Recent News in Heavy Flavor Semileptonic Decays

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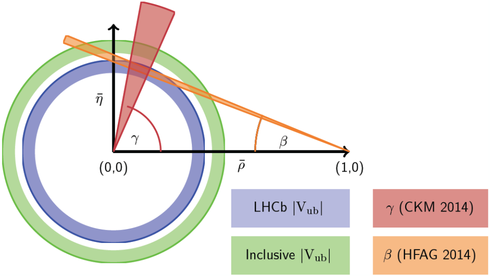
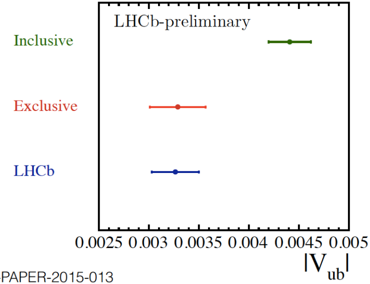
Protoroz 2015, 7.4.2015

Contents

Why Semileptonics?

- Important for Unitarity Triangle Fits





LHCb-PAPER-2015-013

- Provide insight into QCD:
  Form Factors, HQE Matrix Elements
- “Tensions” are taken as hints to “new physics”

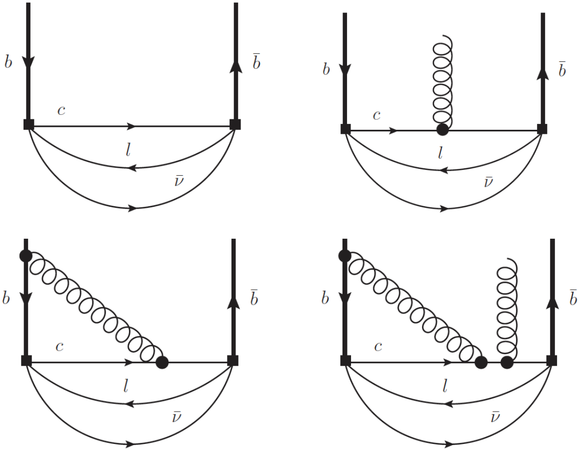
Advances in inclusive V_{cb}

Present state of the $b \to c$ semileptonic Calculations

- Tree level terms up to and including $1/m_b^5$ known
- $\mathcal{O}(\alpha_s)$ and full $\mathcal{O}(\alpha_s^2)$ for the partonic rate known
- $\mathcal{O}(\alpha_s)$ for the μ_π^2/m_b^2 is known
- QCD insprired modelling for the HQE matrix elements
- New: Complete α_s/m_b^2, including the μ_G terms
  Alberti, Gambino, Nandi (arXiv:1311.7381)
  ThM, Pivovarov, Rosenthal (arXiv:1405.5072)
- This was the remaining parametrically largest uncertainty

- Alberti et al.: **Phys.Rev.Lett. 114 (2015) 6, 061802** and **JHEP 1401 (2014) 147**
  - Calculation of the differential rate including the charm mass
  - partially numerical calculation
- ThM, Pivovarov, Rosenthal:
  Phys.Lett. B741 (2015) 290-294
  - Fully analytic calculation
  - limit $m_c \to 0$
  - Possibility to include m_c in a Taylor series
- Results do (not?) agree,
  some more checks in progress

Perform the OPE for $T = i \int d^4x \, T\left[H_{\rm eff}(x) H_{\rm eff}(0)\right]$



- Structure of the result:

$$(\mathrm{Im}\, T)/R_0 = C_0 \left\{ \bar{b} \not{v} b - \frac{\mathcal{O}_\pi}{2m_b^2} \right\} + C_{fin} C_m \frac{\mathcal{O}_G}{2m_b^2}$$

- chromomagnetic moment operator

$$\frac{1}{2M_B} C_m(\mu) \langle B(p_B) | \mathcal{O}_G | B(p_B) \rangle = \Delta m_B^2$$

- Total rate

$$\Gamma(B \to X_c \nu \ell) = \Gamma_b |V_{cb}|^2 \left\{ C_0 \left(1 + \frac{\mu_\pi^2}{2m_b^2} \right) + C_{fin} \frac{3 \Delta m_B^2}{8 m_b^2} \right\}$$

