Cosmology

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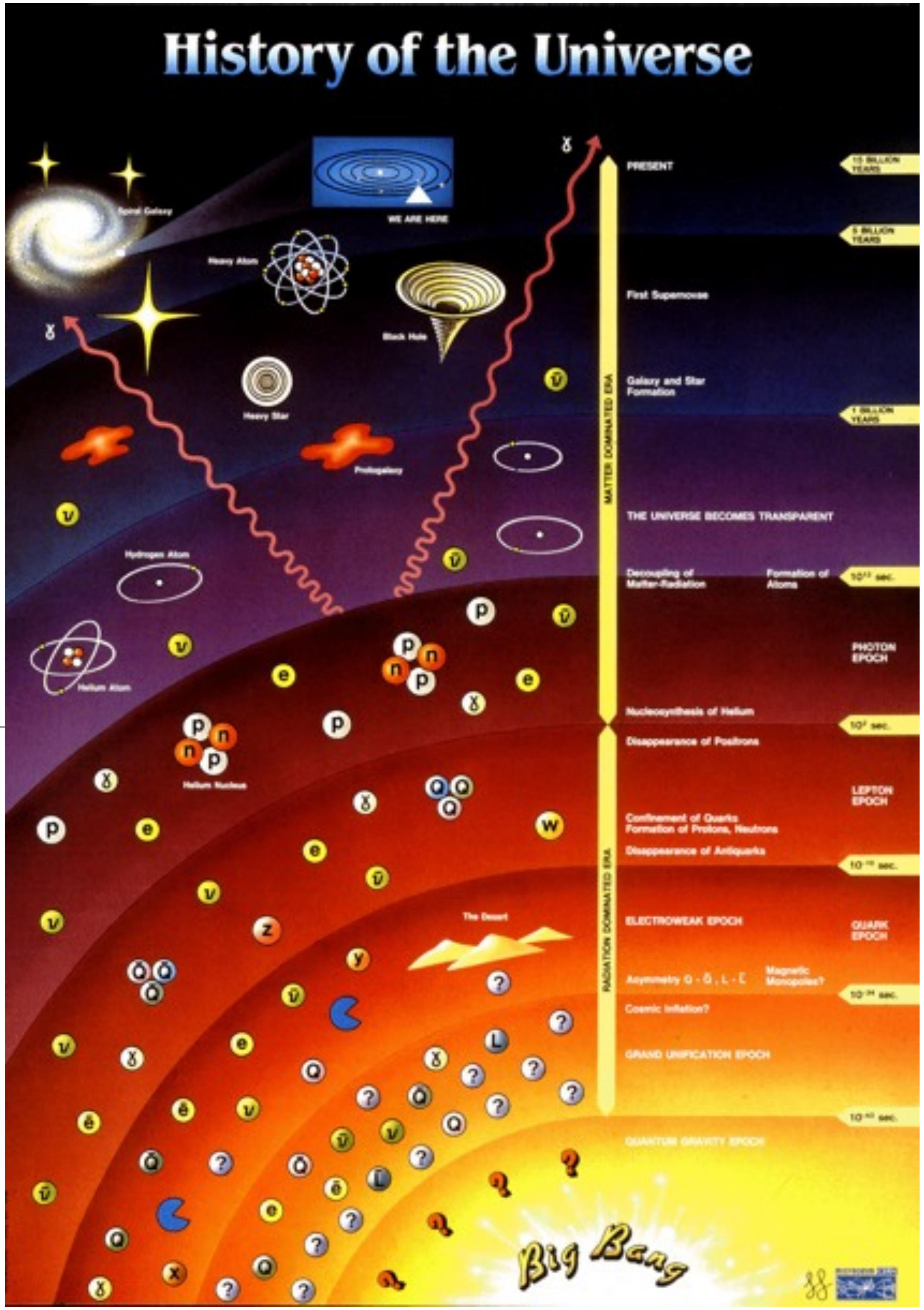


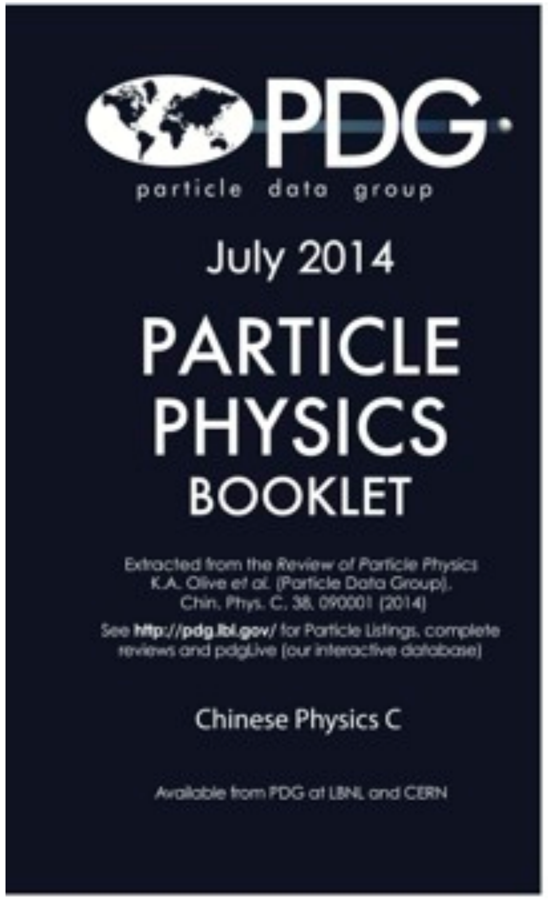
Overview

- Cosmology 1: The homogeneous universe
  - ★The expansion stages of the universe
- Cosmology 2: The inhomogeneous universe
  - ★The cosmic microwave background
  - ★Large scale structure
- Cosmology 3:
  - ★Dark matter
  - ★Dark energy

Cosmology 1

Overview

A good guide for particle physicists: cosmology and astrophysics covered in review sections of the Particle Physics Book(let)



particle data group

July 2014

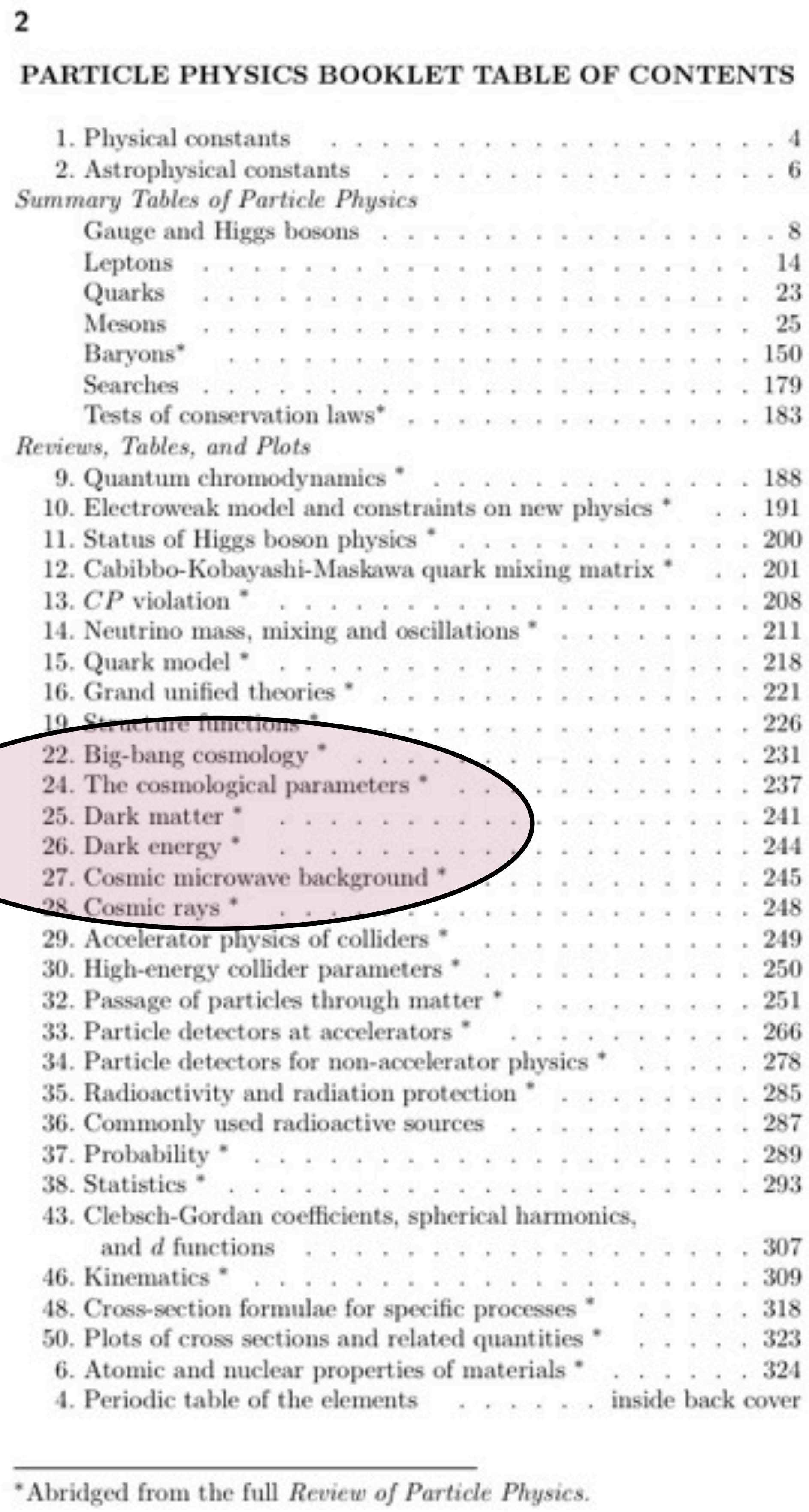
PARTICLE PHYSICS BOOKLET

Extracted from the *Review of Particle Physics*
K.A. Olive *et al.* (Particle Data Group),
Chin. Phys. C, 38, 090001 (2014)

See **http://pdg.lbl.gov/** for Particle Listings, complete reviews and pdgLive (our interactive database)

Chinese Physics C

Available from PDG at LBNL and CERN



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PARTICLE PHYSICS BOOKLET TABLE OF CONTENTS

Summary Tables of Particle Physics

Reviews, Tables, and Plots

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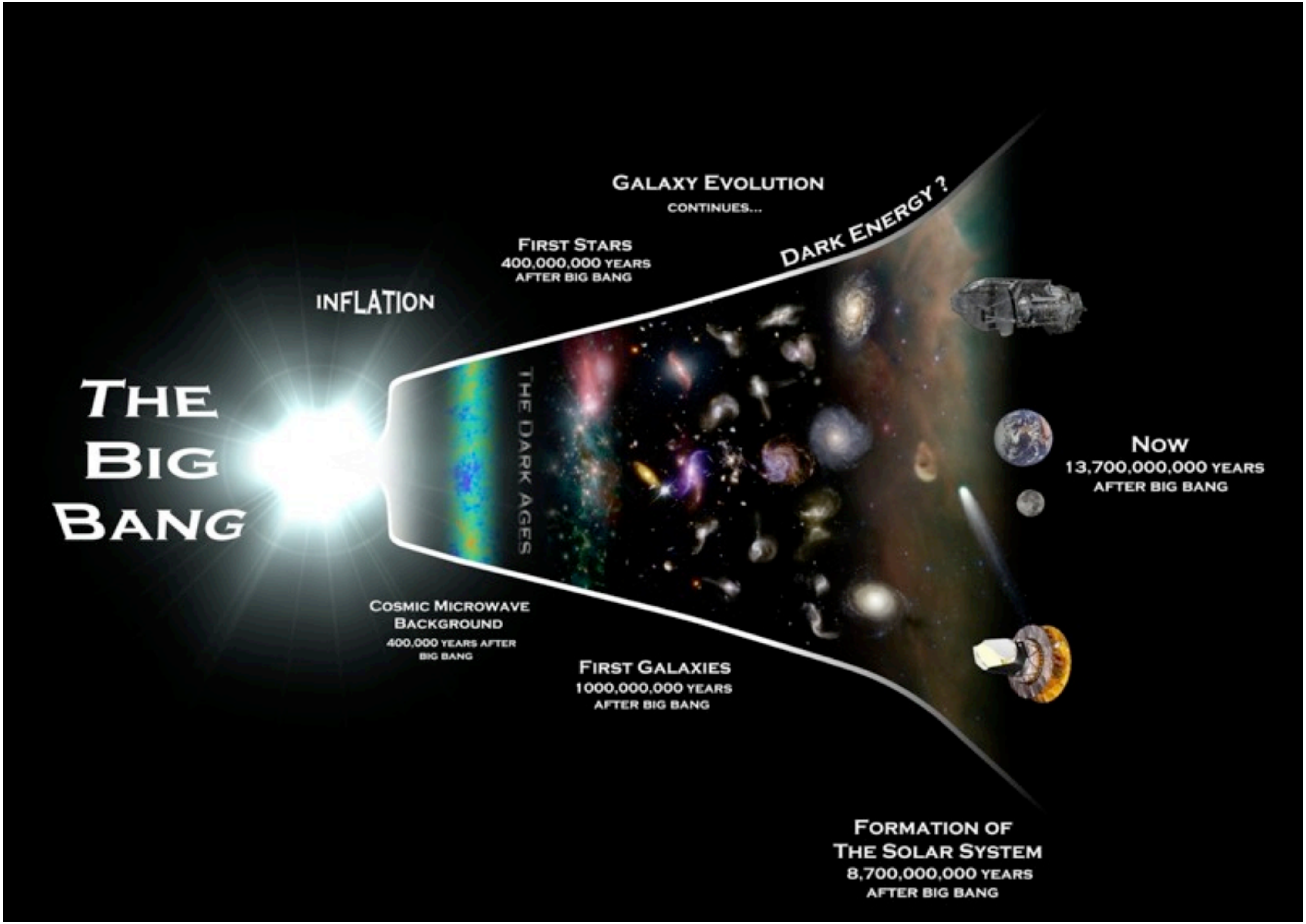
*Abridged from the full *Review of Particle Physics*.

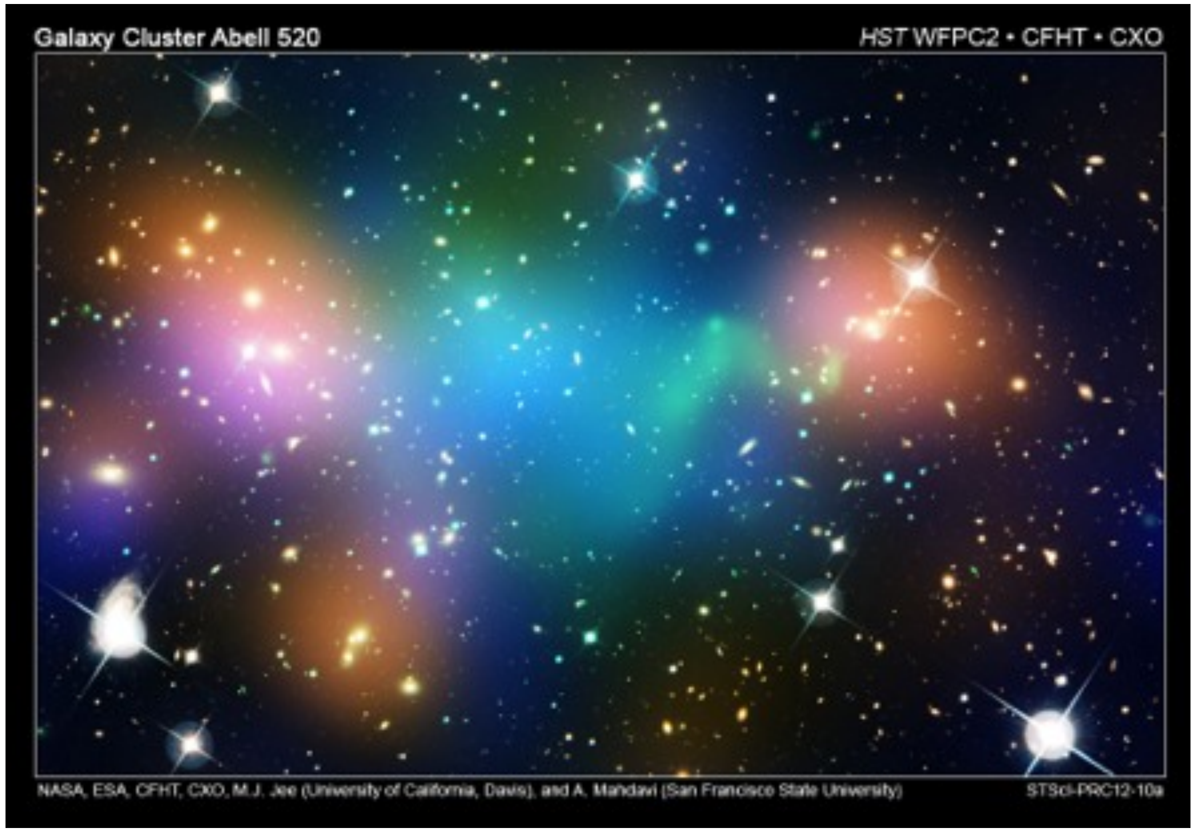
What is cosmology?

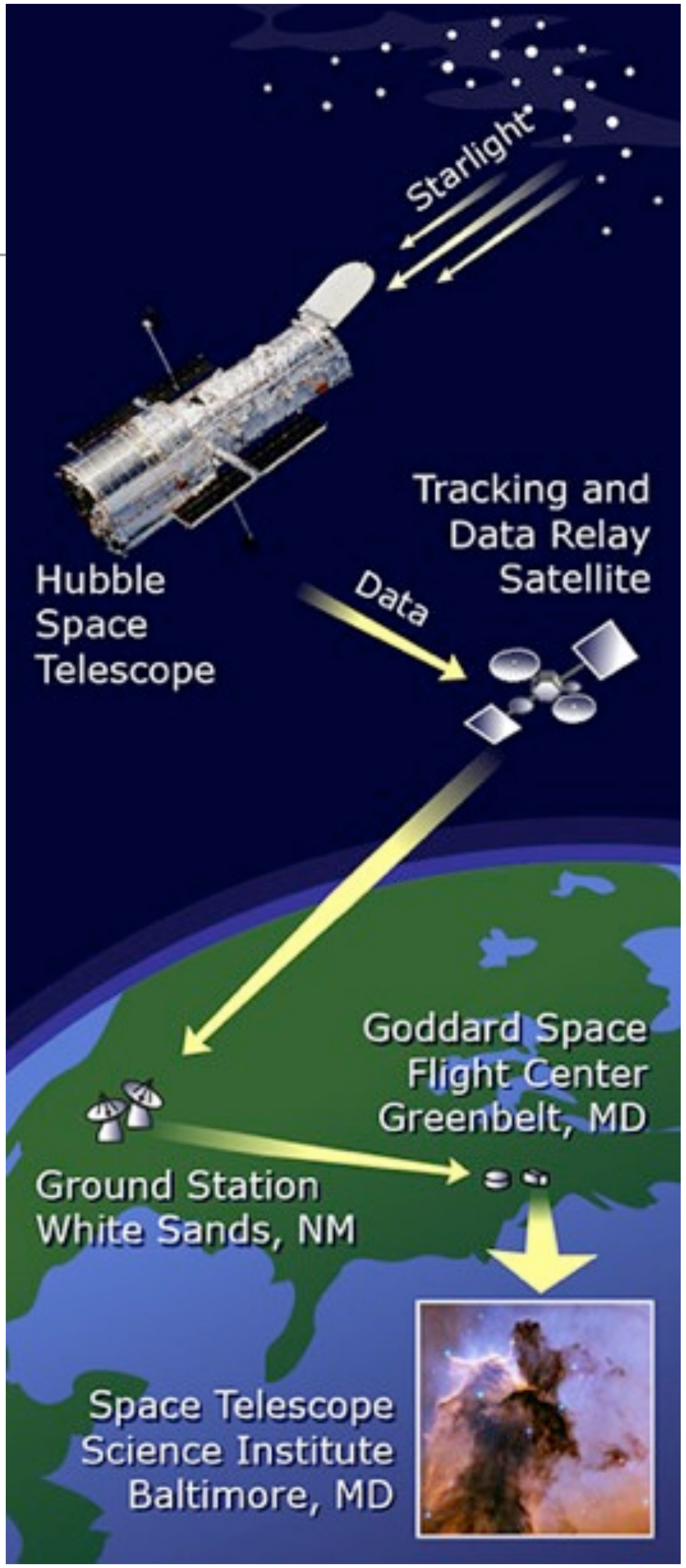
- Its object is the universe as a whole.

- The aim is to understand its origin, evolution and structure.

