Resurrecting the minimal renormalizable supersymmetric SU(5)model

*to appear**

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Is the *minimal renormalizable SUSY SU(5)* really ruled out?

Motivation

The *minimal renormalizable supersymmetric SU(5)* is the simplest GUT model, but . . .

. . . *excluded* according to *Murayama-Pierce '01* , . . .

- *gauge coupling unification* (MSSM @ 2-loops $\longrightarrow m_T \lesssim 1.4 \cdot 10^{15}\,\mathrm{GeV}$)
- *proton decay* ($m_T \gtrsim 2.0 \cdot 10^{17}\,\mathrm{GeV}$)

Assumption:
light SUSY spectrum - 3^{rd} generation *sparticles* $\sim \mathcal{O}(1\,\mathrm{TeV})$, *gauginos* $\sim \mathcal{O}(m_Z)$ ($M_2 \approx 200\,\mathrm{GeV},\ M_3/M_2 \simeq 3.5$).

Goals:
more general superpartner mass spectrum $\longrightarrow$ *phenomenological constraints*?

bound the parameter space of allowed soft $\not{\mathrm{SUSY}}$ terms in MSSM (M_a, a_i, $\tilde{m}_i^2$)?
$\longrightarrow$ learn something about SUSY breaking?

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Outline

Our starting points

1. Why minimal renormalizable SUSY SU(5)?
   * *predictiveness* - probably the only way to test the high scale Yukawas (no SU(5) singlets, small # of parameters $\longrightarrow$ masses calculable)
   * *smallness of terms* $W \supset C \frac{Q_i Q_j Q_k L_l}{M_{Planck}}$; $C \lesssim 10^{-7}$ experimental fact
2. *perturbativity* (of couplings) at least up to the unification scale
3. *soft terms* at the GUT scale *SU(5) invariant* (supergravity mediation)
4. studying the mass scales of the theory (*the effects of running*)
5. correcting the *down-sector quark masses* by generation dependent *supersymmetric thresholds* (a-terms)

BEYOND THE SCOPE of this project

6. flavour structure [the only constraint are *small FCNCs*]
7. neutrino masses (bilinear RPV?)
8. DM (gravitino?)

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Scales in the theory

Relevant scales

weak scale $\sim \mathcal{O}(m_Z)$ $\longrightarrow$ SM particles **(SM)**

$\mathbf{m_{susy}} \approx \sqrt{\mathbf{m_{\tilde{u}^c_3} m_{\tilde{Q}_3}}}$ $\longrightarrow$ **superpartners** **(MSSM)**

$\mathbf{M_{GUT}} \sim \mathcal{O}(\mathbf{10^{16}}\,\mathrm{GeV})$ $\longrightarrow$ **heavy thresholds** **(SU(5))**

Running of model parameters between matching scales (RGEs)

single scale effective theory **2-loop** RGEs $+$ **1-loop** thresholds

Minimality

1. **Higgs sector:**

adjoint representation: $\mathrm{SU}(5) \to \mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$

$$\mathbf{24_H} = \underbrace{(\mathbf{8},\mathbf{1},\mathbf{0})}_{m_8} \oplus \underbrace{(\mathbf{1},\mathbf{3},\mathbf{0})}_{m_3} \oplus \underbrace{(\mathbf{1},\mathbf{1},\mathbf{0})}_{m_1} \oplus \underbrace{(\mathbf{3},\mathbf{2},-\tfrac{\mathbf{5}}{\mathbf{6}}) \oplus (\bar{\mathbf{3}},\mathbf{2},\tfrac{\mathbf{5}}{\mathbf{6}})}_{m_V}$$

fundamental & *antifundamental representation:* $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \to \mathrm{U}(1)_{em}$

$$\mathbf{5_H} = \underbrace{(\mathbf{3},\mathbf{1},-\tfrac{\mathbf{1}}{\mathbf{3}})}_{m_T} \oplus \underbrace{(\mathbf{1},\mathbf{2},\tfrac{\mathbf{1}}{\mathbf{2}})}_{m_H} \quad , \qquad \bar{\mathbf{5}}_\mathbf{H} = \underbrace{(\bar{\mathbf{3}},\mathbf{1},\tfrac{\mathbf{1}}{\mathbf{3}})}_{m_T} \oplus \underbrace{(\mathbf{1},\mathbf{2},-\tfrac{\mathbf{1}}{\mathbf{2}})}_{m_H}$$

Minimality

2. **Gauge sector:**

$$\mathbf{24_g} = (\mathbf{8},\mathbf{1},\mathbf{0}) \oplus (\mathbf{1},\mathbf{3},\mathbf{0}) \oplus (\mathbf{1},\mathbf{1},\mathbf{0}) \oplus \underbrace{(\mathbf{3},\mathbf{2},-\frac{\mathbf{5}}{\mathbf{6}}) \oplus (\overline{\mathbf{3}},\mathbf{2},\frac{\mathbf{5}}{\mathbf{6}})}_{m_V}$$

