A Precision Determination of the CKM Element V_{cb}

An Inclusive Numerical Analysis at $\mathcal{O}(\alpha_s^2, \alpha_s \frac{\Lambda^2}{m_b^2})$

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with

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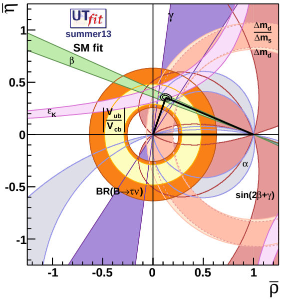
Motivation

- V_{cb} through $(B \to X_c \ell \bar{\nu})$ Important :
  - * Allows us to probe $b \to c$ transition
  - * Constrain NP through FV Processes
  - * Important role in UT Determination
  - - $\frac{|V_{ub}|}{|V_{cb}|}$ ratio gives one of the sides of the CKM unitarity triangle
  - - ϵ_k constraint on $\bar{\rho}$,$\bar{\eta}$ sensitive to $|V_{cb}|$



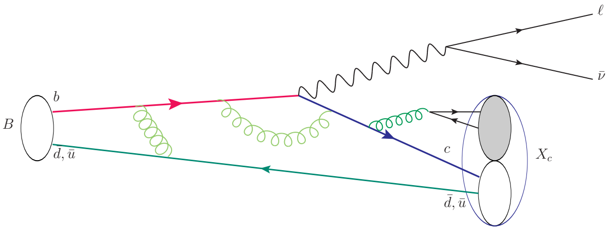
- Determined from both $B \to X_c \ell \bar{\nu}$ and $B \to D^{(*)} \ell \bar{\nu}$

$|V_{cb}|_{inc} = 42.42 \pm 0.86 \times 10^{-3}$ [P.G., C.S. Phys. Rev. D 89, 014022 (2014)]

$|V_{cb}|_{ex} = 39.04 \pm 0.8 \times 10^{-3}$ [FNAL, MILC, Phys. Rev. D 89, 114504 (2014)]

- Near 3 σ discrepancy between inclusive and exclusive $B \to D^* \ell \bar{\nu}$ at zero recoil with form factors from lattice.

Inclusive $B \to X_c \ell \bar{\nu}$ Determination of $|V_{cb}|$



Easy Concept

- Inclusive decays do not depend on the hadronic final state or long-distance dynamics of the B meson factorizes.
- We can perform an OPE to express observables in terms of matrix elements of these local operators.
- The Wilson-Coefficients are perturbative, while matrix elements parameterize the non-perturbative physics.

$$\langle B|\bar{b}(i\vec{D})^2 b|B\rangle_\mu = 2M_B\,\mu_\pi^2(\mu) \qquad \langle B|\bar{b}\tfrac{i}{2}(\sigma\cdot G)b|B\rangle_\mu = 2M_B\,\mu_G^2(\mu)$$

Inclusive $B \to X_c \ell \bar{\nu}$ Determination of $|V_{cb}|$

Operator Product Expansion (OPE)

- Perturbative + Heavy Quark Expansion (HQET)
- Heavy Quark Limit $\to$ partonic decay rate
  - * Corrections to Observables expressed as double expansion in $\mathcal{O}(\alpha_s)$ and $\mathcal{O}(\Lambda_{QCD}/m_b)$
- Express the observables

$$\begin{aligned} M_i \quad = \quad & M_i^{(0)} + M_i^{(\pi,0)} \frac{\mu_\pi^2}{m_b^2} + M_i^{(G,0)} \frac{\mu_G^2}{m_b^2} + M_i^{(D,0)} \frac{\rho_D^3}{m_b^3} + M_i^{(LS,0)} \frac{\rho_{LS}^3}{m_b^3} \\ & + \left(\frac{\alpha_s}{\pi}\right) \left[M_i^{(1)} + M_i^{(\pi,1)} \frac{\mu_\pi^2}{m_b^2} + M_i^{(G,1)} \frac{\mu_G^2}{m_b^2} + M_i^{(D,1)} \frac{\rho_D^3}{m_b^3} + M_i^{(LS,1)} \frac{\rho_{LS}^3}{m_b^3} \right] \\ & + \left(\frac{\alpha_s}{\pi}\right)^2 M_i^{(2)} + \mathcal{O}\left(m_b^{4,5}, \alpha_s^3\right) \end{aligned}$$

