Neutral Meson Mixing

Why should we study this?

- CPV first observed in $K^0 - \bar{K}^0$
- Gives best flavor constraints on NP (as indicated previously)
- Neat phenomena
- Active field

Plan:

- Start with kaon: CPV in mixing (epsilon)
- CPV in decay (epsilon-prime)
- time dependent observables
- CPV in interference of mixing and decay (B)

Leave D-meson standard conventions as homework (same physics, different notation)

What is mixing?

Pictures from:
S. Wandernoth
Rencontres de Moriond 2013

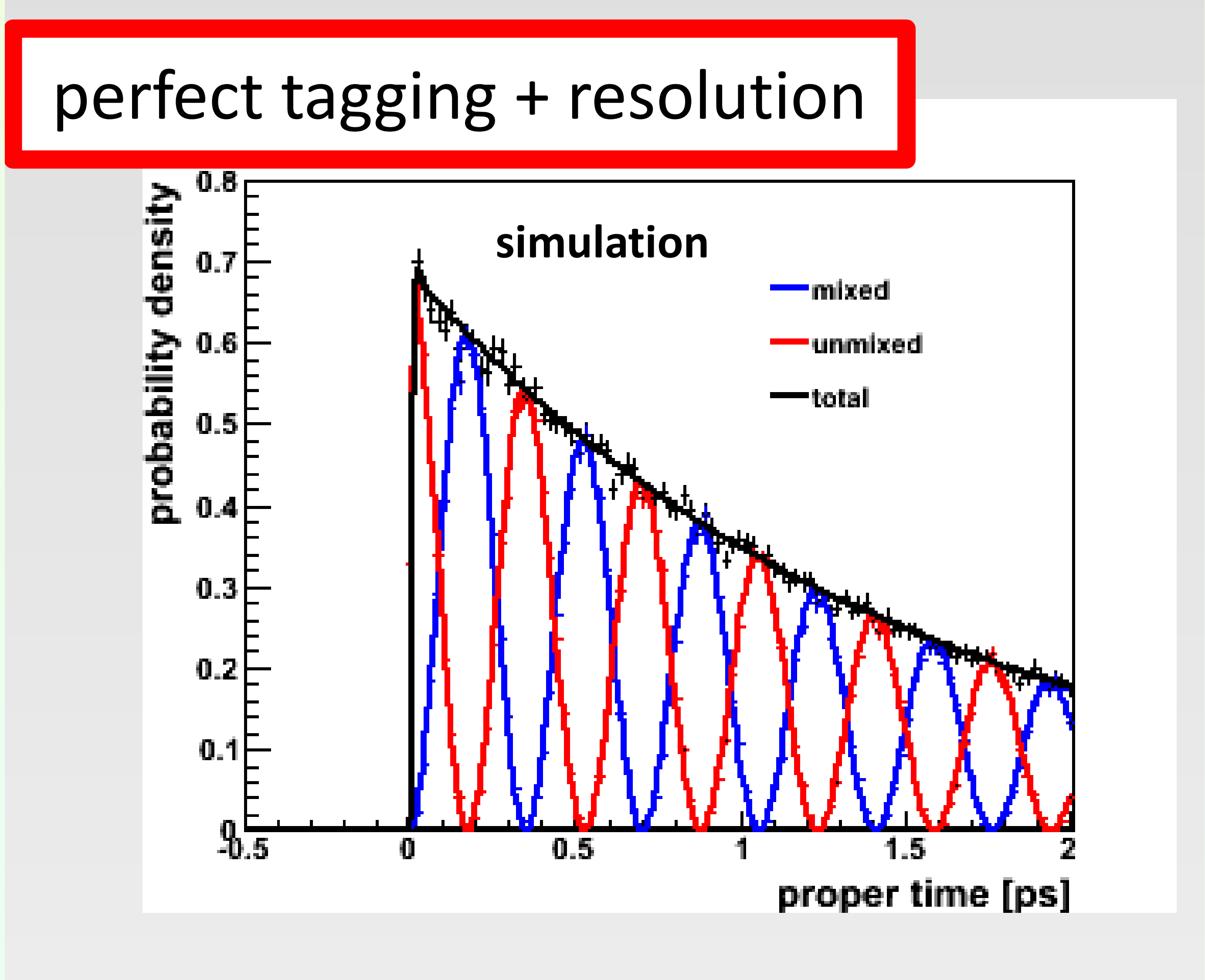
$\bar{B}_s \to D_s^+ \pi^-$

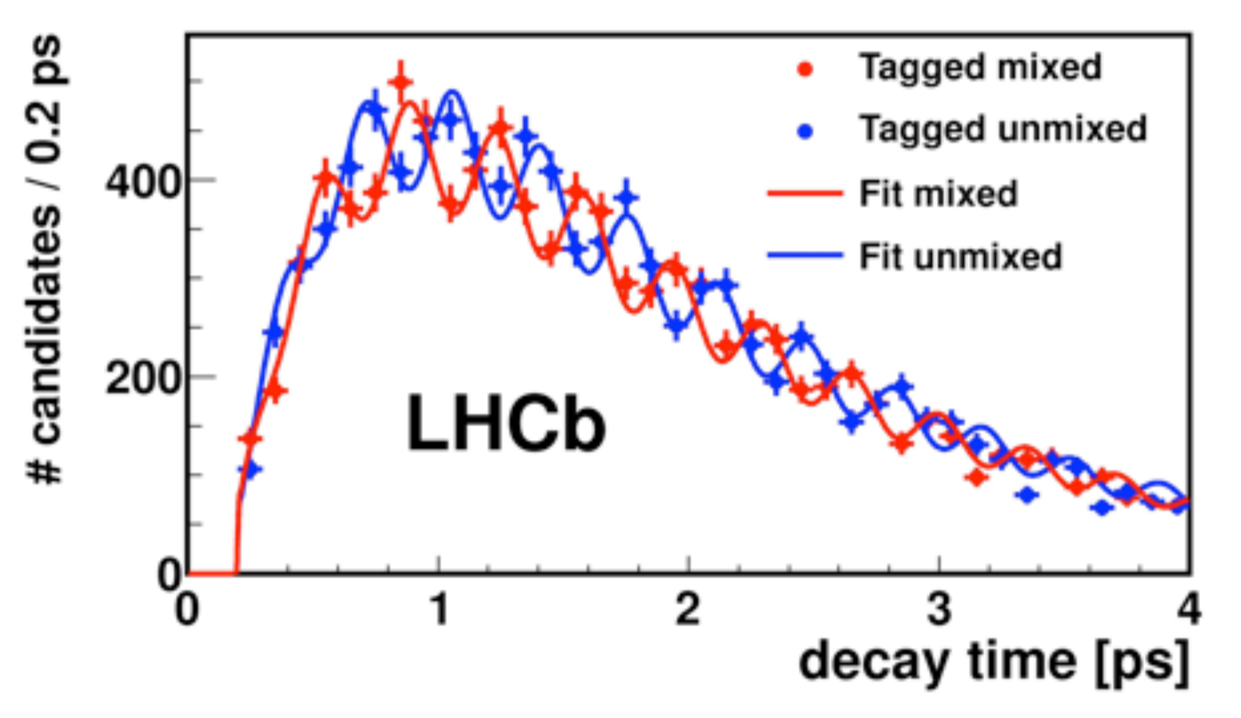
$(\bar{s}b) \to (\bar{s}[c\bar{u}d]) = (\bar{s}c)(\bar{u}d)$