3. **Matter (Yukawa) sector:**

$$\mathbf{10_i} = \underbrace{(\mathbf{3},\mathbf{2},\frac{\mathbf{1}}{\mathbf{6}})}_{m_{\tilde{Q}_i}} \oplus \underbrace{(\overline{\mathbf{3}},\mathbf{1},-\frac{\mathbf{2}}{\mathbf{3}})}_{m_{\tilde{u}^c_i}} \oplus \underbrace{(\mathbf{1},\mathbf{1},\mathbf{1})}_{m_{\tilde{e}^c_i}} \quad , \qquad \overline{\mathbf{5}}_\mathbf{i} = \underbrace{(\overline{\mathbf{3}},\mathbf{1},\frac{\mathbf{1}}{\mathbf{3}})}_{m_{\tilde{d}^c_i}} \oplus \underbrace{(\mathbf{1},\mathbf{2},-\frac{\mathbf{1}}{\mathbf{2}})}_{m_{\tilde{L}_i}}$$

Renormalizability

Higgs sector superpotential

$$W_H = \frac{\mu}{2} Tr\, \mathbf{24_H}^2 + \sqrt{30}\, \frac{\lambda}{3} Tr\, \mathbf{24_H}^3 + \eta\, \mathbf{\overline{5}_H} \left(\mathbf{24_H} + 3 \frac{\langle\sigma\rangle}{\sqrt{30}} \right) \mathbf{5_H}$$

Yukawa sector superpotential

$$W_Y = \mathbf{\overline{5}_i}\, Y_5^{i,j}\, \mathbf{10_j}\, \mathbf{\overline{5}_H} + \frac{1}{8}\, \mathbf{10_i}\, Y_{10}^{i,j}\, \mathbf{10_j}\, \mathbf{5_H} \quad , \quad \text{i=1,2,3}$$

$$\left.\begin{array}{lcl} m_T & = & \frac{5}{\sqrt{30}} \eta \langle\sigma\rangle \\ m_{3,8} & = & = 5\mu = 5\lambda\langle\sigma\rangle \\ m_1 & = & \mu = \lambda\langle\sigma\rangle \\ m_V & = & \frac{5}{\sqrt{30}} g_{GUT} \langle\sigma\rangle \end{array}\right\} \Longrightarrow \textit{perturbativity}\ (m_T, m_{3,8} \lesssim m_V)$$

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Theoretical and experimental constraints

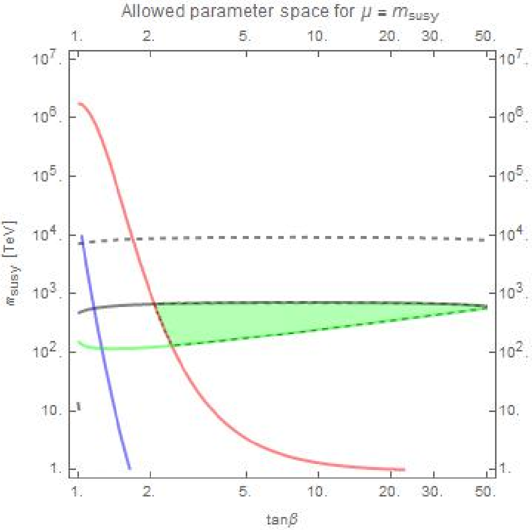
- *Higgs mass* ($m_h \simeq 125.7\,\mathrm{GeV}$)

- correct *down-sector fermion mass* relations (δm_d, δm_s, δm_b)
- *vacuum* (meta)*stability* (UFB 1,2,3 and CCB 1,2,3)

- gauge coupling *unification*
- *perturbativity* ($m_T, m_{3,8} \lesssim m_V \ll M_{Planck}$)
- *proton lifetime* bounds $\tau_p^{exp}(p \to K^+\overline{\nu}) > 2.3 \times 10^{33}\,\mathrm{yrs} \longrightarrow m_T \gtrsim \ldots$,
  $\tau_p^{exp}(p \to \pi^0 e^+) > 13 \times 10^{33}\,\mathrm{yrs} \longrightarrow m_V \gtrsim \ldots$

- *LEP* and *LHC* bounds on sfermion and gaugino masses
  ($m_{\tilde{Q}_{1,2}}, m_{\tilde{g}} \gtrsim 1\,\mathrm{TeV}$; $m_{\tilde{Q}_3}, m_{\tilde{\chi}} \gtrsim 300\,\mathrm{GeV}$)

