

Rare exclusive radiative decays of Z, W and Higgs bosons in QCD factorization

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Zeptometer Scale: Exploring the Origins of
Flavor and Electroweak Symmetry Breaking



Introduction

Obtaining a rigorous control of strong-interaction phenomena in a regime where QCD is strongly coupled is still a challenge to particle physics

- inclusive processes such as $e^+e^- \rightarrow \text{hadrons}$, $B \rightarrow Xl\nu$: quark-hadron duality & local operator-product expansion
- deep-inelastic scattering, collider physics: factorization into partonic cross sections convoluted with **parton distribution functions**
- hard exclusive processes with individual final-state hadrons: **QCD factorization approach**, factorization into partonic rates convoluted with **light-cone distribution amplitudes (LCDAs)**

[Brodsky, Lepage \(1979\); Efremov, Radyushkin \(1980\)](#)

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[Brodsky, Lepage \(1979\); Efremov, Radyushkin \(1980\)](#)

All existing applications of QCD factorization suffer from fact that energy scales are not sufficiently large for power corrections to be negligible

- notoriously difficult to disentangle Λ_{QCD}/Q power corrections from uncertainties related to the LCDAs
- no comprehensive program to determine the LCDAs of hadrons

Introduction

We propose to use **exclusive radiative Z and W decays** into final states containing a single meson as a laboratory to study the QCD factorization approach in a context where power corrections are under control

Price to pay is that the higher the energy release in the process, the smaller the probability for any particular final state is



Enormous rates of electroweak gauge bosons at future, high-luminosity machines present us with new opportunities for **precision electroweak and QCD physics**, which will make such studies possible:

- high-luminosity LHC (3000 fb^{-1}): $\sim 10^{11}$ Z boson and $\sim 5 \cdot 10^{11}$ W bosons
- TLEP, dedicated run at Z pole: $\sim 10^{12}$ Z boson per year
- large samples of W bosons in dedicated runs at WW or $t\bar{t}$ thresholds

Introduction

Our work is motivated by recent investigations of exclusive Higgs decays $h \rightarrow V\gamma$, which were proposed as a way to probe for non-standard Yukawa couplings of the Higgs boson, both diagonal and non-diagonal ones

[Isidori, Manohar, Trott \(2013\)](#)

[Bodwin, Petriello, Stoynev, Velasco \(2013\)](#)

[Kagan *et al.* \(2014\); Bodwin *et al.* \(2014\)](#)

Such measurements are extremely challenging at LHC and future colliders

Observing exclusive radiative decays of Z and W bosons would provide a proof-of-principle that such kind of searches can be performed

Based on:

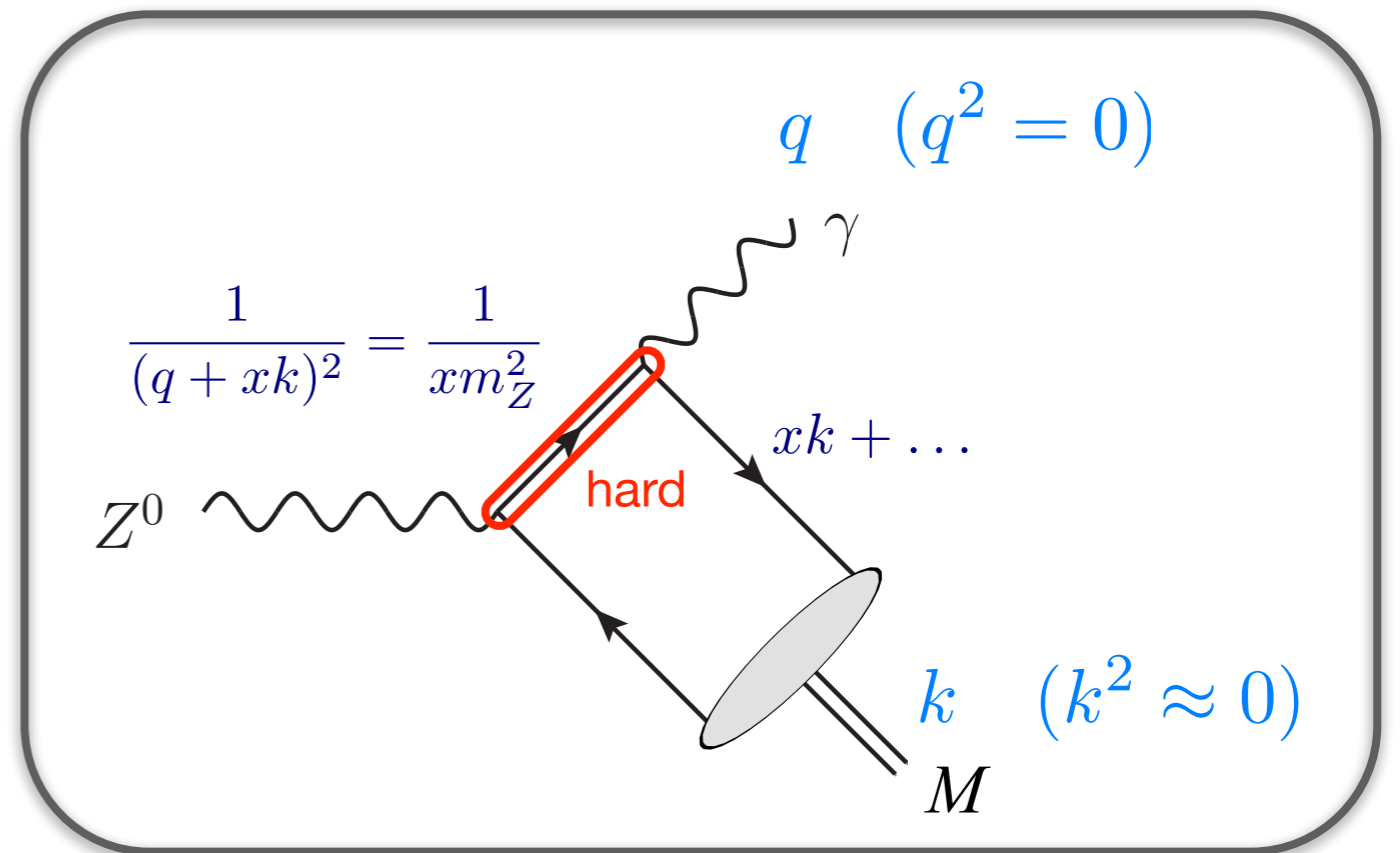
“Exclusive radiative decays of W and Z bosons in QCD factorization”

[Yuval Grossmann, Matthias König, MN \(arXiv:1501.06569 → JHEP\)](#)

+ work in preparation

Physical picture: Exclusive $Z \rightarrow M \gamma$ decays

- the intermediate propagator is highly virtual ($q^2 \sim m_Z^2$) and can be “integrated out”, giving rise to a **hard function** $H(x)$
- field operators for the external quark (and gluon) fields can be separated by **light-like distances**, since $k^2 \approx 0$



At leading power in an expansion in Λ_{QCD}/m_Z , one obtains the **QCD factorization theorem**:

$$\mathcal{A} = -if_M E \int_0^1 dx H_M(x, \mu) \phi_M(x, \mu) + \text{power corrections}$$

decay constant:
extractable from data

hard function:
calculable in PT

LCDA:
non-perturbative hadronic physics

Meson decay constants

Decay constants are the amplitudes for producing a meson out of the vacuum via a local current:

$$\langle P(k) | \bar{q}_1 \gamma^\mu \gamma_5 q_2 | 0 \rangle = -i f_P k^\mu$$

$$\langle V(k, \varepsilon_V) | \bar{q}_1 \gamma^\mu q_2 | 0 \rangle = -i f_V m_V \varepsilon_V^{*\mu}$$

Meson M	f_M [MeV]	Meson M	f_M [MeV]
π	130.4 ± 0.2	D	204.6 ± 5.0
K	156.2 ± 0.7	D_s	257.5 ± 4.6
ρ	212 ± 4	B	186 ± 9
ω	185 ± 5	B_s	224 ± 10
K^*	203 ± 6	J/ψ	403 ± 5
ϕ	231 ± 5	$\Upsilon(1S)$	684 ± 5
		$\Upsilon(4S)$	326 ± 17

$$P^- \rightarrow l^- \bar{\nu}_l$$

$$\tau^- \rightarrow M^- \nu_\tau$$

$$V^0 \rightarrow l^+ l^-$$

lattice QCD

Light-cone distribution amplitudes (LCDAs)

Momentum distribution of partons in a given Fock state of a meson (quark-antiquark, quark-antiquark-gluon, ...):

$$\langle M(k) | \bar{q}(t\bar{n}) \frac{\not{n}}{2} (\gamma_5) [t\bar{n}, 0] q(0) | 0 \rangle = -i f_M E \int_0^1 dx e^{ixt\bar{n}\cdot k} \phi_M(x, \mu)$$

Expansion in **Gegenbauer polynomials** (diagonalizes evolution at LO):

$$\phi_M(x, \mu) = 6x(1-x) \left[1 + \sum_{n=1}^{\infty} a_n^M(\mu) C_n^{(3/2)}(2x-1) \right]$$

- Gegenbauer moments fall off faster than $1/n$ for large n
- odd moments are SU(3)-violating effects
- all moments $a_n^M(\mu) \rightarrow 0$ (except $a_0^M \equiv 1$) in the limit $\mu \rightarrow \infty$
- model predictions obtained using lattice QCD, QCD sum rules and effective field theories (NRQCD, HQET)