- Need information on non-perturbative parameters
- $|V_{cb}|$ extracted from width after HQE parameter fit
- At order $1/m_b^{2,3}$ 6 non-perturbative parameters $(m_{b,c}, \mu_{\pi,G}, \rho_{D,LS})$

Higher Order Corrections

- The accuracy and reliability of the inclusive method depends our ability to control higher order corrections
- Perturbative $\mathcal{O}(\alpha_s^n)$ corrections are known completely to NNLO [Melnikov, Biswas, Czarnecki, Pak, Gambino]
- Higher order non-perturbative power corrections are also calculated $\mathcal{O}(1/m_b^{4,5})$ [Mannel,Turczyk,Uraltsev, 2010]
- Cross-terms from the expansions are complete to $\mathcal{O}(\alpha_s/m_b^2)$ [Becher, Boos, Lunghi, Alberti, Ewerth, Nandi, Gambino]
- $\mathcal{O}(\alpha_s/m_b^3)$ corrections are currently being calculated

Higher Order Corrections : $\mathcal{O}(1/m_b^{4,5})$

- $\mathcal{O}(1/m_b^{4,5})$ [Mannel, Turczyk, Uraltsev 2010/2012, Heinonen, Mannel 2014]
- ...too many parameters for fit

$$
\begin{aligned}
2M_B m_1 &= \langle((\vec{p})^2)^2\rangle & &(...)\\
2M_B m_2 &= \langle g^2\vec{E}^2\rangle & 2M_B m_8 &= \langle(\vec{S}\cdot\vec{B})(\vec{p})^2)\rangle\\
2M_B m_3 &= \langle g^2\vec{B}^2\rangle & 2M_B m_9 &= \langle\Delta(\vec{\sigma}\cdot\vec{B})\rangle
\end{aligned}
$$

Estimated using Lowest Lying State Saturation approximation by truncating $\langle B|\mathcal{O}_1\mathcal{O}_2|B\rangle = \sum_n \langle B|\mathcal{O}_1|n\rangle\langle n|\mathcal{O}_2|B\rangle$

Small changed in $|V_{cb}|$ though, $\frac{\delta V_{cb}}{V_{cb}} \simeq -0.35\%$ [Turczyk, Gambino PRELIM]

- LLSA Corrections have been found, but allowing 80% gaussian deviations from LLSA seem to leave V_{cb} unaffected.

Higher Order Corrections : $\mathcal{O}(\alpha_s/m_b^2)$

[Boos,Becher,Lunghi 2007][Ewerth,Nandi, PG 2009][Alberti,Ewerth,Nandi,PG 2012][Alberti,Nandi,PG 2013]

Concerned most with the Hadronic Tensor :

$$W^{\mu\nu} = \frac{(2\pi)^3}{2m_B} \sum_{X_c} \delta^4(p_b - q - p_X) \langle B | J_L^{\dagger\mu} X_c \rangle \langle X_c | J_L^{\nu} | B \rangle$$

This can be decomposed into :

$$m_b W^{\mu\nu} = -W_1 g^{\mu\nu} + W_2 v^\mu v^\nu + i W_3 \epsilon^{\mu\nu\rho\sigma} v_\rho \hat{q}_\sigma + (\ldots)$$