Theoretical and experimental constraints



Mass of the SM Higgs

For heavy stops the usual MSSM expressions for m_h not accurate

$\longrightarrow$ 2-loop SM running of the *Higgs quartic coupling*

$+$

1-loop *matching* between SM and MSSM RGEs

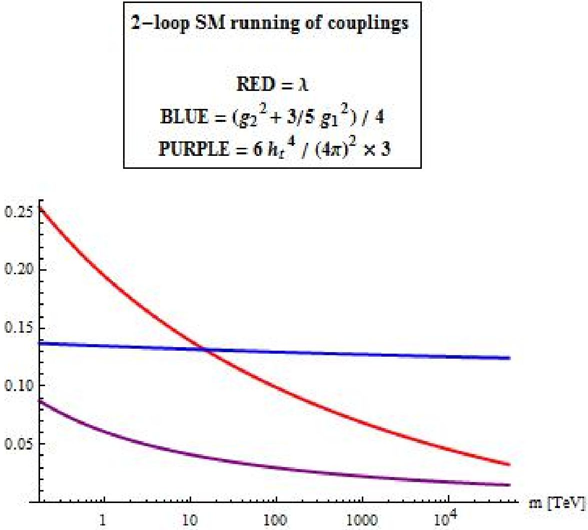
$$\lambda(m_{susy}) = \overbrace{\left(\frac{3}{5}g_1^2(m_{susy}) + g_2^2(m_{susy})\right)\frac{\cos^2(2\beta)}{4}}^{tree-level} +$$

$$+ \underbrace{\frac{6\,(\lambda_t \sin\beta)^4}{(4\pi)^2}\left(\frac{X_t}{m_{susy}}\right)^2\left[1 - \frac{1}{12}\left(\frac{X_t}{m_{susy}}\right)^2\right]}_{\text{1-loop stop-mixing contribution to Higgs mass} \leq \frac{6\,(\lambda_t \sin\beta)^4}{(4\pi)^2}\times 3} + \ldots$$

@ matching scale: $m_{susy} \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}} \approx \sqrt{m_{\tilde{u}_3}(m_{susy}) m_{\tilde{Q}_3}(m_{susy})}$

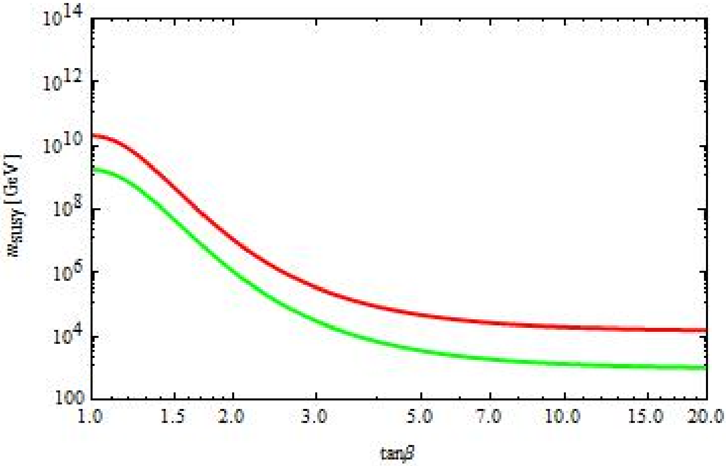
for $X_t \equiv \frac{a_t}{\lambda_t} - \frac{\mu}{\tan\beta}$

Mass of the SM Higgs



Mass of the SM Higgs

For each $\tan\beta$ exist a *minimal* $m_{\rm susy}$ which fits the measured Higgs mass



SU(5) unification

$$\alpha_1(M_{GUT}) = \alpha_2(M_{GUT}) = \alpha_3(M_{GUT}) \equiv \alpha_{GUT}$$

$$\begin{aligned} Y_u(M_{GUT}) &= Y_u^T(M_{GUT}) \\ Y_d(M_{GUT}) &= Y_e^T(M_{GUT}) \end{aligned}$$

$$\begin{aligned} A_u(M_{GUT}) &= A_u^T(M_{GUT}) \\ A_d(M_{GUT}) &= A_e^T(M_{GUT}) \end{aligned}$$

$$M_1(M_{GUT}) = M_2(M_{GUT}) = M_3(M_{GUT}) \equiv M_{1/2}$$

$$\begin{aligned} m_{\tilde{Q}_i}(M_{GUT}) = m_{\tilde{u}_i^c}(M_{GUT}) = m_{\tilde{e}_i^c}(M_{GUT}) &\equiv \tilde{m}_{10_i} \\ m_{\tilde{L}_i}(M_{GUT}) = m_{\tilde{d}_i^c}(M_{GUT}) &\equiv \tilde{m}_{\bar{5}_i} \end{aligned} \qquad (i = 1, 2, 3)$$

All the *splittings* within SU(5) representations are only due to *running* !