$B_s \to D_s^- \pi^+$

$(\bar{b}s) \to ([\bar{c}u\bar{d}]s) = (\bar{c}s)(\bar{d}u)$

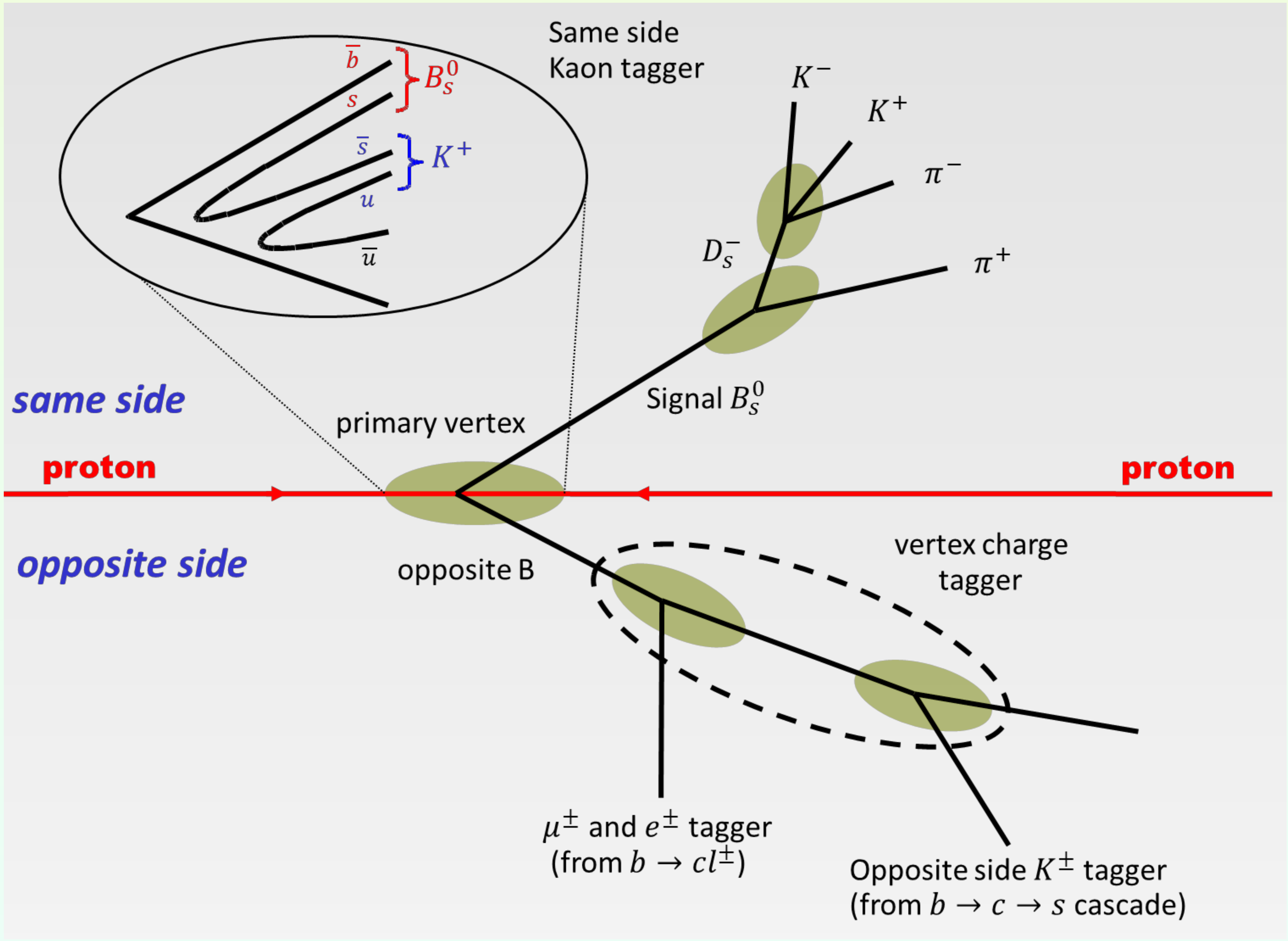
"Unmixed:" same as starting state (anti-B_s)





$\bar{B}_s \to D_s^+ \pi^-$

Tagging:



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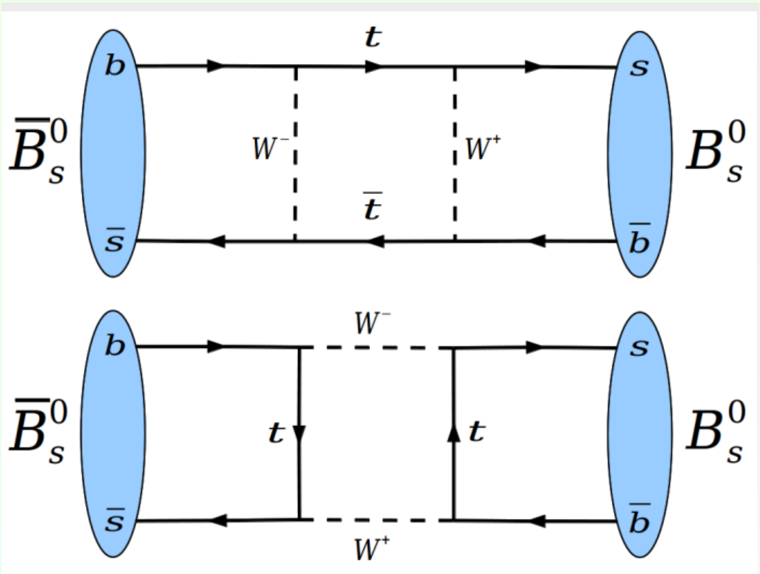
Very roughly, we guess $\bar{B}_s \to B_s \to \bar{B}_s \to B_s \to \cdots$

$$|\bar{B}_s(t)\rangle = e^{-\frac{1}{2}\Gamma t}\left[\cos(\omega t)|\bar{B}_s\rangle + \sin(\omega t)|B_s\rangle\right]$$

But why?