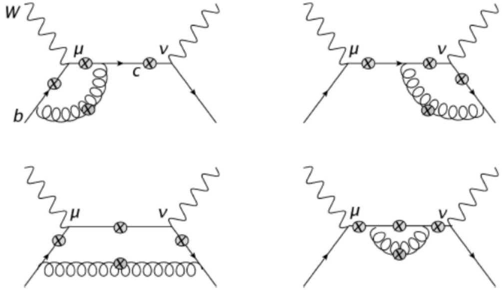
Where each W_i can be double-expanded :

$$W_i = W_i^{(0)} + \frac{\mu_{\pi,G}^2}{2m_b^2} W_i^{(\pi,G,0)} + \frac{C_F \alpha_s}{\pi} \left[W_i^{(1)} + \frac{\mu_{\pi,G}^2}{2m_b^2} W_i^{(\pi,G,1)} \right]$$

A useful check are the RPI relations [Manohar 2010] for $W_i^{\pi,n}$, for example to all orders :

$$W_3^{(\pi,n)} = \frac{5}{3} \hat{q}_0 \frac{dW_3^{(n)}}{d\hat{q}_0} - \frac{\hat{q}^2 - \hat{q}_0^2}{3} \frac{dW_3^{(n)}}{d\hat{q}_0}$$

Matching at $\mathcal{O}(\alpha_s)$



- Diagrams contributing to $\mathcal{O}(\alpha_s/m_b^2)$, (x) denotes a gluon insertion for symmetrization of HQET operators (split μ_π, μ_G)

$$\frac{2i}{\pi}\int d^4x\, e^{-iq\cdot x}\, T[J_L^{\dagger\mu}(x)J_L^\nu(0)] = \sum_i c^{(i)\mu\nu}_{\{\alpha\}}(v,q)\, O_i^{\{\alpha\}}(0)$$

- Expansion around on-shell b-quark matched onto HQET local operators
- Analytic formulae obtained, RPI satisfied, but unlike μ_π, μ_G gets ren.

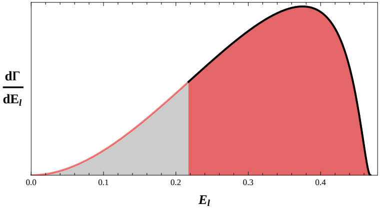
Relevant Observables

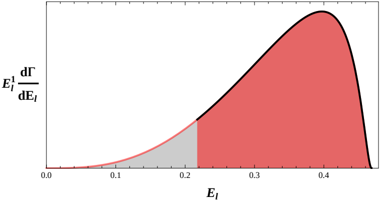
Leptonic Moments

$$\langle E_\ell^n \rangle_{E_\ell > E_{cut}} = \frac{\int_{E_{cut}}^{E_{max}} E_\ell^n \frac{d\Gamma}{dE_\ell} dE_\ell}{\int_{E_{cut}}^{E_{max}} dE_\ell \frac{d\Gamma}{dE_\ell}}$$

$$R^*(E_{cut}) = \frac{\int_{E_{cut}}^{E_{max}} \frac{d\Gamma}{dE_\ell} dE_\ell}{\int_0^{E_{max}} \frac{d\Gamma}{dE_\ell} dE_\ell}$$

$\langle E_\ell^1 \rangle, \langle E_\ell^2 \rangle, \langle E_\ell^3 \rangle$ Highly Correlated





Central Leptonic Moments

$$\ell_1(E_{cut}) = \langle E_\ell \rangle_{E_\ell > E_{cut}}$$

$$\ell_{2,3}(E_{cut}) = \langle (E_\ell - \langle E_\ell \rangle)^{2,3} \rangle_{E_\ell > E_{cut}}$$

Hadronic Moments

$$\langle (M_X^2)^n \rangle_{E_\ell > E_{cut}} = \frac{\int_{E_{cut}}^{E_{max}} (M_X^2)^n \frac{d\Gamma}{dM_X^2} dM_x^2}{\int_{E_{cut}}^{E_{max}} \frac{d\Gamma}{dM_X^2} dM_x^2}$$

$$h_1(E_{cut}) = \langle M_X^2 \rangle_{E_\ell > E_{cut}}$$

$$h_{2,3}(E_{cut}) = \langle (M_X^2 - \langle M_X^2 \rangle)^{2,3} \rangle_{E_\ell > E_{cut}}$$