Gauge coupling Unification

No unification in MSSM $\longrightarrow$ high-energy thresholds m_T, m_8, m_3, m_V required

single scale (m_{susy}) MSSM **2-loop** RGEs + **1-loop** thresholds

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$$\Big[\overbrace{m_V^2}^{\substack{m_T^2\\ \wr\wedge}}(m_3 m_8)^{1/2}\Big]^{1/3} = M_{GUT} \times \exp\left[\frac{\pi}{18}\left(5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1}\right)_{2-loop}(M_{GUT})\right]$$

$$\times\left(\frac{m_{susy}^2}{m_{\tilde{w}} m_{\tilde{g}}}\right)^{1/9} \underbrace{\prod_{i=1}^{3}\left(\frac{m_{\tilde{u}_i^c} m_{\tilde{e}_i^c}}{m_{\tilde{Q}_i}^2}\right)^{1/36}}_{\substack{\wr|\\ \mathcal{O}(1)}}$$

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$$m_T^0 \simeq 2\times10^{15}\,\mathrm{GeV} \times \left(\frac{m_{susy}}{1\,\mathrm{TeV}}\right)^{5/6}$$

$$m_T = \overbrace{M_{GUT} \times \exp\left[\frac{5\pi}{6}\left(-\alpha_1^{-1}+3\alpha_2^{-1}-2\alpha_3^{-1}\right)_{2-loop}(M_{GUT})\right]}^{\equiv\, m_T^0}$$

$$\times \underbrace{\left(\frac{m_3}{m_8}\right)^{5/2}}_{=\,1} \underbrace{\left(\frac{m_{\tilde{w}}}{m_{\tilde{g}}}\right)^{5/3}}_{\uparrow\; m_{\tilde{g}} \approx m_{susy}} \underbrace{\prod_{i=1}^{3}\left(\frac{m_{\tilde{Q}_i}^4}{m_{\tilde{u}_i^c}^3 m_{\tilde{e}_i^c}} \frac{m_{\tilde{L}_i}^2}{m_{\tilde{d}_i^c}^2}\right)^{1/12}}_{\gtrsim\, \mathcal{O}(1)} \left(\frac{m_{\tilde{h}}^4 m_A}{m_{susy}^5}\right)^{1/6}$$

Light m_T mediates too fast proton decay.

Large m_{susy} poses the opposite problem: m_T can be too heavy (perturbativity).

Gauge coupling Unification

$$|\mu|^2 = \frac{m^2_{H_d} - m^2_{H_u}\tan^2\beta}{\tan^2\beta - 1} - \frac{m_Z^2}{2} \quad \text{(tree-level EWSB condition at } m_{susy}\text{)}$$

$$m_{\tilde{h}} = |\mu|$$

$$m_A = \sqrt{(\mu^2 + m^2_{H_d})(1 + 1/\tan^2\beta)} \approx \sqrt{(m^2_{H_d} - m^2_{H_u})\frac{\tan^2\beta + 1}{\tan^2\beta - 1}}$$

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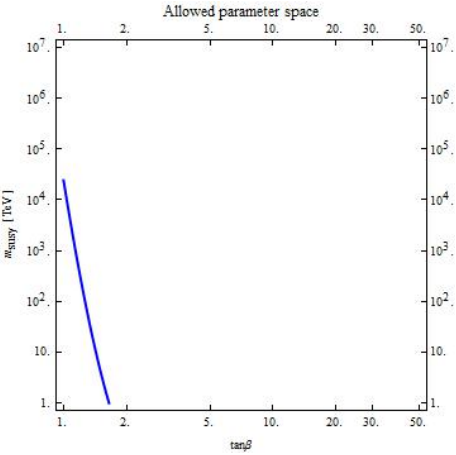
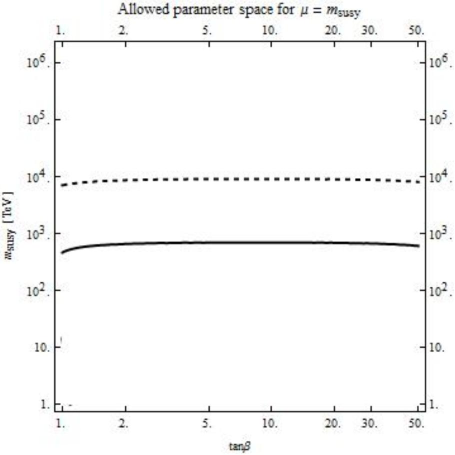
Perturbativity

