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A minimal supersymmetric E_6 Unified Theory

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Outline

- Why E_6? Some properties.
- Our E_6 model: the minimal realistic setup.
  $\{3 \times 27_F\} + \{27 + \overline{27} + 351' + \overline{351'} + 78\}$
- Vacuum, DT splitting.
- Yukawa sector:
  - analysis of low-energy mass matrices,
  - 2 generation fit.
- Conclusions.

Why E_6?

- **Unified group** criteria:
  1. Simple group (true unification),
  2. $\mathrm{SU}(3)_C \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \subset G$,
  3. Contains chiral (complex) representations.

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- Candidates for GUT (above criteria):
  1. $\mathrm{SU}(N)$ for $N > 4$,

     e.g. $\mathrm{SU}(5)$.
  2. $\mathrm{SO}(4N+2)$ for $N > 1$,

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  3. E_6.

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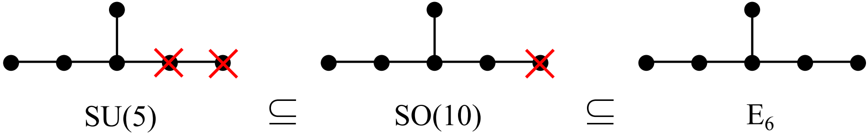
  3. E_6.
- No chiral representations: G_2, F_4, E_7, E_8, $\mathrm{Sp}(2n)$.

Properties of E_6

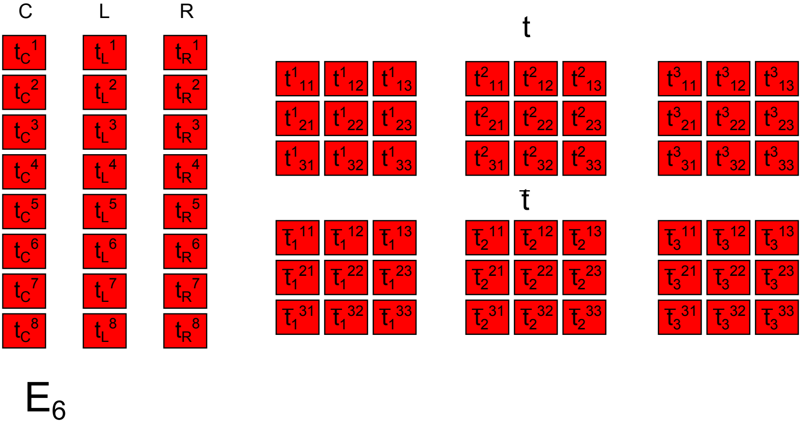


- Dimension: 78,
  Rank: 6.
- Has complex representations.
- Some maximal subgroups:
  $\mathrm{SO}(10) \times \mathrm{U}(1)$,
  $\mathrm{SU}(3)_C \times \mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$.
- Lowest dimensional reps:

  $27, \overline{27}, 78, 351, \overline{351}, 351', \overline{351'}, 650.$

Subgroup chain — generator level

- E_6 subgroups



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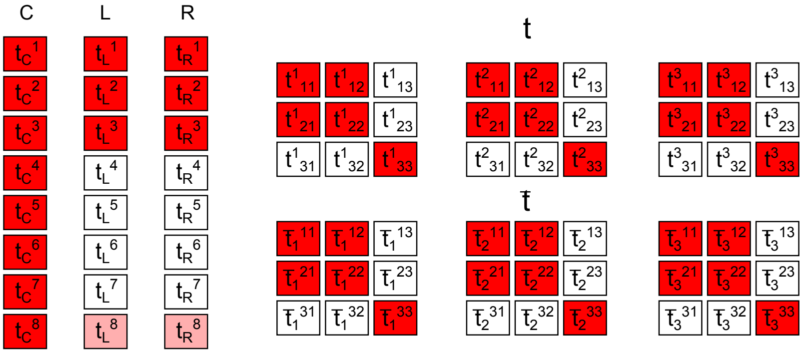

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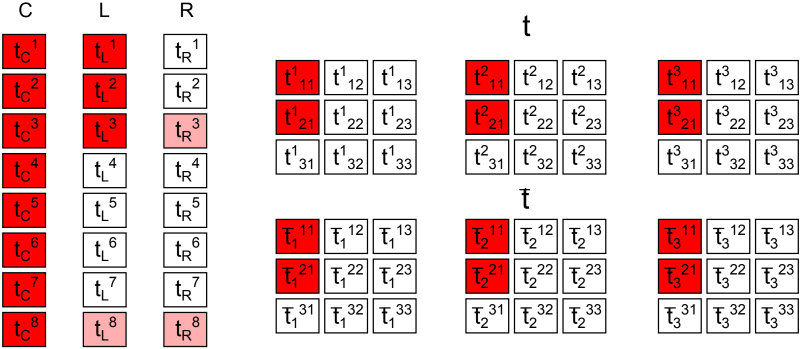