Experimental Observables

- Observables as $F_x(E_{cut}, m_c^2/m_b^2)$
- Express $m_b, \mu_{\pi,G}, \rho_{D,LS}$ in the "kinetic scheme" with a cutoff $\mu_{kin} = 1 GeV$
- Additionally employ both kinetic and $\overline{\mathrm{MS}}$ definitions for m_c
- $\alpha_s(m_b = 4.6 GeV) = 0.22$
  $\alpha_s \pm 0.005 \rightarrow \delta m_b < 1 MeV$
- Additional Constraints:

Hyperfine Splitting

$$M_{B^*} - M_B = \frac{2}{3}\frac{\mu_G^2}{m_b} + O\left(\frac{\alpha_s \mu_G^2}{m_b}, \frac{1}{m_b^2}\right)$$

$$\mu_G^2 = (0.35 \pm 0.07)\,\mathrm{GeV}^2$$

Heavy Quark Sum Rules

$$\rho_{LS}^3 = (-0.15 \pm 0.10)\,\mathrm{GeV}^3$$

	Experiment	Values of $E_{cut}(\mathrm{GeV})$
R^*	BaBar	0.6, 1.2, 1.5
ℓ_1	BaBar	0.6, 0.8, 1, 1.2, 1.5
ℓ_2	BaBar	0.6, 1, 1.5
ℓ_3	BaBar	0.8, 1.2
h_1	BaBar	0.9, 1.1, 1.3, 1.5
h_2	BaBar	0.8, 1, 1.2, 1.4
h_3	BaBar	0.9, 1.3
R^*	Belle	0.6, 1.4
ℓ_1	Belle	1, 1.4
ℓ_2	Belle	0.6, 1.4
ℓ_3	Belle	0.8, 1.2
h_1	Belle	0.7, 1.1, 1.3, 1.5
h_2	Belle	0.7, 0.9, 1.3
$h_{1,2}$	CDF	0.7
$h_{1,2}$	CLEO	1, 1.5
$\ell_{1,2,3}$	DELPHI	0
$h_{1,2,3}$	DELPHI	0

Note:

- Semileptonic moments are sensitive to a linear combination of m_c and m_b: poor individual accuracy
- Generally use photon energy moments in $B \to X_s\gamma$
- Much better m_c determination from e^+e^- sum rules, Lattice QCD

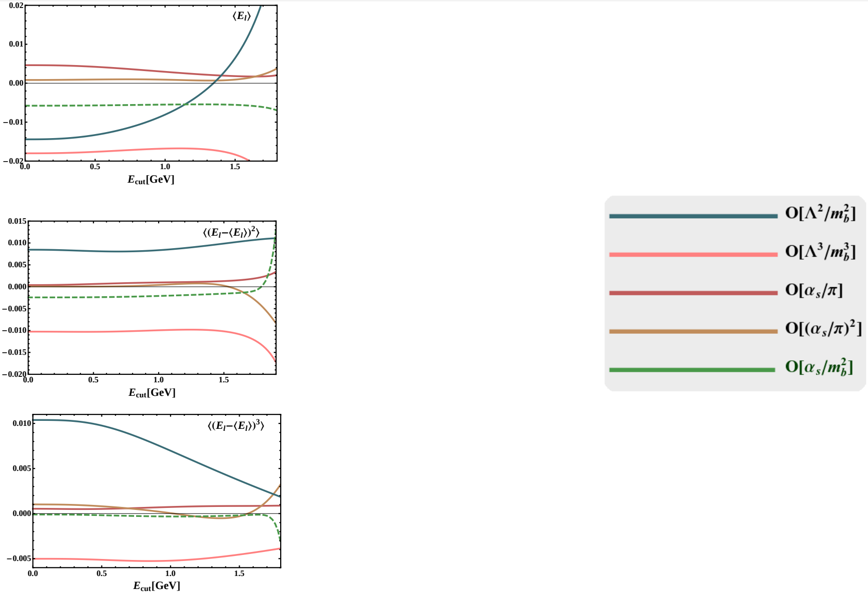
New Contributions:

$$M_i \quad = (\dots) + \left(\frac{\alpha_s}{\pi}\right)\left[\, M_i^{(\pi,1)}\frac{\mu_\pi^2}{m_b^2} + M_i^{(G,1)}\frac{\mu_G^2}{m_b^2}\right]$$

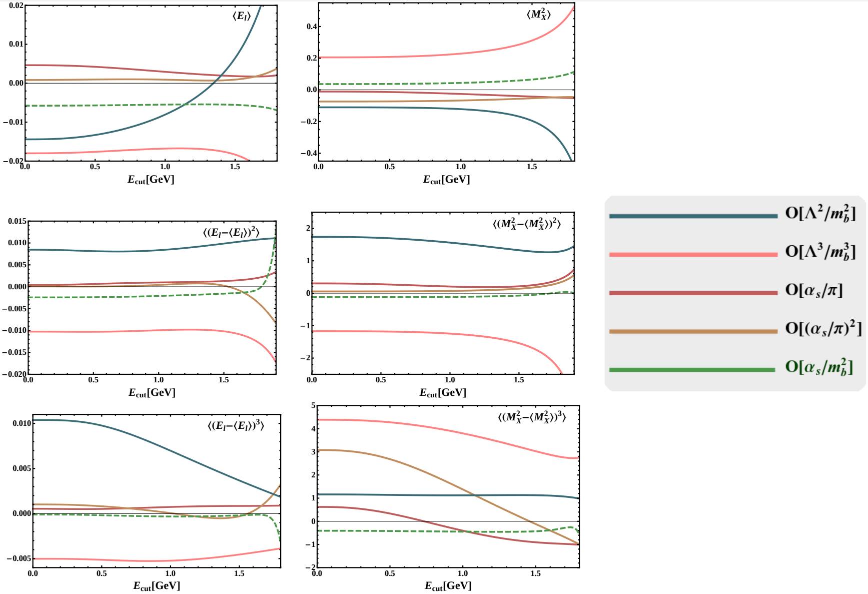
A. Alberti, P. Gambino and S. Nandi [JHEP, 1 (2014)]
A. Alberti, T. Ewerth, P. Gambino and S. Nandi [Nucl. Phys. B 870, 16 (2013)]
T. Becher, H. Boos, E. Lunhgi [JHEP 0712 (2007) 062]

Large cancellation in ℓ_1, ℓ_2, ℓ_3
Requires high precision : Avoid divergences

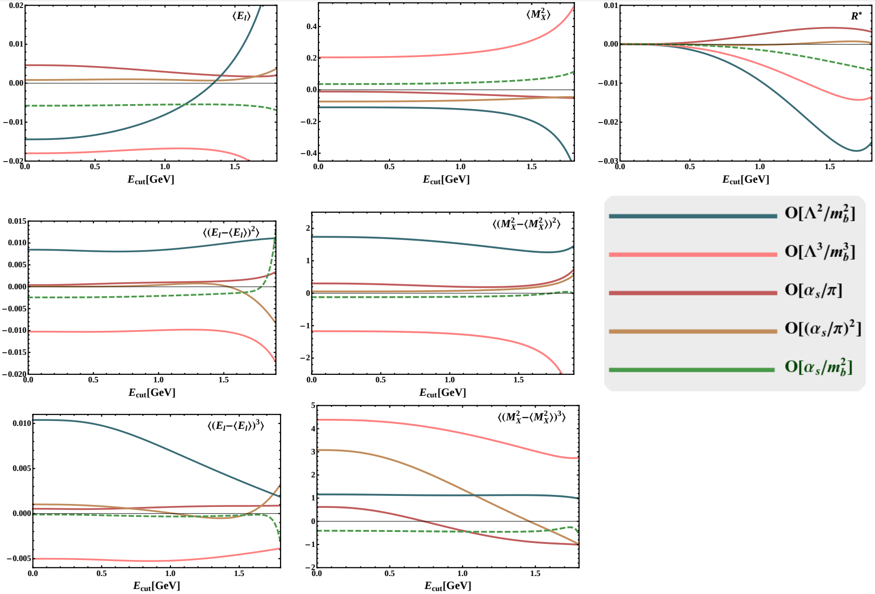
New Contributions $\mathcal{O}(\alpha_s/m_b^2)$: I1, I2 ,I3