[Ball, Braun \(1996\); Ball et al. \(2006, 2007\)](#)
[Arthur et al. \(2010\)](#)
[Braguta, Likhoded, Luchinsky \(2006\)](#)
[Grozin, MN \(1996\)](#)

RG evolution effects

RG evolution from μ_0 up to the **electroweak scale** changes the shapes of the LCDAs significantly, as they approach closer to the asymptotic form $\phi_M(x, \mu \rightarrow \infty) = 6x(1-x)$

Evolution of moments:

$$a_n^M(\mu) = \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)} \right)^{\gamma_n/2\beta_0} a_n^M(\mu_0)$$

positive and increasing with n

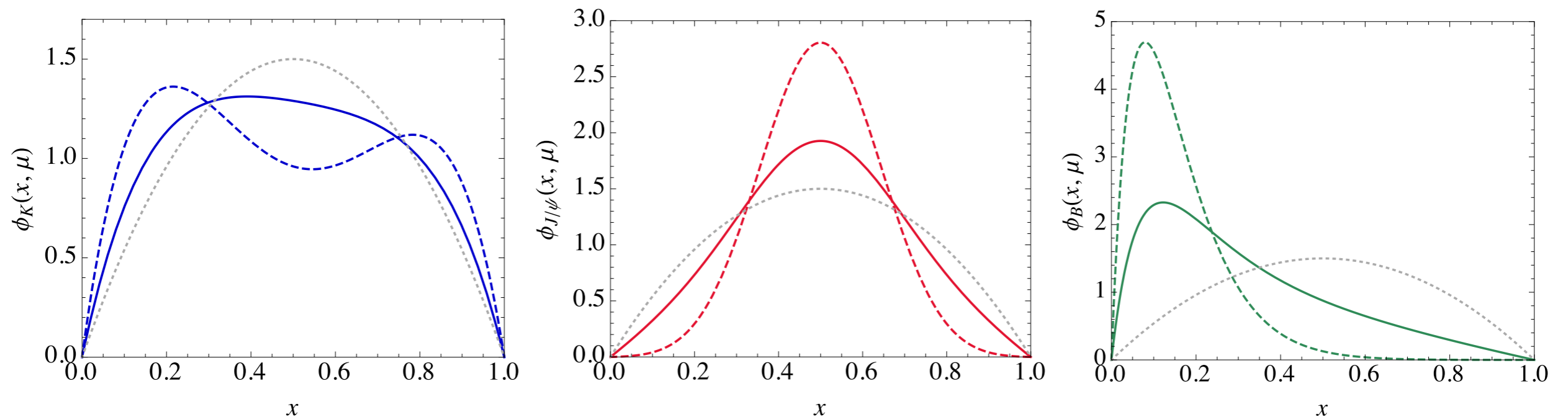


Figure 3: RG evolution of the LCDAs of the kaon (left), the J/ψ meson (middle) and the B meson (right) from a low scale $\mu_0 = 1 \text{ GeV}$ (dashed lines) to a high scale $\mu = m_Z$ (solid lines). The dotted grey line shows the asymptotic form $6x(1-x)$ for comparison.

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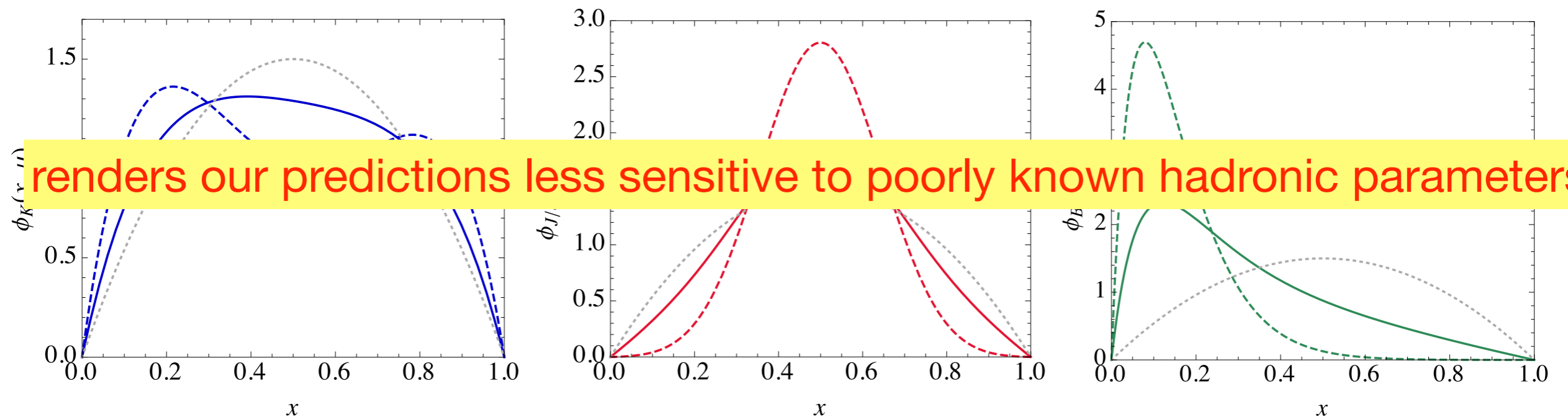


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Exclusive radiative decays $Z \rightarrow M\gamma$

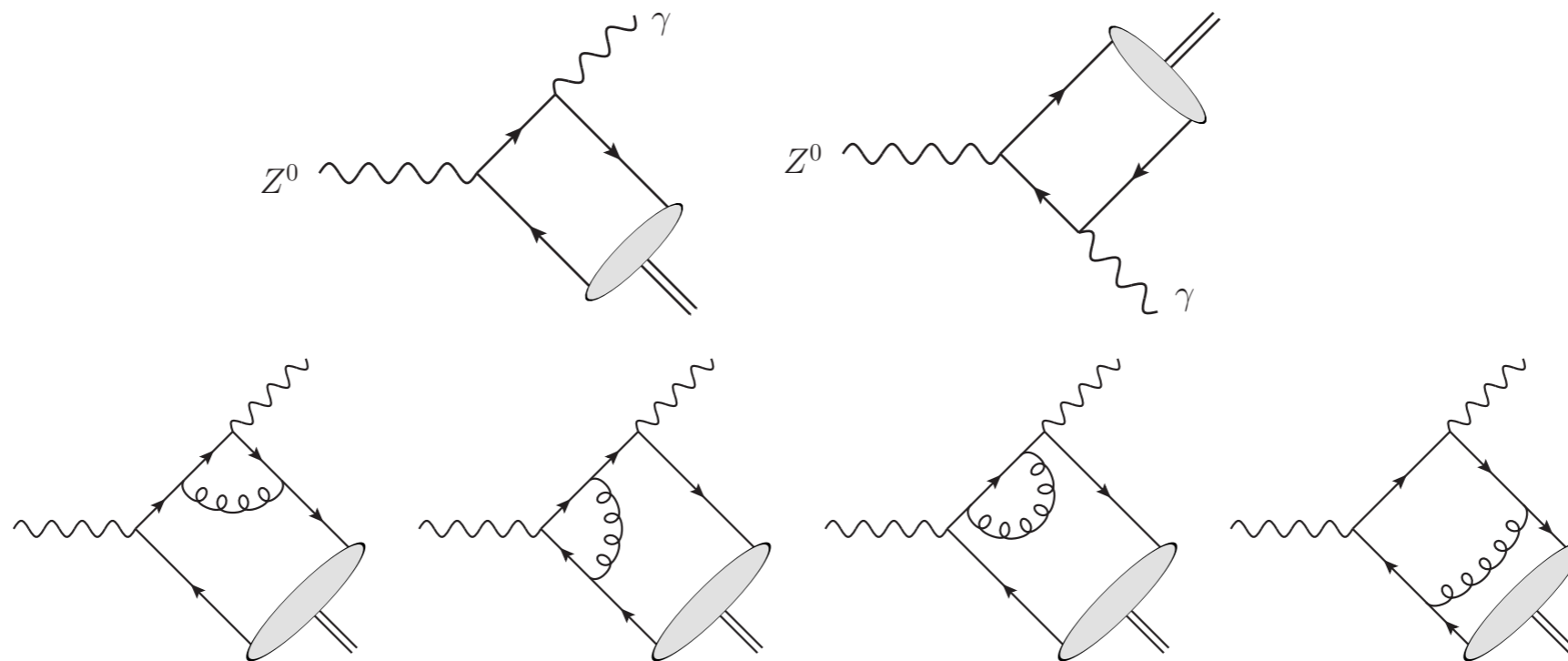
Factorization of the decay amplitude

Form-factor decomposition of the decay amplitude:

$$i\mathcal{A}(Z \rightarrow M\gamma) = \pm \frac{egf_M}{2 \cos \theta_W} \left[i\epsilon_{\mu\nu\alpha\beta} \frac{k^\mu q^\nu \epsilon_Z^\alpha \epsilon_\gamma^{*\beta}}{k \cdot q} F_1^M - \left(\epsilon_Z \cdot \epsilon_\gamma^* - \frac{q \cdot \epsilon_Z k \cdot \epsilon_\gamma^*}{k \cdot q} \right) F_2^M \right]$$

At leading power, the Z-boson (and the photon) have transverse polarization, while a final-state vector meson is longitudinally polarized