- We get ($\rho = m_c^2/m_b^2$)

$$C_{fin} = -3 + \Delta_G^{(0)}(\rho) + \frac{\alpha_s}{\pi}\Delta_G^{(1)}(\rho) + \frac{\alpha_s}{\pi}\left\{ C_A\left(\frac{31}{18} - \frac{\pi^2}{9}\right) + C_F\left(\frac{43}{144} - \frac{19\pi^2}{36}\right)\right\}$$

- $\Delta_G^{(0)}(\rho)$ is known

$$\Delta_G^{(0)}(\rho) = 8\rho - 24\rho^2 + 24\rho^3 - 5\rho^4 - 12\rho^2 \ln \rho$$

- $\Delta_G^{(1)}(\rho)$ is such that $\Delta_G^{(1)}(0) = 0$
- Numerically known from Alberti et al.

- Numerically (in the $m_c \to 0$ case:

$$C_{fin} = -3(1 + 1.56\frac{\alpha_s}{\pi})$$

This has a "normal" size

- The corresponding shift in V_{cb} is

$$\frac{\Delta |V_{cb}|}{|V_{cb}|} = 4.67\frac{\alpha_s}{\pi}\frac{3\Delta m_B^2}{8m_b^2}\frac{1}{2(1+\Delta_0^{(0)}(\rho))} \sim +0.3\%$$

Perspectives

- Fully analytic calculation underway
  - for moments
  - for the m_c dependence in a Taylor series
  - including the full mass dependence
- Complete comparison of the two results.

Decays into D^{**} states

R. Klein, ThM, F. Shahriaran, D. van Dyk,
arXiv:1503.00569

Decays into D^{**} states

- $B \to D$ and $B \to D^*$ exhaust about 75% of the inclusive $b \to c$ rate
- Aside from non-resonant $B \to D\pi$:
  Decays into D^{**} states
- ... mainly the orbitally excited states

Orbitally excited states

Make use of Heavy Quark Symmetry:

- Spin Symmetry Doublets of orbitally excited states, labelled by the total j of the light degrees of freedom:

$$\begin{pmatrix} |D(0^+)\rangle \\ |D(1^+)\rangle \end{pmatrix} \quad j = 1/2 \qquad \text{and} \qquad \begin{pmatrix} |D^*(1^+)\rangle \\ |D^*(2^+)\rangle \end{pmatrix} \quad j = 3/2$$

- Masses in the $m_c \to \infty$ limit:

$$M(D(0^+)) = M(D(1^+)) = m_c + \bar{\Lambda}_{1/2}$$
$$M(D^*(1^+)) = M(D^*(2^+)) = m_c + \bar{\Lambda}_{3/2}$$

- $\bar{\Lambda}_{3/2} - \bar{\Lambda}_{1/2}$ does not scale with m_c!

$1/m_c$ corrections

- Kinetic energy and choromomagnetic moment:

$$\mathcal{H}_{1/m} = \frac{1}{2m_c}\bar{c}(iD_\perp)^2 c + \frac{g_s}{2m_c}\bar{c}(\vec{\sigma}\cdot\vec{B})c$$

- Kinetic energy is spin independent and is absorbed into the mass definition

$$M_j = m_c + \bar{\Lambda}_j + \frac{1}{2m_c}\mu_\pi^2$$

Mass shift is independent of j

- Chromomagnetic moment depends on the spins:

$$\vec{K} = \vec{J} + \vec{\sigma} = \vec{L} + \vec{s} + \vec{\sigma} \qquad \begin{cases} \vec{L}\text{: Orbital Angular Momentum} \\ \vec{s}\text{: Light Quark Spin} \\ \vec{\sigma}\text{: Heavy Quark Quark Spin} \end{cases}$$

- Simple parametrization: The gyro-chromo-magnetic factors of the orbital motion and of the light quark spin are different:

$$\vec{B} \sim \alpha' \vec{L} + \beta' \vec{s} = \alpha \vec{J} + \beta \vec{s}$$