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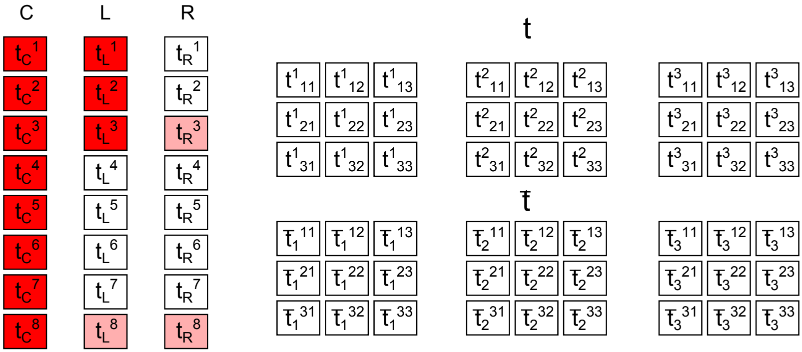

$$\mathrm{E}_6 \supset \mathrm{SO}(10) \supset \mathrm{SU}(5) \supset \mathrm{SM}$$

Minimal model considerations

- $\mathrm{E}_6 \to \mathrm{SO}(10)$ decompositions:

$$\mathbf{27} = \mathbf{16} + 10 + 1, \tag{1}$$

$$\mathbf{351'} = \mathbf{126} + \overline{144} + 54 + 16 + 10 + 1, \tag{2}$$

$$\mathbf{78} = \mathbf{45} + 16 + \overline{16} + 1. \tag{3}$$

- Minimal SUSY $\mathrm{SO}(10)$ Yukawa sector (Clark,Kuo,Nakagawa; Aulakh,Mohapatra):

$$16^i_F 16^j_F (Y^{ij}\, 10 + \tilde{Y}^{ij}\, \overline{126}). \tag{4}$$

The E_6 analogue:

$$27^i_F 27^j_F (Y^{ij}\, 27 + \tilde{Y}^{ij}\, \overline{351'}). \tag{5}$$

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- $\mathrm{E}_6 \to \mathrm{SM}$ OK: $\{27 + \overline{27} + 351' + \overline{351'}\}$.
  DT splitting: $D \sim (1, 2, +1/2)$, $T \sim (3, 1, -1/3)$.
  Fine tuning **not possible** (strange!).
- If we add 78: can fine-tune, Yukawa sector stays minimal.

Model Setup

- **Renormalizable supersymmetric** E_6 model.
- Matter content:
  - Fermionic sector: $\{27^1_F, 27^2_F, 27^3_F\}$.
    Contains the SM fermions.
  - Breaking sector: $\mathrm{E}_6 \to \mathrm{SM}$ one-step breaking.
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Model Setup

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- Superpotential:

$$
\begin{aligned}
W = {} & m_{27}\, 27 \cdot \overline{27} + m_{351'}\, 351' \cdot \overline{351'} + m_{78}\, 78^2 \\
& + \lambda_1\, 351'^3 + \lambda_2\, \overline{351'}^3 + \lambda_3\, \overline{351'} \cdot 27^2 + \lambda_4\; 351' \cdot 27^2 + \lambda_5\, 27^3 + \lambda_6\, \overline{27}^3 \\
& + \lambda_7\, 27 \cdot 78 \cdot \overline{27} + \lambda_8\, 351' \cdot 78 \cdot \overline{351'} \\
& + \tfrac{1}{2} 27^i_F \cdot 27^j_F \cdot \left(Y^{ij}_{27}\, 27 + Y^{ij}_{351'}\, \overline{351'} \right).
\end{aligned} \tag{6}
$$

Vacuum and DT splitting

- Vacuum:
  - $\langle 27_F \rangle = 0$.
  - The $\{27 + \overline{27} + 351' + \overline{351'} + 78\}$ model: $2 + 2 + 5 + 5 + 5 = 19$ SM singlets.
  - EOM: 19 F-terms, 4 D-terms.

$$F_i(\phi_k) := \partial W/\partial\phi_i, \qquad D^a(\phi_k, \phi_k^\dagger) := -g \sum_i \phi_i^\dagger \hat{t}^a \phi_i. \tag{7}$$