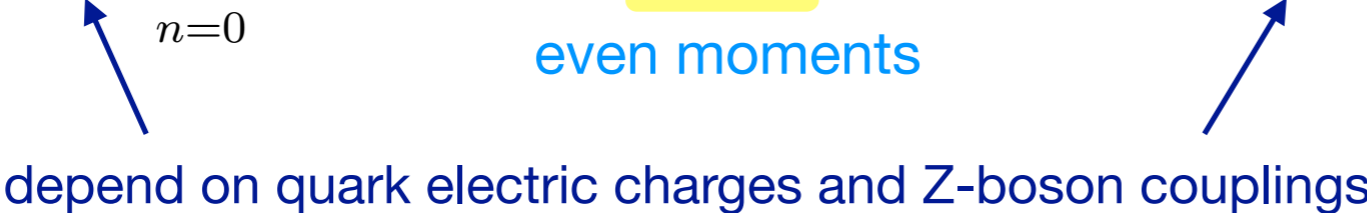
Diagrams at LO and NLO:



Factorization of the decay amplitude

Form factors are related to overlap integrals of hard functions with LCDAs and can be expressed in terms of Gegenbauer moments:

$$F_1^M = Q_M \sum_{n=0}^{\infty} C_{2n}^{(+)}(m_Z, \mu) a_{2n}^M(\mu), \quad F_2^M = -Q'_M \sum_{n=0}^{\infty} C_{2n+1}^{(-)}(m_Z, \mu) a_{2n+1}^M(\mu)$$

 even moments odd moments

Factorization of the decay amplitude

Form factors are related to overlap integrals of hard functions with LCDAs and can be expressed in terms of Gegenbauer moments:

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even moments
odd moments

depend on quark electric charges and Z-boson couplings

Hard functions in moment space:

$$C_n^{(\pm)}(m_V, \mu) = 1 + \frac{C_F \alpha_s(\mu)}{4\pi} c_n^{(\pm)}\left(\frac{m_V}{\mu}\right) + \mathcal{O}(\alpha_s^2)$$

with:

$$c_n^{(\pm)}\left(\frac{m_V}{\mu}\right) = \left[\frac{2}{(n+1)(n+2)} - 4H_{n+1} + 3 \right] \left(\ln \frac{m_V^2}{\mu^2} - i\pi \right) + 4H_{n+1}^2 - \frac{4(H_{n+1} - 1) \pm 1}{(n+1)(n+2)} + \frac{2}{(n+1)^2(n+2)^2} - 9$$

→ large logs are resummed to all orders by choosing $\mu \sim m_Z$

Factorization of the decay amplitude

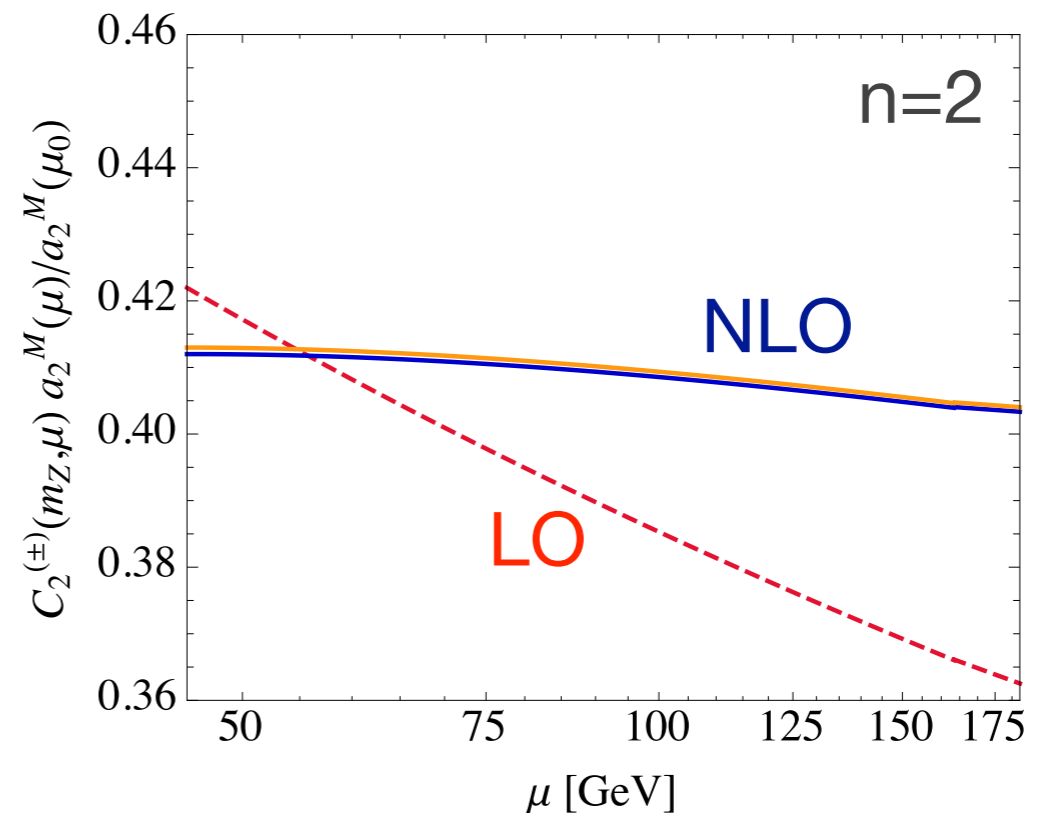
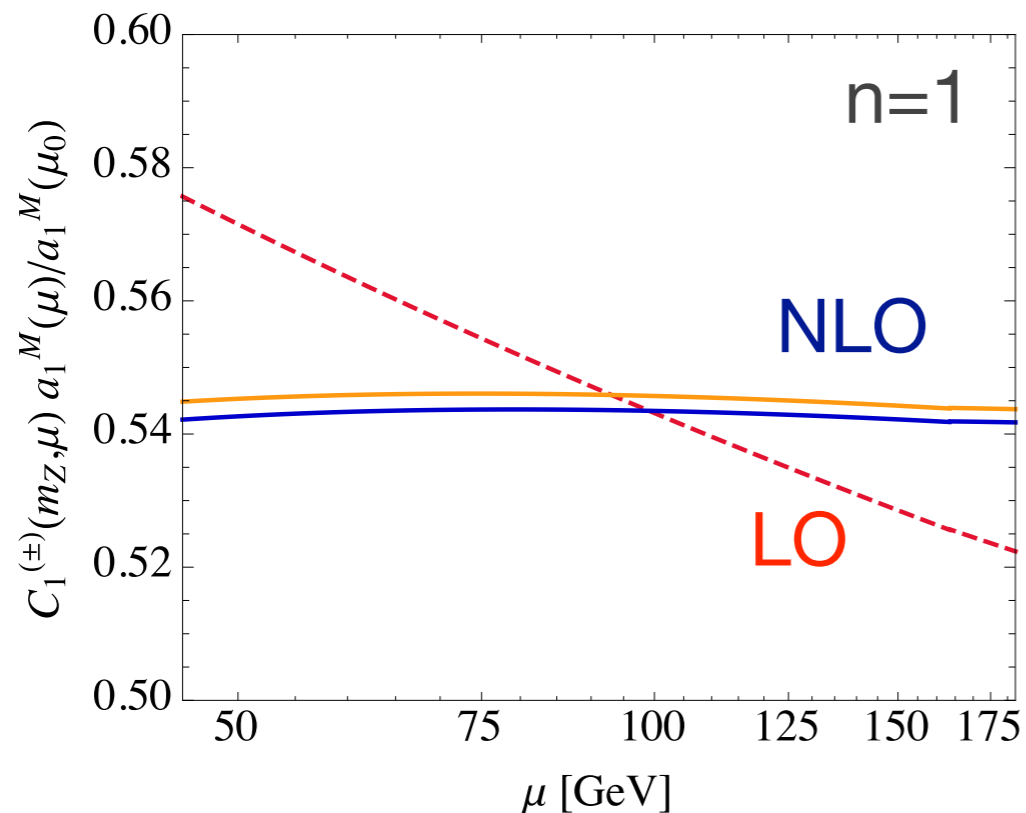
For flavor-diagonal neutral mesons all odd moments vanish:

$$\begin{aligned} \text{Re } F_1^M &= \mathcal{Q}_M \left[0.94 + 1.05 a_2^M(m_Z) + 1.15 a_4^M(m_Z) + 1.22 a_6^M(m_Z) + \dots \right] \\ &= \mathcal{Q}_M \left[0.94 + 0.41 a_2^M(\mu_0) + 0.29 a_4^M(\mu_0) + 0.23 a_6^M(\mu_0) + \dots \right]. \end{aligned}$$

$$F_2^M = 0$$

strongly reduced sensitivity to hadronic parameters!

Each term in the sum is formally scale independent:



Power-suppressed corrections

Power-suppressed contributions to the decay amplitudes **with given helicities** are organized in an expansion in powers of $(\Lambda_{\text{QCD}}/m_Z)^2$ for light mesons and $(m_M/m_Z)^2$ for mesons containing heavy quarks

These corrections are **tiny**, of order 10^{-4} for light mesons and at most 1% for the heaviest meson we will consider — the $\Upsilon(1S)$

The QCD factorization approach thus allows for precise predictions, which are limited only by our incomplete knowledge of the LCDAs

This opens up the possibility for a beautiful program of electroweak precision physics and precision tests of the SM and the QCD factorization approach!