- The Spin dependent part of the Hamiltonian becomes (schematically)

$$H_{1/m} = \int d^3\vec{x}\, \frac{g_s}{2m_c} \bar{c}(\vec{\sigma} \cdot \vec{B})c = g(\vec{J} \cdot \vec{\sigma}) + g'(\vec{s} \cdot \vec{\sigma})$$

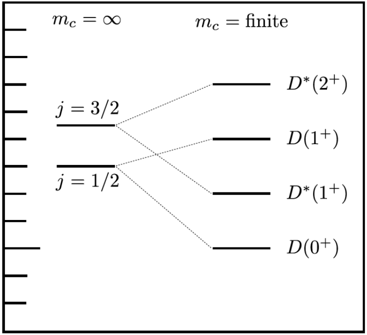
- $|D^*(2^+)\rangle$ and $|D(0^+)\rangle$ are Eigenstates of H

$$H|D^*(2^+)\rangle = \left(M_{3/2} + \frac{3}{4}g + \frac{1}{4}g'\right)|D^*(2^+)\rangle$$
$$H|D(0^+)\rangle = \left(M_{1/2} - \frac{3}{4}g + \frac{1}{4}g'\right)|D(0^+)\rangle$$

- There is a mixing between the two 1^+ states:

$$H|D(1^+)\rangle = \left(M_{1/2} + \frac{1}{4}g - \frac{1}{12}g'\right)|D(1^+)\rangle + \frac{\sqrt{2}}{3}g'\,|D^*(1^+)\rangle$$
$$H|D^*(1^+)\rangle = \left(M_{3/2} - \frac{5}{4}g - \frac{5}{12}g'\right)|D^*(1^+)\rangle + \frac{\sqrt{2}}{3}g'\,|D(1^+)\rangle$$

- Parametrically, the mixing is $g' \sim 1/m_c$, however, it is numerically of the same order as $\bar{\Lambda}_{3/2} - \bar{\Lambda}_{1/2}$
- Expect a significant mixing!

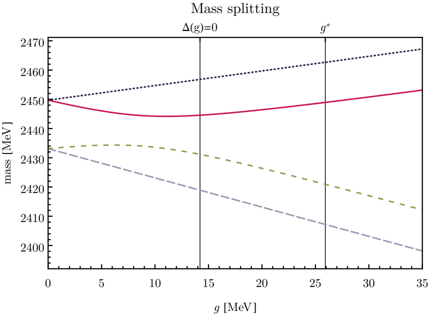


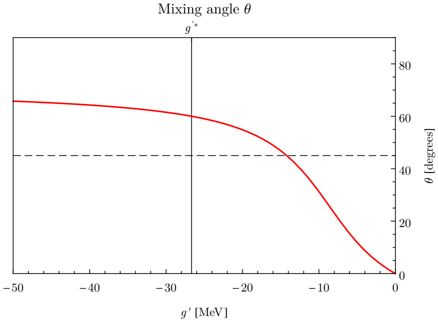
State	Mass [MeV]	Width [MeV]
$D(0^+)$	2318 ± 29	267 ± 40
$D(1^+)$	2421.4 ± 0.6	27.4 ± 2.5
$D^*(1^+)$	2427 ± 40	384 ± 120
$D^*(2^+)$	2462.6 ± 0.6	49 ± 1.3

- Assignment of the states:
  $j = 1/2$ are wide, $j = 3/2$ are narrow.

$$|D_1(1^+)\rangle = \cos\theta\,|D(1^+)\rangle + \sin\theta\,|D^*(1^+)\rangle$$
$$|D_2(1^+)\rangle = -\sin\theta\,|D(1^+)\rangle + \cos\theta\,|D^*(1^+)\rangle$$