Yukawas: $\lambda_i(M_{GUT}) \lesssim 1$

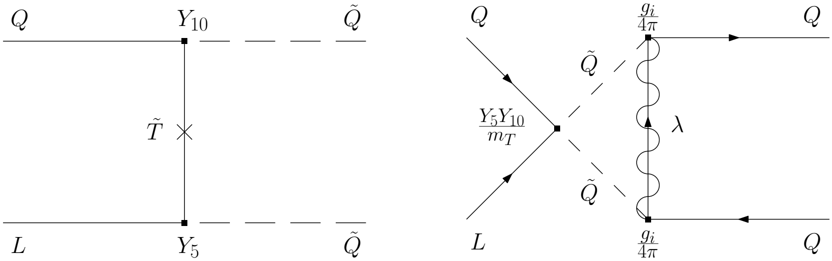


Heavy thresholds:
$m_T, m_{3,8} \lesssim m_V \ll M_{Planck}$



Proton decay bounds

Dimension 5 *colour Higgs triplet exchange* operators get *dressed* by *winos* which leads to Weinberg type 4-fermion effective operators



Dominant proton decay channel is $p \to K^+\bar{\nu}$ that scales approximately as

$$\tau(p \to K^+\bar{\nu}) \propto \left(\frac{\tan\beta}{1+\tan^2\beta}\right)^2 m_{susy}^2\, m_T^2 \propto \left(\frac{\tan\beta}{1+\tan^2\beta}\right)^2 m_{susy}^{11/3}$$

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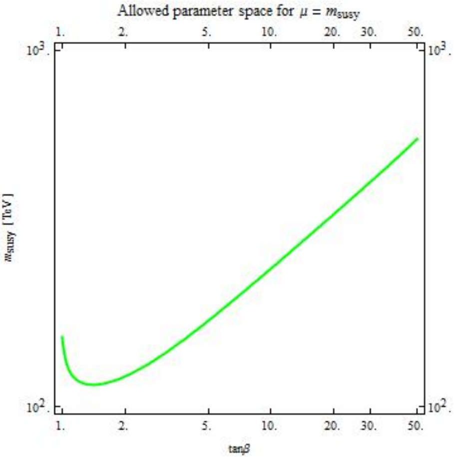

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Mismatch between measured and SU(5)-respecting masses

$$\mathbf{y_e}, \mathbf{y}_\mu, \mathbf{y}_\tau : \quad m_Z \xrightarrow{SM} m_{susy} \xrightarrow{MSSM} M_{GUT} \quad \begin{pmatrix} \text{no susy threshold corr.} \\ \alpha_2 \text{ instead of } \alpha_3 \text{ depen.} \end{pmatrix}$$

minimal renormalizable SU(5) model $\longrightarrow \begin{cases} m_e(M_{GUT}) = m_d(M_{GUT}) \\ m_\mu(M_{GUT}) = m_s(M_{GUT}) \\ m_\tau(M_{GUT}) = m_b(M_{GUT}) \end{cases}$

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$\left.\begin{array}{l} m_e(m_Z)/m_d(m_Z) \\ m_\mu(m_Z)/m_s(m_Z) \\ m_\tau(m_Z)/m_b(m_Z) \end{array}\right\} =$ **wrong** $\longrightarrow$ **threshold corrections needed**

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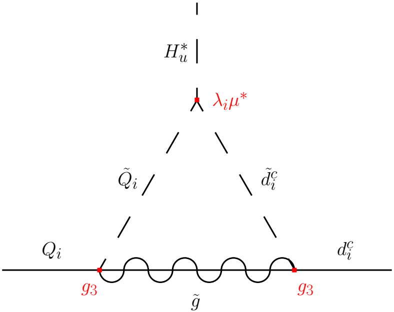
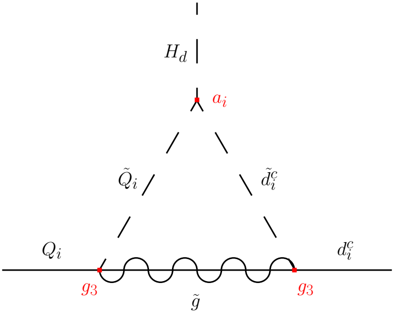
Correcting light fermion masses with a-terms

Diagrams for the finite corrections to the quark Yukawa couplings.





$$W_{MSSM} \supset \mu\, \mathbf{H_u}\, \mathbf{H_d} + \lambda_i\, \mathbf{Q_i}\, \mathbf{d_i^c}\, \mathbf{H_d}$$

$$\mathcal{L}_{soft} \supset a_i\, \tilde{Q}_i\, \tilde{d}_i^c\, H_d$$

$$\frac{\Delta m_{d_i}}{m_{d_i}} = -\frac{2\alpha_3}{3\pi}\, m_{\tilde{g}}\, X_{d_i}\, I_3(m_{\tilde{g}}^2, m_{\tilde{d}_{L_i}}^2, m_{\tilde{d}_{R_i}}^2) + \frac{\lambda_t^2}{16\pi^2}\, \mu\, X_t\, \tan\beta\, I_3(\mu^2, m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2)\, \delta_{i3}$$