Exclusive radiative decays of Z bosons

Predictions for branching ratios including detailed error estimates:

Decay mode	Branching ratio	asymptotic	LO
$Z^0 \rightarrow \pi^0 \gamma$	$(9.80_{-0.14}^{+0.09} \mu \pm 0.03_f \pm 0.61_{a_2} \pm 0.82_{a_4}) \cdot 10^{-12}$	7.71	14.67
$Z^0 \rightarrow \rho^0 \gamma$	$(4.19_{-0.06}^{+0.04} \mu \pm 0.16_f \pm 0.24_{a_2} \pm 0.37_{a_4}) \cdot 10^{-9}$	3.63	5.68
$Z^0 \rightarrow \omega \gamma$	$(2.82_{-0.04}^{+0.03} \mu \pm 0.15_f \pm 0.28_{a_2} \pm 0.25_{a_4}) \cdot 10^{-8}$	2.48	3.76
$Z^0 \rightarrow \phi \gamma$	$(1.04_{-0.02}^{+0.01} \mu \pm 0.05_f \pm 0.07_{a_2} \pm 0.09_{a_4}) \cdot 10^{-8}$	0.86	1.49
$Z^0 \rightarrow J/\psi \gamma$	$(8.02_{-0.15}^{+0.14} \mu \pm 0.20_f \pm 0.39_{-0.36} \sigma) \cdot 10^{-8}$	10.48	6.55
$Z^0 \rightarrow \Upsilon(1S) \gamma$	$(5.39_{-0.10}^{+0.10} \mu \pm 0.08_f \pm 0.11_{-0.08} \sigma) \cdot 10^{-8}$	7.55	4.11
$Z^0 \rightarrow \Upsilon(4S) \gamma$	$(1.22_{-0.02}^{+0.02} \mu \pm 0.13_f \pm 0.02_{-0.02} \sigma) \cdot 10^{-8}$	1.71	0.93
$Z^0 \rightarrow \Upsilon(nS) \gamma$	$(9.96_{-0.19}^{+0.18} \mu \pm 0.09_f \pm 0.20_{-0.15} \sigma) \cdot 10^{-8}$	13.96	7.59

Table 4: Predicted branching fractions for various $Z \rightarrow M\gamma$ decays, including error estimates due to scale dependence (subscript “ μ ”) and the uncertainties in the meson decay constants (“ f ”), the Gegenbauer moments of light mesons (“ a_n ”), and the width parameters of heavy mesons (“ σ ”). See text for further explanations.

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LO PT @ $\mu_0 = 1 \text{ GeV}$

asymptotic LCDAs ($a_N^M \rightarrow 0$)

Exclusive radiative decays of Z bosons

Predictions for branching ratios including detailed error estimates:

Decay mode	Branching ratio	
$Z^0 \rightarrow \pi^0 \gamma$	$(9.80^{+0.09}_{-0.14} \mu \pm 0.03_f \pm 0.61_{a_2} \pm 0.82_{a_4}) \cdot 10^{-12}$	ATLAS analysis: arXiv:1501.03276 $< 2.6 \cdot 10^{-6}$ $< 3.4 \cdot 10^{-6}$
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Comparison with existing predictions

When all Gegenbauer moments are neglected, i.e. $\phi_M(x) = 6x(1-x)$, we obtain for the decay rates:

$$\Gamma(Z^0 \rightarrow M^0 \gamma) \Big|_{\text{asympt}} = \frac{\alpha m_Z f_M^2}{6v^2} Q_M^2 \left[1 - \frac{10}{3} \frac{\alpha_s(m_Z)}{\pi} \right]$$

→ agrees with a formula for $Z^0 \rightarrow P^0 \gamma$ in [Arnellos, Marciano, Parsa \(1982\)](#)

Manohar obtained an estimate for the $Z^0 \rightarrow \pi^0 \gamma$ rate using a **local OPE**, which is too small by a factor $(2/3)^2 = 4/9$ (understood ✓) [Manohar \(1990\)](#)

[Huang and Petriello \(2014\)](#) performed a calculation of some $Z^0 \rightarrow V^0 \gamma$ decay rates using NRQCD and an approach similar to ours, finding:

$$B_{SM}(Z \rightarrow J/\psi + \gamma) = (9.96 \pm 1.86) \times 10^{-8} \quad (\checkmark)$$

$$B_{SM}(Z \rightarrow \Upsilon(1S) + \gamma) = (4.93 \pm 0.51) \times 10^{-8} \quad (\checkmark)$$

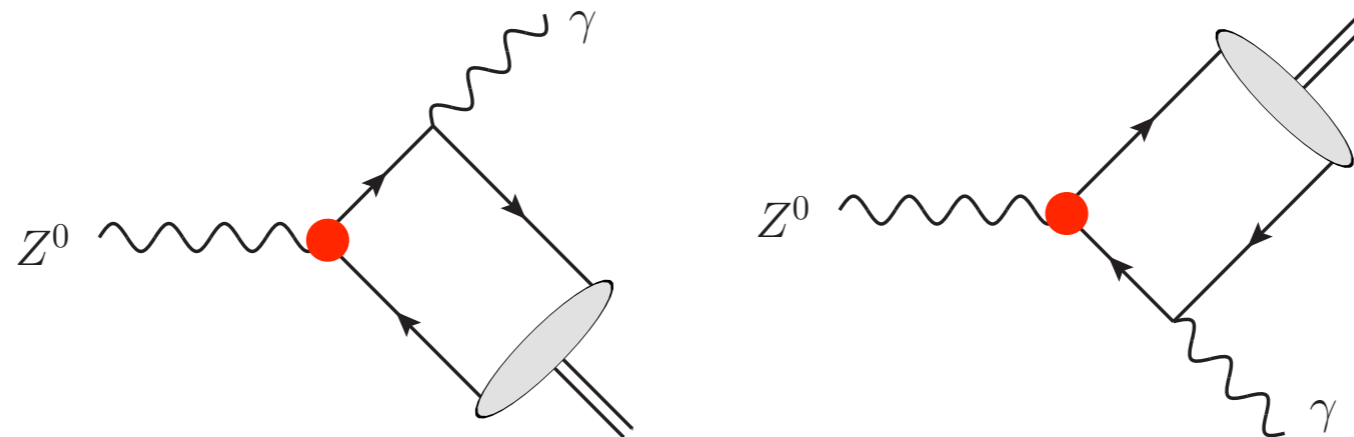
$$B_{SM}(Z \rightarrow \phi + \gamma) = (1.17 \pm 0.08) \times 10^{-8} \quad (\checkmark)$$



Exclusive radiative decays as BSM probes

Radiative Z decays as a BSM probe

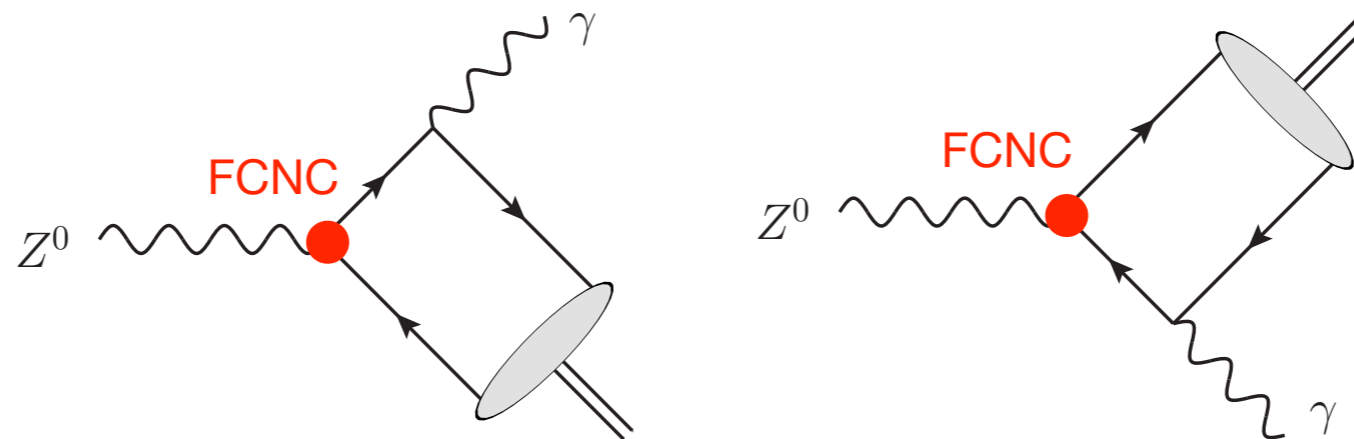
Predictions for branching ratios test Z-boson couplings to quarks:



- at LEP, $|a_b|$ and $|a_c|$ have been measured to 1% accuracy, but no accurate direct determinations of the light-quark couplings have been performed
- using our predictions, one could measure $|a_s|$, $|a_d|$ and $|a_u|$ to about 6%

Radiative Z decays as a BSM probe

Predictions for branching ratios with non-standard FCNC Z-couplings:

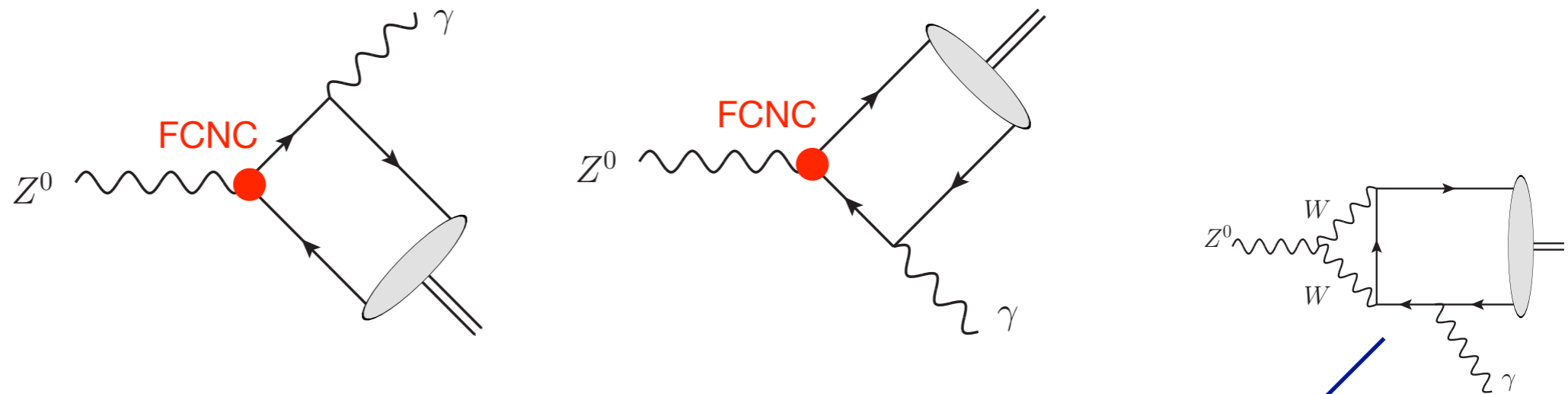


Decay mode	Branching ratio	SM background
$Z^0 \rightarrow K^0 \gamma$	$[(7.70 \pm 0.83) v_{sd} ^2 + (0.01 \pm 0.01) a_{sd} ^2] \cdot 10^{-8}$	$\frac{\lambda}{\sin^2 \theta_W} \frac{\alpha}{\pi} \sim 2 \cdot 10^{-3}$
$Z^0 \rightarrow D^0 \gamma$	$[(5.30^{+0.67}_{-0.43}) v_{cu} ^2 + (0.62^{+0.36}_{-0.23}) a_{cu} ^2] \cdot 10^{-7}$	$\frac{\lambda}{\sin^2 \theta_W} \frac{\alpha}{\pi} \sim 2 \cdot 10^{-3}$
$Z^0 \rightarrow B^0 \gamma$	$[(2.08^{+0.59}_{-0.41}) v_{bd} ^2 + (0.77^{+0.38}_{-0.26}) a_{bd} ^2] \cdot 10^{-7}$	$\frac{\lambda^3}{\sin^2 \theta_W} \frac{\alpha}{\pi} \sim 8 \cdot 10^{-5}$
$Z^0 \rightarrow B_s \gamma$	$[(2.64^{+0.82}_{-0.52}) v_{bs} ^2 + (0.87^{+0.51}_{-0.33}) a_{bs} ^2] \cdot 10^{-7}$	$\frac{\lambda^2}{\sin^2 \theta_W} \frac{\alpha}{\pi} \sim 4 \cdot 10^{-4}$

Table 6: Branching fractions for FCNC transitions $Z \rightarrow M\gamma$, which could arise from physics beyond the Standard Model. The different theoretical uncertainties have been added in quadrature. The last column shows our estimates for the irreducible Standard Model background up to which one can probe the flavor-changing couplings v_{ij} and a_{ij} . Here $\lambda \approx 0.2$ is the Wolfenstein parameter.

Radiative Z decays as a BSM probe

Predictions for branching ratios with non-standard FCNC Z-couplings:



Decay mode	Branching ratio	SM background
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Radiative Z decays as a BSM probe

Indirect upper bounds on FCNC couplings from neutral-meson mixing:

$ \operatorname{Re}[(v_{sd} \pm a_{sd})^2] $	$< 2.9 \cdot 10^{-8}$	$ \operatorname{Re}[(v_{sd})^2 - (a_{sd})^2] $	$< 3.0 \cdot 10^{-10}$
$ \operatorname{Im}[(v_{sd} \pm a_{sd})^2] $	$< 1.0 \cdot 10^{-10}$	$ \operatorname{Im}[(v_{sd})^2 - (a_{sd})^2] $	$< 4.3 \cdot 10^{-13}$
$ (v_{cu} \pm a_{cu})^2 $	$< 2.2 \cdot 10^{-8}$	$ (v_{cu})^2 - (a_{cu})^2 $	$< 1.5 \cdot 10^{-8}$
$ (v_{bd} \pm a_{bd})^2 $	$< 4.3 \cdot 10^{-8}$	$ (v_{bd})^2 - (a_{bd})^2 $	$< 8.2 \cdot 10^{-9}$
$ (v_{bs} \pm a_{bs})^2 $	$< 5.5 \cdot 10^{-7}$	$ (v_{bs})^2 - (a_{bs})^2 $	$< 1.4 \cdot 10^{-7}$

These imply:

[Bona et al. \(2007\)](#); [Bertone et al. \(2012\)](#)
[Carrasco et al. \(2013\)](#)

$$|v_{sd}| < 8.5 \cdot 10^{-5}, \quad |v_{cu}| < 7.4 \cdot 10^{-5}, \quad |v_{bd}| < 1.0 \cdot 10^{-4}, \quad |v_{bs}| < 3.7 \cdot 10^{-4}$$

If these indirect bounds are used, the $Z \rightarrow P\gamma|_{\text{FCNC}}$ branching ratios are pushed to below 10^{-14} , which makes them unobservable

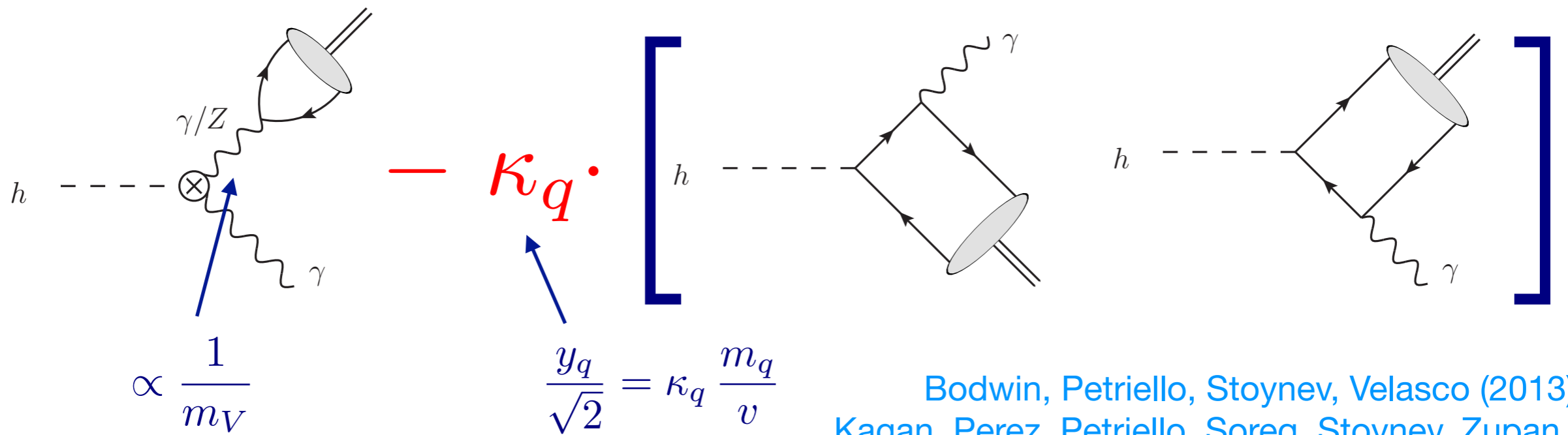
However, the **direct bounds** obtainable using our method are **model independent** and should be seen as **complementary** to the indirect ones!



**Radiative decays $h \rightarrow V\gamma$ as probes of
light-quark Yukawa couplings**

Two competing $h \rightarrow V\gamma$ decay topologies

Decay amplitudes are governed by the **destructive interference** of an “indirect” $h \rightarrow \gamma\gamma^*/\gamma Z^* \rightarrow \gamma V$ pole contribution and a “direct” contribution proportional to the quark **Yukawa coupling**, which can be calculated using QCD factorization



Contribution of the **on-shell $h \rightarrow \gamma\gamma$ amplitude**, which is sensitive to new physics via $\kappa_W, \kappa_t, \kappa_b, \kappa_\tau, \kappa_{\gamma\gamma} \dots$, can be eliminated by considering a **ratio of decay rates**

Two competing $h \rightarrow V\gamma$ decay topologies

Ratio of branching fractions:

$$\frac{\text{Br}(h \rightarrow V\gamma)}{\text{Br}(h \rightarrow \gamma\gamma)} = \frac{\Gamma(h \rightarrow V\gamma)}{\Gamma(h \rightarrow \gamma\gamma)} = \frac{8\pi\alpha^2(m_V)}{\alpha} \frac{Q_q^2 f_V^2}{m_V^2} \left| 1 - \kappa_q \Delta_V - \delta_V \right|^2$$