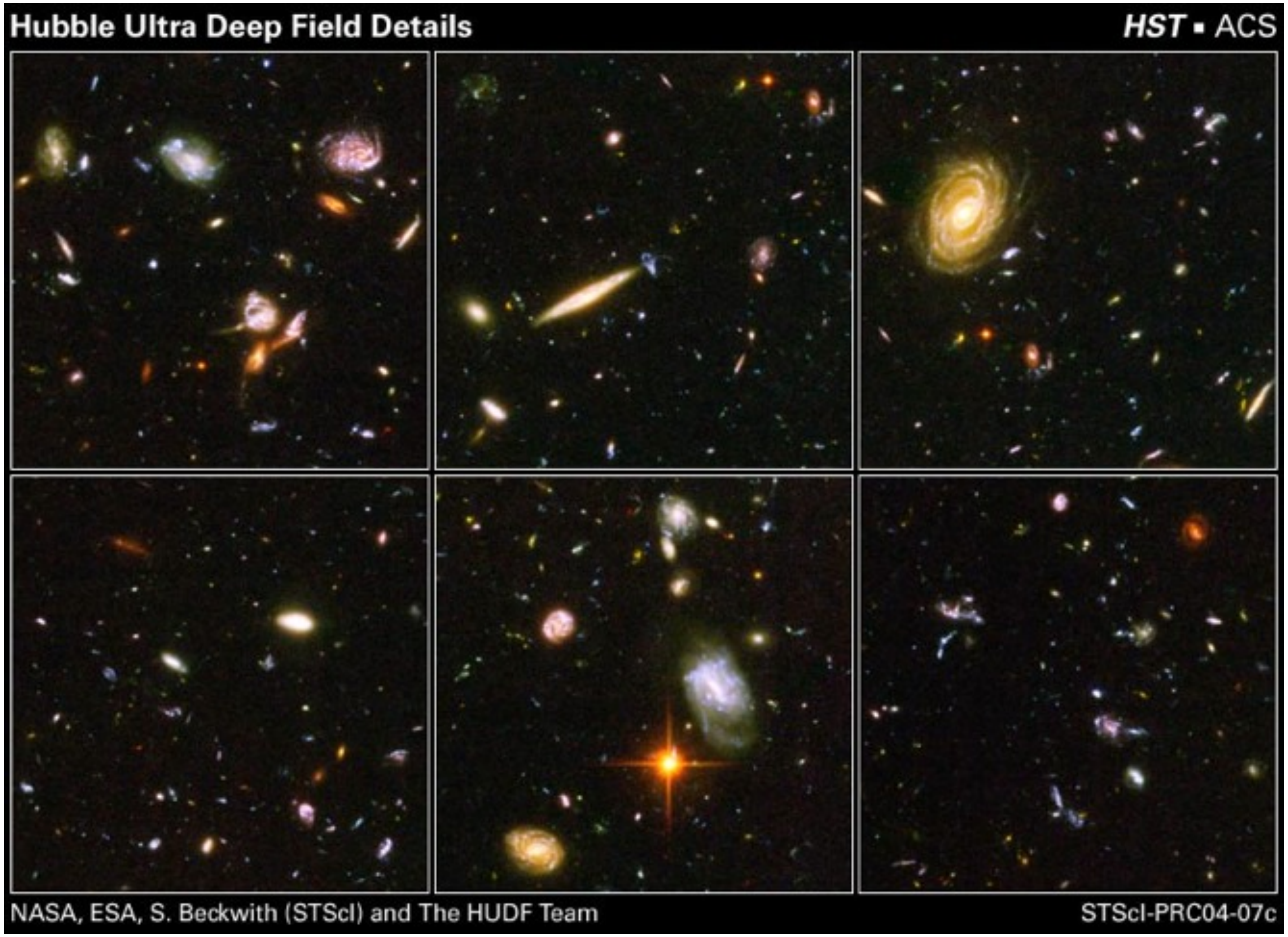


Observations

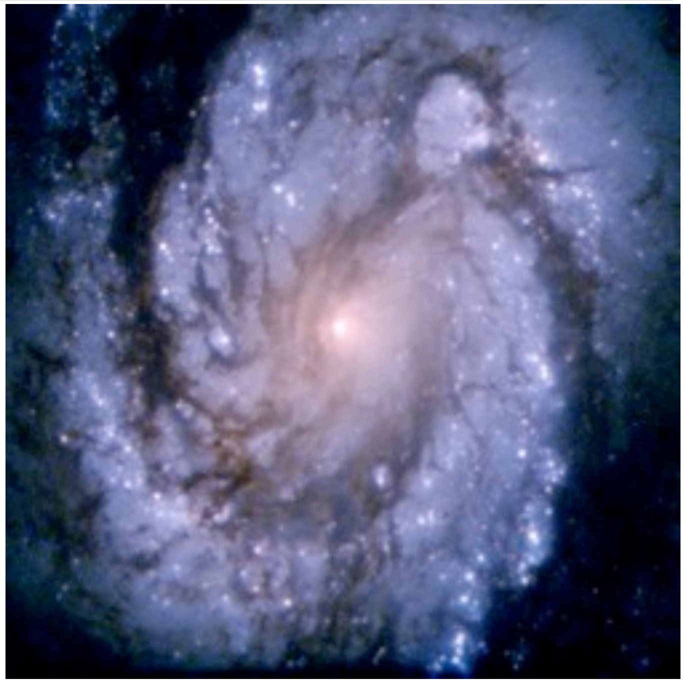


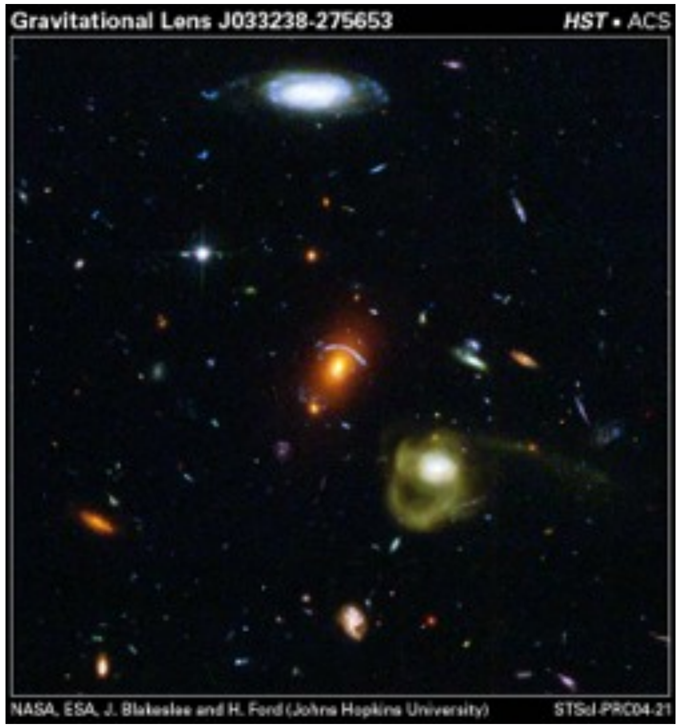


Observations



Spiral Galaxy M100

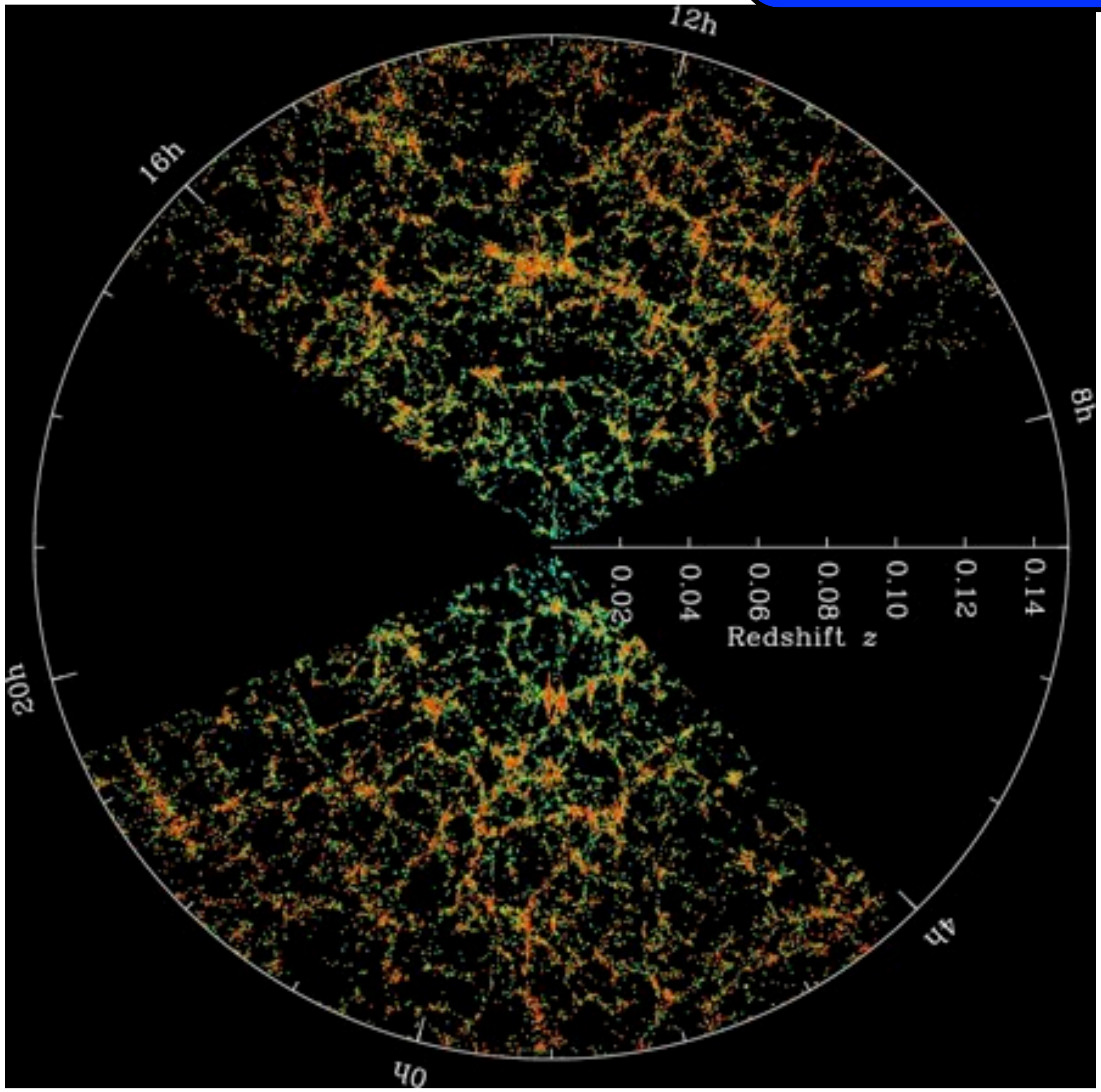




Observations

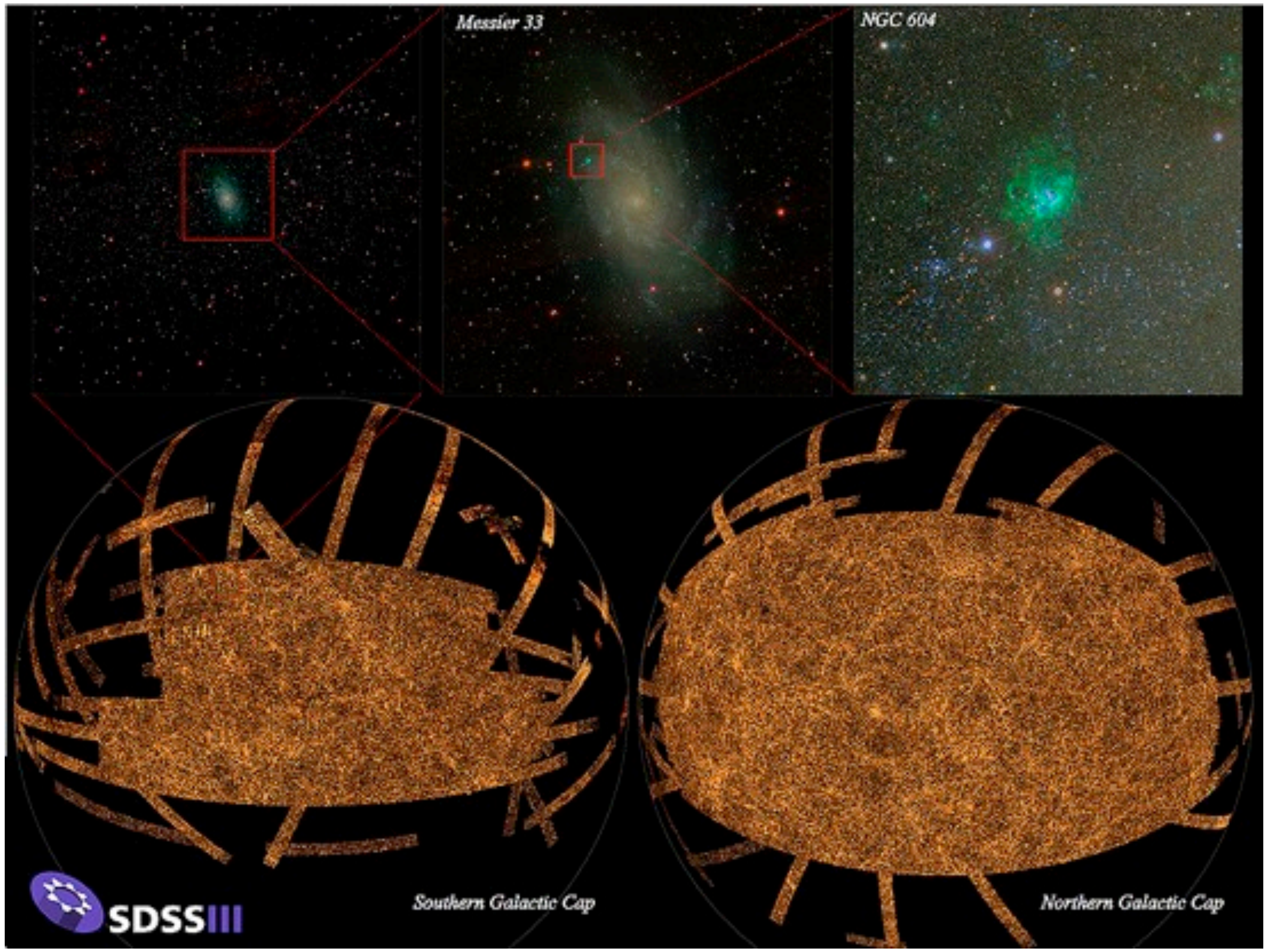
Sloan Digital Sky Survey

Galaxy Surveys



Slices through the SDSS 3-dimensional map of the distribution of galaxies. Earth is at the center, and each point represents a galaxy, typically containing about 100 billion stars. Galaxies are colored according to the ages of their stars, with the redder, more strongly clustered points showing galaxies that are made of older stars. The outer circle is at a distance of two billion light years. The region between the wedges was not mapped by the SDSS because dust in our own Galaxy obscures the view of the distant universe in these directions. Both slices contain all galaxies within -1.25 and 1.25 degrees declination.

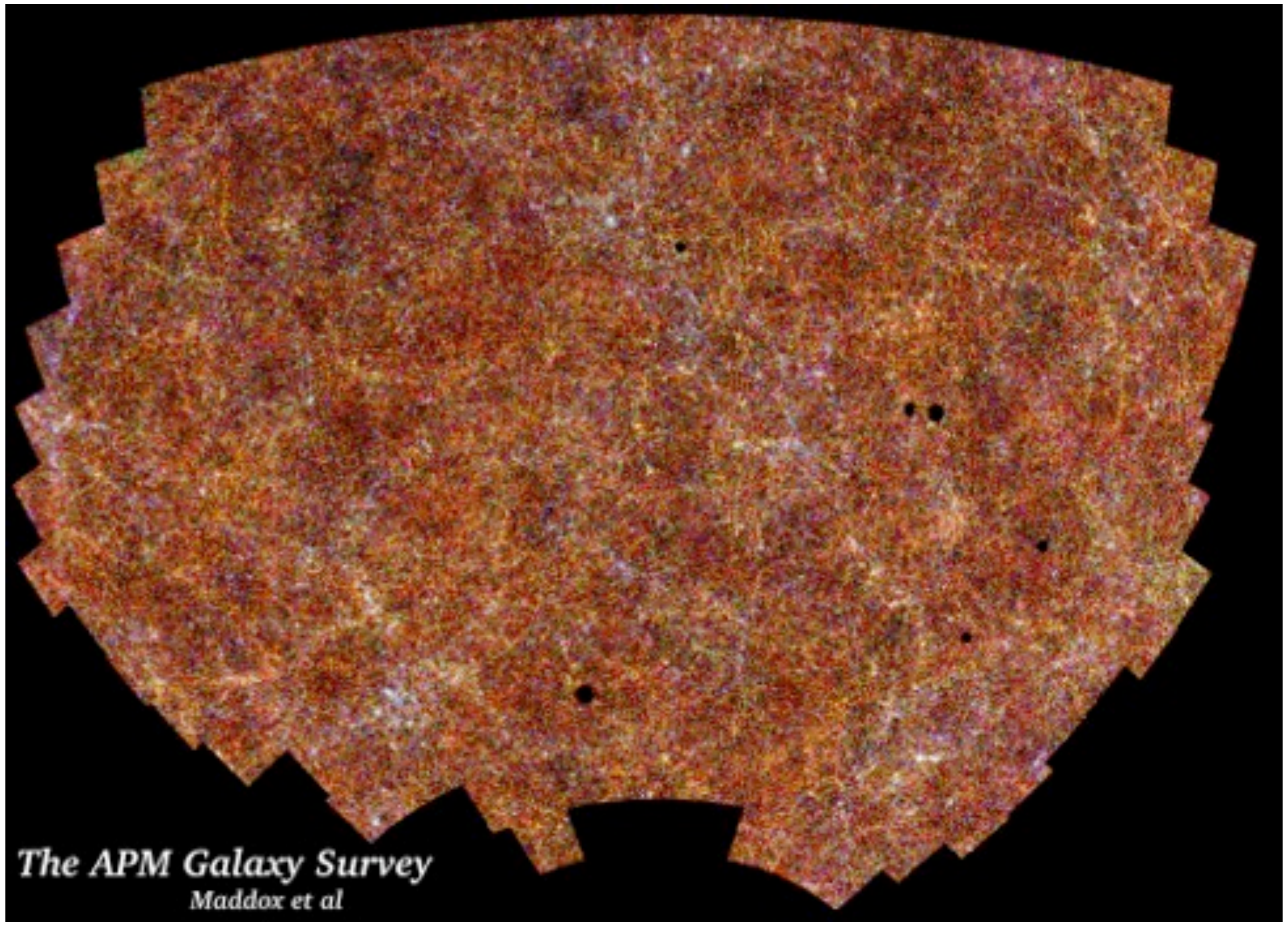
Credit: M. Blanton and the Sloan Digital Sky Survey. **(2000-2008)**



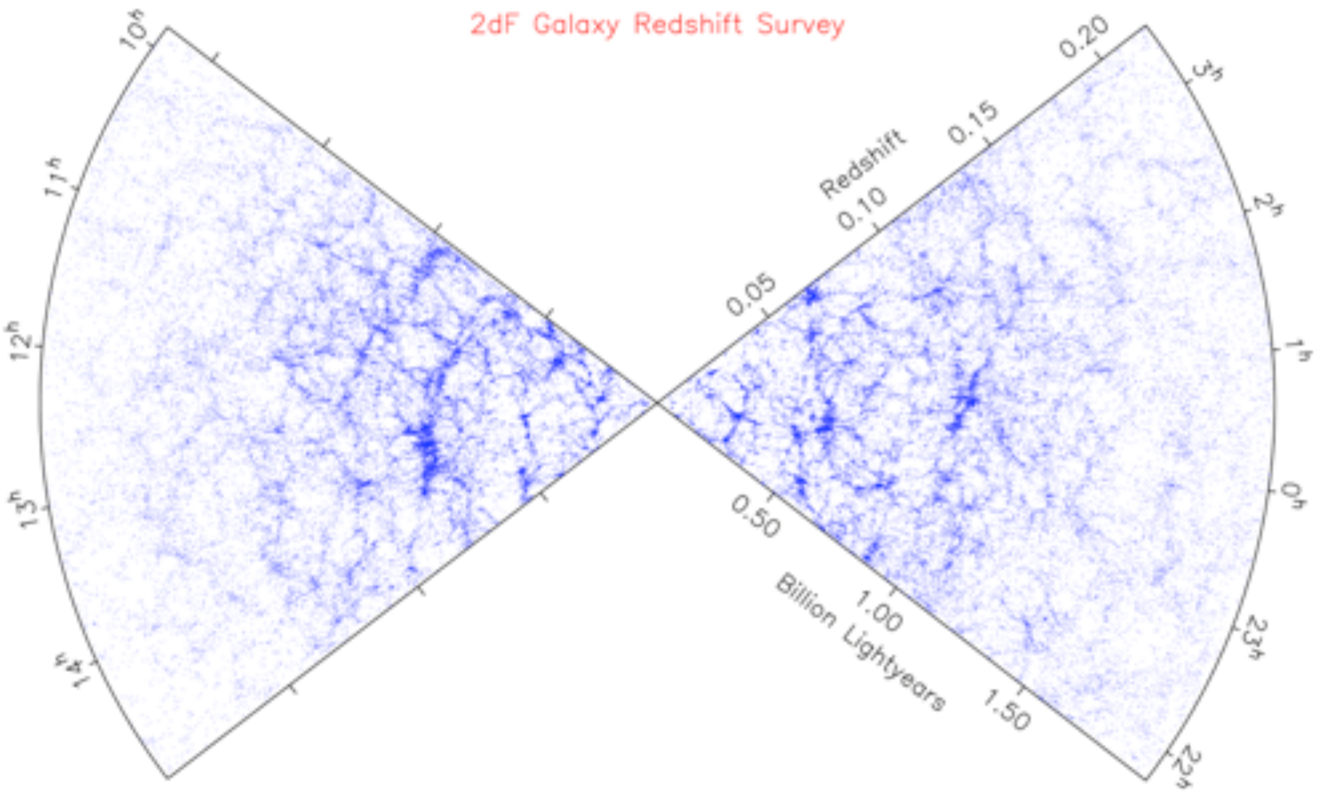
The bottom panel shows the sky coverage of the final SDSS imaging survey, including data from SDSS I, II, and III.SDSS imaging covered slightly more than 1/3 of the sky, concentrated in the northern and southern Galactic caps (above and below the plane of the galaxy). In this image, stripes are radiating out from these caps; these stripes are areas imaged by the SEGUE survey, extending toward the plane of the Milky Way. Each orange dot in this map is a galaxy.The sequence of zooms in the upper panels zeroes in on the star-forming nebula NGC 604 in the nearby (2.5 million light years) galaxy Messier 33. In all, the SDSS imaging map shown here contains more than a trillion pixels, each one imaged in five colors.

Credit: M. Blanton and the SDSS-III collaboration

Observations



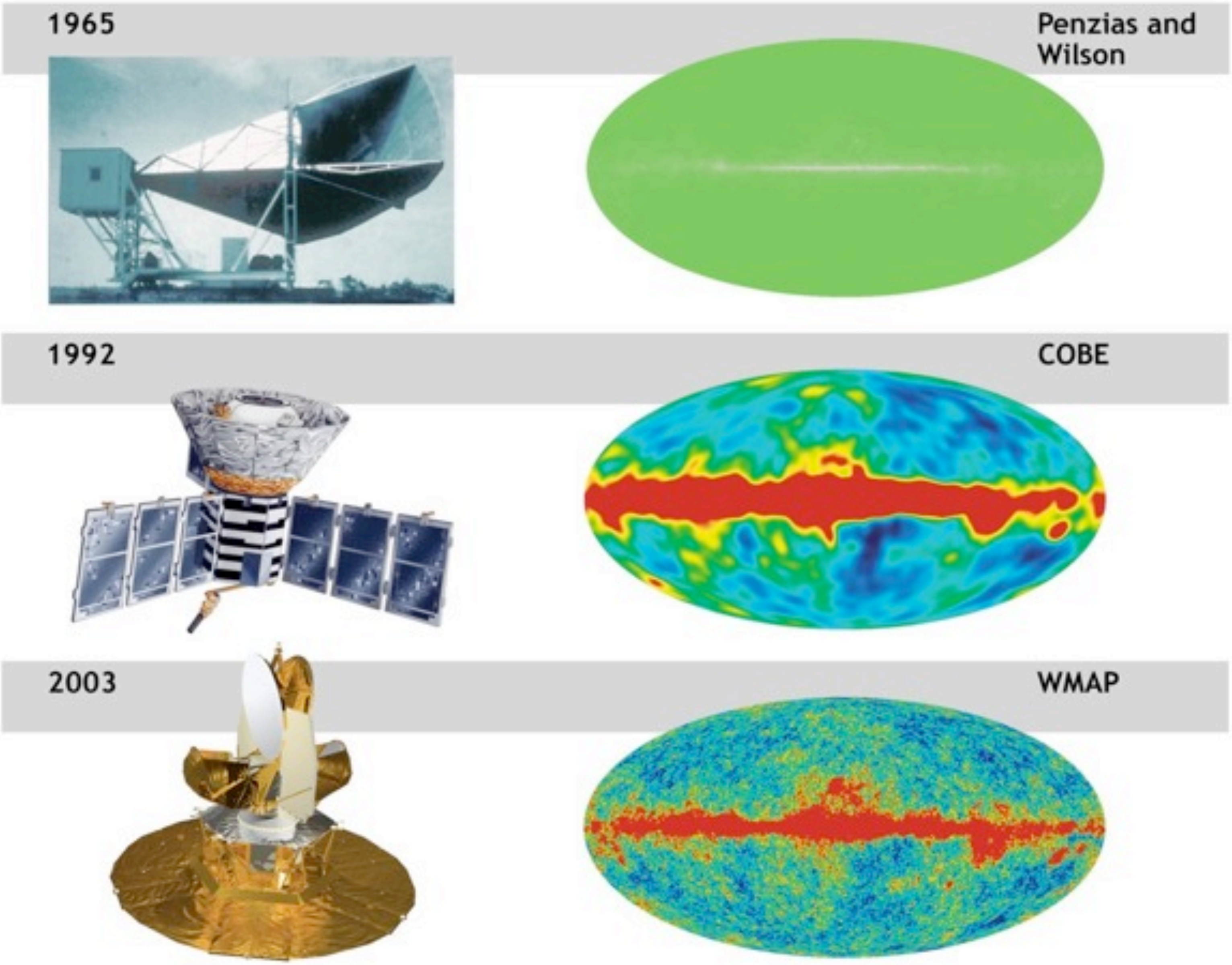
The **Automated Plate Measurement (APM) Galaxy Survey** was one of two surveys (the other being the Edinburgh–Durham Southern Galaxy Catalogue) carried out in the late 1980s and early 1990s



2dF Galaxy Redshift Survey (Two-degree-Field Galaxy Redshift Survey), redshift survey conducted by the Anglo-Australian Observatory (AAO) with the 3.9m Anglo-Australian Telescope between 1997 and 2002.

