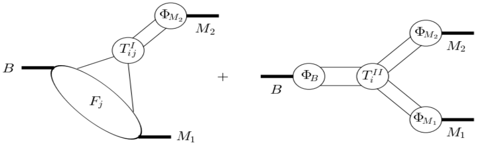

$$\mathcal{O}_i \to \int dz\, T_i(z)[\bar{h}_v \Gamma \xi_{\bar{n}}][\bar{\chi}_n(zn)\Gamma' \chi_n(0)]$$

$$\langle M_{\bar{n}} M_n | \mathcal{O}_i | B \rangle = \langle M_{\bar{n}} | \bar{h}_v \Gamma \xi_{\bar{n}} | B \rangle \times \int dz\, T_i(z) \langle M_n | \bar{\chi}_n(zn) \Gamma' \chi_n(0) | 0 \rangle$$

$$\sim F^{B\to M}\, T_i \otimes \phi_M$$

## Factorization Formula

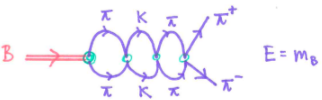
$$\langle M_1 M_2 | \mathcal{O}_i | B \rangle =$$

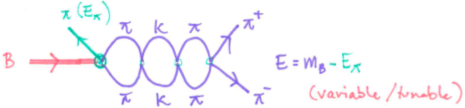



[Beneke, Buchalla, Neubert, Sachrajda, Bauer, Pirjol, Rothstein Stewart, ...]

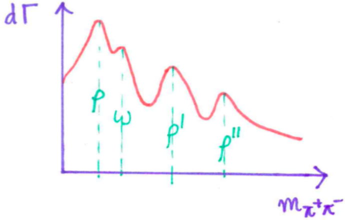
# Three-body non-leptonics: Motivation

- Very abundant
- Rich opportunities for phenomenology + study of QCD effects
- Kinematics not fixed (as in two-body) → dependence in $E$ can be exploited




- Quasi-two-body are a particular case of multi-body decays
  - Consistent study of quasi-two-body
  - Finite-width effects
  - What is quasi-two-body anyway??
  - Simplest multi-body is three-body




→ From now on we focus on $B^- \to \pi^-\pi^+\pi^-$

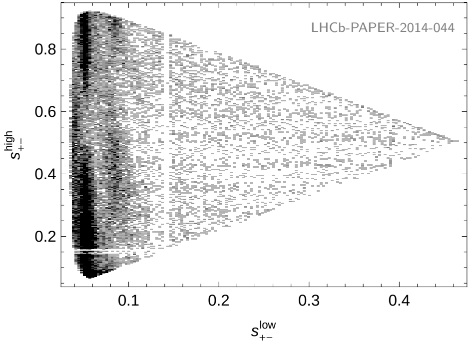
# Kinematics

$B^-(p) \to \pi^-(k_1)\pi^+(k_2)\pi^-(k_3)$

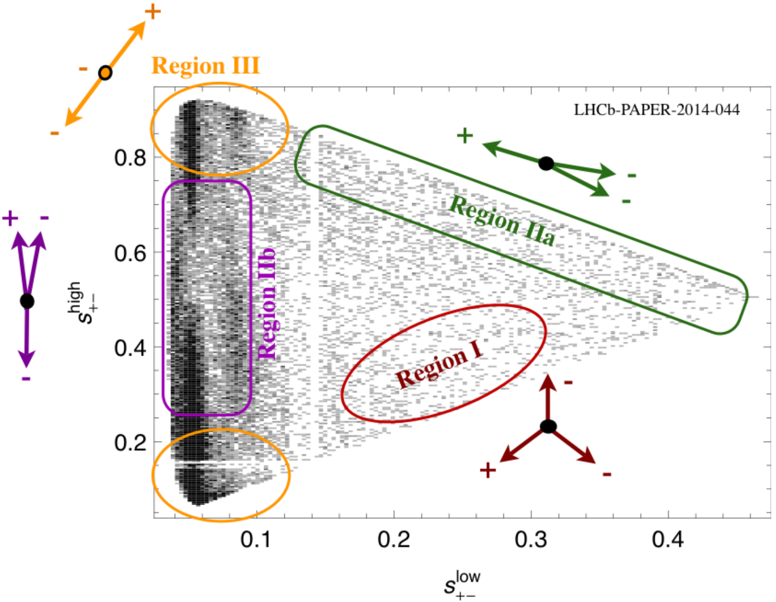
Kinematics completely specified in terms of 2 invariants:

$$p^2 = m_B^2, \quad k_i^2 = 0, \quad s_{ij} \equiv \frac{(k_i + k_j)^2}{m_B^2}, \quad s_{12} + s_{13} + s_{23} = 1$$

For example $s_{12}$ and $s_{23}$:

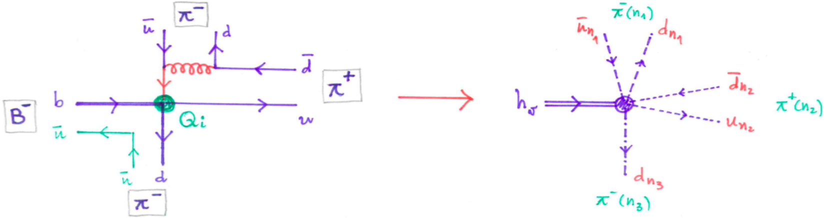


# Regions of phase space



# Region I: Center

⋆ Three collinear directions $n_1$, $n_2$, $n_3$, disconnected at the leading power.




$$\langle \pi^-_{n_1} \pi^+_{n_2} \pi^-_{n_3} | \mathcal{O}_i | B \rangle = \langle \pi^-_{n_3} | \bar{d}_{n_3} \Gamma_3 h_v | B \rangle$$

$$\times \int du dv \, T_i(u,v) \langle \pi^-_{n_1} | \bar{d}_{n_1}(\bar{u}) \Gamma_1 u_{n_1}(u) | 0 \rangle \langle \pi^+_{n_2} | \bar{u}_{n_2}(\bar{v}) \Gamma_2 d_{n_2}(v) | 0 \rangle$$

$$\sim F^{B\to\pi} \, T_i \otimes \phi_\pi \otimes \phi_\pi$$

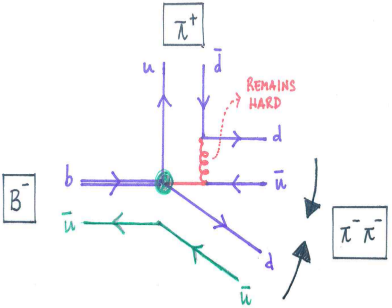
⋆ Power $(1/m_b^2)$ & $\alpha_s$ suppressed with respect to two-body.

⋆ At leading order/power/twist all convolutions are finite → factorization ✓

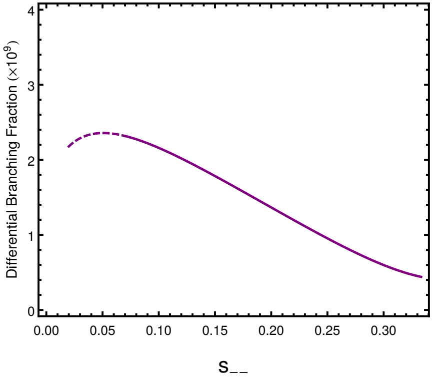
# Region I: Extrapolating towards ($\pi^-\pi^-$) Edge

⋆ No resonances → perturbative result should be reasonable (regular).