- "Level Crossing": The mixing angle is larger than 45°





$$g \sim g',\ \theta \sim 60^\circ$$

$B \to D^{**} \ell \bar{\nu}$

Channel	GI	VD	CCCN	ISGW
$m_c \to \infty$				
$\mathcal{B}(B^- \to D(0^+)\ell\bar{\nu})$	$4.7 \cdot 10^{-4}$	$1.8 \cdot 10^{-4}$	$3.7 \cdot 10^{-5}$	$1.0 \cdot 10^{-3}$
$\mathcal{B}(B^- \to D(1^+)\ell\bar{\nu})$	$6.4 \cdot 10^{-4}$	$2.5 \cdot 10^{-4}$	$4.9 \cdot 10^{-5}$	$1.4 \cdot 10^{-3}$
$\mathcal{B}(B^- \to D^*(1^+)\ell\bar{\nu})$	$4.4 \cdot 10^{-3}$	$2.9 \cdot 10^{-3}$	$4.0 \cdot 10^{-3}$	$4.7 \cdot 10^{-3}$
$\mathcal{B}(B^- \to D^*(2^+)\ell\bar{\nu})$	$7.4 \cdot 10^{-3}$	$4.9 \cdot 10^{-3}$	$6.7 \cdot 10^{-3}$	$8.0 \cdot 10^{-3}$
$\mathcal{B}(B^- \to D^{**}\ell\bar{\nu})$	1.3%	0.82%	1.1%	1.5%
$\frac{\mathcal{B}(B^- \to D^*(1^+)\ell\bar{\nu})}{\mathcal{B}(B^- \to D(1^+)\ell\bar{\nu})}$	6.9	11	80	3.4
m_c finite				
$\mathcal{B}(B^- \to D_L\ell\bar{\nu})$	$3.0 \cdot 10^{-3}$	$2.1 \cdot 10^{-3}$	$3.0 \cdot 10^{-3}$	$3.2 \cdot 10^{-3}$
$\mathcal{B}(B^- \to D_H\ell\bar{\nu})$	$2.3 \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$	$1.3 \cdot 10^{-3}$	$3.1 \cdot 10^{-3}$
$\frac{\mathcal{B}(B^- \to D_L\ell\bar{\nu})}{\mathcal{B}(B^- \to D_H\ell\bar{\nu})}$	1.3	1.6	2.3	1.0

- GI: Godfrey, Isgur (1985);
- VD: Veseli, Dunietz (1996);
- CCCN: Cea, Colangelo, Cosmai, Nardulli (1991);
- ISGW: Isgur, Scora, Grinstein, Wise (1989)

$$\frac{\mathcal{B}(B^- \to D^*(1^+)\ell\nu)}{\mathcal{B}(B^- \to D(1^+)\ell\nu)} \approx 2.2$$

Heavy Flavour Conserving Decays

S. Faller, ThM, arXiv:1503.06088

Why Heavy Flavour Conserving Decays?

- Weak decays of light quarks inside heavy hadrons
- May be an interesting QCD lab:
  Light quarks moving in the color-background field of the heavy quark
- Unfortunately heavily phase-space suppressed:

$$w = v \cdot v' = \frac{M^2 + m^2 - q^2}{2Mm} ,$$

and

$$1 \leq w \leq w_{\max} = \frac{M^2 + m^2}{2Mm} = 1 + \frac{(M-m)^2}{2Mm} \sim 1 ,$$

Decay	Δm [MeV]	$J^P \to J'^{P'}$	$s_\ell \to s'_\ell$	Quark Transition
		Semi-electronic decays		
$D^+ \to D^0 e^+ \nu$	4.8	$0^- \to 0^-$	$1/2 \to 1/2$	$d \to u$
$D_s^+ \to D^0 e^+ \nu$	103.5	$0^- \to 0^-$	$1/2 \to 1/2$	$s \to u$
$B_s^0 \to B^- e^+ \nu$	87.5	$0^- \to 0^-$	$1/2 \to 1/2$	$s \to u$
$B_s^0 \to B^{*-} e^+ \nu$	41.6	$0^- \to 1^-$	$1/2 \to 1/2$	$s \to u$
$\Xi_c^0 \to \Lambda_c^+ e^- \bar{\nu}$	184.4	$1/2^+ \to 1/2^+$	$0 \to 0$	$s \to u$
$\Xi_c^0 \to \Sigma_c^+ e^- \bar{\nu}$	18.0	$1/2^+ \to 1/2^+$	$0 \to 0$	$s \to u$
$\Xi_c^+ \to \Sigma_c^{*++} e^- \bar{\nu}$	13.8	$1/2^+ \to 3/2^+$	$0 \to 1$	$s \to u$
$\Omega_c^0 \to \Xi_c^+ e^- \bar{\nu}$	227.4	$1/2^+ \to 1/2^+$	$1 \to 0$	$s \to u$
$\Omega_c^0 \to \Xi_c'^+ e^- \bar{\nu}$	119.7	$1/2^+ \to 1/2^+$	$1 \to 1$	$s \to u$
$\Omega_c^0 \to \Xi_c^{*+} e^- \bar{\nu}$	49.3	$1/2^+ \to 3/2^+$	$1 \to 1$	$s \to u$
$\Xi_b^- \to \Lambda_b^0 e^- \bar{\nu}$	175.4	$1/2^+ \to 1/2^+$	$0 \to 0$	$s \to u$
$\Omega_b^- \to \Xi_b^0 e^- \bar{\nu}$	255.7	$1/2^+ \to 1/2^+$	$1 \to 0$	$s \to u$
$\Omega_b^- \to \Xi_b^{*0} e^- \bar{\nu}$	99.5	$1/2^+ \to 3/2^+$	$1 \to 1$	$s \to u$