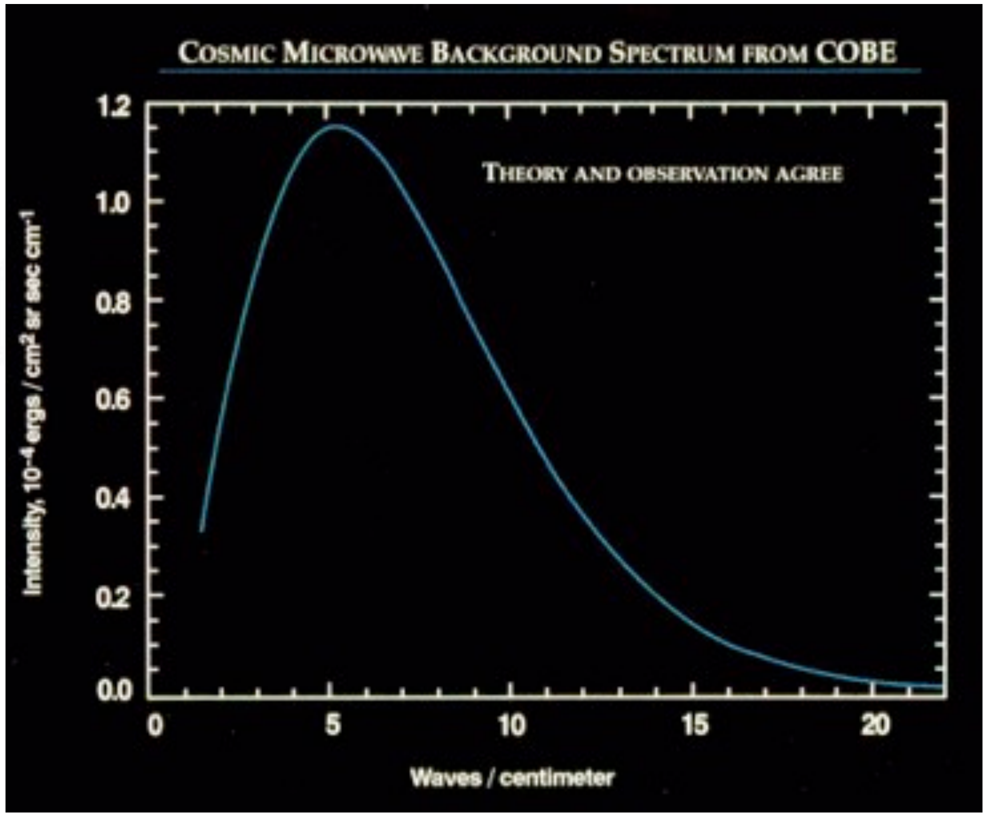
Observations

Cosmic Microwave Background



Penzias and Wilson discovered the remnant afterglow from the Big Bang.

COBE first discovered the patterns in the afterglow.

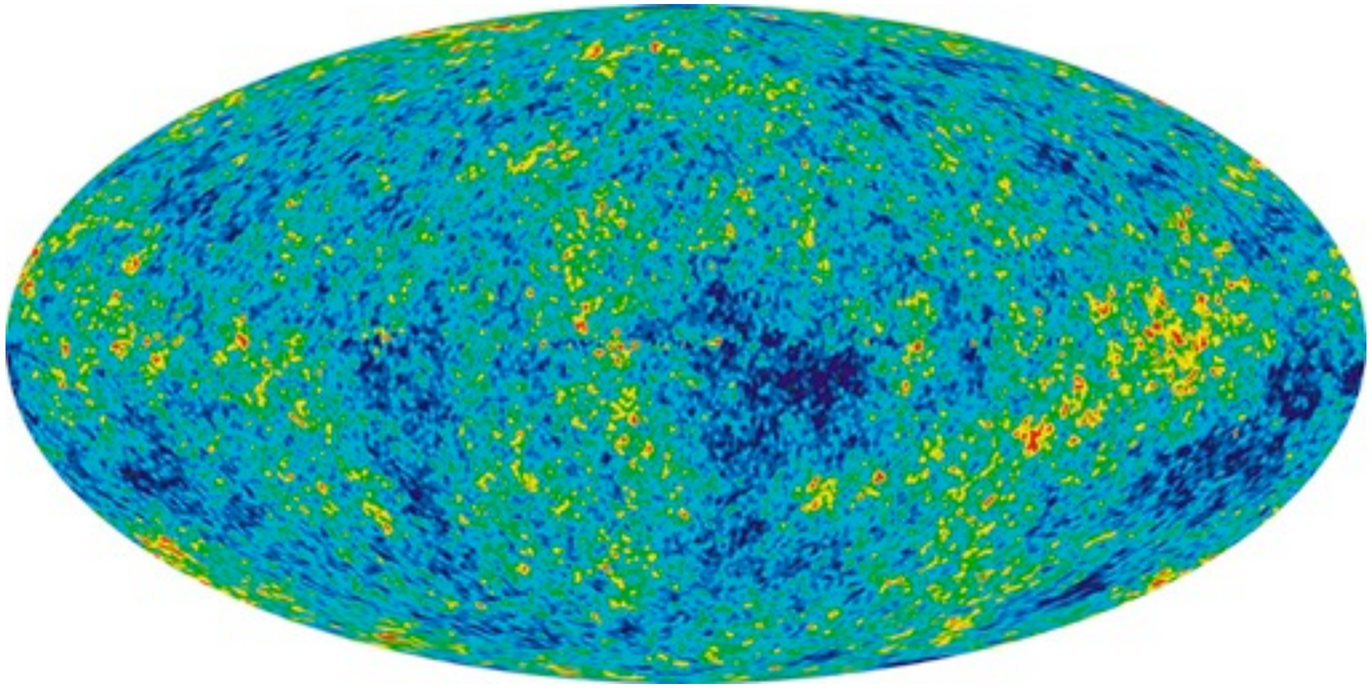


NASA

$$T_0 = 2.725K$$

Observations

Cosmic Microwave Background



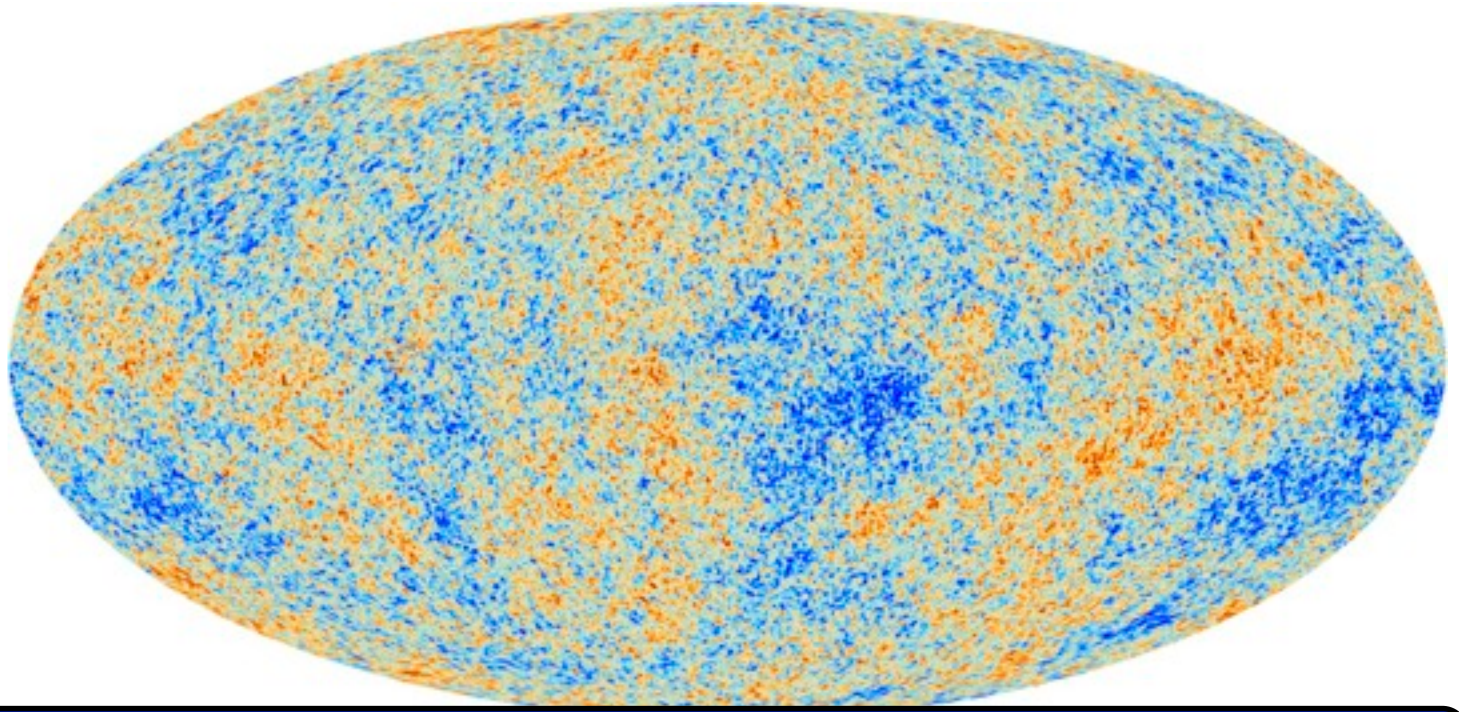
WMAP 9 (2012)

Credit: NASA / WMAP Science Team

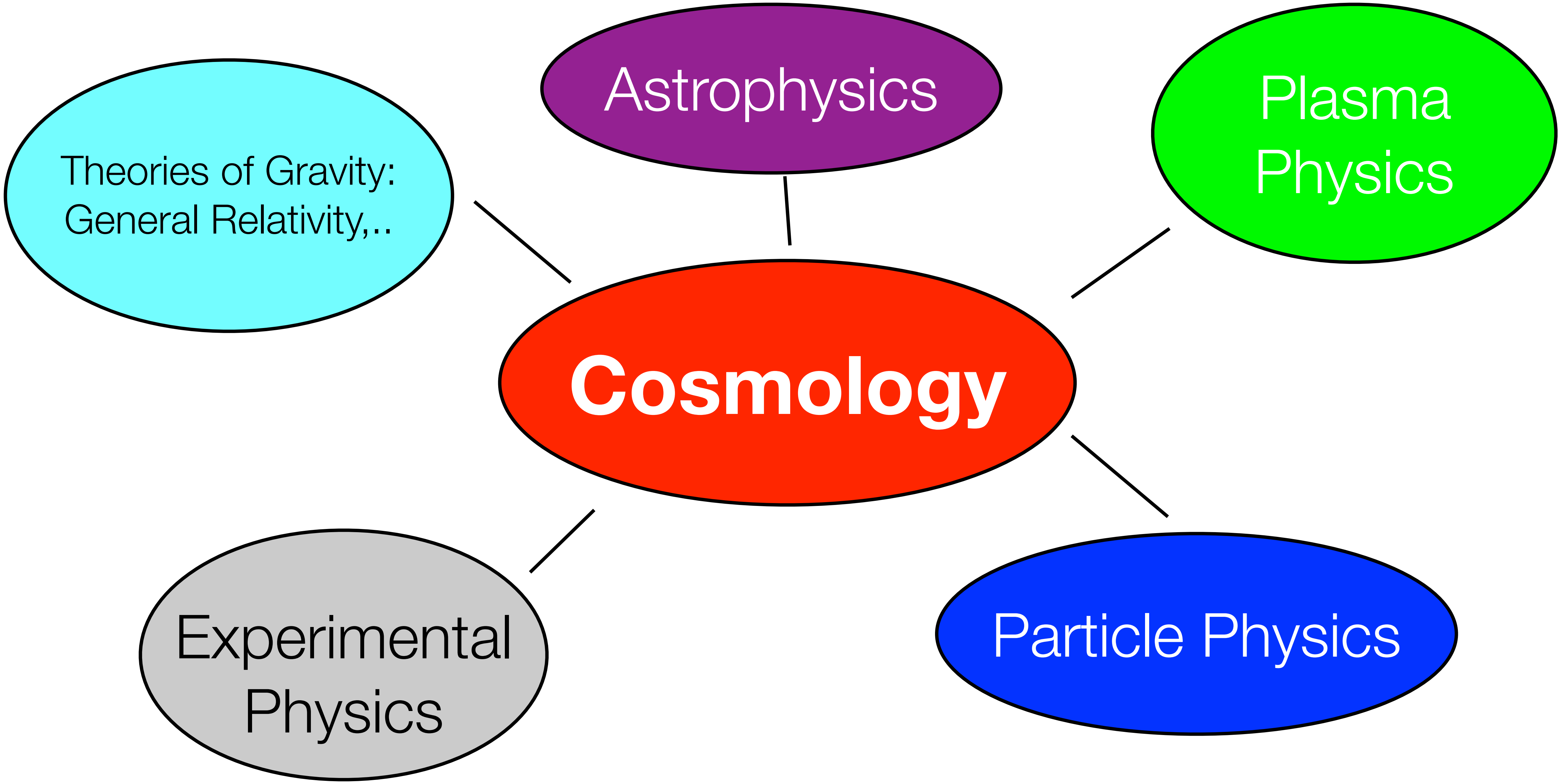
The detailed, all-sky picture of the infant universe created from nine years of WMAP data. The image reveals 13.77 billion year old temperature fluctuations (shown as color differences) that correspond to the seeds that grew to become the galaxies. The signal from the our Galaxy was subtracted using the multi-frequency data. This image shows a temperature range of ± 200 microKelvin.

Planck 2013

Date: 21 Mar 2013
Satellite: Planck
Depicts: Cosmic Microwave Background
Copyright: ESA, Planck Collaboration



What is cosmology?



Theories of Gravity:
General Relativity,..

Astrophysics

Plasma Physics

Cosmology

Experimental Physics

Particle Physics

Cosmology 1

- The expanding universe

- General relativity: space-time is curved or deformed by the presence of matter or energy distribution.

- *Recall*: Special Relativity

➡ The invariant interval between two points X y Y with coordinates (t,x,y,z) and (t+dt,x+dx,y+dy,z+dz) is given by

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$

➡ ds is invariant under coordinate transformations. An observer in a different inertial sistem using coordinates (x’,y’,z’,t’) also measures ds.

Cosmology 1

- General Relativity (A. Einstein 1915)

geometry

matter

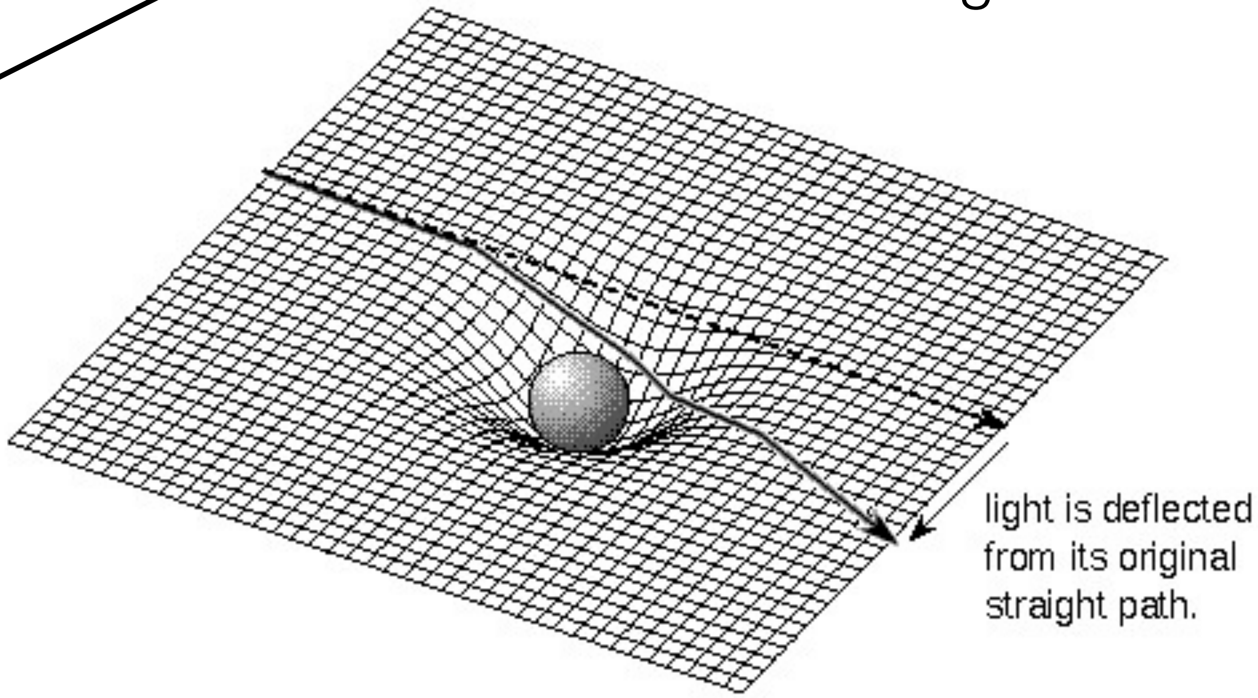
$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}$$

cosmological constant

Ricci tensor and scalar:

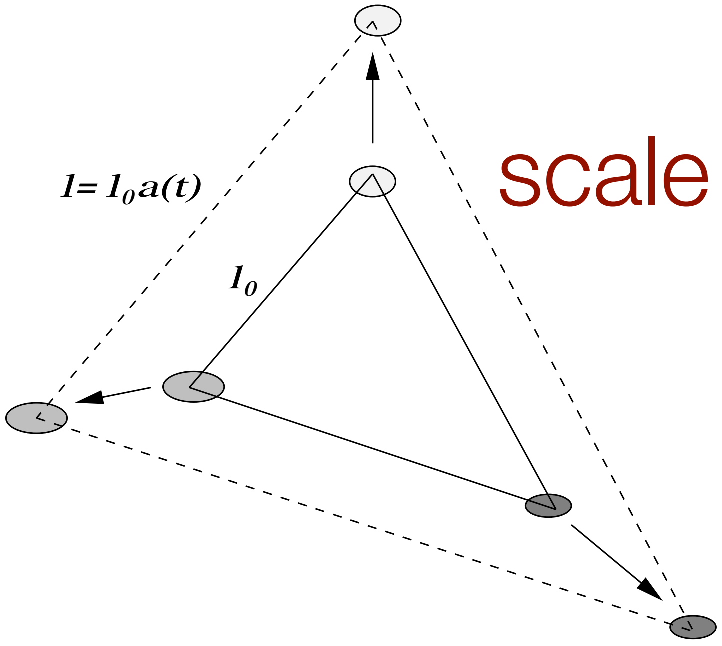
constructed from metric $g_{\mu\nu}$ and its first and second derivatives



Cosmology 1

- Friedmann-Robertson-Walker models

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right]$$

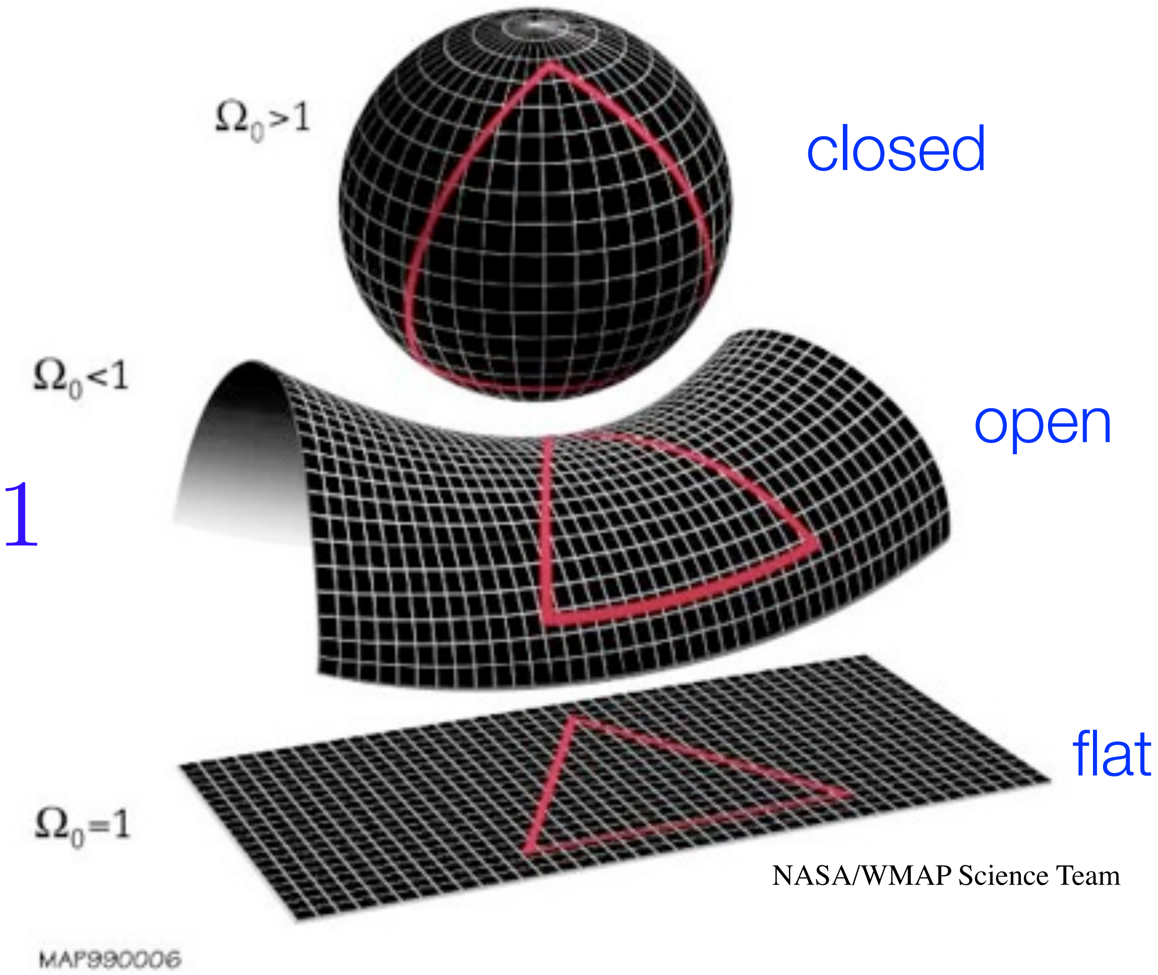


scale factor

$$k = 1$$

$$k = -1$$

$$k = 0$$



NASA/WMAP Science Team

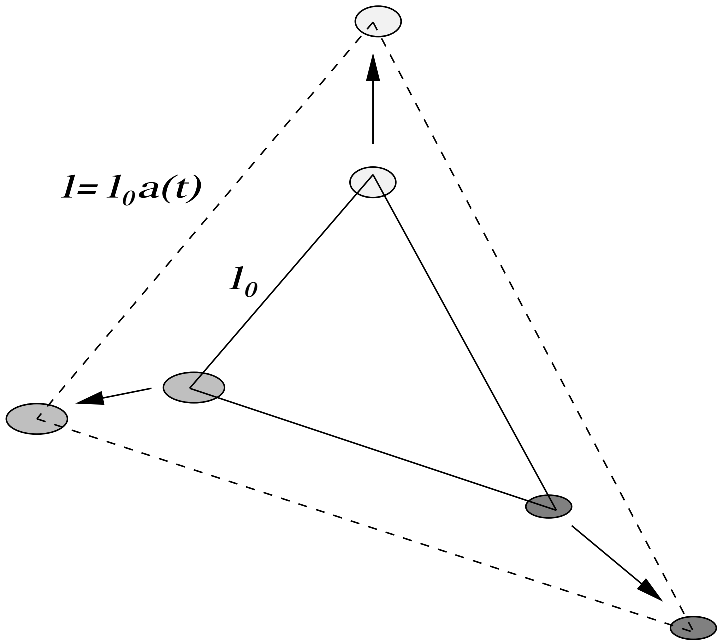
homogeneous and isotropic:

the same everywhere and in every direction

Cosmological principle

Cosmology 1

- Coordinate (comoving) distance vs. physical distance



$$d_p = \int_0^r \frac{a\,dr'}{(1-kr'^2)^{\frac{1}{2}}} = a(t)f(r)$$

time dependent

recession velocity

$$\boxed{v_r = \dot{a}f(r) = \frac{\dot{a}}{a}d_p(t)}$$

holds locally

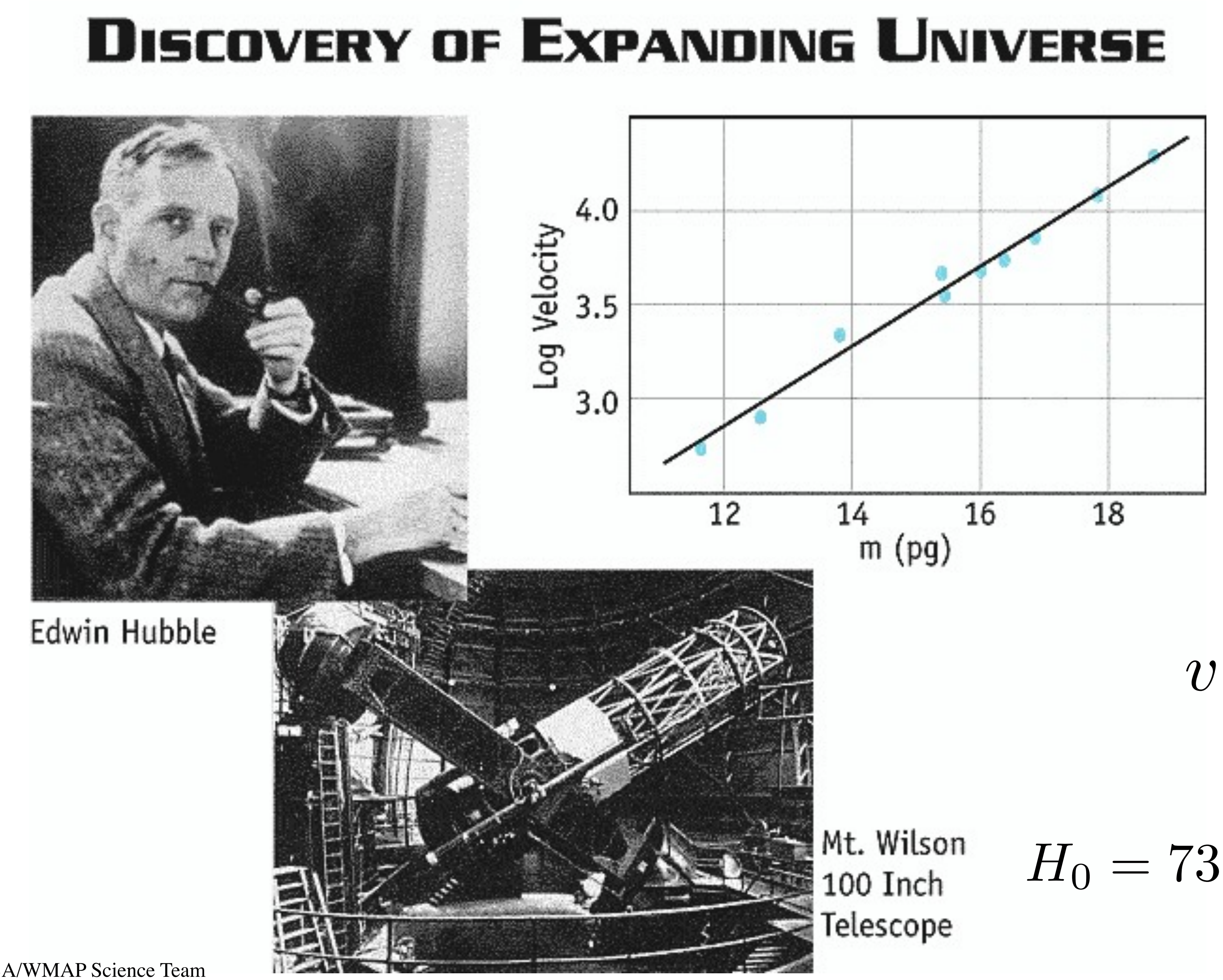
Hubble law

$$\boxed{H(t) \equiv \frac{\dot{a}}{a}}$$

Hubble parameter

Cosmology 1

- Expansion of the universe



NASA/WMAP Science Team

In 1929 Hubble found galaxies outside the Milky Way moving away, with speed proportional to their distance.

$$v = H_0 d_p$$

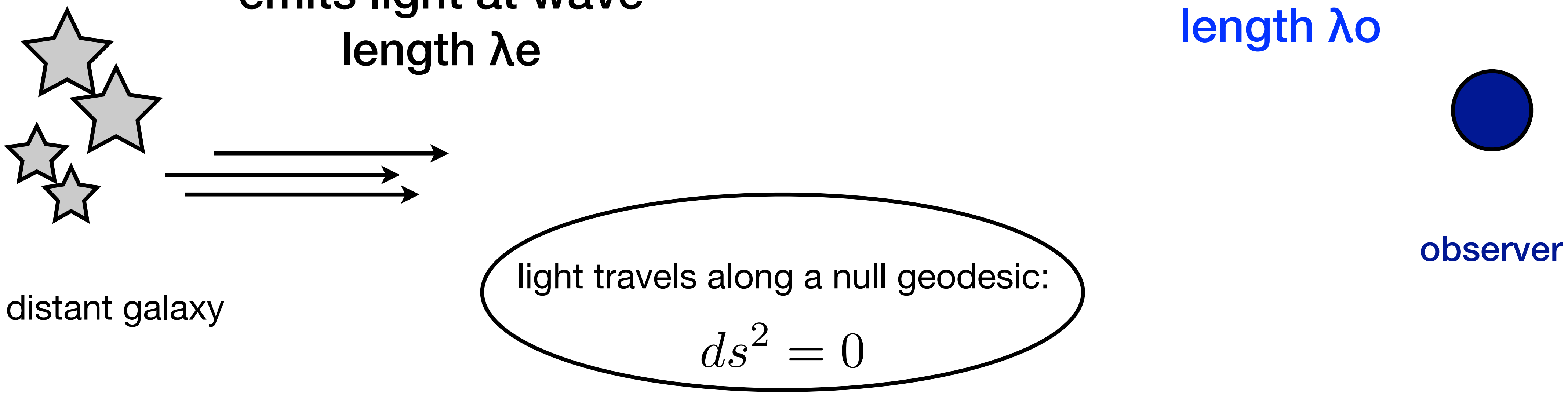
$$H_0 = 73.8 \pm 2.4 \mathrm{kms}^{-1}\mathrm{Mpc}^{-1}$$

Riess et al. 2011

Cosmology 1

- **Redshift**

- Cosmological redshift caused by expansion of the universe.



emits light at wave length λe

receives light at wave length λo

distant galaxy

light travels along a null geodesic:

$$ds^2 = 0$$

observer

$$\Rightarrow \quad \int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_0^{r_1} \frac{dr}{(1 - kr^2)^{\frac{1}{2}}} = f(r_1)$$

Cosmology 1

- Light emitted at a time $t_1 + \delta t_1$ reaches the detector at a time $t_0 + \delta t_0$. Since $f(r_1)$ is a constant and the source is at a fixed position

$$\int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_{t_1+\delta t_1}^{t_0+\delta t_0} \frac{dt}{a(t)} \Rightarrow \int_{t_1}^{t_1+\delta t_1} \frac{dt}{a(t)} = \int_{t_0}^{t_0+\delta t_0} \frac{dt}{a(t)}$$

- For small δt ($\lambda = c\delta t \ll ct$) and a(t)~const. during the time interval of integration $\Rightarrow \frac{\delta t_1}{a(t_1)} = \frac{\delta t_0}{a(t_0)}$ and with $\lambda_1 \sim c\delta t_1, \lambda_0 \sim c\delta t_0$

light emitted light received

$$\Rightarrow \frac{\lambda_1}{\lambda_0} = \frac{a(t_1)}{a(t_0)}$$

Cosmology 1

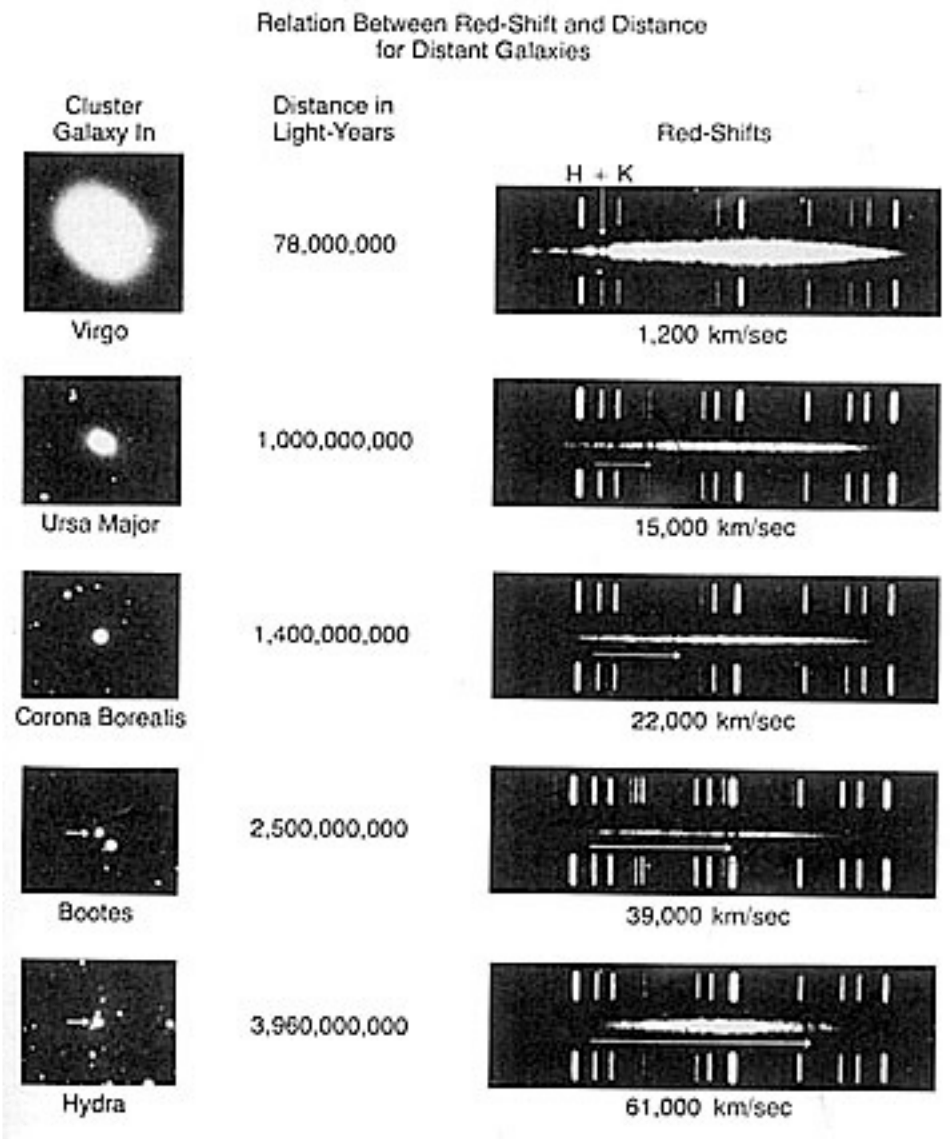
- The redshift z is defined by the ratio of the wavelengths of the emitted and received light:

$$\boxed{1 + z \equiv \frac{\lambda_0}{\lambda_1} = \frac{a(t_0)}{a(t_1)}}$$

- If the scale factor *grows* → *redshift*.
- If the scale factor *becomes smaller* → *blueshift.*
- Spectra of distant galaxies show redshift → expanding universe

Cosmology 1

- Spectra of galaxies at different distances.



terrestial reference spectrum

spectrum of galaxy

terrestial reference spectrum

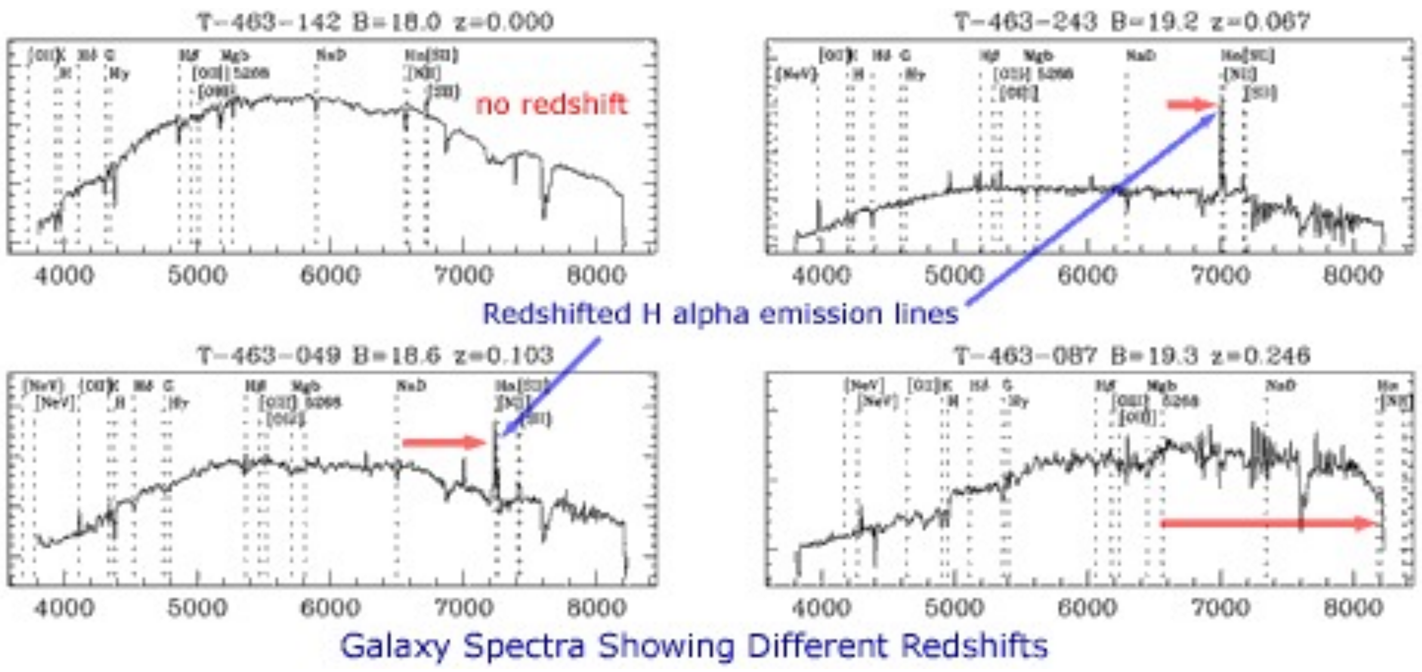
H, K: Ca lines.
In the galaxy spectrum:
absorption lines

in the reference spectrum:
emission lines.

From J. Silk, The Big Bang

Cosmology 1

-



Credit: Adapted from data in *The 2dF Galaxy Redshift Survey*
Four different galactic spectra from the 2dF Galaxy Redshift Survey. Note the differing redshifts of the galaxies. The top left galaxy is at zero redshift whilst the bottom right one has a redshift of $z = 0.246$. The other two galaxies show prominent Hα emission lines suggesting active star formation in them. These Hα lines have been redshifted beyond the rest frame value of 6563 Ångstroms.

Australia Telescope National Facility

Cosmology 1

- Luminosity distance D_L

- Distances of astronomical objects cannot be directly measured.

- Distant objects are observed by the light they emit.

- Define distance measures which are directly observable.

- One of these is the luminosity distance D_L .

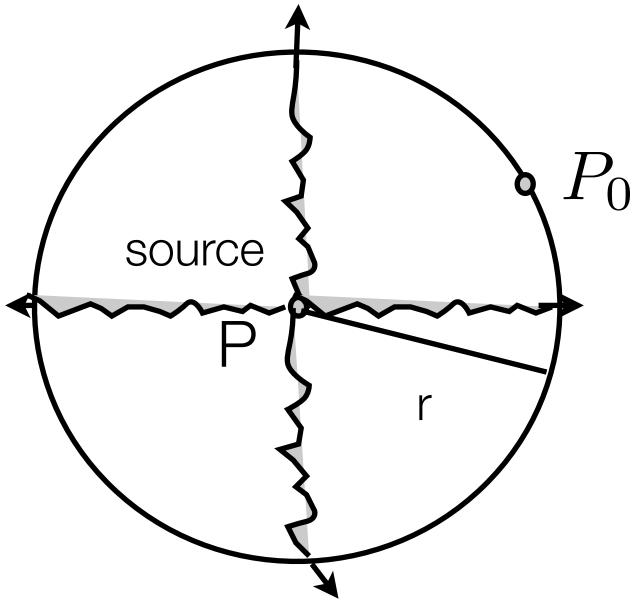
Cosmology 1

$$\boxed{D_L = \left(\frac{L}{4\pi\ell}\right)^{\frac{1}{2}}}$$

- The luminosity distance is defined by where L is the absolute luminosity and ℓ the visible luminosity, that is the received one.
- Say the source is at a point P, at a coordinate distance r at some time t. The observer is at a point P_0 and makes the obervation at a time t_0 .
- L=energy flux =energy/time
- ℓ=energy flux density=energy/(time x surface)

Cosmology 1

- surface of sphere= $4\pi a_0^2 r^2$



- wavelength of photons is redshifted: energy $E = h\nu = \dfrac{hc}{\lambda}$

- relation between rates of emission and reception of photons: $\dfrac{(\delta t_o)^{-1}}{(\delta t_e)^{-1}} = \dfrac{a(t)}{a_0}$

- $$L = \frac{E_e}{\delta t_e} = \frac{hc}{\lambda_e \delta t_e}$$

$$\ell = \frac{E_o}{\delta t_o 4\pi a_0^2 r^2} = \frac{hc}{\lambda_o \delta t_o 4\pi a_0^2 r^2}$$

$$\Rightarrow \quad \frac{L}{4\pi\ell} = a_0^2 r^2 \left(\frac{a_0}{a(t)}\right)^2 = a_0^2 r^2 (1+z)^2$$

Cosmology 1

➡luminosity distance $D_L = a_0 r \frac{a_0}{a(t)}$

➡ with $r = a_0^{-1} H_0^{-1} \left[z - \frac{1}{2}(1+q_0) z^2 + ... \right]$

$$\Rightarrow \quad D_L = H_0^{-1} \left[z - \frac{1}{2}(1+q_0) z^2 + ... \right] (1+z)$$

$$\Rightarrow \quad \boxed{D_L \simeq H_0^{-1} \left[z + \frac{1}{2}(1-q_0) z^2 + ... \right]}$$

Cosmology 1

- Modern version of the Hubble diagram

D_L : luminosity distance

absolute luminosity

$$D_L = \left(\frac{L}{4\pi\ell}\right)^{\frac{1}{2}}$$

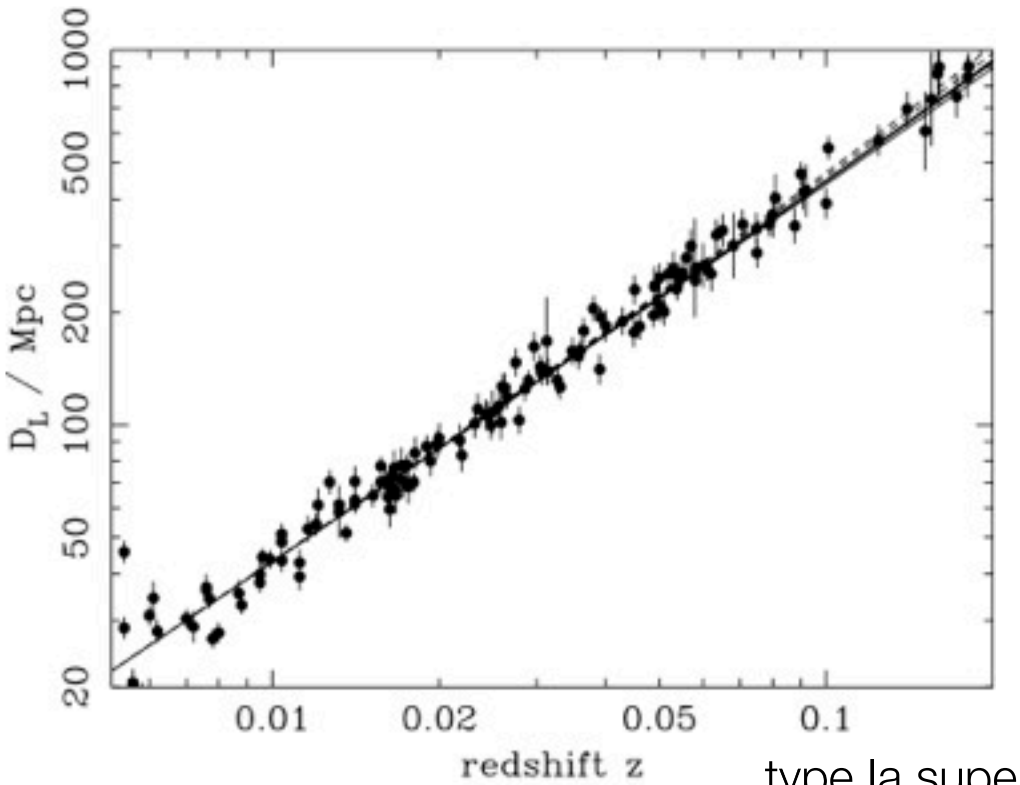
visible luminosity

Different version of Hubble law

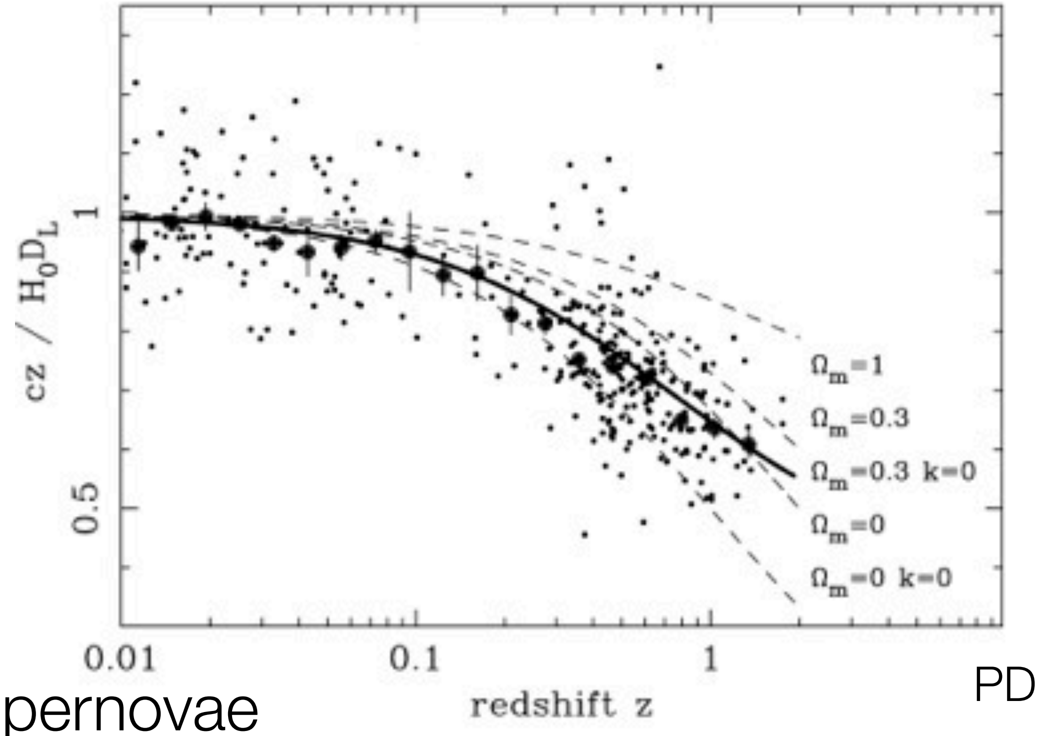
$$H_0 D_L = z + \frac{1}{2}(1-q_0)z^2 + ...$$