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    Complete classification of vacua not done (yet).
- DT splitting:
  - M_D is 12×12,
    M_T is 13×13.
  - $\mathrm{E}_6 \to \mathrm{SM}$: a would-be Goldstone mode present in each.
  - Fine-tuning is possible: extra massless mode in M_D, not in M_T.
    Use λ_5 or λ_6.
  - The low-energy Higgs mode: v_i, $\bar{v}_i$.

$$\sum_i |v_i|^2 = v_u^2, \quad \sum_i |\bar{v}_i|^2 = v_d^2, \quad v_u^2 + v_d^2 = (246\,\mathrm{GeV})^2. \tag{8}$$

Yukawa sector

- What does the 27_F contain:

$$27 = \underbrace{16}_{Q+L+u^c+d^c+e^c+\nu^c} + \underbrace{10}_{L'+L'^c+d'+d'^c} + \underbrace{1}_{s}. \qquad (9)$$

- Exotics:
  - Presence of vector-like down-quarks and leptons in each family.
  - 2 right-handed neutrinos per family.

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- Exotics:
  - Presence of vector-like down-quarks and leptons in each family.
  - 2 right-handed neutrinos per family.
- **2** Yukawa terms: the 78 not involved here

$$\tfrac{1}{2} 27^i_F \cdot 27^j_F \cdot \left(Y^{ij}_{27}\, 27 + Y^{ij}_{\overline{351'}}\, \overline{351'} \right) \qquad (10)$$

- Up quark sector:

$$u^T \left(-v_1\, Y_{27} + \left(\tfrac{1}{2\sqrt{10}} v_5 - \tfrac{1}{2\sqrt{6}} v_7\right) Y_{\overline{351}'} \right) u^c \qquad (11)$$

Yukawa sector 2

- Down-quark and charged lepton sectors:

$$\begin{pmatrix} d^{cT} & d'^{cT} \end{pmatrix} M_d \begin{pmatrix} d \\ d' \end{pmatrix} + \begin{pmatrix} e^{T} & e'^{T} \end{pmatrix} M_e \begin{pmatrix} e^c \\ e'^c \end{pmatrix}$$

General form in the vacuum:

$$\begin{pmatrix} \bar{v}_2 Y_{27} + (\bar{v}_4 + \alpha \bar{v}_8) Y_{\overline{351}'} & \langle 27 \rangle_2 Y_{27} + \cancel{\beta \langle \overline{351'} \rangle_5 Y_{\overline{351}'}} \\ \bar{v}_3 Y_{27} + (\bar{v}_9 + \alpha \bar{v}_{11}) Y_{\overline{351}'} & \beta \langle \overline{351'} \rangle_4 Y_{\overline{351}'} + \cancel{\langle 27 \rangle_1 Y_{27}} \end{pmatrix}$$

- Mixing $16 \leftrightarrow 10$.
  One combination at M_{GUT}, the other at m_W.
  Low energy: heavy mode can be rotated away.

Yukawa sector — 3

- Neutrinos: Type I + Type II seesaw.

$$\begin{pmatrix}\nu\\ \nu'\end{pmatrix}^T \begin{pmatrix} v_1 Y_{27} + (v_5+v_7) Y_{\overline{351'}} & v_6 Y_{\overline{351'}} & \langle 27\rangle_2 Y_{27}\\ v_{10} Y_{\overline{351'}} & v_1 Y_{27} + v_5 Y_{\overline{351'}} & \langle\overline{351'}\rangle_4 Y_{\overline{351'}}\end{pmatrix}\begin{pmatrix}\nu^c\\ s\\ \nu'^c\end{pmatrix}$$
$$+\begin{pmatrix}\nu^c\\ s\\ \nu'^c\end{pmatrix}^T\begin{pmatrix}\langle\overline{351'}\rangle_1 Y_{\overline{351'}} & 0 & \bar{v}_3 Y_{27} + \bar{v}_9 Y_{\overline{351'}}\\ 0 & \langle\overline{351'}\rangle_3 Y_{\overline{351'}} & \bar{v}_2 Y_{27} + \bar{v}_4 Y_{\overline{351'}}\\ \bar{v}_3 Y_{27} + \bar{v}_9 Y_{\overline{351'}} & \bar{v}_2 Y_{27} + \bar{v}_4 Y_{\overline{351'}} & 0\end{pmatrix}\begin{pmatrix}\nu^c\\ s\\ \nu'^c\end{pmatrix}$$
$$+\begin{pmatrix}\nu\\ \nu'\end{pmatrix}^T\begin{pmatrix}\Delta_1 Y_{\overline{351'}} & \Delta_2 Y_{\overline{351'}}\\ \Delta_2 Y_{\overline{351'}} & \Delta_3 Y_{\overline{351'}}\end{pmatrix}\begin{pmatrix}\nu\\ \nu'\end{pmatrix}. \tag{12}$$

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- At low energy:

$$R\Big(\frac{v^2}{\langle\overline{351'}\rangle_1}(\cdots) + \frac{v^2}{\langle\overline{351'}\rangle_3}(\cdots) + \Delta(\cdots)\Big) R^T \tag{13}$$

- Can take small $\langle\overline{351'}\rangle_1$ or $\langle\overline{351'}\rangle_3$ without an intermediate scale.