New Contributions $\mathcal{O}(\alpha_s/m_b^2)$: h1, h2 ,h3



New Contributions $\mathcal{O}(\alpha_s/m_b^2)$: R^*



Theoretical Error and Correlation Issues

- Perform High-Precision fits for use in HFAG Fortran Routine
- Functional dependence on E_{cut} is important
- Theoretical uncertainty assigned :

  $M_i(1\,\text{GeV})$ and $M_i(1.1\,\text{GeV})$ are very close and *highly correlated*

  * Previous fits assumed 100% correlation : too strong
  * Dependence of observable on E_{cut} would be free from theoretical uncertainty

Standard Global Fit Theoretical Correlation Scenarios

A : 100% Correlation between M_i at different E_{cut}

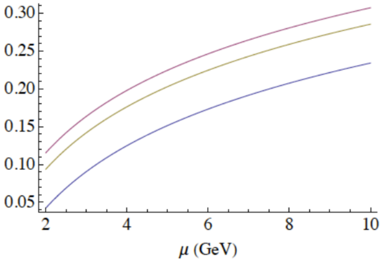
B : Correlations from varying theoretical parameters

C : Constant scale factor (0.97/100 MeV Steps)

D : Functional Scale Factor ($E_0 \approx 1.75\,\text{GeV}, \Delta \approx 0.25\,\text{GeV}$) *

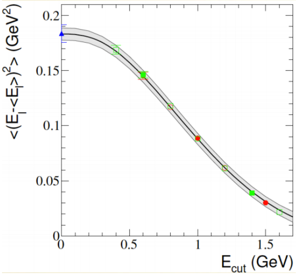
$$\xi(E_{cut}) = 1 - \frac{1}{2}\exp^{-\frac{E_0 - E_{cut}}{\Delta}}$$

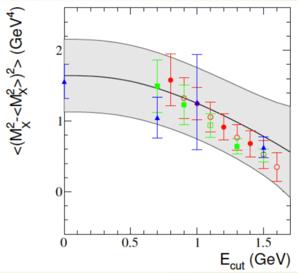
Residual Scale Dependence



- Renormalization of $\mu_G(\mu)$ and $\mathcal{O}(\alpha_s \mu_G^2/m_b^2)$ lead to residual scale dependence
- Relative NLO correction to the coefficients of $\mu_G(\mu)$ in the width (blue), first (red) and second central (yellow) leptonic moments as a function of the renormalization scale μ.
- The lower the scale, the smaller the corrections