Redshift

$$1+z = \frac{\lambda_{obs}}{\lambda_{em}} = \frac{a_0}{a(t)}$$



type Ia supernovae



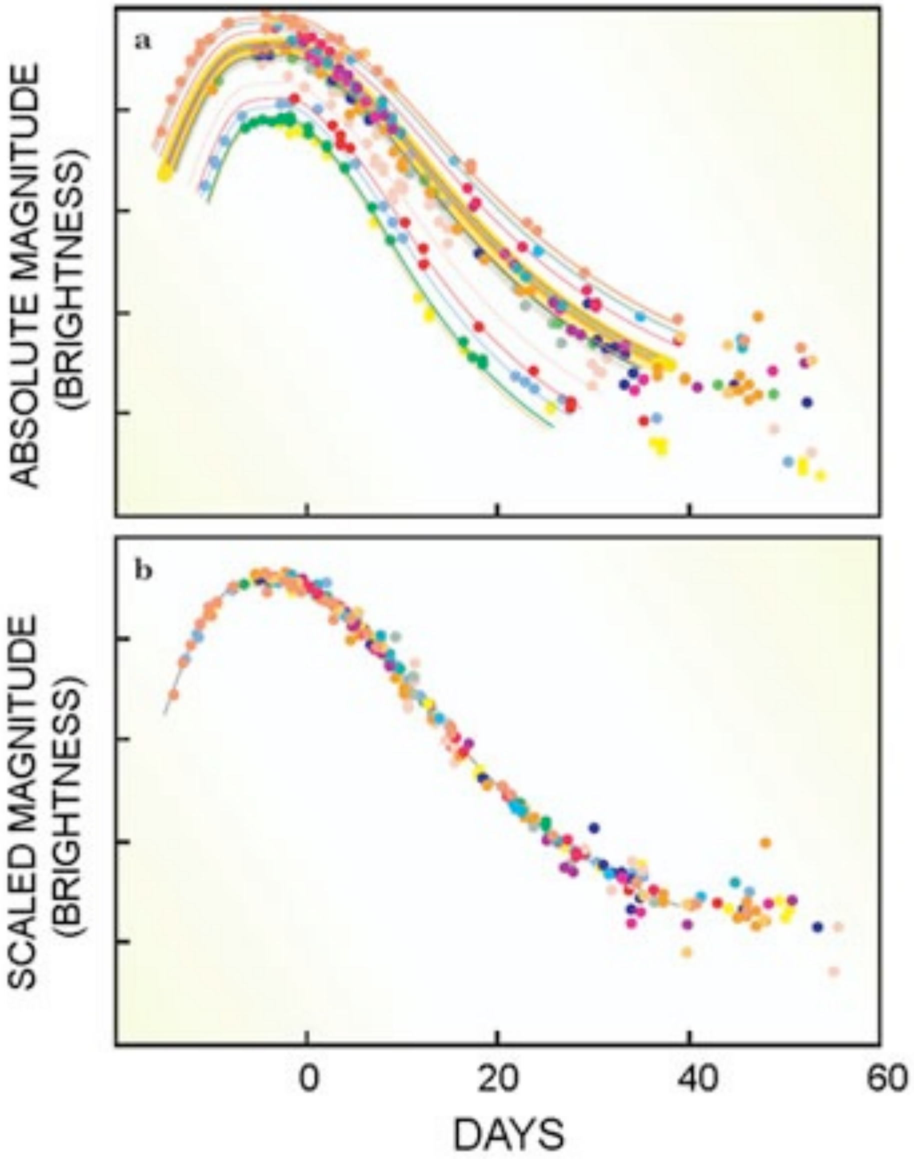
Type Ia Supernovae

PDG 2012

Cosmology 1

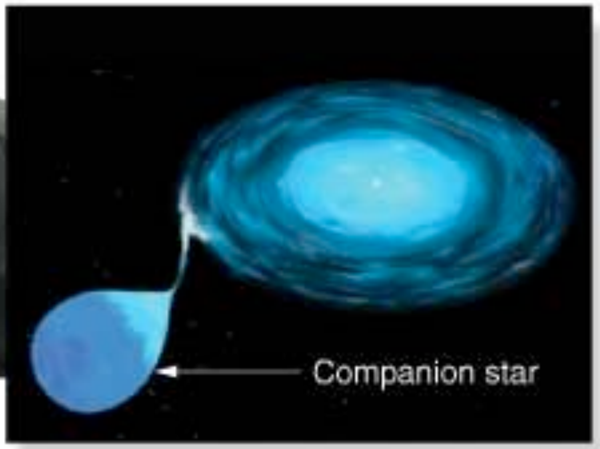
- Type Ia supernovae as standard candles

light curve



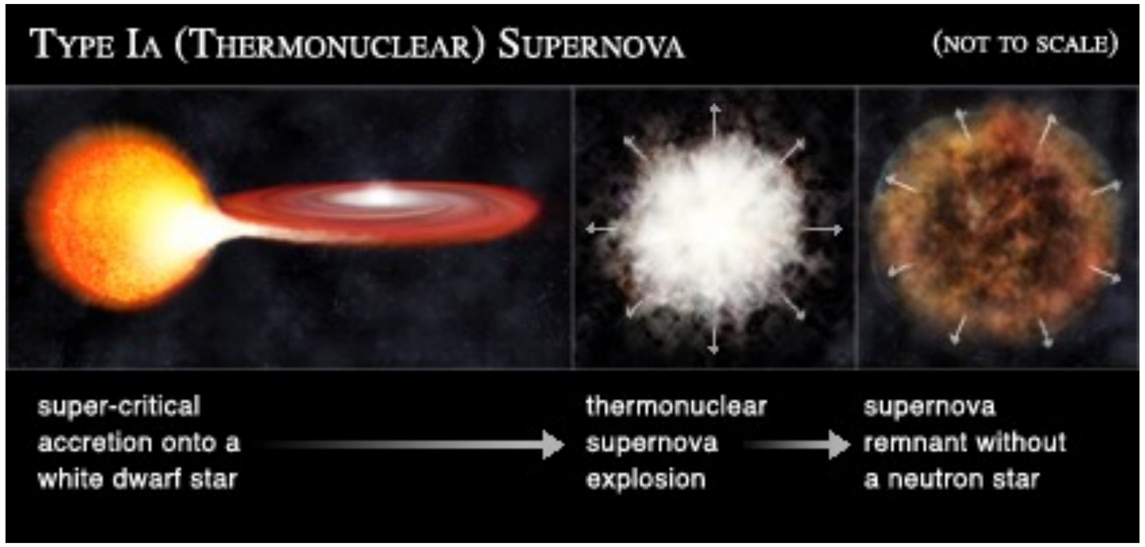
Perlmutter; Hamuy et al.

Lawrence Livermore National Laboratory

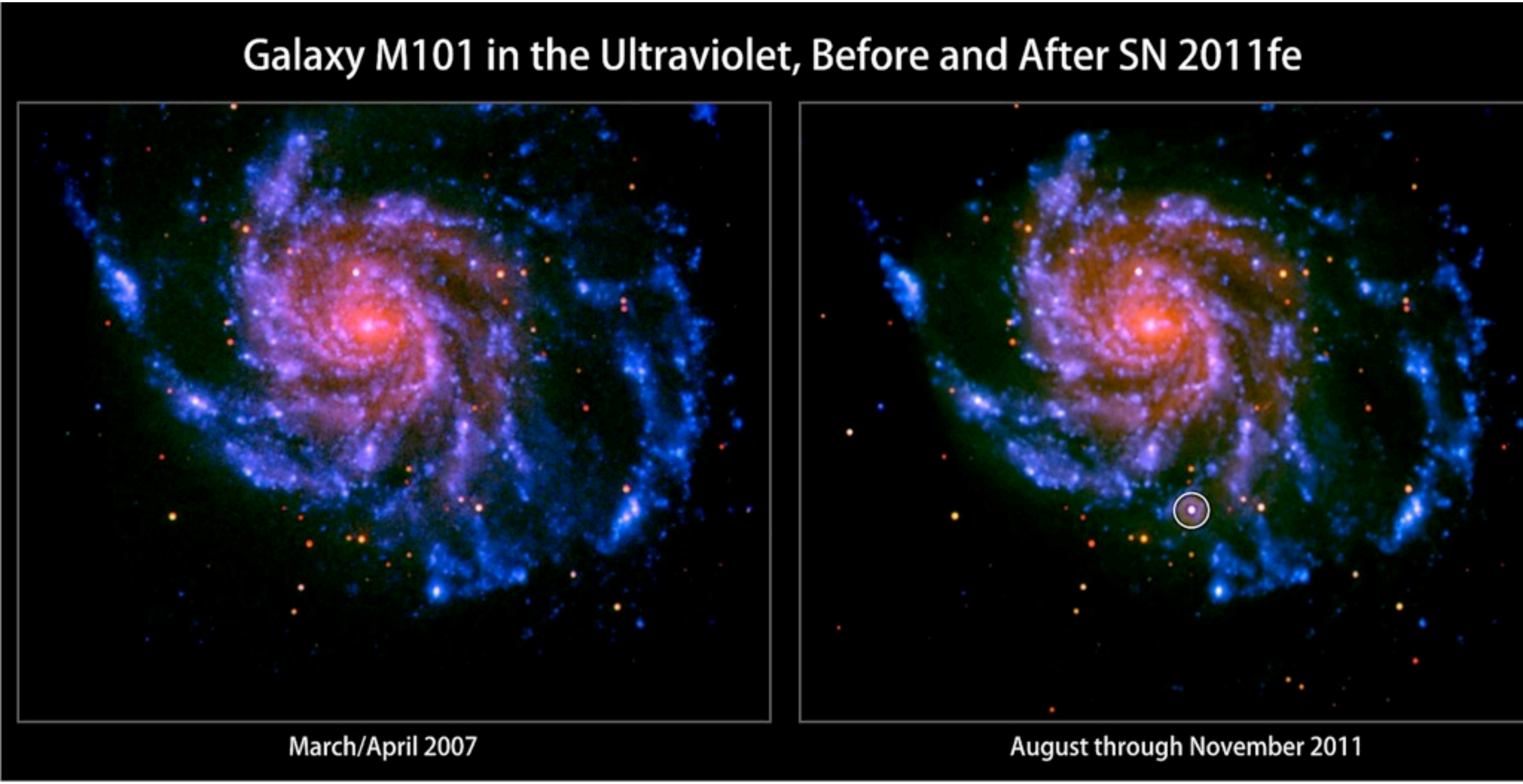


(Foreground) An artist’s conception of a Type Ia supernova precursor system depicts a companion star accreting material through an accretion disk onto a dead stellar core composed of carbon and oxygen, called a white dwarf. (Drawing courtesy of Space Telescope Science Institute and NASA.) (Background) In 1931, astrophysicist Subrahmanyan Chandrasekhar showed theoretically that a white dwarf would explode as a supernova when its mass exceeded what is now called the Chandrasekhar limit (1.38 times the mass of our Sun). In 1983, Chandrasekhar, along with William Alfred Fowler, was awarded the Nobel Prize in Physics in part for this work.

Illustration: NASA/CXC/M.Weiss



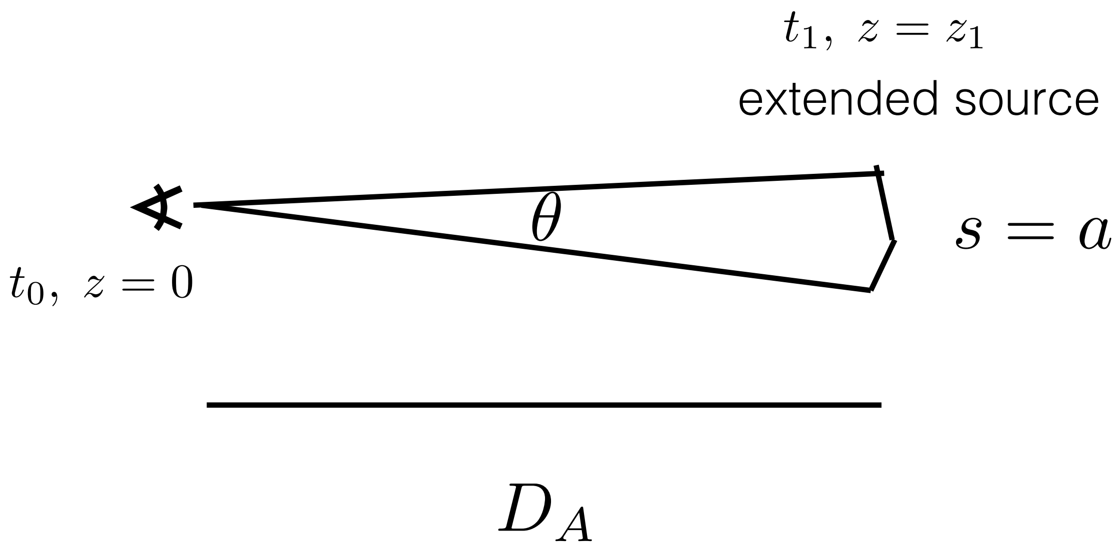
Cosmology 1



These images from Swift's Ultraviolet/Optical Telescope (UVOT) show the nearby spiral galaxy M101 before and after the appearance of SN 2011fe (circled, right), which was discovered on Aug. 24, 2011. At a distance of 21 million light-years, it was the nearest Type Ia supernova since 1986 but appeared too late for inclusion in the published studies. Left: View constructed from images taken in March and April 2007. Right: The supernova was so bright that most UVOT exposures were short, so this view includes imagery from August through November 2011 to better show the galaxy. Credit: NASA/Swift/Peter Brown, Univ. of Utah

Cosmology 1

- Another distance measure is the angular diameter distance D_A .



$t_1, \; z = z_1$

extended source

θ

$t_0, \; z = 0$

D_A

$s = a(t_1) r_1 \theta$

FRW metric in spherical coord.

proper distance perpendicular to the line of sight (along the arc)

define diameter distance D_A so that θ is given by the usual relation of euclidean geometry: $\theta = s/D_A$

$$\boxed{D_A = a(t_1) r_1}$$

- Luminosity distance $D_L = a_0 r \frac{a_0}{a(t)}$

➡ $$\boxed{\frac{D_A}{D_L} = (1+z)^{-2}}$$

More relevant for graviational lensing and CMB anisotropies.

Cosmology 1

- Friedmann equations

Hubble parameter

$$H = \frac{\dot{a}}{a}$$

Evolution of matter

$$\boxed{\dot{\rho} + 3H(\rho + p) = 0}$$

Equation of state

$$p = w\rho$$

radiation

$$w = \frac{1}{3}$$

matter

$$w = 0$$

DYNAMICS
OF THE FRW
MODELS

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p)$$

energy density

pressure

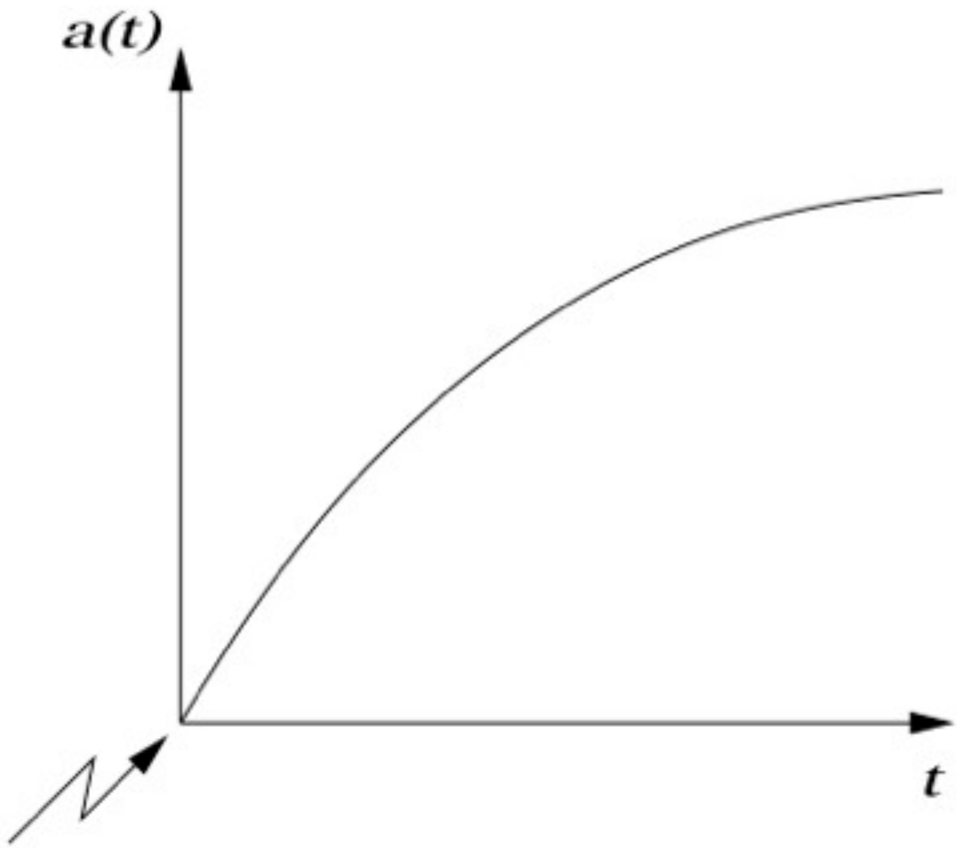
PERFECT
FLUID

Cosmology 1

- Typical solutions for radiation and matter in flat universe (k=0)

$\rho \sim a^{-4}, a \sim t^{\frac{1}{2}}$

$\rho \sim a^{-3}, a \sim t^{\frac{2}{3}}$



BIG BANG

singularity

Cosmology 1

- Density parameter

$$\Omega \equiv \frac{\rho}{\rho_c} \qquad \rho_c \equiv \frac{3H^2}{8\pi G}$$

Friedmann equation takes the form:

$$\frac{k}{H^2 a^2} = \Omega - 1$$

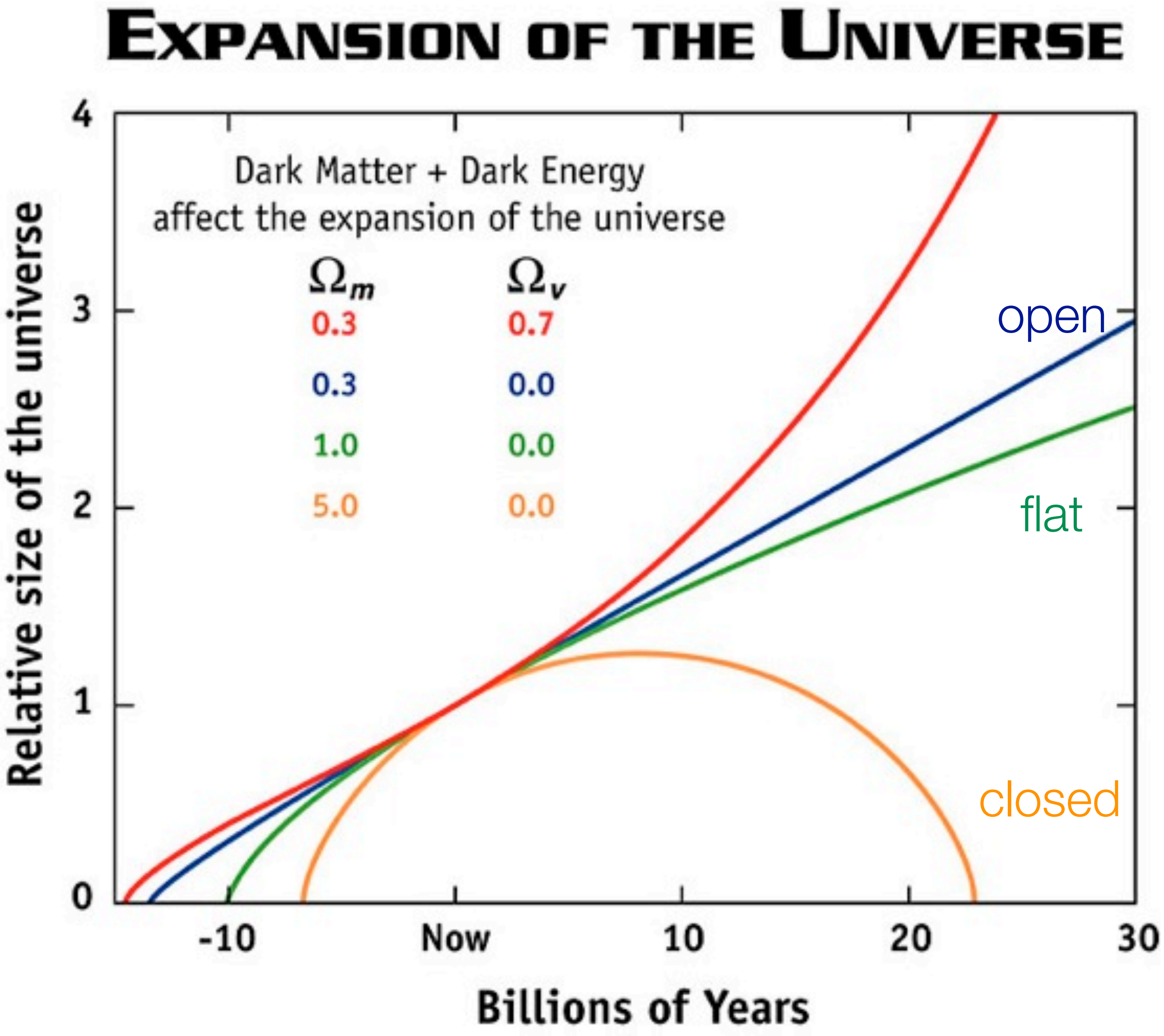
- Cosmic inventory

$$H^2 = \frac{8\pi G}{3}\rho_R + \frac{8\pi G}{3}\rho_M - \frac{k}{a^2} + \frac{\Lambda}{3}$$

what contributes
to the effective
energy density:
radiation (R),
matter (M) etc.