Decay	Δm [MeV]	$J^P \to J'^{P'}$	$s_\ell \to s'_\ell$	Quark Transition
		Semi-muonic decays		
$\Xi_c^0 \to \Lambda_c^+ \mu^- \bar{\nu}$	151.2	$1/2^+ \to 1/2^+$	$0 \to 0$	$s \to u$
$\Omega_c^0 \to \Xi_c^+ \mu^- \bar{\nu}$	201.4	$1/2^+ \to 1/2^+$	$1 \to 0$	$s \to u$
$\Omega_c^0 \to \Xi_c'^+ e^- \bar{\nu}$	56.1	$1/2^+ \to 1/2^+$	$1 \to 1$	$s \to u$
$\Xi_b^- \to \Lambda_b^0 \mu^- \bar{\nu}$	140.0	$1/2^+ \to 1/2^+$	$0 \to 0$	$s \to u$
$\Omega_b^- \to \Xi_b^0 \mu^- \bar{\nu}$	232.8	$1/2^+ \to 1/2^+$	$1 \to 0$	$s \to u$
		Pionic decays		
$\Xi_c^0 \to \Lambda_c^+ \pi^-$	184.4	$1/2^+ \to 1/2^+$	$0 \to 0$	$s \to u$
$\Xi_c^+ \to \Lambda_c^+ \pi^0$	181.3	$1/2^+ \to 1/2^+$	$0 \to 0$	$s \to u$
$\Omega_c^0 \to \Xi_c^+ \pi^-$	227.4	$1/2^+ \to 1/2^+$	$1 \to 0$	$s \to u$
$\Omega_c^0 \to \Xi_c^0 \pi^0$	224.3	$1/2^+ \to 1/2^+$	$1 \to 0$	$s \to u$
$\Xi_b^- \to \Lambda_b^0 \pi^-$	175.4	$1/2^+ \to 1/2^+$	$0 \to 0$	$s \to u$
$\Xi_b^0 \to \Lambda_b^0 \pi^0$	173.6	$1/2^+ \to 1/2^+$	$0 \to 0$	$s \to u$
$\Omega_b^- \to \Xi_b^0 \pi^-$	255.7	$1/2^+ \to 1/2^+$	$1 \to 0$	$s \to u$
$\Omega_b^- \to \Xi_b^- \pi^0$	253.9	$1/2^+ \to 1/2^+$	$1 \to 0$	$s \to u$

Mesonic Decays

- We need the matrix elements close to $v = v'$

$$\langle H_f(p')|\bar{q}'\gamma_\mu q|H_i(p)\rangle = \sqrt{Mm}(v+v')_\mu \Phi_+(w) + \dots ,$$
$$\langle H_f^*(p',\epsilon)|\bar{q}'\gamma_\mu\gamma_5 q|H_i(p)\rangle = i\sqrt{Mm}(w+1)\epsilon_\mu^* \Phi_{A_1}(w) + \dots ,$$

- Normalization statement from light quark flavor symmetry

$$\Phi_+(w=1) = 1$$

- No normalization for the axial current, but we assume that $\Phi_{A_1}(1)$ is also close to unity