Vacuum (meta)stability

1. *absolute vacuum stability* $\longrightarrow$ our vacuum is NOT a *global minimum*

UFB 1,2,3 $\longrightarrow$ $m^2_{H_u} > 0$

CCB 1,2,3 $\longrightarrow$ $|a_{d_i}| \not\lesssim \lambda_i \sqrt{3\left(m^2_{H_d} + m^2_{\tilde{Q}_i} + m^2_{\tilde{d}^c_i}\right)}$

2. *vacuum metastability* $\longrightarrow$ our vacuum only a *local minimum*, but its *lifetime* longer than the age of the Universe

CCB 1,2*,3* $\longrightarrow$ $|a_{d_i}| \lesssim \sqrt{m^2_{H_d} + m^2_{\tilde{Q}_i} + m^2_{\tilde{d}^c_i}}$

$$S \gtrsim 400$$

**more complicated situation, numerical analysis required*

Solving RGEs

$\mathbf{g_a}, \mathbf{y_i}, \lambda$ @ 2-loops
$\mathbf{M_a}, \mathbf{a_i}, \mathbf{\tilde{m}_i^2}$ @ 1-loop
15 free parameters in the soft sector ($1 \times M_a$, $6 \times a_i$, $8 \times \tilde{m}_i^2$)

SCANNING the parameter space

Problem: system of entangled differential equations
$\longrightarrow$ numerically demanding

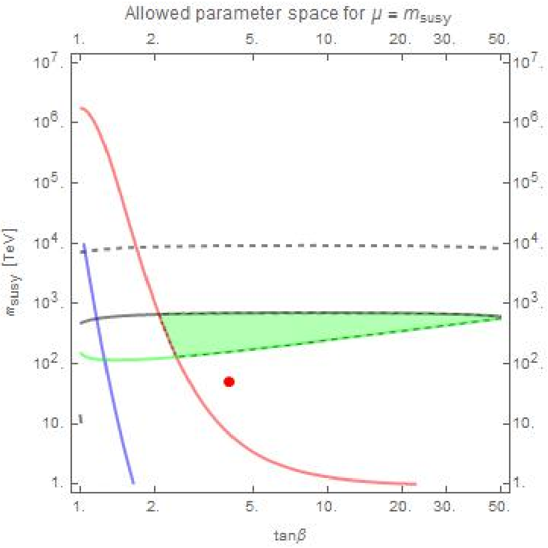
Solution: "disentangle" the equations
$\longrightarrow$ solve them in a specific order [1. g_a, y_i, λ, 2. M_a, 3. a_i, 4. $\tilde{m}_i^2$]

Problem: boundary conditions defined at various scales connecting different quantities
@ $\mathbf{M_{GUT}}$: SU(5) unification ($M_a, a_i, \tilde{m}_i^2$)
@ $\mathbf{m_{susy}}$: no tachyons ($\tilde{m}_i^2 > 0$), EWSB ($\mu^2 > 0$), Higgs matching ($M_a, a_i, \tilde{m}_i^2, \mu^2$), fermion mass corrections ($M_a, a_i, \tilde{m}_i^2, \mu^2$)

Solution: solve (approximate) RGEs analytically
$\longrightarrow$ hierarchy of Yukawas

INPUT:

$$\begin{aligned} \tan\beta &= 4.0 \\ m_{susy} &= 50.0\,\mathrm{TeV} \\ M_{GUT} &= 2.0\times 10^{16}\,\mathrm{GeV} \\ sgn(\mu) &= + \end{aligned}$$



A-terms:

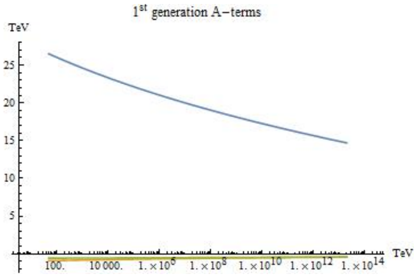
at $\mathbf{M_{GUT}}$

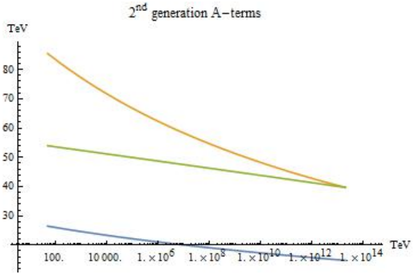
$$
\begin{aligned}
a_t(M_{GUT}) &= 14.6\,\mathrm{TeV} \\
a_b(M_{GUT}) &= 118.9\,\mathrm{TeV} \\
a_\tau(M_{GUT}) &= 118.9\,\mathrm{TeV} \\
\\
a_c(M_{GUT}) &= 14.8\,\mathrm{TeV} \\
a_s(M_{GUT}) &= 39.7\,\mathrm{TeV} \\
a_\mu(M_{GUT}) &= 39.7\,\mathrm{TeV} \\
\\
a_u(M_{GUT}) &= 14.7\,\mathrm{TeV} \\
a_d(M_{GUT}) &= -0.4\,\mathrm{TeV} \\
a_e(M_{GUT}) &= -0.4\,\mathrm{TeV}
\end{aligned}
$$

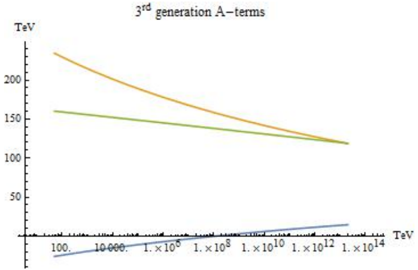
at $\mathbf{m_{susy}}$

$$
\begin{aligned}
a_t(m_{susy}) &= -25.9\,\mathrm{TeV} \\
a_b(m_{susy}) &= 234.4\,\mathrm{TeV} \\
a_\tau(m_{susy}) &= 160.2\,\mathrm{TeV} \\
\\
a_c(m_{susy}) &= 26.5\,\mathrm{TeV} \\
a_s(m_{susy}) &= 85.5\,\mathrm{TeV} \\
a_\mu(m_{susy}) &= 54.0\,\mathrm{TeV} \\
\\
a_u(m_{susy}) &= 26.5\,\mathrm{TeV} \\
a_d(m_{susy}) &= -0.9\,\mathrm{TeV} \\
a_e(m_{susy}) &= -0.6\,\mathrm{TeV}
\end{aligned}
$$

A-terms:


1st generation A−terms
TeV
25
20
15
10
5
100.
10 000.
1. × 10^6
1. × 10^8
1. × 10^10
1. × 10^12
1. × 10^14
TeV


2nd generation A−terms
TeV
80
70
60
50
40
30
100.
10 000.
1. × 10^6
1. × 10^8
1. × 10^10
1. × 10^12
1. × 10^14
TeV


3rd generation A−terms
TeV
200
150
100
50
100.
10 000.
1. × 10^6
1. × 10^8
1. × 10^10
1. × 10^12
1. × 10^14
TeV

Soft masses (1):

at $\mathbf{M_{GUT}}$

$$M_{1/2} = 34.0\,\mathrm{TeV}$$

$$m_{H_u,GUT} = 105.0\,\mathrm{TeV}$$
$$m_{H_d,GUT} = 525.0\,\mathrm{TeV}$$

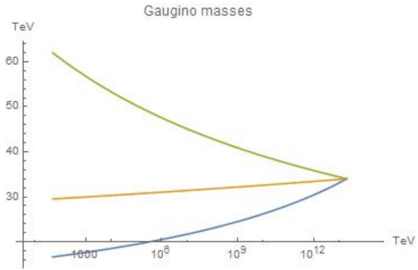
at $\mathbf{m_{susy}}$

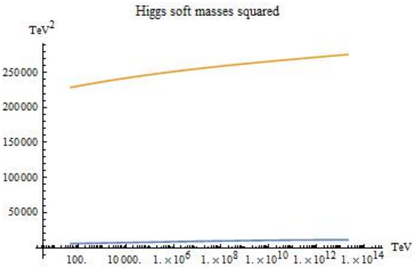
$$M_1(m_{\tilde{g}}) = m_{\tilde{b}} = 16.7\,\mathrm{TeV}$$
$$M_2(m_{\tilde{g}}) = m_{\tilde{w}} = 29.6\,\mathrm{TeV}$$
$$M_3(m_{\tilde{g}}) = m_{\tilde{g}} = 62.0\,\mathrm{TeV}$$

$$m_{H_u}(m_{susy}) = 75.9\,\mathrm{TeV}$$
$$m_{H_d}(m_{susy}) = 478.1\,\mathrm{TeV}$$

$$\mu(m_{susy}) = 95.4\,\mathrm{TeV}$$
$$m_A(m_{susy}) = 502.6\,\mathrm{TeV}$$

Soft masses (1):