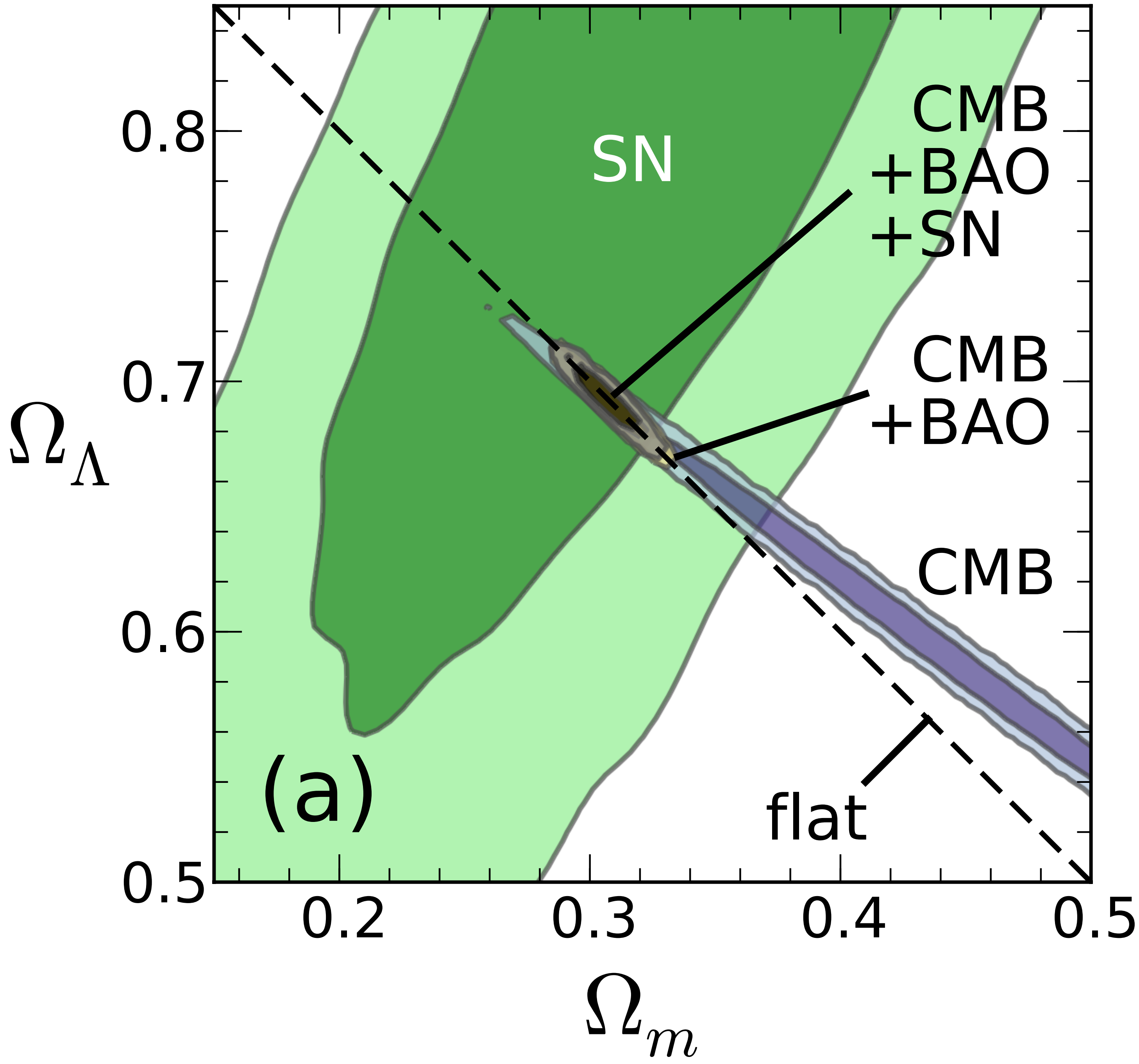
$$\Rightarrow \qquad \boxed{\Omega_R + \Omega_M - \Omega_k + \Omega_\Lambda = 1}$$

Cosmology 1



NASA/WMAP Science Team

Cosmology 1

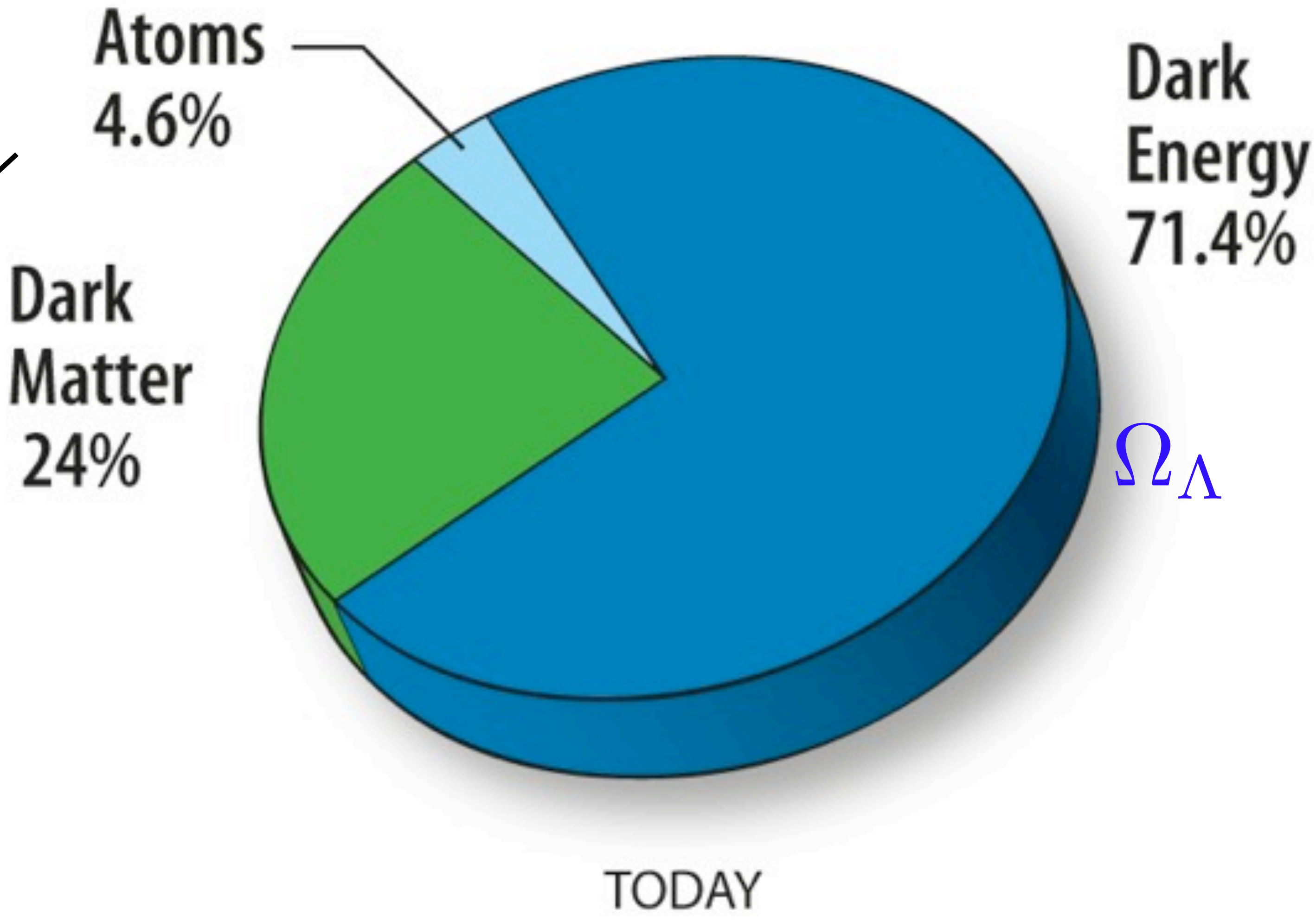


PDG 2014

Cosmology 1

- Cosmological parameters

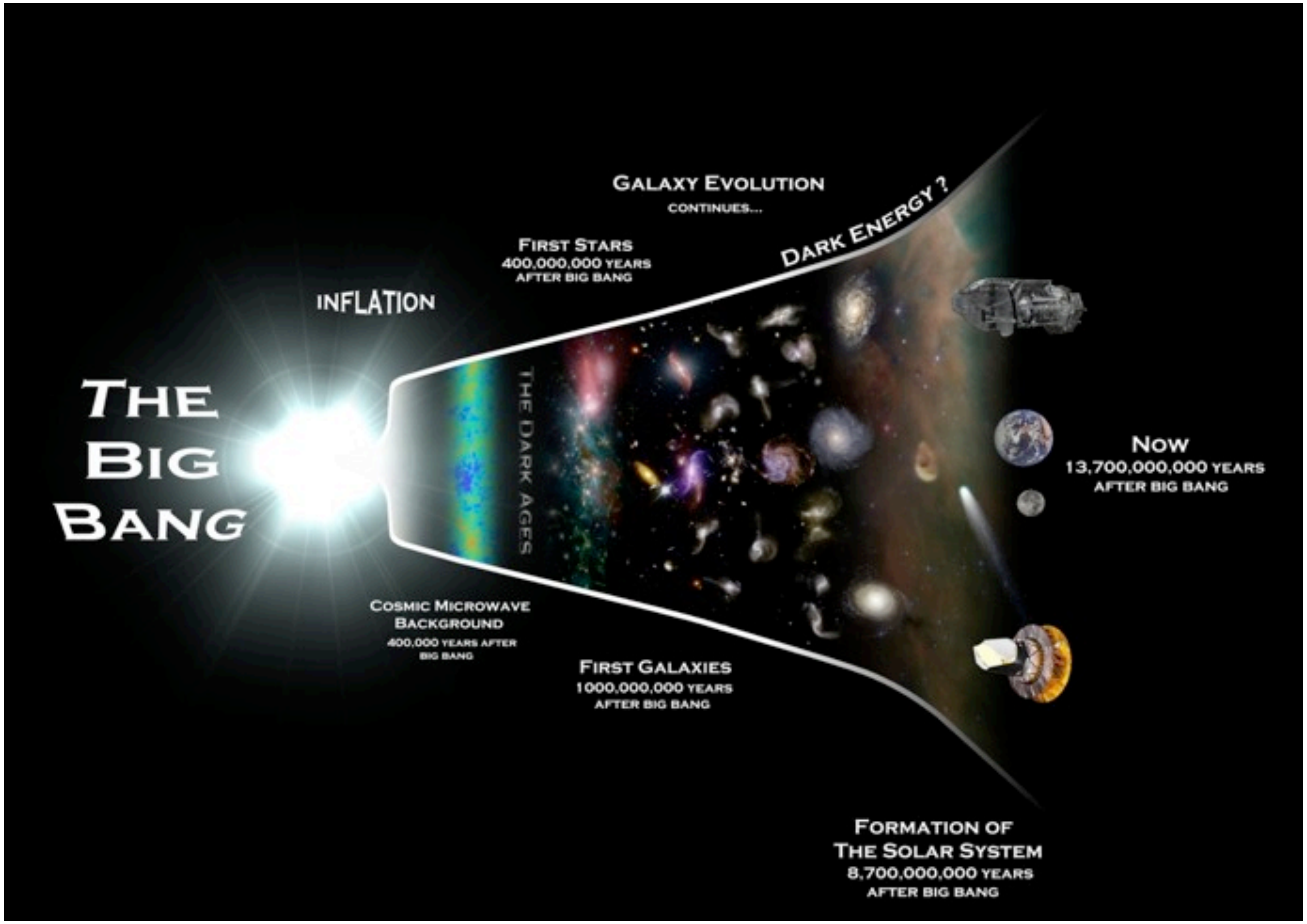
$$\Omega_M = \Omega_b + \Omega_{cdm}$$



Ω_Λ

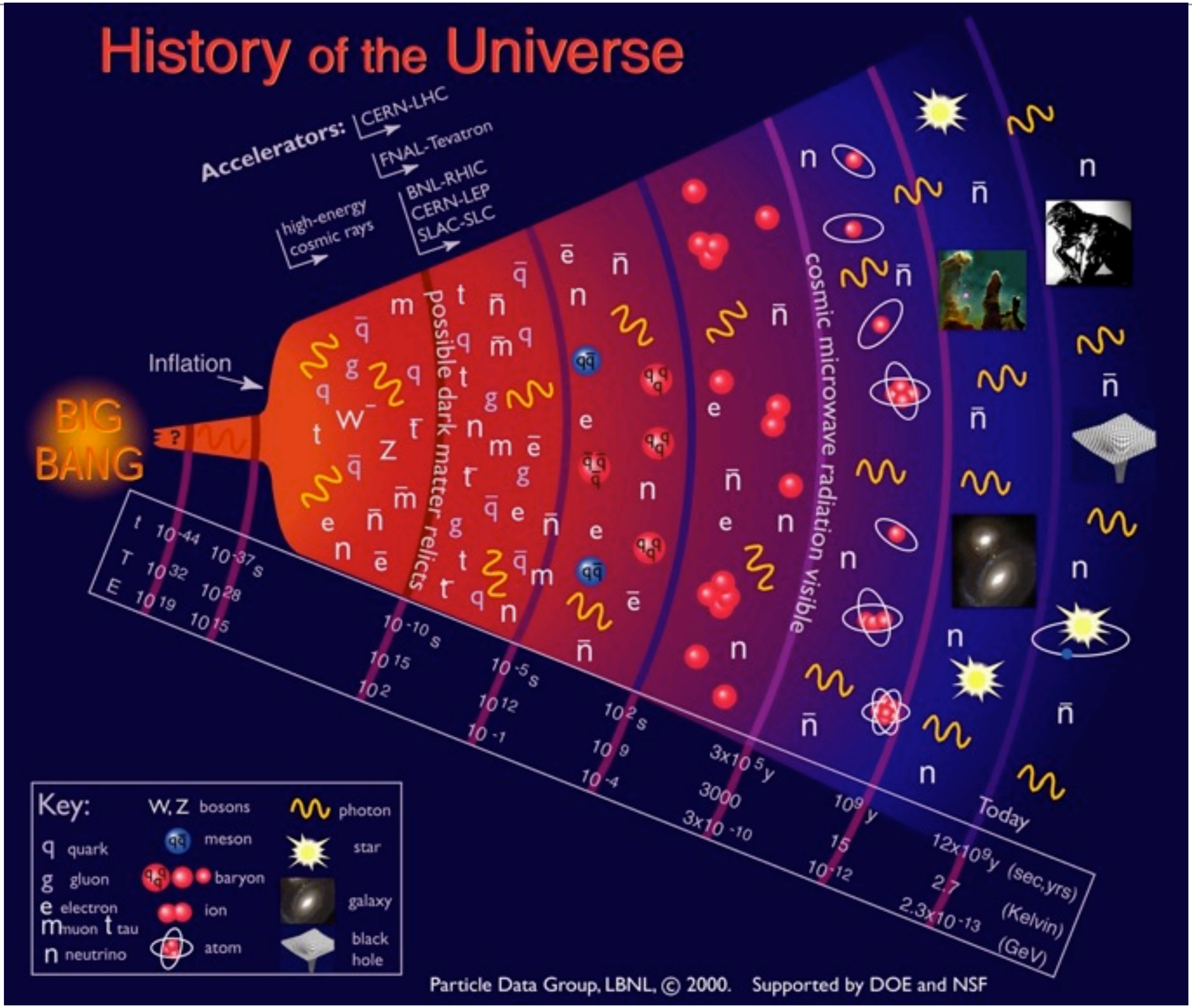
Cosmology 1

- Brief thermal history of the universe



NASA

Cosmology 1


History of the Universe
Accelerators:
CERN-LHC
FNAL-Tevatron
BNL-RHIC
CERN-LEP
SLAC-SLC
high-energy cosmic rays
Inflation
BIG BANG
?
possible dark matter relicts
cosmic microwave radiation visible
t 10^-44 10^-37 s
T 10^32 10^28
E 10^19 10^15
10^-10 s
10^15
10^2
10^-5 s
10^12
10^-1
10^2 s
10^9
10^-4
3x10^5 y
3000
3x10^-10
10^9 y
15
10^-12
Today
12x10^9 y (sec,yrs)
2.7 (Kelvin)
2.3x10^-13 (GeV)
Key:
q quark
g gluon
e electron
m muon
t tau
n neutrino
W,Z bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL, © 2000. Supported by DOE and NSF

Cosmology 1

- Brief thermal history of the universe

- For times before $t \sim 10^{-43}\ \mathrm{s}$ (Planck epoch): nothing is really known: need quantum corrections to general relativity (theory of quantum gravity ???)

- $t > 10^{-43}\ \mathrm{s}$ (corresponds to a temperature $T \sim 10^{19}\ \mathrm{GeV}$: plasma of relativistic particles including quarks, leptons, gauge bosons, Higgs bosons)

- At temperatures $100 - 300\ \mathrm{MeV}$, at a time $t \sim 10^{-5}\ \mathrm{s}$ takes place the quark-gluon phase transition: free quarks and gluons combine to barions y mesons.

Cosmology 1

- At temperatures $T \sim 10 - 0.1\ \mathrm{MeV}$, at a time $t \sim 10^{-2} - 10^{2}\ \mathrm{s}$

  ➡ **primordial nucleosynthesis** takes place:

  ★production of light elements: hidrogen, helium-4...

  ★earliest time to test standard model of cosmology.

  ➡during this epoch (standard model, light) neutrinos decouple (stop interacting with the rest of the particles)

Cosmology 1

- At a temperature $T \sim \mathrm{eV}$, at a time $t \sim 10^{11}\ \mathrm{s}$ takes place the epoch of matter-radiation equality: moment when the density of matter and radiation energy densities are the same.

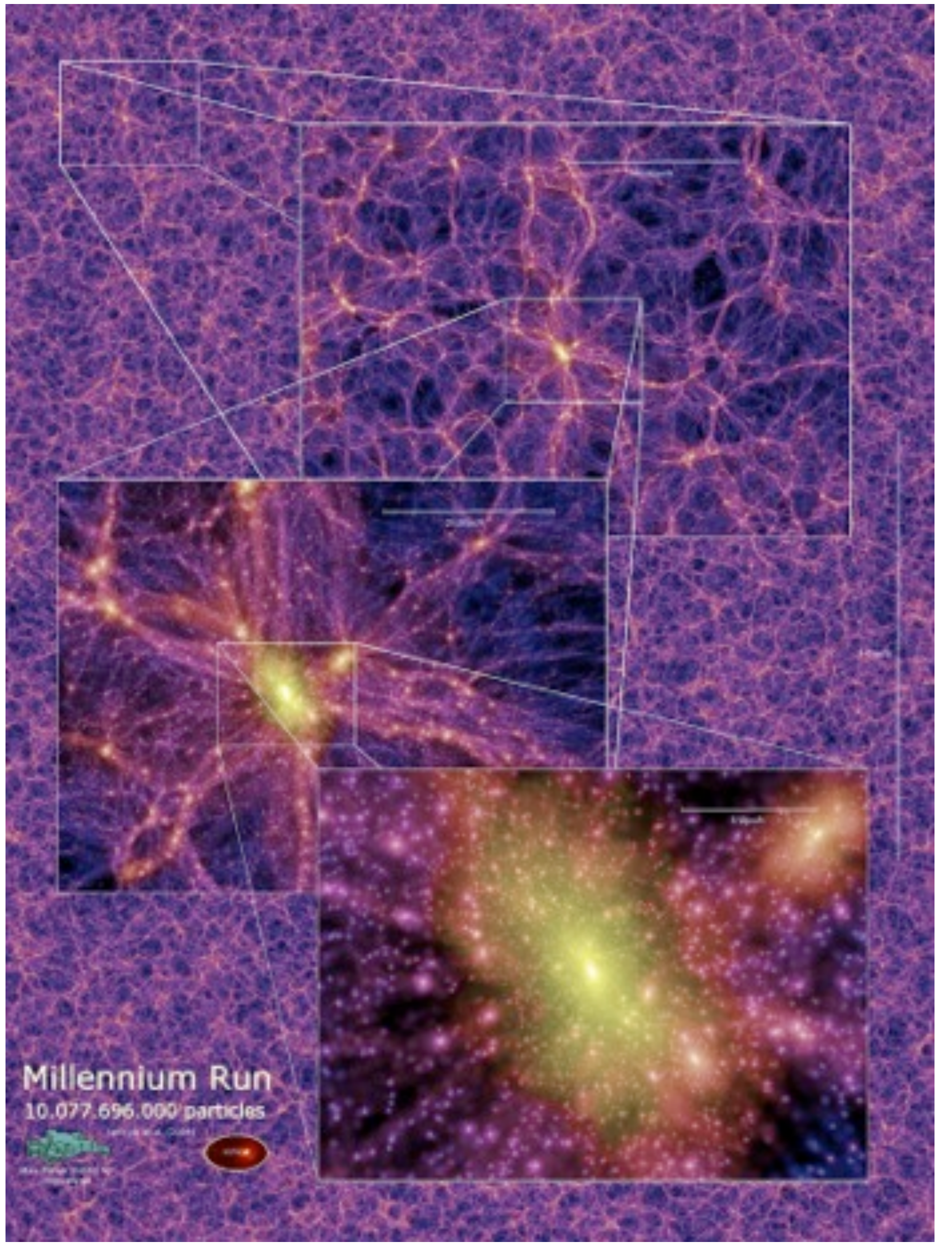
➡ Ends the epoch radiation domination, starts the matter dominated era.

- At a time $t \sim 10^{12} - 10^{13}\ \mathrm{s}$ nearly all electrons and protons recombine to form neutral hydrogen (*recombination*). Shortly afterwards the universe becomes transparent to radiation. Radiation, that is the photons decouple from matter ⇨ origin of ***cosmic microwave background*** (CMB).

- Temperature fluctuations in the CMB caused by density fluctuations present before photon decoupling ⇨ **test physics of the very early universe**

Cosmology 1

- At a time $t \sim 10^{16} - 10^{17}\ \mathrm{s}$: nonlinear evolution, gravitational instability → galaxy formation



Cosmology 1

- Thermodynamics of the early universe

- Early on the universe is dense and hot and its matter content is dominated by radiation dominated equation of state.