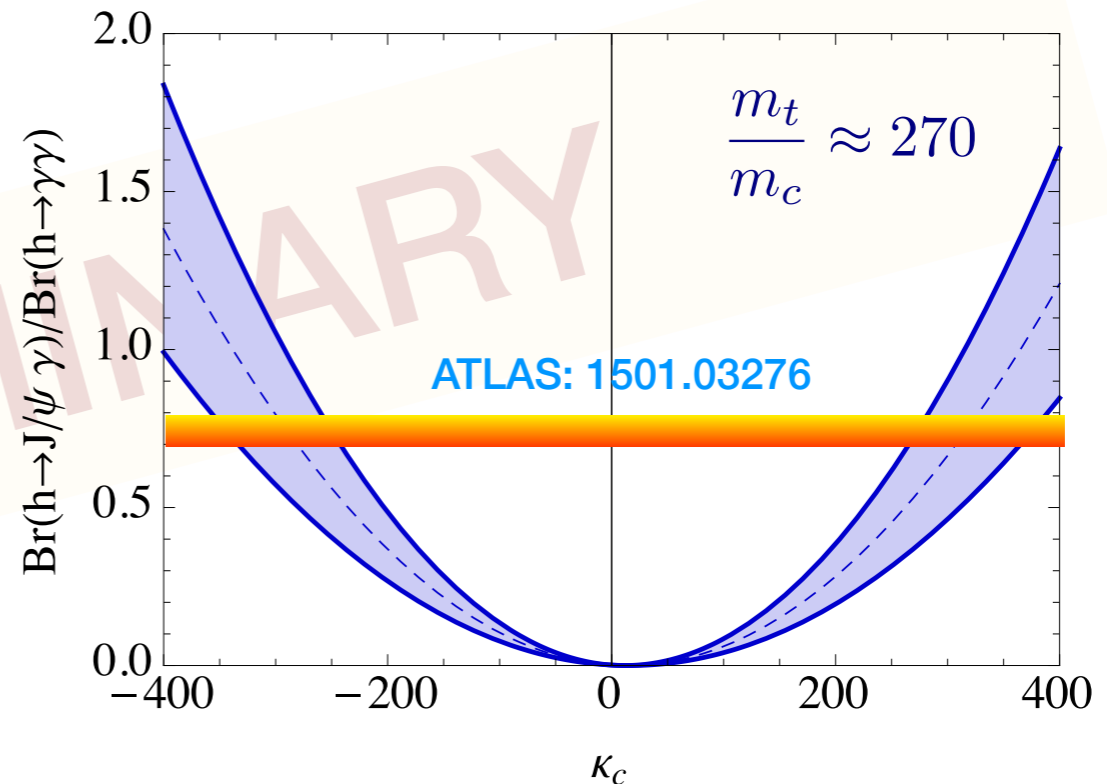
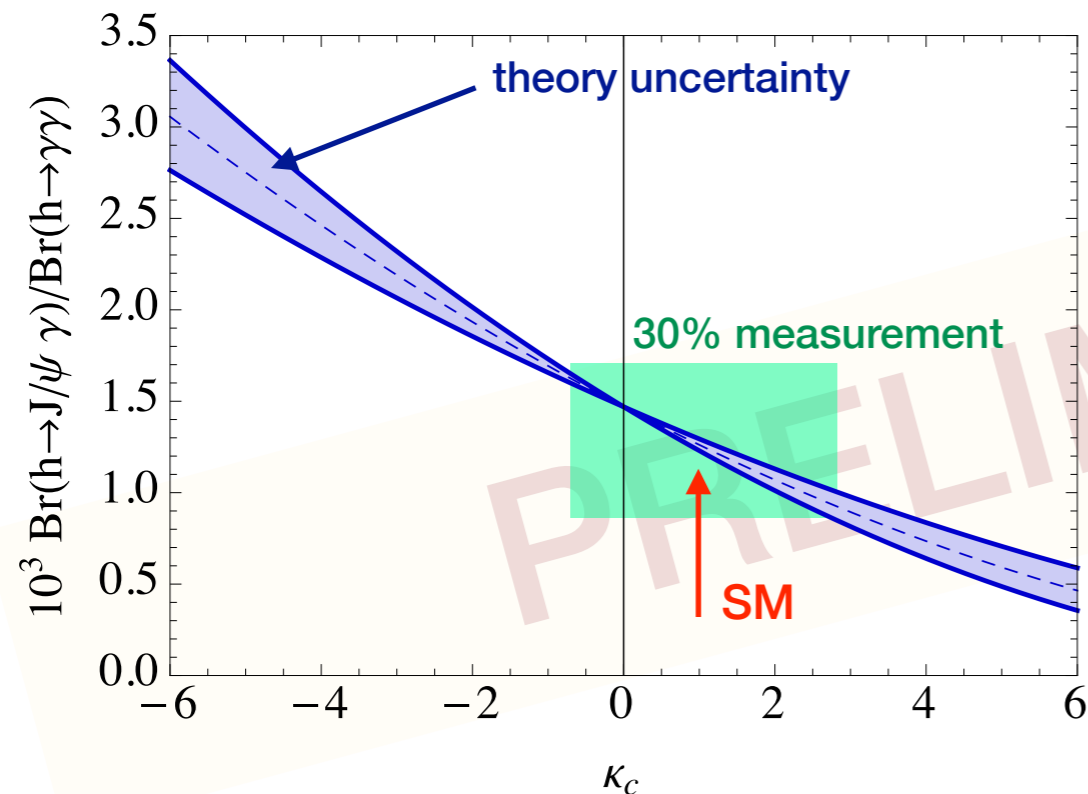
König, MN (in preparation)

Advantages:

- leading term predicted without theoretical uncertainties
- effects of $h \rightarrow \gamma Z^* \rightarrow \gamma V$ amplitude and off-shellness of the intermediate boson are power suppressed: $\delta_V \sim m_V^2/m_{Z,h}^2$ very small even for $\Upsilon(1S)$
- ratio of branching ratios is insensitive to the unknown total Higgs width

Predictions for $h \rightarrow J/\psi \gamma$

König, MN (in preparation)



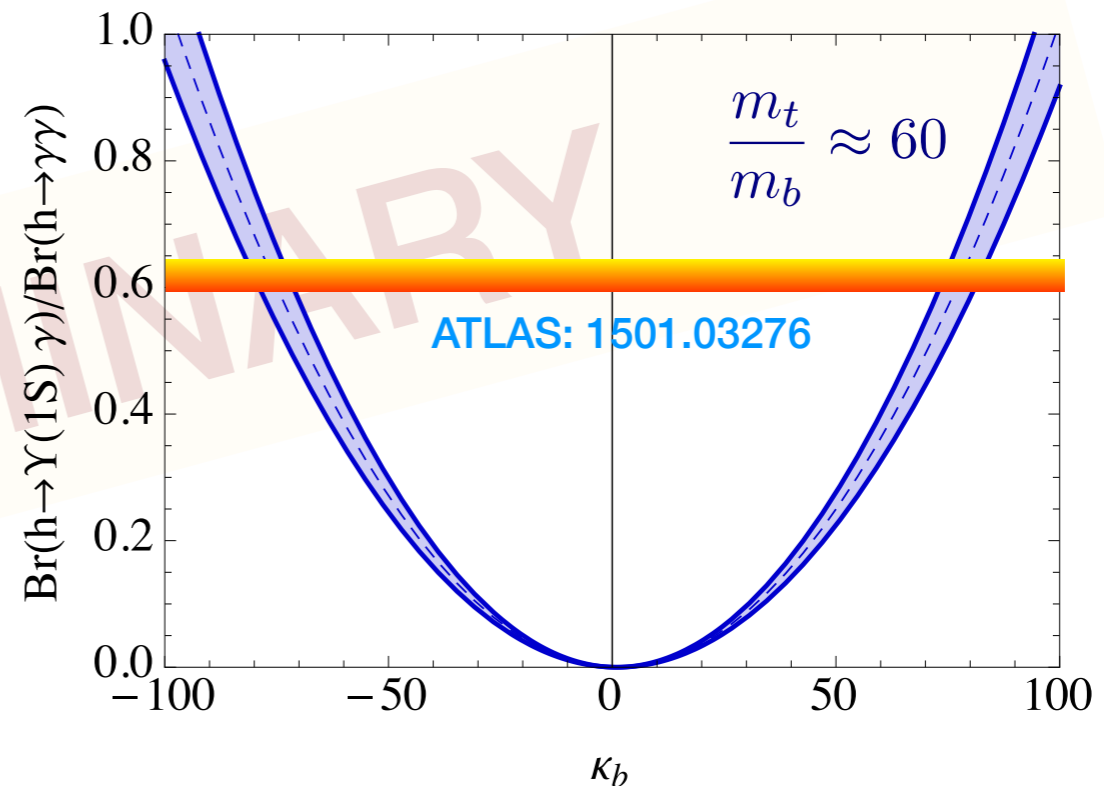
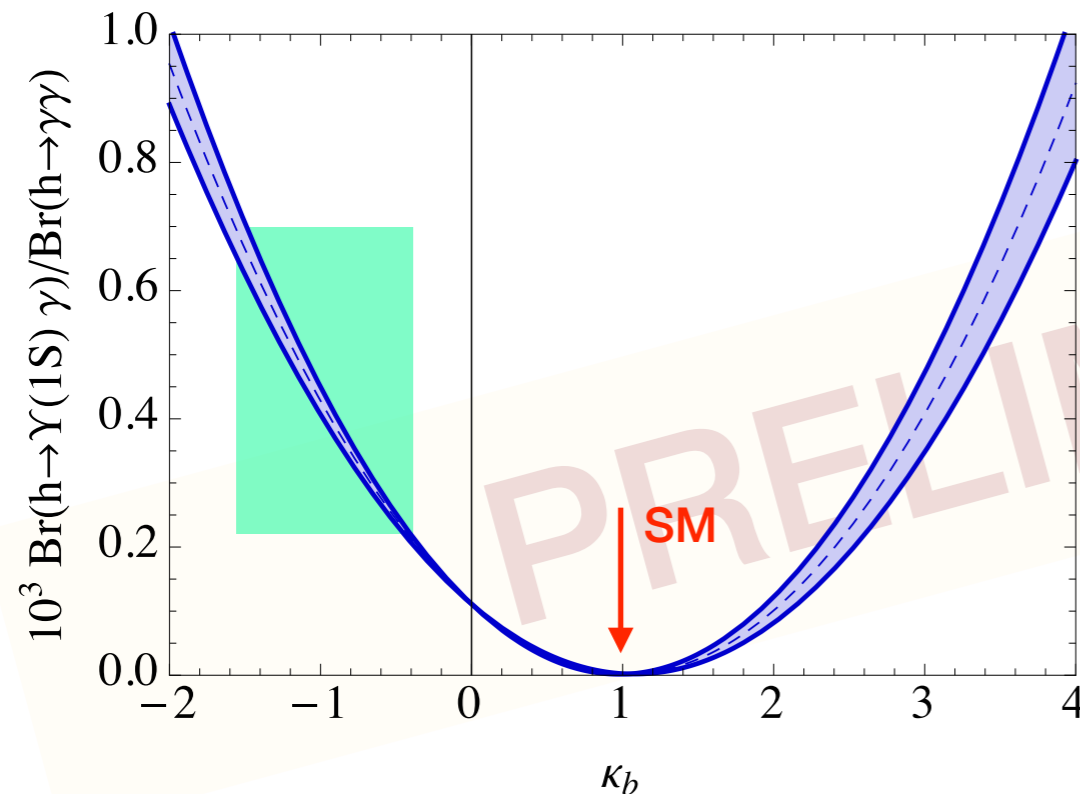
Features:

- SM branching ratio $\sim 3 \cdot 10^{-6}$ challenging [also: Bodwin, Chung, Ee, Lee, Petriello (2014)]
- 30% measurement would constrain κ_c to lie within -0.8 and +2.8
- present ATLAS bound suggests that charm quark likely couples more weakly to the Higgs boson than the top quark 😊

[also: Perez, Soreq, Stamou, Tobioka (2015)]

Predictions for $h \rightarrow Y(1S) \gamma$

König, MN (in preparation)

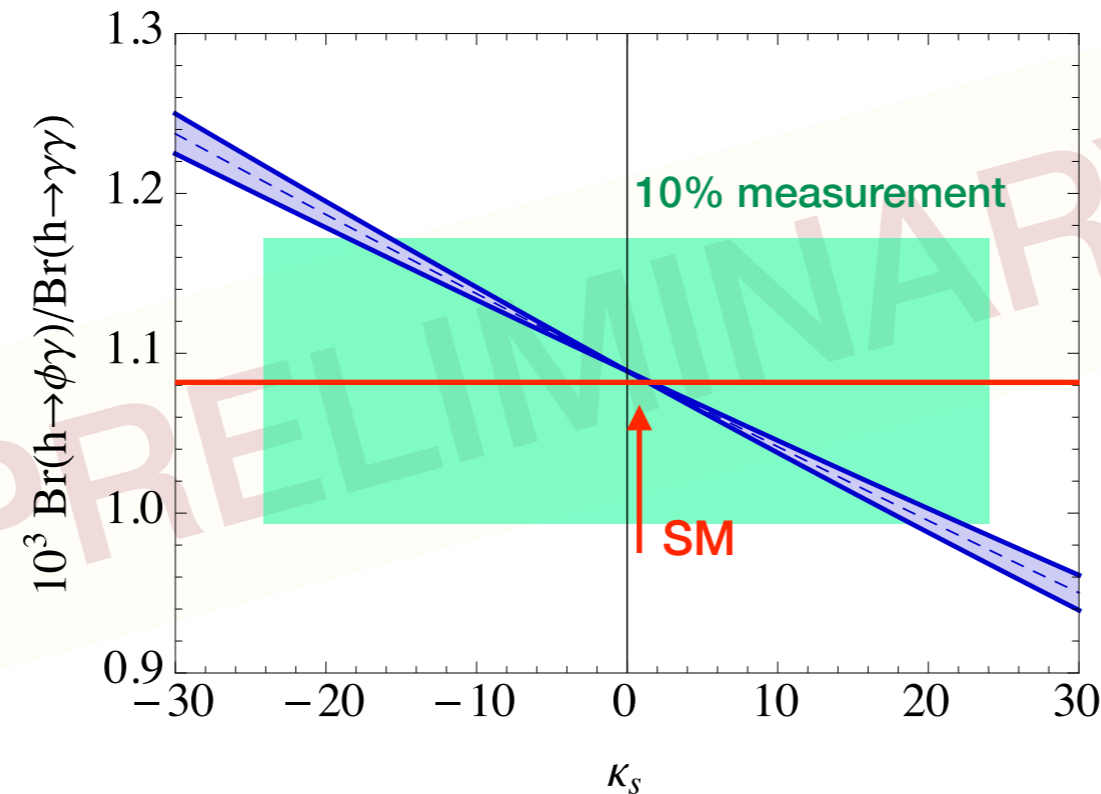


Features:

- SM branching ratio $\sim 5 \cdot 10^{-10}$ hopeless [also: Bodwin, Chung, Ee, Lee, Petriello (2014)]
- may be possible to probe the interesting region where $\kappa_b \approx -1$, for which the branching fraction would be $\sim 10^{-6}$
- present ATLAS bound suggests that bottom quark likely couples more weakly to the Higgs boson than the top quark 😊

Predictions for $h \rightarrow \phi \gamma$

König, MN (in preparation)



Features:

- SM branching ratio $\sim 2.5 \cdot 10^{-6}$ very challenging [also: Kagan *et al.* (2014)]
- 10% measurement would be required to constrain κ_s to the region between -20 and +20, where the strange quark couples less than half as strongly to the Higgs boson than the bottom quark



Conclusions

Summary

Predicted branching ratios with theory errors added in quadrature:

Decay mode	Branching ratio	Decay mode	Branching ratio
$Z^0 \rightarrow \pi^0 \gamma$	$(9.80 \pm 1.03) \cdot 10^{-12}$	$W^\pm \rightarrow \pi^\pm \gamma$	$(4.00 \pm 0.83) \cdot 10^{-9}$
$Z^0 \rightarrow \rho^0 \gamma$	$(4.19 \pm 0.47) \cdot 10^{-9}$	$W^\pm \rightarrow \rho^\pm \gamma$	$(8.74 \pm 1.91) \cdot 10^{-9}$
$Z^0 \rightarrow \omega \gamma$	$(2.82 \pm 0.41) \cdot 10^{-8}$	$W^\pm \rightarrow K^\pm \gamma$	$(3.25 \pm 0.69) \cdot 10^{-10}$
$Z^0 \rightarrow \phi \gamma$	$(1.04 \pm 0.12) \cdot 10^{-8}$	$W^\pm \rightarrow K^{*\pm} \gamma$	$(4.78 \pm 1.15) \cdot 10^{-10}$
$Z^0 \rightarrow J/\psi \gamma$	$(8.02 \pm 0.45) \cdot 10^{-8}$	$W^\pm \rightarrow D_s \gamma$	$(3.66^{+1.49}_{-0.85}) \cdot 10^{-8}$
$Z^0 \rightarrow \Upsilon(1S) \gamma$	$(5.39 \pm 0.16) \cdot 10^{-8}$	$W^\pm \rightarrow D^\pm \gamma$	$(1.38^{+0.51}_{-0.33}) \cdot 10^{-9}$
$Z^0 \rightarrow \Upsilon(4S) \gamma$	$(1.22 \pm 0.13) \cdot 10^{-8}$	$W^\pm \rightarrow B^\pm \gamma$	$(1.55^{+0.79}_{-0.60}) \cdot 10^{-12}$

- for Z decays, one can trigger on high-energy photon and muons
- estimate that one can get several hundreds of $J/\psi \gamma$ events at LHC
- ideas for reconstructing $(\rho, \omega, \phi) + \gamma$ exists [Kagan et al. \(2014\)](#)
- reconstructing W decays at LHC is more challenging [Mangano, Melia \(2014\)](#)
- a Z-factory could measure most modes with good precision!

Summary

- ★ With precise measurements of branching ratios, one can extract — in a model-independent way — information about LCDAs (sums over even and odd moments at a scale $\mu \sim m_Z$)
- ★ It will also be possible to perform a series of novel new-physics searches
- ★ Exclusive radiative decays of Higgs bosons can be used to probe in a direct way the Yukawa couplings of the Higgs to light quarks

The physics case for studying these very rare decays is compelling!

The challenge is to make it possible to observe them!