- Total rates for semi-electonic decays

$$\Gamma^{0^- \to 0^- e\bar{\nu}} = \frac{G_F^2}{60\pi^3} |V_{\rm CKM}|^2 (M-m)^5$$

$$\Gamma^{0^- \to 1^- e\bar{\nu}} = \frac{G_F^2}{20\pi^3} |V_{\rm CKM}|^2 (M-m)^5 |\Phi_A(1)|^2$$

- Strong phase space supression $(M-m)^5/M^5$

Mode	Decay Rate [GeV]	Branching Ratio
$D^+ \to D^0 e^+ \nu$	1.72×10^{-25}	2.71×10^{-13}
$D_s^+ \to D^0 e^+ \nu$	4.40×10^{-20}	3.34×10^{-8}
$B_s^0 \to B^- e^+ \nu$	1.90×10^{-20}	4.37×10^{-8}
$B_s^0 \to B^{*-} e^+ \nu$	1.38×10^{-21}	3.17×10^{-9}

Baryonic Decays

- What are the form factors close to $v = v'$?
- Heavy Quark Limit:
  - Heavy Quark = "cannon ball": Velocity is unchanged
  - Heavy Quark spin decouples: Spin is unchanged
- "Λ-like" Baryons: Light degrees of freedom are 0^+

$$\langle \Xi_H(v,s) \, |\bar{s}\gamma_\mu u| \, \Lambda_H(v',s')\rangle = \bar{u}_\Xi(v,s) u_\Lambda(v',s') B(w) (v+v')_\mu + \dots \, ,$$

$$\langle \Xi_H(v,s) \, |\bar{s}\gamma_\mu \gamma_5 u| \, \Lambda_H(v',s')\rangle = 0 + \dots$$

- $B(w)$: Vector form factor of the $0^+ \to 0^+$ transition
- $B(w=1) = 1$ from light quark symmetries
- No axial vector form factors for the $0^+ \to 0^+$ transition

- "Σ-like" Baryons: Light degrees of freedom are 1^+
- $\Sigma \to \Lambda$ transition: $1^+ \to 0^+$

$$\langle \Xi_H(v,s) | \bar{s}\gamma_\mu\gamma_5 u | \Sigma_H(v',s') \rangle = \bar{u}_i(v,s) u_f(v',s') \epsilon_\mu A(w) + \dots ,$$
$$\langle \Xi_H(v,s) | \bar{s}\gamma_\mu u | \Sigma_H(v',s') \rangle = 0 + \dots ,$$

- $A(w)$: Axial vector form factor of $1^+ \to 0^+$
- Vector form factor vanishes
- Decompose $1/2 \otimes 1$ into $1/2$ and $3/2$

$$\psi_\mu^{(3/2)} = \epsilon'_\nu \left[\delta_\mu^\nu - \frac{1}{3}(\gamma_\mu + v'_\mu)\gamma^\nu \right] u_f(v',s') = R_\mu^{\Sigma,3/2}(v',s') ,$$
$$\psi_\mu^{(1/2)} = \epsilon'_\nu \left[\frac{1}{3}(\gamma_\mu + v'_\mu)\gamma^\nu \right] u_f(v',s') = \frac{1}{\sqrt{3}}(\gamma_\mu + v'_\mu)\gamma_5 u^{\Sigma,1/2}(v',s')$$

- Finally: $\Sigma \to \Sigma$ transitions: $1^+ \to 1^+$

$$\left\langle \Omega_H(v,s) \left| \bar{s}\gamma_\mu u \right| \Xi_H^{(\prime,*)}(v',s') \right\rangle = \bar{u}_i(v,s)u_f(v',s')(\epsilon^* \cdot \epsilon')(v_\mu + v'_\mu)C(w) + \dots ,$$

$$\left\langle \Omega_H(v,s) \left| \bar{s}\gamma_\mu\gamma_5 u \right| \Xi_H^{(\prime,*)}(v',s') \right\rangle = 0 + \dots .$$

- $C(w)$ Vector Form factor for the $1^+ \to 1^+$ transition
- $C(w=1) = 1$ due to light quark symmetries
- No axial vector form factor
- ... Decompose into the $1/2$ and $3/2$ components ...