- Thermal equilibrium is established by rapid interactions of particles in comparison to the time scale of expansion of the universe.

Cosmology 1

- Thermodynamics in equilibrium

- The number density n, the energy density ρ, the pressure P of a gas of particles without strong interaction with g internal degrees of freedom is given in terms of the distribution function $f(\vec{p})$ in phase space by:

$$\boxed{\begin{aligned} n &= \frac{g}{(2\pi)^3}\int f(\vec{p})d^3p \\ \rho &= \frac{g}{(2\pi)^3}\int E(\vec{p})f(\vec{p})d^3p \\ P &= \frac{g}{(2\pi)^3}\int \frac{|\vec{p}|}{3E}f(\vec{p})d^3p \end{aligned}}$$

where

$$E^2 = |\vec{p}|^2 + m^2$$

Cosmology 1

- In (kinetic) equilibrium the distribution function f corresponds to Fermi-Dirac o Bose-Einstein statistics, respectively:

$$f(\vec{p}) = [\exp((E-\mu)/T) \pm 1]^{-1}$$

μ. chemical
potential

+1: Fermi-Dirac
-1: Bose-Einstein

- If the particle species are in chemical equilibrium, and the species i interacts with the species j,k,l: i+j ⟷ k+l

⇒

$$\mu_i + \mu_j = \mu_k + \mu_l$$

Cosmology 1

- $\mu_\gamma = 0$
- Particle-anti particle pairs annihilate to photons $\Longrightarrow$ $\mu_p = -\mu_{\bar{p}}$

Cosmology 1

- Relativistic limit $T \gg m, T \gg \mu$

fermions

$$\rho = \frac{7}{8}\frac{\pi^2}{30} g T^4$$

$$n = \frac{3}{4}\frac{\zeta(3)}{\pi^2} g T^3$$

bosons

$$\rho = \frac{\pi^2}{30} g T^4$$

$$n = \frac{\zeta(3)}{\pi^2} g T^3$$

$$p = \frac{1}{3}\rho$$

$$\zeta(3) = 1.20206...$$

Cosmology 1

- Non relativistic limit $m \gg T$

fermions, bosons

$$n = g\left(\frac{mT}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{m-\mu}{T}}$$

$$\rho = nm$$

$$p = nT \ll \rho = nm$$

Cosmology 1

- Write total energy density and pressure of all species in equilibrium in terms of photon temperature T:

$$\rho_R = T^4 \sum_{i=all\ species} \left(\frac{T_i}{T}\right)^4 \frac{g_i}{2\pi^2} \int_{x_i}^{\infty} \frac{(u^2 - x_i^2)^{\frac{1}{2}} u^2 du}{\exp(u - y_i) \pm 1}$$

$$p_R = T^4 \sum_{i=all\ species} \left(\frac{T_i}{T}\right)^4 \frac{g_i}{6\pi^2} \int_{x_i}^{\infty} \frac{(u^2 - x_i^2)^{\frac{3}{2}} u^2 du}{\exp(u - y_i) \pm 1}$$

where $u \equiv \frac{E}{T_i}, \quad x_i \equiv \frac{m_i}{T_i} \quad y_i \equiv \frac{\mu_i}{T_i}$

Cosmology 1

- Since the energy density and pressure of a non relativistic species (ρ=mn, P=nT) it is a good approximation only to include contributions from relativistic species in the sum

$$\rho_R = \frac{\pi^2}{30} g_* T^4 \qquad\qquad p_R = \frac{1}{3}\rho_R = \frac{\pi^2}{90} g_* T^4$$

g_* counts the total number of effectively massless degrees of freedom,

$m_i \ll T$

$$g_* = \sum_{i=bosons} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{i=fermions} g_i \left(\frac{T_i}{T}\right)^4$$

depends on temperature

Cosmology 1

- g_* function of temperature T, since sum includes massive species $m_i \ll T$.

- For $T \ll 1\mathrm{MeV}$: the only relativistic species: 3 species of light neutrinos and the photon.

    ✴ Taking into account

$$T_\nu = \left(\frac{4}{11}\right)^{\frac{1}{3}} T_\gamma$$

⇒

$$g_*(T \ll \mathrm{MeV}) = 2 \times \frac{7}{8} \times 3 \left(\frac{4}{11}\right)^{\frac{4}{3}} + 2 = 3.36$$

$\nu, \bar{\nu}$ γ

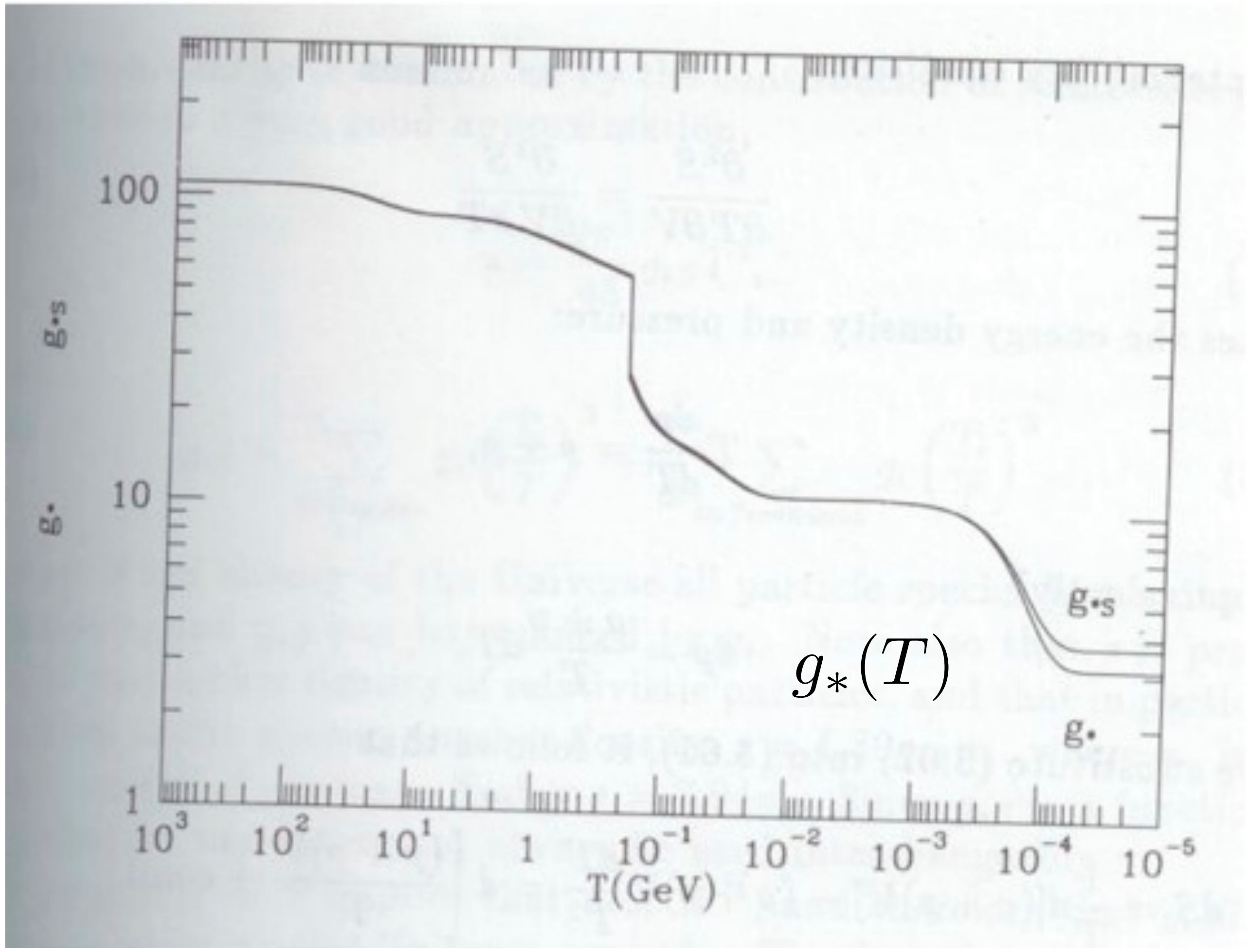
Cosmology 1

- $100\mathrm{Mev} > T > 1\mathrm{MeV}$ also electrons and positrons are relativistic:

$$g_* = 2 + \frac{7}{8}(2 + 2 + 2 \times 3) = \frac{43}{4} = 10.75$$

$$\gamma \qquad e^- \quad e^+ \qquad \nu, \bar{\nu}$$

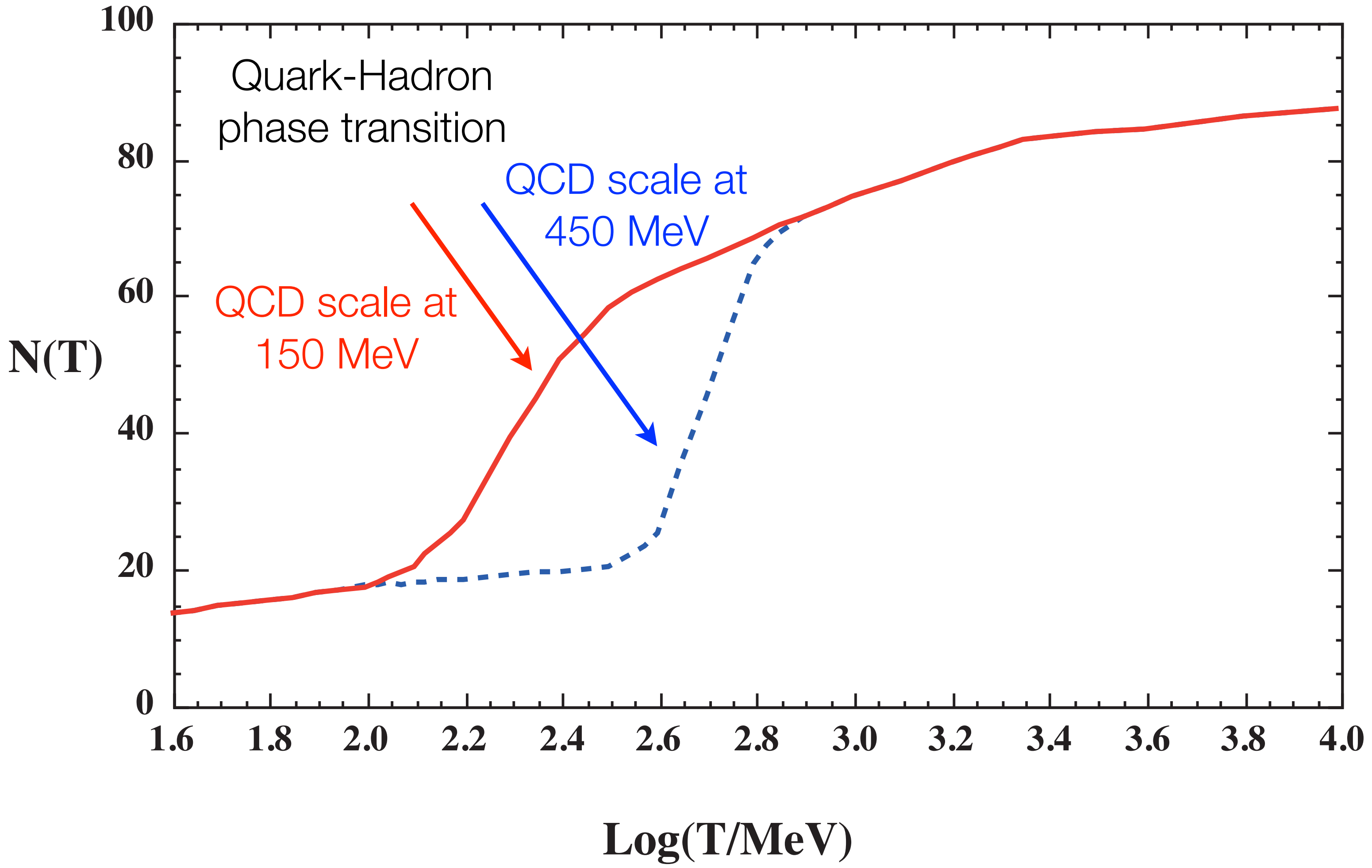
- $T > 300\mathrm{GeV}$ all species of the standard model are relativistic.



E. Kolb, M. Turner “The Early Universe”

Cosmology 1

- Effective number of relativistic degrees of freedom



Quark-Hadron phase transition

QCD scale at 150 MeV

QCD scale at 450 MeV

g_*

PDG 2014

Cosmology 1

- During the radiation dominated epoch $\rho = \rho_R$

$$g_* \sim const., p_R = \frac{\rho_R}{3}, a(t) \propto t^{\frac{1}{2}}$$

$$H^2 = \frac{8\pi}{3M_P^2}\frac{\pi^2}{30}g_* T^4$$

$$\boxed{H = 1.66 g_*^{\frac{1}{2}}\frac{T^2}{M_P}}$$

Planck mass

$$M_P = 1.22 \times 10^{19}\ \mathrm{GeV}$$

Cosmology 1

- Primordial nucleosynthesis - Big bang nucleosynthesis (BBN)
- Production of light elements in the early universe.
- Predictions of the abundances of the light elements D, He-3,He-4, Li-7 synthesized at the end of the first three minutes are in good agreement with observations.
- BBN provides powerful constraints on possible deviations from the standard cosmology and on new physics beyond the Standard Model.

Cosmology 1

- Important quantity: ratio of number density of neutrons over number density of protons n/p

- At very high temperatures (very early on) ($T \sim a^{-1}$) $T \gg 1\mathrm{MeV}$

$$\frac{n}{p} = 1$$

weak
interactions
very effective

- In thermal equilibrium

$$\frac{n}{p} = e^{-\frac{Q}{T}}$$

$$Q = m_n - m_p = 1.293\mathrm{MeV}$$

Cosmology 1

- At around $T_D = 0.7\mathrm{MeV}$ the neutron-proton interconversion rate

$$\Gamma_{n\leftrightarrow p} \sim G_F^2 T^5$$

drops below the expansion rate $H \sim \sqrt{g_* G_N} T^2$

g_* counts relativistic degrees of freedom (e.g. at temperatures much larger than 1 MeV, electrons, positrons, 3 families of neutrinos + antineutrinos, photons contribute)

n/p freezes out at around 0.7 MeV: $$\frac{n}{p} = \frac{1}{6}$$

$$\frac{n}{p} = e^{-\frac{Q}{T}}$$

Cosmology 1

- After freeze-out the free neutrons can decay via n->p+e+v, thereby reducing n/p.
- Free neutron life time $\tau = 885.7 \pm 0.8s$
- After time t number density of neutrons given by $n_n(0)e^{-\frac{t}{\tau}}$
- Final n/p: $$\frac{n}{p} = \frac{1}{7}$$
- Nearly all neutrons end up in helium-4.

Cosmology 1

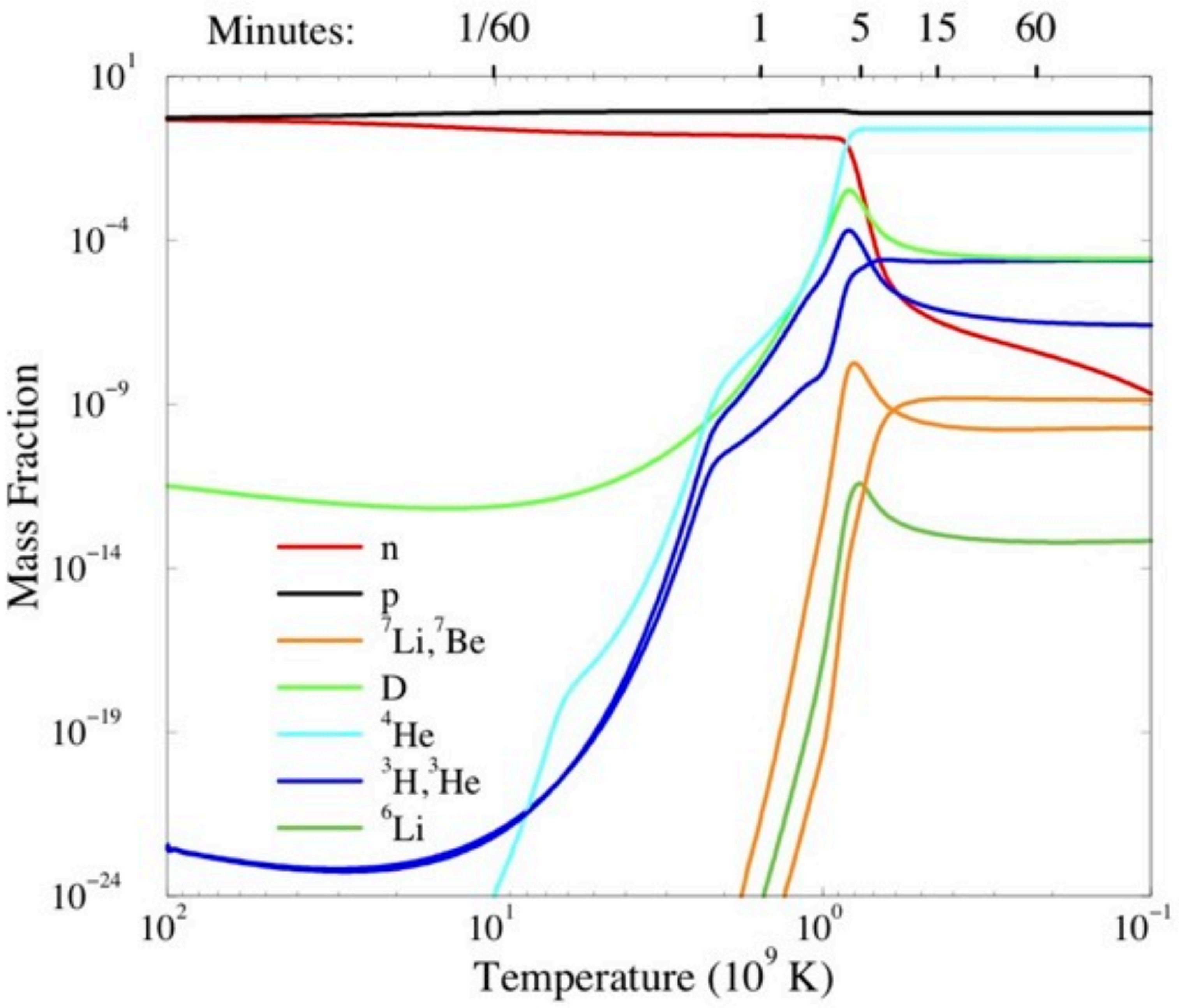
- The nucleosynthesis chain begins with the formation of deuterium in the process p(n,γ)D.

- Only 2-body reactions, such as D(p,γ)He-3, He-3(D,p)He-4, are important because the density by this time has become rather low.

- Nearly all neutrons end up bound in the most stable light element He-4.

- Heavier nuclei do not form in any significant quantity both because of the absence of stable nuclei with mass number 5 or 8 (which impedes nucleosynthesis via nHe-4, pHe-4 or He-4He-4 reactions), and the large Coulomb barriers for reactions such as He-3(He-4,γ)Li-7 and He-3(He-4,γ)Be-7.

Cosmology 1



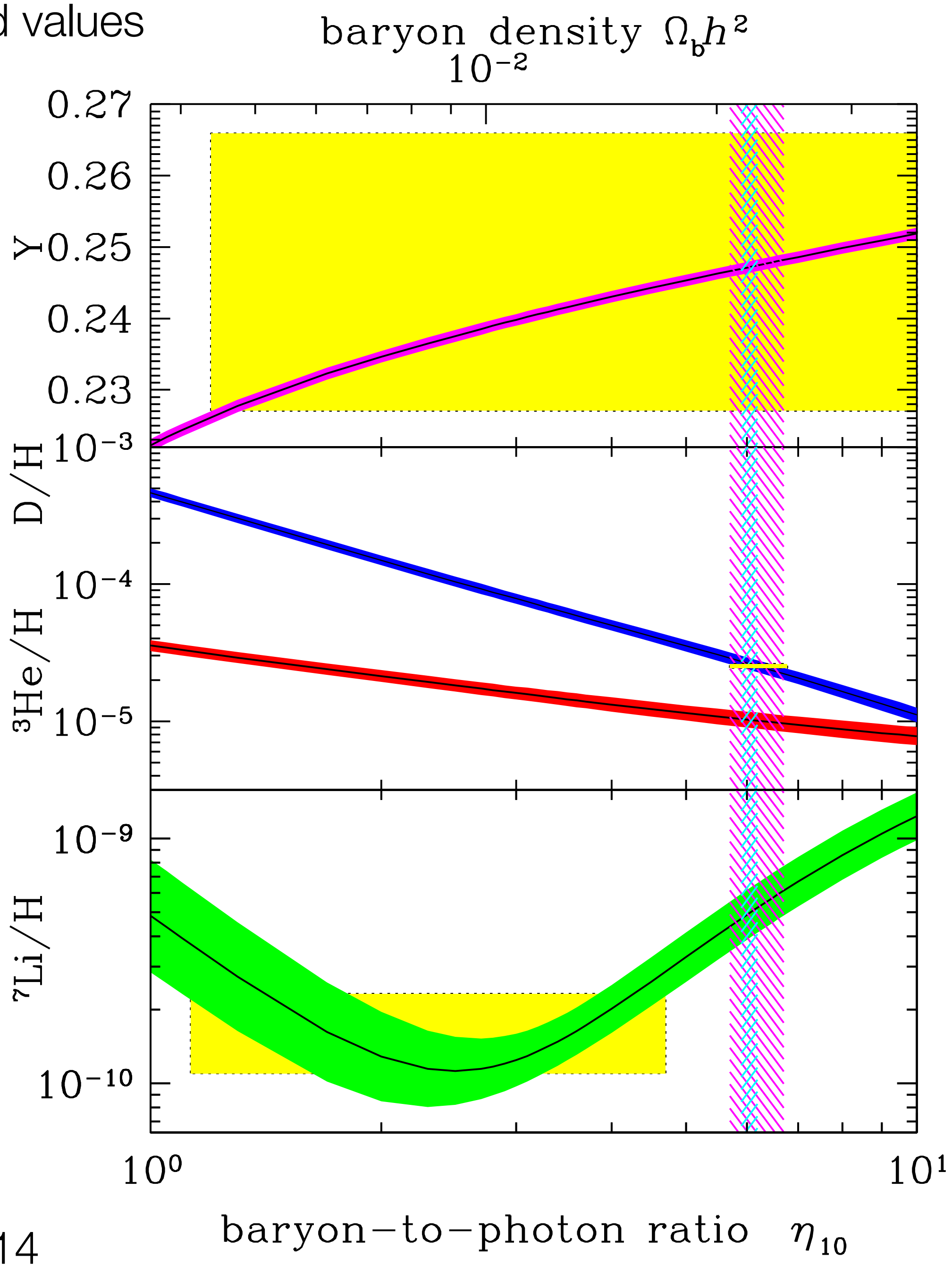
Burles, Nollett, Turner (1999)

Cosmology 1

- Primordial mass fraction of helium-4, Y_p,

$$Y_p = \frac{2(n/p)}{1 + n/p} \simeq 0.25$$

Boxes indicate observed values



PDG 2014

observed value:

Yp = 0.2465 ± 0.0097.

Observations of primordial abundances:

•BBN theory predicts the universal abundances of D, 3He, 4He, and 7Li, which are essentially fixed by t ~ 180 s.