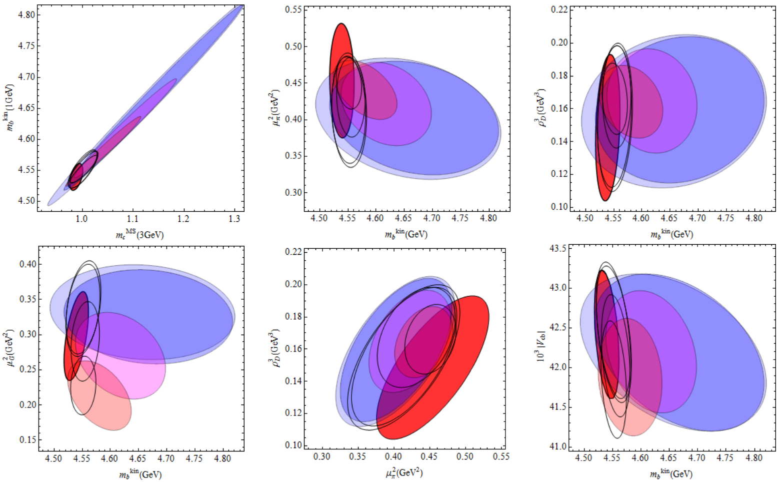
Theoretical Errors





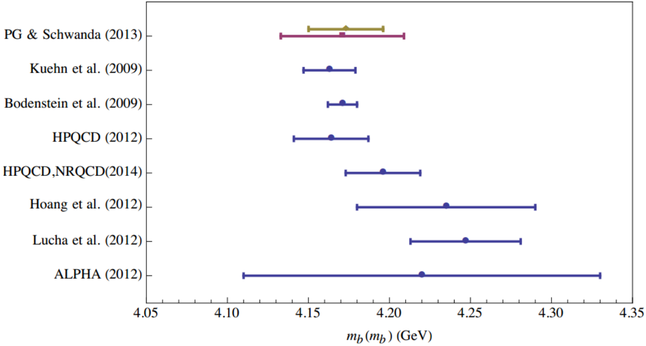
- Theoretical errors are generally *dominant* in the fits.
- Estimated in a conservative way by mimicking higher orders, varying the parameters by fixed amounts.
- **Quark-Hadron duality violation**, expected to be suppressed, would appear as an inconsistency in the fit.

Parameter Fits and Correlations



Two-dimensional projections of the fits performed with different assumptions for the theoretical correlations. The orange, magenta, blue, light blue 1-sigma regions correspond to scenarios A,B,C,D ($\Delta = 0.25$GeV), respectively. The red corresponds to scenario D with $\mathcal{O}(\alpha_s/m_b^2)$ corrections.

Numerical Results : Bottom Mass



- Gives $m_b^{kin}(1GeV) = 4.553(20)GeV$, independent of theoretical error
- Scheme translation error : $m_b^{kin}(1GeV) = m_b(m_b) + 0.37(3)GeV$

Numerical Results

	m_b^{kin}	m_c	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$\mathrm{BR}_{c\ell\nu}(\%)$	$10^3\,|V_{cb}|$
$\mathcal{O}(\alpha_s^2, m_b^{-2})$	4.541	0.987	0.414	0.154	0.340	-0.147	10.65	42.42
$\overline{m}_c(3\mathrm{GeV})$	0.023	0.013	0.078	0.045	0.066	0.098	0.16	0.86
	m_b^{kin}	m_c	μ_π^2	ρ_D^3	μ_G^2	ρ_{LS}^3	$\mathrm{BR}_{c\ell\nu}(\%)$	$10^3\,|V_{cb}|$
$\mathcal{O}(\alpha_s^2, \alpha_s m_b^{-2})$	4.553	0.987	**0.465**	0.170	**0.332**	-0.150	10.65	42.21
$\overline{m}_c(3\mathrm{GeV})$	0.020	0.013	0.068	0.038	0.062	0.096	0.16	0.78

New Inclusive Fit : $|V_{cb}|_{inc} = (42.21 \pm 0.78) \times 10^{-3}$

- Significant increase μ_π^2, decrease in μ_G^2
- Decreased theoretical error on inclusive $|V_{cb}|$ [Schwanda, Gambino, 1307.4551]
- Result reinforces our confidence in the inclusive method

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Progress Aim :

- Incorporate $\mathcal{O}(\alpha_s/m_b^3)$: calculation currently in progress
- Review of *higher power corrections*, unlikely to shift V_{cb}, but should be understood
- No closer to solving exclusive vs. inclusive $3\,\sigma\,(8\%)$ tension.

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