Yukawa sector fit

- Fit of the 2nd and 3rd generation masses and mixing angles. [in $\mathbb{R}$]
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Yukawa sector fit

- Fit of the 2nd and 3rd generation masses and mixing angles. [in $\mathbb{R}$]
- Complication: low-energy Yukawa sector is non-linear.
- Procedure, how the low energy masses are computed:
  1. Choose values for $\mathcal{L}$ parameters except λ_5.
     (3 ms, 8 λs, 2 symmetric Yukawas: 16 parameters total).
  2. Compute the *GUT* scale VEVs through vacuum solution.
  3. Compute the vs and $\bar{v}$s by fine-tuning λ_5.
  4. Use all the above to compute the low energy mass matrices.
  5. Extract masses and mixing angles (9 total).
- Masses and angles at M_{GUT} taken from (Xing, et. al, 2007). Assumed $\tan\beta = 10$. Take the errors $\sigma_i = 10\% \cdot y_i$.

Yukawa sector fit — Lagrangian parameters

parameter	point 1	point 2
$m_{351'}$ [GeV]	1.17×10^{16}	-4.17×10^{15}
m_{27} [GeV]	-2.92×10^{14}	-1.64×10^{15}
m_{78} [GeV]	-6.19×10^{16}	-5.04×10^{15}
λ_1	-6.49×10^{-1}	-8.91×10^{-2}
λ_2	5.66×10^{-2}	-1.24×10^{-1}
λ_3	-1.5×10^{-1}	2.78×10^{-1}
λ_4	-7.38×10^{-5}	-5.04×10^{-4}
λ_5	-1.58×10^{-3}	1.50×10^{-1}
λ_6	-7.59×10^{-3}	3.06×10^{-2}
λ_7	-4.23×10^{-1}	9.55×10^{-1}
λ_8	5.08	1.16
$(Y_{27})_{11}$	-7.23×10^{-1}	1.87
$(Y_{27})_{12}$	7.03×10^{-1}	-1.09
$(Y_{27})_{22}$	-6.76×10^{-1}	6.30×10^{-1}
$(Y_{\overline{351'}})_{11}$	-3.71×10^{-1}	7.39×10^{-1}
$(Y_{\overline{351'}})_{12}$	0	0
$(Y_{\overline{351'}})_{22}$	3.63×10^{-1}	-2.58×10^{-1}

Yukawa sector fit — masses and mixing angles

quantity	experiment	parameter point 1		parameter point 2	
	y_i	$f_i(x)$	χ_i	$f_i(x)$	χ_i
m_c	0.236	0.226	-0.432	0.205	-1.30
m_t	92.2	94.0	$+0.193$	105	$+1.38$
θ_{cb}	0.0409	0.0358	-1.24	0.0378	-0.757
m_s	0.013	0.0144	$+1.06$	0.146	$+1.20$
m_b	0.79	0.79	$+0.0021$	0.781	-0.112
m_μ	0.0599	0.0613	$+0.241$	0.0664	$+1.08$
m_τ	1.02	0.867	-1.51	1.03	$+0.056$
m_{ν_3}		0.135		0.0824	
m_{ν_2}		0.126		0.0659	
$(m_{\nu_3}^2 - m_{\nu_2}^2)10^3$	2.32	2.45	$+0.568$	2.24	-0.325
$\sin^2\theta_{23}$	0.386	0.343	-1.12	0.327	-1.52
$\hat{\chi}^2$			0.76		1.03

Mass units in GeV, neutrinos in eV.

Conclusions

We considered the E_6 model
$\{3 \times 27_F\} + \{27 + \overline{27} + 351' + \overline{351'} + 78\}$.

- It is the minimal model with all the ingredients to be realistic.
- At least one vacuum looks to be realistic.
  2 generation rough fit performed.
- Unknowns:
  - 3 generation fit.
  - Threshold corrections at M_{GUT}.
  - Other vacua.
- E_6 model building: the familiar features of GUTs are present (such as seesaw, etc.), but things are more complicated and interconnected.