- Total rates for semi electronic decays

$$\Gamma^{1/2^+ \to 3/2^+}_{0^+ \to 1^+} = \frac{G_F^2 \, |V_{\rm CKM}|^2}{30\pi^3} (M - m)^5 |A(1)|^2 \,,$$

$$\Gamma^{1/2^+ \to 1/2^+}_{0^+ \to 1^+} = \frac{1}{2} \Gamma^{1/2^+ \to 3/2^+}_{0^+ \to 1^+}$$

$$\Gamma^{1/2^+ \to 1/2^+}_{1^+ \to 1^+} = \frac{G_F^2 \, |V_{\rm CKM}|^2}{15\pi^3} (M - m)^5 \,,$$

$$\Gamma^{1/2^+ \to 3/2^+}_{1^+ \to 1^+} = \mathcal{O}([M - m]^7)$$

Mode	Decay Rate [GeV]	Branching Ratio
$\Xi_c^0 \to \Lambda_c^+ e^- \bar{\nu}$	7.91×10^{-19}	1.35×10^{-7}
$\Xi_c^0 \to \Sigma_c^+ e^- \bar{\nu}$	6.97×10^{-24}	1.19×10^{-12}
$\Xi_c^+ \to \Sigma_c^{++} e^- \bar{\nu}$	3.74×10^{-24}	1.26×10^{-12}
$\Omega_c^0 \to \Xi_c^+ e^- \bar{\nu}$	2.26×10^{-18}	2.36×10^{-7}
$\Omega_c^0 \to \Xi_c'^+ e^- \bar{\nu}$	3.63×10^{-19}	3.81×10^{-8}
$\Omega_c^0 \to \Xi_c^{*+} e^- \bar{\nu}$	1.49×10^{-29}	1.57×10^{-18}
$\Xi_b^- \to \Lambda_b^0 e^- \bar{\nu}$	6.16×10^{-19}	1.46×10^{-6}
$\Omega_b^- \to \Xi_b^0 e^- \bar{\nu}$	4.05×10^{-18}	6.78×10^{-6}
$\Omega_b^- \to \Xi_b^{*0} e^- \bar{\nu}$	3.27×10^{-28}	5.47×10^{-16}

Mode	Decay Rate [GeV]	Branching Ratio
$\Xi_c^0 \to \Lambda_c^+ \mu^- \bar{\nu}$	1.3×10^{-19}	2.3×10^{-8}
$\Omega_c^0 \to \Xi_c^+ \mu^- \bar{\nu}$	7.1×10^{-19}	7.4×10^{-8}
$\Omega_c^0 \to \Xi_c^{+\prime} \mu^- \bar{\nu}$	1.0×10^{-21}	1.1×10^{-10}
$\Xi_b^- \to \Lambda_b^0 \mu^- \bar{\nu}$	9.1×10^{-20}	2.2×10^{-7}
$\Omega_b^- \to \Xi_b^0 \mu^- \bar{\nu}$	1.7×10^{-18}	2.8×10^{-6}

- There are also pionic decays possible
- ... may as well be an interesting QCD lab.

$\Lambda_b \to \Lambda_c$ Form Factors

ThM, D. van Dyk, SI-HEP-2015-14

(all numbers preliminary)

$\Lambda_b \to \Lambda_c$ form factors

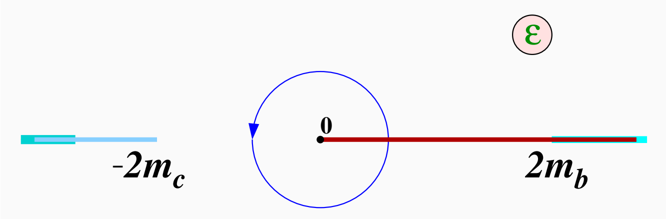
- Recent V_{ub} determination from LHCb based on $\Lambda_b \to p\ell\bar{\nu}$ is normalized to $\Lambda_b \to \Lambda_c\ell\bar{\nu}$
- Relies heavily on Lattice QCD Form Factors for both $\Lambda_b \to p$ and $\Lambda_b \to \Lambda_c$ (W. Detmold, C. Lehner, S. Meinel (2015))
- Can we say something using continuum methods?
- $\to$ Zero Recoil Sum Rule for $\Lambda_b \to \Lambda_c$ form factors