BACKUP SLIDES

Light-cone distribution amplitudes (LCDAs)

Model predictions based on **QCD sum rules & lattice QCD** ($\mu_0 = 1 \text{ GeV}$):

Ball, Braun (1996); Ball *et al.* (2006, 2007)
Arthur *et al.* (2010)

Meson M	f_M [MeV]	$a_1^M(\mu_0)$	$a_2^M(\mu_0)$
π	130.4 ± 0.2	0	0.29 ± 0.08
K	156.2 ± 0.7	-0.07 ± 0.04	0.24 ± 0.08
ρ	212 ± 4	0	0.17 ± 0.07
ω	185 ± 5	0	0.15 ± 0.12
K^*	203 ± 6	-0.06 ± 0.04	0.16 ± 0.09
ϕ	231 ± 5	0	0.23 ± 0.08

Model estimate suggest than higher moments (n=6 and higher) for light mesons are tiny; will use $a_4^M(\mu_0) \in [-0.15, 0.15]$ to estimate such effects

Bakulev, Passek-Kumericki, Schroers, Stefanis (2001)
Bakulev, Mikhailov, Stefanis (2003)

Light-cone distribution amplitudes (LCDAs)

Heavy quarkonia:

NRQCD matrix element
Braguta, Likhoded, Luchinsky (2006)

$$\int_0^1 dx (2x - 1)^2 \phi_M(x, \mu_0) = \frac{\langle v^2 \rangle_M}{3} + \mathcal{O}(v^4)$$

- simple model function:

$$\phi_M(x, \mu_0) = N_\sigma \frac{4x(1-x)}{\sqrt{2\pi\sigma}} \exp\left[-\frac{(x - \frac{1}{2})^2}{2\sigma^2}\right]; \quad \sigma^2 = \frac{\langle v^2 \rangle_M}{12}$$

Heavy-light mesons:

$$\int_0^1 dx \frac{\phi_M(x, \mu_0)}{x} \equiv \frac{m_M}{\lambda_M(\mu_0)} + \dots$$

HQET matrix element
Grozin, MN (1996)

- simple mode function:

$$\phi_M(x, \mu_0) = N_\sigma \frac{x(1-x)}{\sigma^2} \exp\left(-\frac{x}{\sigma}\right); \quad \sigma = \frac{\lambda_M(\mu_0)}{m_M}$$

Light-cone distribution amplitudes (LCDAs)

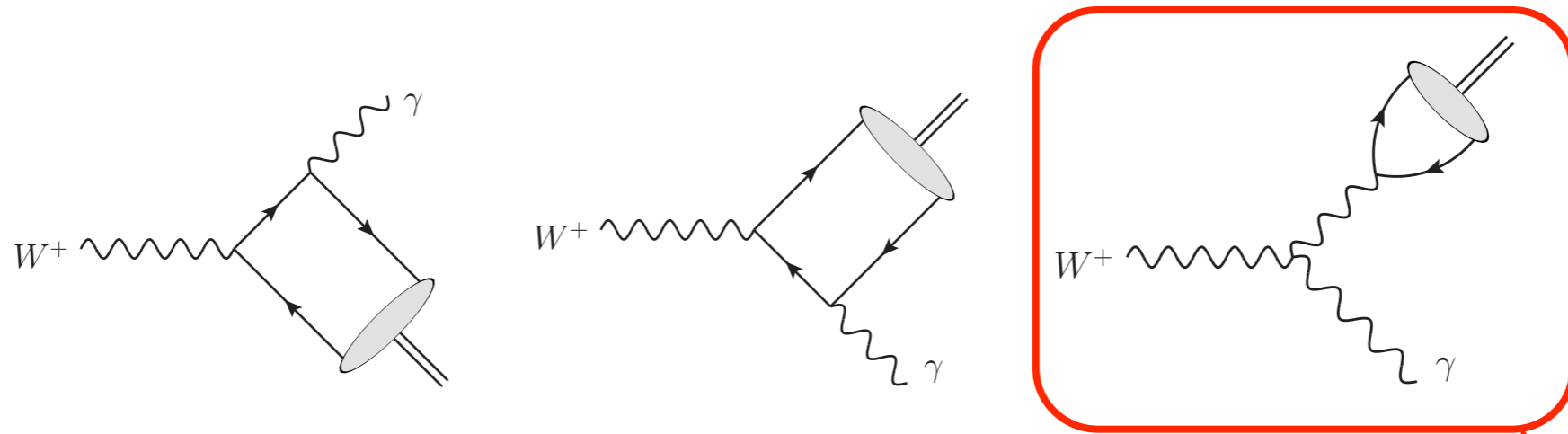
Input parameters for heavy mesons:

Meson M	f_M [MeV]	λ_M [MeV]	$\langle v^2 \rangle$	σ
D	204.6 ± 5.0	460 ± 110	–	0.246 ± 0.059
D_s	257.5 ± 4.6	550 ± 150	–	0.279 ± 0.076
B	186 ± 9	460 ± 110	–	0.087 ± 0.021
B_s	224 ± 10	550 ± 150	–	0.102 ± 0.028
J/ψ	403 ± 5	–	0.30 ± 0.15	0.158 ± 0.040
$\Upsilon(1S)$	684 ± 5	–	0.10 ± 0.05	0.091 ± 0.023
$\Upsilon(4S)$	326 ± 17	–	0.10 ± 0.05	0.091 ± 0.023

- first $n \sim 1/\sigma$ Gegenbauer moments are important for heavy mesons

Exclusive radiative decays of W bosons

Situation is analogous, but the trilinear $WW\gamma$ vertex gives rise to an additional (local) contribution:



Form-factor decomposition:

$$i\mathcal{A}(W^+ \rightarrow M^+ \gamma) = \pm \frac{egf_M}{4\sqrt{2}} V_{ij} \left(i\epsilon_{\mu\nu\alpha\beta} \frac{k^\mu q^\nu \epsilon_W^\alpha \epsilon_\gamma^{*\beta}}{k \cdot q} F_1^M - \epsilon_W^\perp \cdot \epsilon_\gamma^{\perp*} F_2^M \right)$$

Explicit results:

$$F_1^M = \sum_{n=0}^{\infty} \left[C_{2n}^{(+)}(m_W, \mu) a_{2n}^M(\mu) - 3C_{2n+1}^{(+)}(m_W, \mu) a_{2n+1}^M(\mu) \right]$$

$$F_2^M = -2 + \sum_{n=0}^{\infty} \left[3C_{2n}^{(-)}(m_W, \mu) a_{2n}^M(\mu) - C_{2n+1}^{(-)}(m_W, \mu) a_{2n+1}^M(\mu) \right]$$

Exclusive radiative decays of W bosons

Predictions for branching ratios including detailed error estimates:

Decay mode	Branching ratio	asymptotic	LO
$W^\pm \rightarrow \pi^\pm \gamma$	$(4.00^{+0.06}_{-0.11} \mu \pm 0.01_f \pm 0.49_{a_2} \pm 0.66_{a_4}) \cdot 10^{-9}$	2.45	8.09
$W^\pm \rightarrow \rho^\pm \gamma$	$(8.74^{+0.17}_{-0.26} \mu \pm 0.33_f \pm 1.02_{a_2} \pm 1.57_{a_4}) \cdot 10^{-9}$	6.48	15.12
$W^\pm \rightarrow K^\pm \gamma$	$(3.25^{+0.05}_{-0.09} \mu \pm 0.03_f \pm 0.24_{a_1} \pm 0.38_{a_2} \pm 0.51_{a_4}) \cdot 10^{-10}$	1.88	6.38
$W^\pm \rightarrow K^{*\pm} \gamma$	$(4.78^{+0.09}_{-0.14} \mu \pm 0.28_f \pm 0.39_{a_1} \pm 0.66_{a_2} \pm 0.80_{a_4}) \cdot 10^{-10}$	3.18	8.47
$W^\pm \rightarrow D_s \gamma$	$(3.66^{+0.02}_{-0.07} \mu \pm 0.12_{\text{CKM}} \pm 0.13_f^{+1.47}_{-0.82} \sigma) \cdot 10^{-8}$	0.98	8.59
$W^\pm \rightarrow D^\pm \gamma$	$(1.38^{+0.01}_{-0.02} \mu \pm 0.10_{\text{CKM}} \pm 0.07_f^{+0.50}_{-0.30} \sigma) \cdot 10^{-9}$	0.32	3.42
$W^\pm \rightarrow B^\pm \gamma$	$(1.55^{+0.00}_{-0.03} \mu \pm 0.37_{\text{CKM}} \pm 0.15_f^{+0.68}_{-0.45} \sigma) \cdot 10^{-12}$	0.09	6.44

Table 5: Predicted branching fractions for various $W \rightarrow M\gamma$ decays, including error estimates due to scale dependence and the uncertainties in the CKM matrix elements, the meson decay constants and the LCDAs. The notation is the same as in Table 4. See text for further explanations.

Exclusive radiative decays of W bosons

When all Gegenbauer moments are neglected, i.e. $\phi_M(x) = 6x(1-x)$, we obtain for the decay rates:

$$\Gamma(W^\pm \rightarrow M^\pm \gamma)|_{\text{asympt}} = \frac{\alpha m_W f_M^2}{24v^2} |V_{ij}|^2 \left[1 - \frac{17}{3} \frac{\alpha_s(m_W)}{\pi} \right]$$

→ agrees with a formula for $W^\pm \rightarrow P^\pm \gamma$ in [Arnellos, Marciano, Parsa \(1982\)](#)

Using Manohar's approach, [Mangano and Melia \(2014\)](#) obtained an estimate for the $W^\pm \rightarrow \pi^\pm \gamma$ rate, which is too small by a factor 2/9 (understood ✓)

In some very old papers, the authors claimed that the $W, Z \rightarrow P\gamma$ rates are **enhanced by several orders of magnitude** due to an unsuppressed contribution $\sim 1/f_P$ from the axial anomaly. [Jacob, Wu \(1989\)](#); [Keum, Pham \(1993\)](#)

We find that such claims are false!