⋆ Regularity also expected from absence of soft propagators in QCD:




⋆ We confirm this expectation:

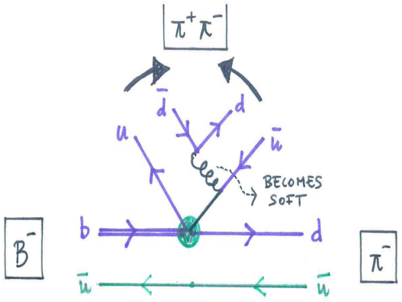



⋆ Asymptotic result: $$\frac{d\Gamma}{ds_{--}\,ds_{+-}} \simeq 0.84\,\Gamma_0\, f_+(m_B^2/2)^2 + \mathcal{O}(s_{--})$$

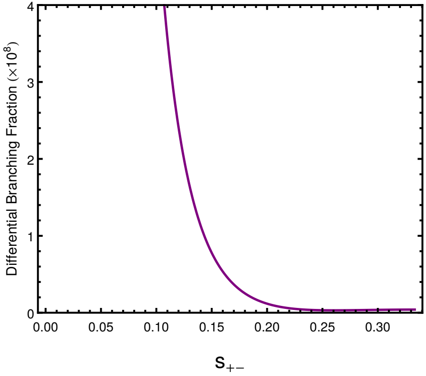
# Region I: Extrapolating towards $(\pi^+\pi^-)$ Edge

⋆ Resonances $(\rho, \omega, \rho', \ldots)$ → perturbative result should break down.

⋆ Non-regularity also expected from presence of soft propagators in QCD:



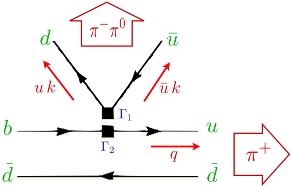

⋆ We confirm this expectation:

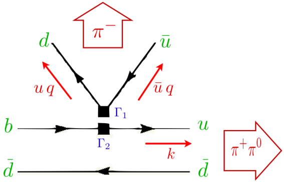



⋆ Asymptotic result: $$\frac{d\Gamma}{ds_{+-}\,ds_{--}} \simeq \frac{0.38}{s_{+-}}\,\Gamma_0\, f_+(0)^2 + \text{regular}$$

# Region IIb: New non-perturbative input in resonant edges

- Breakdown of factorization at resonant edges requires new NP functions.
- 3-body decay remsembles 2-body, but with new $(\pi\pi)$ "compound object":




- Operators are the same as in 2-body, but final states different:

$$\begin{aligned}
\langle \pi^-_{\bar n}\pi^+_{\bar n}\pi^-_{n}|\mathcal{O}|B\rangle \;&=\; \langle \pi^-_{n}|\bar h_v \Gamma \xi_n|B\rangle \times \int dz\, T_1(z) \langle \pi^-_{\bar n}\pi^+_{\bar n}|\bar\chi_{\bar n}(z\bar n)\Gamma' \chi_{\bar n}(0)|0\rangle \\
&+\; \langle \pi^-_{\bar n}\pi^+_{\bar n}|\bar h_v \Gamma \xi_{\bar n}|B\rangle \times \int dz\, T_2(z) \langle \pi^-_{n}|\bar\chi_{\bar n}(zn)\Gamma' \chi_{n}(0)|0\rangle \\
&\sim\; F^{B\to\pi}\, T_1 \otimes \phi_{\pi\pi} + F^{B\to\pi\pi}\, T_2 \otimes \phi_\pi
\end{aligned}$$

- New Non-perturbative input:
  - Generalized Distribution Amplitudes (GDAs) [Diehl, Polyakov, Gousset, Pire...]
  - Generalized Form Factors (GFFs) [Faller, Feldmann, Khodjamirian, Mannel, van Dyk...]

# GDAs from data

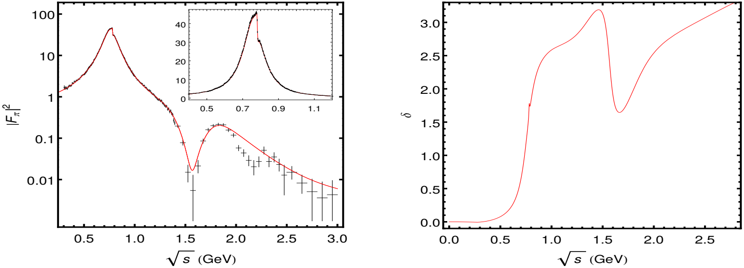
- Definition: $[s = (k_1 + k_2)^2,\ k_1 = \zeta k_{12},\ k_2 = \bar{\zeta} k_{12}]$

$$\phi^q_{\pi\pi}(z,\zeta,s) = \int \frac{dx^-}{2\pi} e^{iz(k_{12}^+ x^-)} \langle \pi^+(k_1)\pi^-(k_2)|\bar{q}(x^- n_-)\not{n}_+ q(0)|0\rangle$$