Zero Recoil Sum Rule

- Start from

$$T(\omega) = \frac{1}{3} \int d^4x\, e^{i\,(v\cdot x)\omega}$$
$$\langle \Lambda_b(P) | \mathcal{T}\{ \bar{b}_v(x) \gamma_\mu \gamma_5 c_v(x)\, \bar{c}_v(0) \gamma^\mu \gamma_5 b_v(0) \} | \Lambda_b(P) \rangle$$

- compute the contour integral $I_0(\mu) = -\frac{1}{2\pi i} \oint_{|\varepsilon|=\mu} T(\varepsilon)\, \mathrm{d}\varepsilon$



- Inserting a complete set of states:
  Lowest state is Λ_c
- Form factor definition: (T. Feldmann , M. Yip (2011))

$$\langle \Lambda_c(v', s') | \bar{c} \gamma_5 \gamma_\mu b | \Lambda_b(v, s) \rangle = \bar{u}_{\Lambda_c}(v', s') \gamma_5 \left[g_0(w)(M_{\Lambda_b} + M_{\Lambda_c}) \frac{q^\mu}{q^2} \right.$$

$$+ g_+(w) \frac{M_{\Lambda_b} - M_{\Lambda_c}}{s_-} \left(M_{\Lambda_b} v_\mu + M_{\Lambda_c} v'_\mu - (M^2_{\Lambda_b} - M^2_{\Lambda_c}) \frac{q^\mu}{q^2} \right)$$

$$\left. + g_\perp(w) \left(\gamma_\mu - \frac{2 M_{\Lambda_c} M_{\Lambda_b}}{s_+} (v_\mu - v'_\mu) \right) \right] u_{\Lambda_b}(v, s)$$

- Zero Recoil Sum Rule:

$$I_0(\epsilon_M) = \frac{1}{3} \left[2 |g_\perp(1)|^2 + |g_+(1)|^2 \right] + \text{ inelastic}$$

$$= \xi^{\text{pert}}(\epsilon_M, \mu) - \Delta_{1/m^2}(\epsilon_M, \mu) - \Delta_{1/m^3}(\epsilon_M, \mu) + \cdots$$

- Perturbative Contribution

  ($\mathcal{O}(\alpha_s)$, computed in Wilsonian Cut Off Scheme)

$$\xi^{\rm pert}(\epsilon_M = \mu = 0.75\,{\rm GeV}) = 0.970 \pm 0.02$$

- Nonperturbative Contributions

$$\Delta_{1/m^2} = \frac{\mu_\pi^2(\Lambda_b)}{4}\left(\frac{1}{m_c^2} + \frac{1}{m_b^2} + \frac{2}{3m_b m_c}\right)$$

$$\Delta_{1/m^3} = \frac{\rho_D^3(\Lambda_b)}{4m_c^3} + \frac{\rho_D^3(\Lambda_b)}{12m_b}\left(\frac{1}{m_c^2} + \frac{3}{m_b^2} + \frac{1}{m_b m_c}\right)$$

- Note $\mu_G = \rho_{LS} = 0$ and $\mu_\pi^2(\Lambda_b) \sim \mu_\pi^2(B)$

- ... or as an inequality

$$\frac{1}{3}\left[2|g_\perp(1)|^2 + |g_+(1)|^2\right] \leq \xi^{\rm pert}(\epsilon_M,\mu) - \Delta_{1/m^2}(\epsilon_M,\mu) - \Delta_{1/m^3}(\epsilon_M,\mu) + \cdots$$

- Numerically we have (Preliminary)

$$\frac{1}{3}\left[2|g_\perp(1)|^2 + |g_+(1)|^2\right] \leq 0.86$$

- ... to be compared to the lattice number
(W. Detmold, C. Lehner, S. Meinel (2015))

$$\frac{1}{3}\left[2|g_\perp(1)|^2 + |g_+(1)|^2\right] = 0.824 \pm 0.020$$

Conclusions

- The V_{ub} Problem seems to remain with us
  - Scrutinize the inclusive methodology
  - Can it be BSM physics?
- There is also a “tension” in V_{cb}, but not as severe (personal view!)
- Λ_b semileptonic decays become important.