Soft masses (2):

at $\mathbf{M_{GUT}}$

$$\tilde{m}_{10_1} = 79.0\,\mathrm{TeV}$$

$$\tilde{m}_{\bar{5}_1} = 150.0\,\mathrm{TeV}$$

$$\tilde{m}_{10_2} = 79.0\,\mathrm{TeV}$$

$$\tilde{m}_{\bar{5}_2} = 150.0\,\mathrm{TeV}$$

at $\mathbf{m_{susy}}$

$$m_{\tilde{Q}_1}(m_{susy}) = 104.7\,\mathrm{TeV}$$
$$m_{\tilde{u}_1^c}(m_{susy}) = 15.6\,\mathrm{TeV}$$
$$m_{\tilde{e}_1^c}(m_{susy}) = 136.7\,\mathrm{TeV}$$
$$m_{\tilde{L}_1}(m_{susy}) = 129.7\,\mathrm{TeV}$$
$$m_{\tilde{d}_1^c}(m_{susy}) = 170.3\,\mathrm{TeV}$$

$$m_{\tilde{Q}_2}(m_{susy}) = 98.8\,\mathrm{TeV}$$
$$m_{\tilde{u}_2^c}(m_{susy}) = 15.5\,\mathrm{TeV}$$
$$m_{\tilde{e}_2^c}(m_{susy}) = 131.1\,\mathrm{TeV}$$
$$m_{\tilde{L}_2}(m_{susy}) = 126.8\,\mathrm{TeV}$$
$$m_{\tilde{d}_2^c}(m_{susy}) = 163.0\,\mathrm{TeV}$$

Soft masses (2):

at $\mathbf{M_{GUT}}$

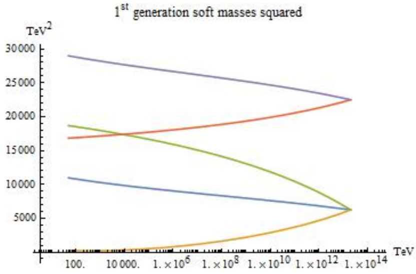
$$\tilde{m}_{10_3} = 124.3\,\mathrm{TeV}$$

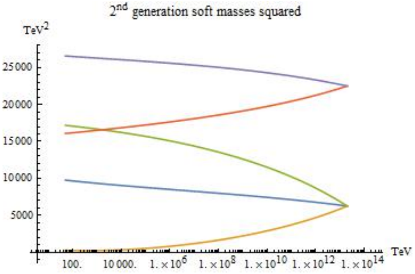
$$\tilde{m}_{\bar{5}_3} = 139.5\,\mathrm{TeV}$$

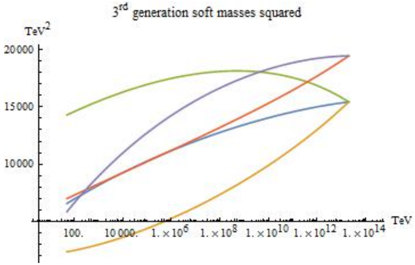
at $\mathbf{m_{susy}}$

$$\begin{aligned}
m_{\tilde{Q}_3}(m_{susy}) &= 81.1\,\mathrm{TeV}\\
m_{\tilde{u}^c_3}(m_{susy}) &= 48.5\,\mathrm{TeV}\\
m_{\tilde{e}^c_3}(m_{susy}) &= 119.6\,\mathrm{TeV}\\
m_{\tilde{L}_3}(m_{susy}) &= 83.7\,\mathrm{TeV}\\
m_{\tilde{d}^c_3}(m_{susy}) &= 76.5\,\mathrm{TeV}
\end{aligned}$$

Soft masses (2):







Phenomenology:

Spectrum

* **Higgs mass** ✓
* **Fermion masses** ✓
* **NO tachyons** ✓

Unification of gauge couplings ✓

$$m_T = 8.5 \times 10^{16}\,\mathrm{GeV}$$
$$m_{8,3} = 5.1 \times 10^{13}\,\mathrm{GeV}$$
$$m_V = 8.6 \times 10^{16}\,\mathrm{GeV}$$
$$\alpha_{GUT}^{-1} = 24.7$$

Perturbativity ✓

Proton decay ✓

$$\tau_p(p^+ \to K^+\bar{\nu}) = 5.4 \times 10^{33}\,\mathrm{yrs} = 2.3 \times \tau_p^{exp}$$

(for $\phi_1 = \phi_2 = \phi_3 = 0$)

Vacuum stability

* absolute stability ✗
* metastability ✓

$$S_{min} = 1793$$

Summary

POINTS TO TAKE HOME:

* minimal renormalizable SUSY SU(5) works
* large a-terms
* heavy spectrum $\longrightarrow$ probably $\exists$ a lower bound

TO DO:

* scan of the soft ~~SUSY~~ parameter space

Thank you for your attention!