- Normalization (local correlator):

$$\int dz\, \phi_{\pi\pi}(z,\zeta,s) = (2\zeta - 1) F_\pi(s) \quad \text{(pion time-like FF)}$$

- $F_\pi(s)$: Data (BaBar) + Theory ($\chi PT$, $R\chi PT$, Asymptotics...)

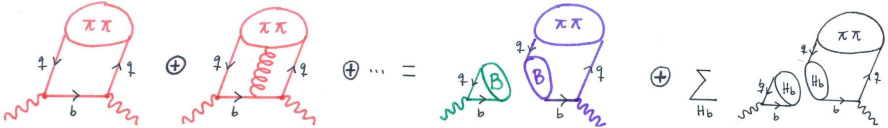


# Sum-rule for GFF

⋆ *Time-like* $B \to \pi\pi$ form factor $F_t(\zeta, s)$: [the only one relevant here]

$$\langle \pi^+(k_1)\pi^-(k_2)|\, \bar{u} \not{k}_3 P_{L,R} b \,|B^-(p)\rangle = \mp \frac{m_\pi}{2} F_t(\zeta, s)$$

⋆ GFF can be related to GDA via a Light-Cone Sum Rule:




$$\phi_{\pi\pi} \quad \sim \quad f_B \cdot F^{B\to\pi\pi} \quad + \quad \text{duality}$$

⋆ At the end of the day [Khodjamirian, Hambrock]

$$F_t(\zeta, s) = \frac{m_b^2}{\sqrt{2}\hat{f}_B m_\pi} \int_{u_0}^1 \frac{du}{u} \exp\left[\frac{(1 + s\bar{u})m_B^2}{M^2} - \frac{m_b^2}{uM^2}\right] \phi_{\pi\pi}(u, \zeta, s)$$

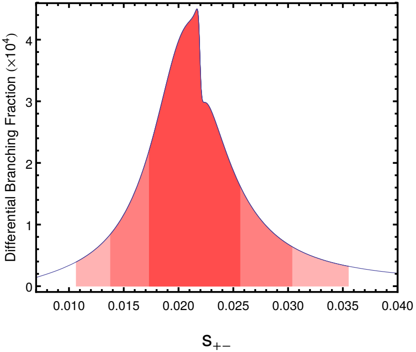
# First application: $BR(B \to \rho\pi)$

⋆ Leading order amplitude:

$$\mathcal{A}|_{s_{+-}\ll 1} = \frac{G_F}{\sqrt{2}}\left[4m_B^2 f_0(s_{+-})(2\zeta-1)F_\pi(s_{+-})(a_2+a_4) + f_\pi m_\pi (a_1-a_4) F_t(\zeta, s_{+-})\right]$$

⋆ Integrating around the $\rho$:

$$BR(B^- \to \rho\,\pi^-) \simeq \int_0^1 ds_{++} \int_{s_\rho^-}^{s_\rho^+} ds_{+-}\ \frac{\tau_B\, m_B |\mathcal{A}|^2}{32(2\pi)^3}$$




with $s_\rho^\pm = (m_\rho \pm n\Gamma_\rho)^2/m_B^2$

$BR(B^+ \to \rho\pi^+) \simeq 9.4 \cdot 10^{-6} \quad (n = 0.5)$

$BR(B^+ \to \rho\pi^+) \simeq 12.8 \cdot 10^{-6} \quad (n = 1)$

$BR(B^+ \to \rho\pi^+) \simeq 14.1 \cdot 10^{-6} \quad (n = 1.5)$

$BR(B^+ \to \rho\pi^+)_{\text{EXP}} = (8.3 \pm 1.2) \cdot 10^{-6}$

$BR(B^+ \to \rho\pi^+)_{\text{QCDF}} = (11.9^{+7.8}_{-6.1}) \cdot 10^{-6}$

# Center-Edge Correspondence

- Some contributions in Center correspond to pert. limit of GDAs at Edge.
- Some contributions in Center correspond to pert. limit of GFFs at Edge.
- Some contributions in Center are power suppressed at Edge.