•Abundances usually observed at much later epochs, after stellar nucleosynthesis has begun.

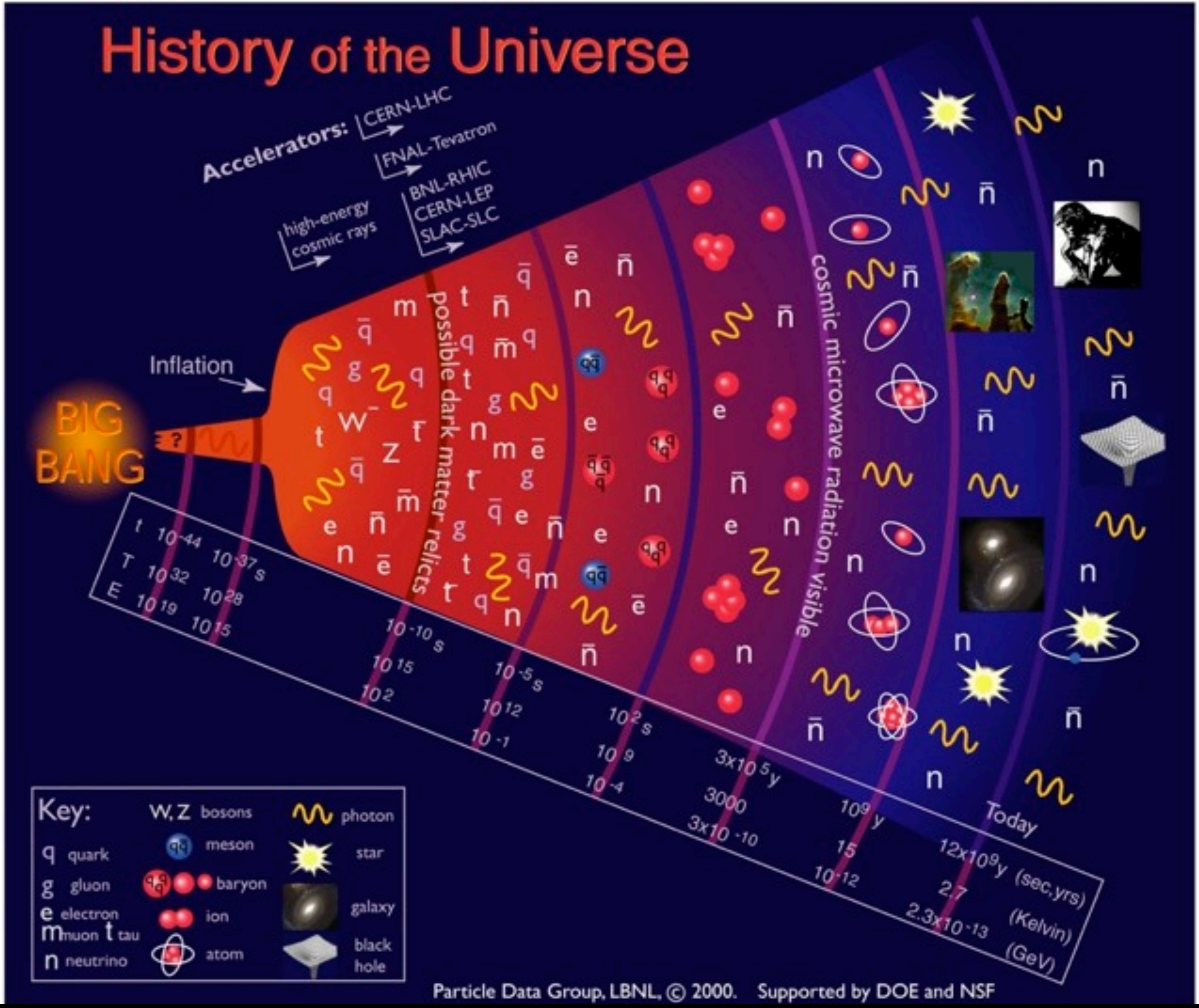
•Ejected remains of this stellar processing can alter the light element abundances from their primordial values, and also produce heavy elements such as C, N, O, and Fe ('metals').

•Thus find **astrophysical sites with low metal abundances**, in order to measure light element abundances which are closer to primordial.

Cosmology 1

- Successes: Expansion, Abundances of light elements, cosmic microwave background

Big Bang Model: Successes and Shortcomings



Particle Data Group, LBNL, © 2000. Supported by DOE and NSF

Cosmology 1

- Problems

★ Flatness problem $\Omega_0 = \Omega_m + ... + \Omega_\Lambda = 1.006 \pm 0.007$

$$|\Omega - 1| = \frac{|k|}{a^2 H^2} \sim t^{\frac{2}{3}}$$

matter
dominated
universe

Assuming $0.01 < \Omega_0 < 10$ today, so at the time of nucleosynthesis at 1s

$$|\Omega - 1|_{BBN} = |\Omega - 1|_0 \left(\frac{t_0}{t_{BBN}}\right)^{-\frac{2}{3}} \Longrightarrow |\Omega - 1|_{BBN} < 10^{-11}$$

★ fine tuning of initial conditions ★

Cosmology 1

- Horizon problem

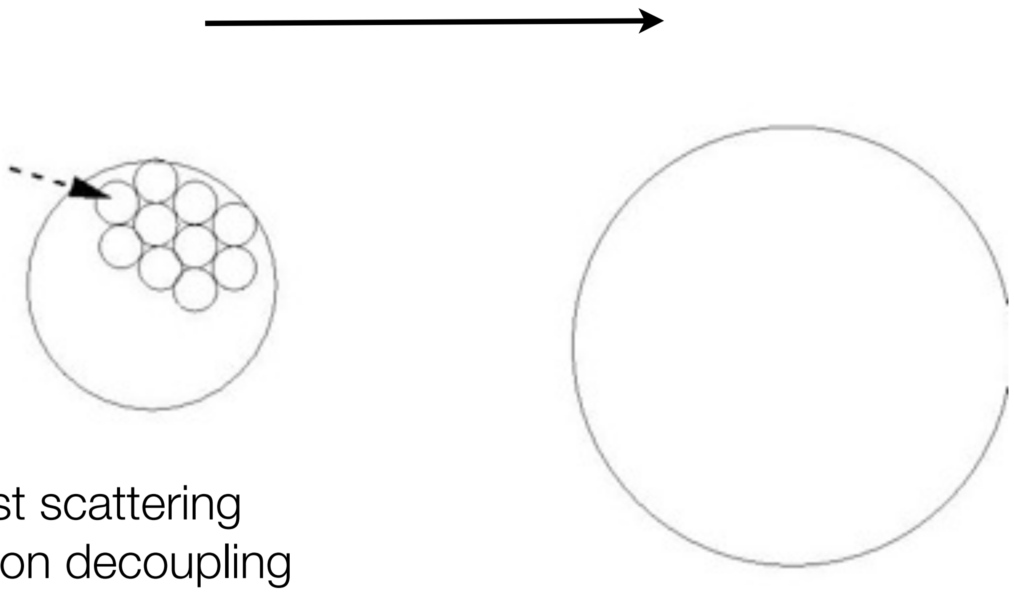
Speed of light is finite and so domains which can have been in causal contact have finite size.

Physical horizon distance

$$d_H(t) = a(t) \int_{t_i}^{t} \frac{dt}{a(t)} \sim H(t)^{-1}$$

Two points separated by more than this distance have not been in causal contact.

FRW universe, radiation, matter dominated



time

causal domain

last scattering
photon decoupling
CMB forms

today

How can the CMB be so isotropic?
Temperature fluctuations regions separated by 180° $\frac{\Delta T}{T} \sim 10^{-5}$

Cosmology 1

- Large scale structure formation → ***Cosmology 2***

- FRW is exactly homogeneous and isotropic.

  - How to explain the small temperature fluctuations in the CMB or CMB anisotropies?

  - How to explain galaxy formation? Small density fluctuations needed.

Cosmology 1

- Inflation does not replace standard cosmology. It is an era **before** the era of standard cosmology.

Guth 1981
Albrecht, Steinhardt 1982
Linde 1982, 1983

- Accelerated expansion of the universe

$$\ddot{a} > 0 \quad \Longleftrightarrow \quad \text{inflation}$$

- Cosmological constant

$$p = -\rho \qquad a = a_0 e^{Ht} \qquad H = const.$$

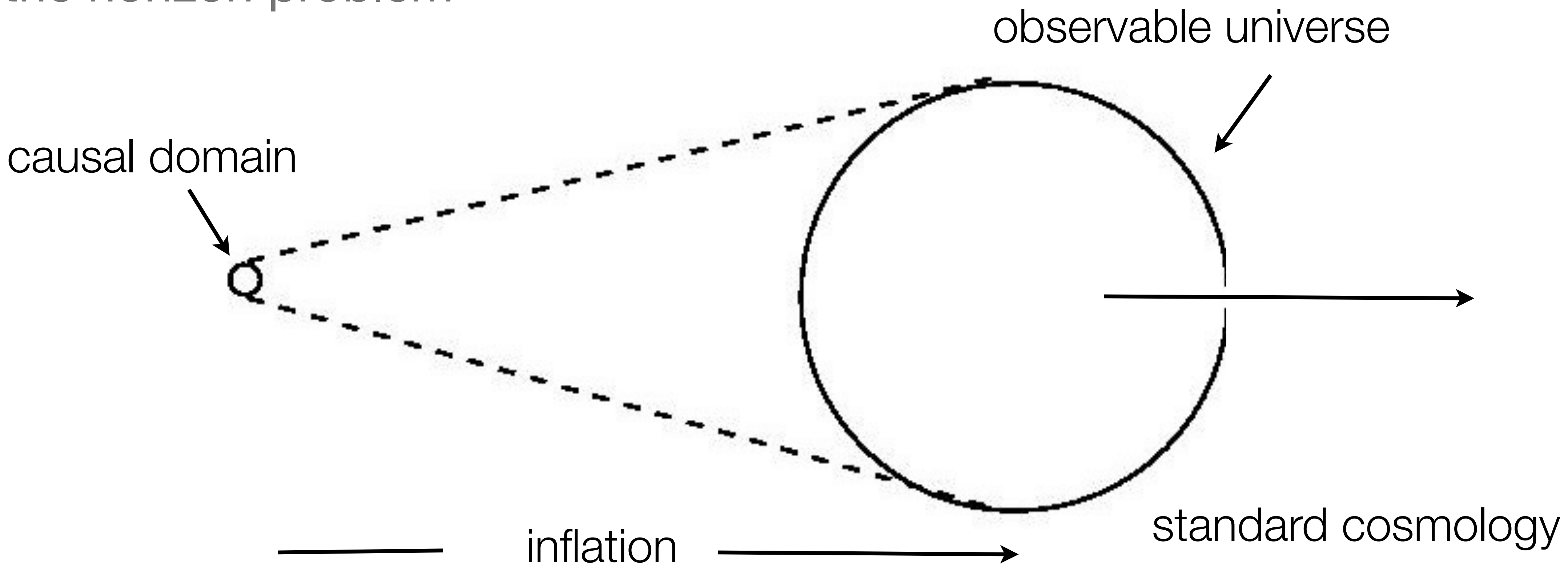
Cosmology 1

Inflation

- Solution to the flatness problem

$$|\Omega - 1| = \frac{|k|}{a^2H^2} \sim |k|e^{-2Ht} \to 0$$

- Solution to the horizon problem



Cosmology 1

- Solution to problem of fluctuations:
    - ✤ Quantum fluctuations of scalar field which become classical perturbations.

Cosmology 1

- Implementation of inflation

★ Scalar field ϕ

★ Recall

$$\frac{\ddot{a}}{a} = -\frac{4\pi}{3M_P^2}(\rho + 3p)$$

$$\frac{\ddot{a}}{a} > 0 \Leftrightarrow \rho + 3p < 0$$

★ Lagrangian

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \underset{\nwarrow}{V(\phi)}^{\text{potential}}$$

Cosmology 1

- Energy density and pressure

$$\rho_\phi = \frac{\dot{\phi}^2}{2} + V(\phi)$$

$$p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)$$

- Klein-Gordon equation

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{dV}{d\phi}$$

- Friedmann equation

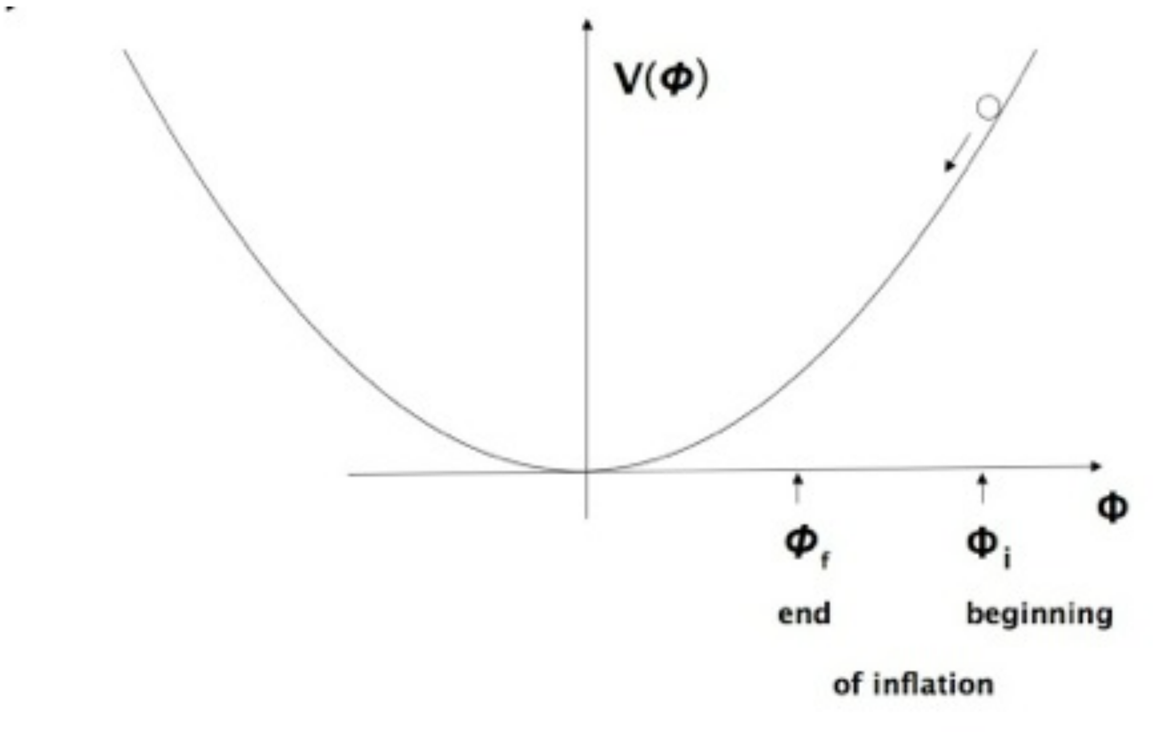
$$H^2 = \frac{8\pi}{3M_P^2}\left(\frac{\dot{\phi}^2}{2} + V(\phi)\right)$$

Cosmology 1

- Slow roll approximation

$$\cancel{\ddot{\phi}} + 3H\dot{\phi} = -\frac{dV}{d\phi}$$

$$H^2 = \frac{8\pi}{3M_P^2}\left(\cancel{\frac{\dot{\phi}^2}{2}} + V(\phi)\right)$$



Chaotic inflation (Linde 1984)

$$V(\phi) = \frac{1}{2}m^2\phi^2$$

Cosmology 1

- Slow roll parameters

$$\epsilon(\phi) = \frac{M_P^2}{16\pi}\left(\frac{V'}{V}\right)^2$$

$$\eta = \frac{M_P^2}{8\pi}\frac{V''}{V}$$

Condition for slow roll inflation

$$\epsilon \ll 1$$

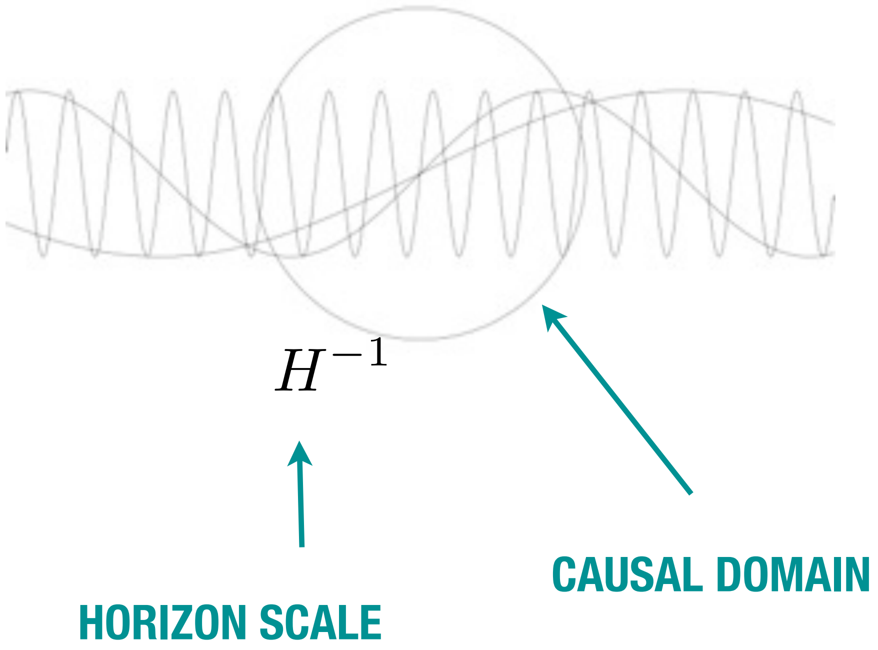
$$|\eta| \ll 1$$

- Number of e-folds: measures the amount of inflation $N(t) = \ln\frac{a(t_{fin})}{a(t)}$
- To solve flatness problem etc. $N \sim 55 - 65$

Cosmology 1

$$\phi(t,\vec{x}) = \phi(t) + \delta\phi(t,\vec{x}) \qquad \delta\phi(t,\vec{x}) = \sum_{\vec{k}} \delta\phi_{\vec{k}}(t) e^{i\vec{k}\cdot\vec{x}}$$

$$\delta\ddot{\phi}_{\vec{k}} + 3H\delta\dot{\phi}_{\vec{k}} + \left(\frac{k}{a}\right)^2 \delta\phi_{\vec{k}} = 0$$



- Fluctuations “freeze” on superhorizon scales, treat as classical contribution to classical values of inflaton field.

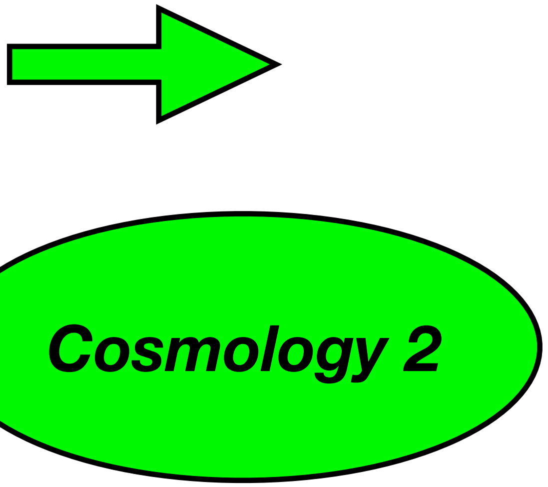
- Quantum fluctuations induce classical fluctuations with amplitude

$$|\delta\phi| \simeq \frac{H}{2\pi}$$

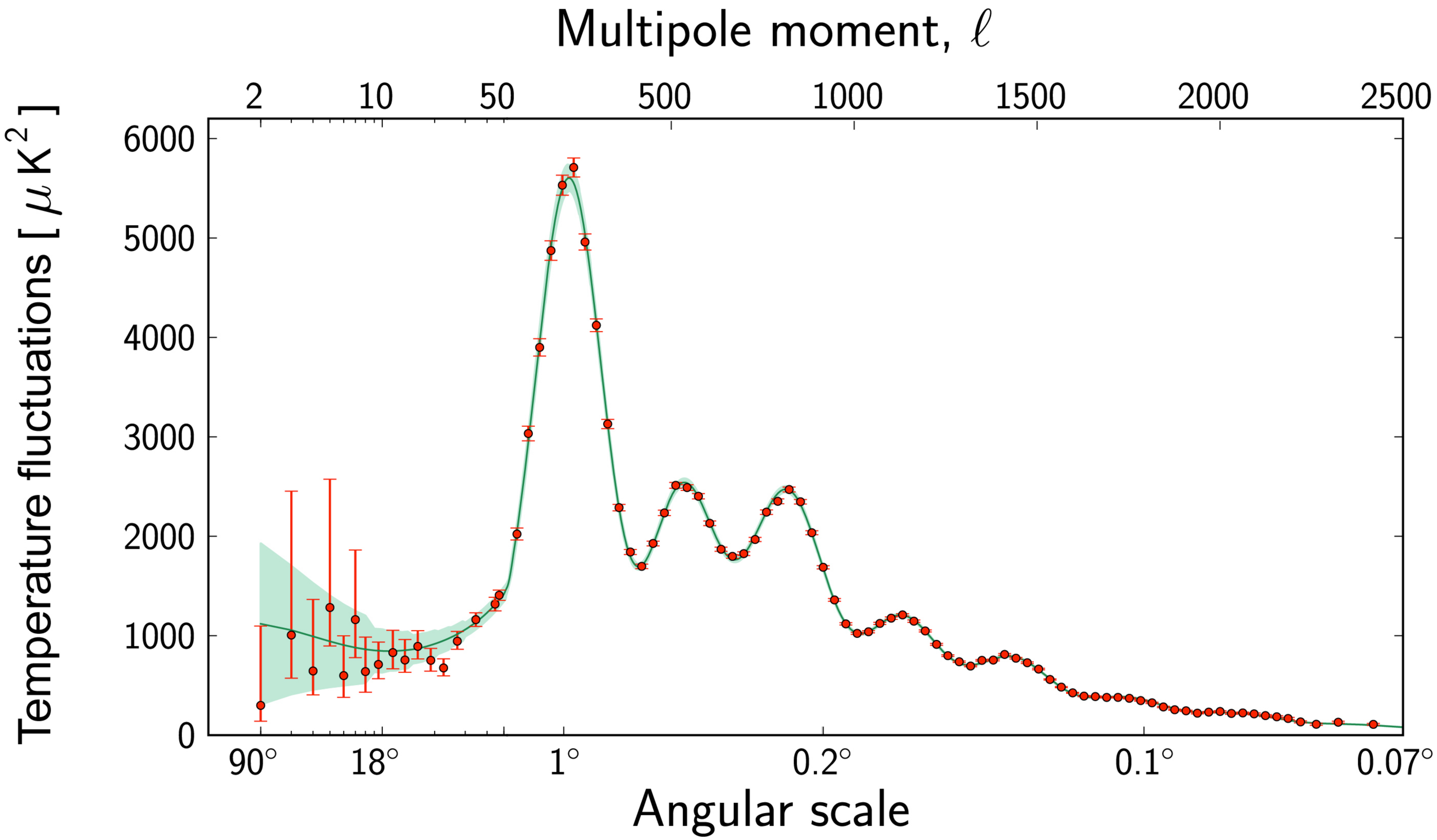
- Phases of each wave are random. Sum of all waves at a given point fluctuates, described by Brownian motion in all directions.

Cosmology 1

- Induces a curvature perturbation observable in the CMB



Cosmology 2



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