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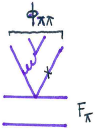
<tr>
<td>Center</td>
<td>
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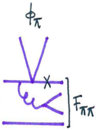
<td>
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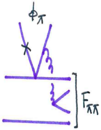
<td>
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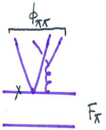
<td>
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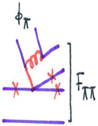
<td>
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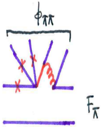
<td>
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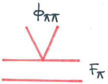
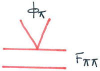
<tr>
<td>Edge</td>
<td>

Leading</td>
<td>

Leading</td>
<td>"non-factorizable"<br>Power-suppressed</td>
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<td>6-quark Operator<br>Power-suppressed</td>
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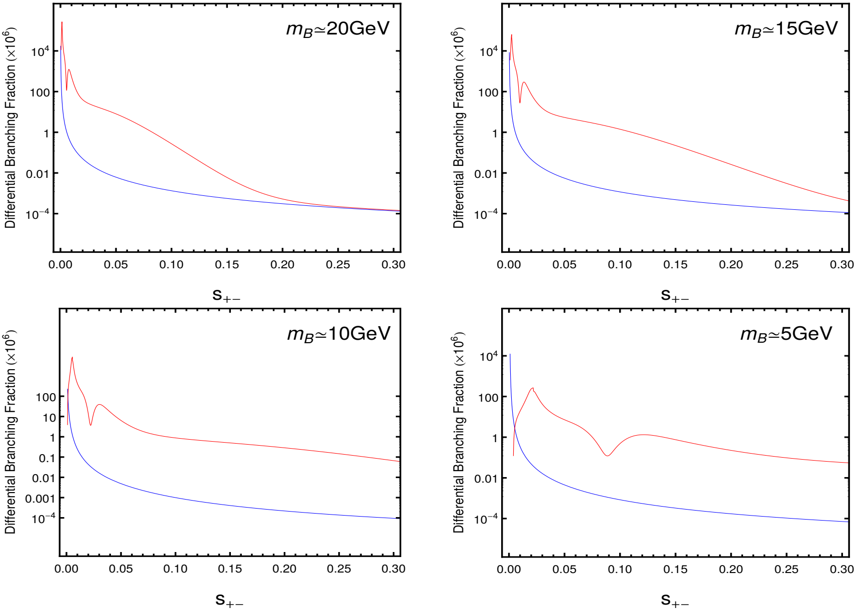
</tr>
</table>

- Should reproduce Center from Edge up to power suppressed terms.

# Merging Regions: How large should $m_B$ be? ($\phi_{\pi\pi}$ term)



# Summary

- Three-body decays are interesting for phenomenology and for QCD aspects & factorization issues.
- Different regions of phase space $\rightarrow$ different factorization properties.
- Center: same factorization as two-body, but power & $\alpha_s$ suppressed.
- Edges similar to two-body, but new non-perturbative functions (GDAs, GFFs).
- GDAs and GFFs can be obtained from data:
  - GDAs: $\gamma\gamma \rightarrow \pi\pi$, $\tau \rightarrow \pi\pi$..., $B \rightarrow D\pi\pi$ etc
  - GFF: $B \rightarrow \pi\pi\ell\nu$, $B \rightarrow \pi\pi\ell\ell$, ...
- Probably no perturbative center for realistic $m_B$, but still interesting.
- Can go beyond quasi-two body.
- This is just a first glimpse: many more possibilities to explore...