This is very small (weak interaction at 1-loop, suppressed by CKM) but important for eigenstates:

$$i\frac{d}{dt}\begin{pmatrix}\bar{B}_s(t)\\ B_s(t)\end{pmatrix} = M\begin{pmatrix}1 & \epsilon\\ \epsilon & 1\end{pmatrix}\begin{pmatrix}\bar{B}_s(t)\\ B_s(t)\end{pmatrix} \qquad \Rightarrow \qquad \bar{B}_s(t) = e^{-iMt}\left[\cos(\epsilon M t)\bar{B}_s(0) - i\sin(\epsilon M t)B_s(0)\right]$$

$\begin{pmatrix}1 & \epsilon\\ \epsilon & 1\end{pmatrix}$ has eigenvalues $1 \pm \epsilon$ and eigenvectors $\begin{pmatrix}1\\ \pm 1\end{pmatrix}$

Mixing: formalism

Weisskopf-Wigner
Neutral mesons, at rest

Analyze all at once: $X^0 = K^0, D^0, B^0, B_s$

$$P|X^0\rangle = -|X^0\rangle \qquad P|\bar{X}^0\rangle = -|\bar{X}^0\rangle$$
$$C|X^0\rangle = |\bar{X}^0\rangle \qquad C|\bar{X}^0\rangle = |X^0\rangle$$

$$CP|X^0\rangle = -|\bar{X}^0\rangle \qquad CP|\bar{X}^0\rangle = -|X^0\rangle$$

$$\mathbf{H} = \mathbf{M} - \frac{i}{2}\mathbf{\Gamma} = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}$$

$\mathbf{M}^\dagger = \mathbf{M}$ $\qquad |1\rangle = |X^0\rangle$

$\mathbf{\Gamma}^\dagger = \mathbf{\Gamma}$ $\qquad |2\rangle = |\bar{X}^0\rangle$

$\mathbf{H}^\dagger \neq \mathbf{H}$ (unstable)

We have insisted on CPT : $\quad (CPT)^{-1}\,\mathbf{H}\,(CPT) = \mathbf{H}^\dagger \quad \Rightarrow H_{11} = H_{22}$

(If you want to test CPT you relax this)

CP-invariance $\Rightarrow M_{12}^* = M_{12}, \Gamma_{12}^* = \Gamma_{12}$ $\qquad$ CPV if $\quad \mathrm{Im} M_{12} \neq 0$ or $\mathrm{Im}\Gamma_{12} \neq 0$

$$\mathbf{H} = \mathbf{M} - \frac{i}{2}\mathbf{\Gamma} = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix}$$

Define eigenvalues

$$M_{X_{\substack{H\\L}}} - \tfrac{i}{2}\Gamma_{X_{\substack{H\\L}}} = M - \tfrac{i}{2}\Gamma \pm \tfrac{1}{2}(\Delta M - \tfrac{i}{2}\Delta\Gamma)$$

eigenvectors

$$|X_{\substack{H\\L}}\rangle = p|X^0\rangle \pm q|\bar{X}^0\rangle$$

Note that for $q = p$ $\quad CP|X_{\substack{H\\L}}\rangle = \mp|X_{\substack{H\\L}}\rangle$

Solving:

$$\frac{p}{q} = 2\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{\Delta M - \frac{i}{2}\Delta\Gamma} = \frac{1}{2}\frac{\Delta M - \frac{i}{2}\Delta\Gamma}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}$$

It follows that:

$$(\Delta M)^2 - \frac{1}{4}(\Delta\Gamma)^2 = 4|M_{12}|^2 - |\Gamma_{12}|^2$$

$$\Delta M \Delta\Gamma = 4\mathrm{Re}(M_{12}\Gamma_{12}^*)$$

For Kaons, it is common to define "Long" and "Short" (instead of Heavy and Light):

$$M_{K_{\substack{L\\S}}} - \frac{i}{2}\Gamma_{K_{\substack{L\\S}}} = M - \frac{i}{2}\Gamma \pm \frac{1}{2}(\Delta M - \frac{i}{2}\Delta\Gamma)$$

$$|K_{\substack{L\\S}}\rangle = \frac{1}{\sqrt{2(1+|\epsilon|^2)}}\left[(1+\epsilon)|K^0\rangle \pm (1-\epsilon)|\bar{K}^0\rangle\right]$$

$\epsilon = 0 \Rightarrow CP|K_L\rangle = -|K_L\rangle$
$CP|\pi\pi\rangle_{\ell=0} = |\pi\pi\rangle_{\ell=0}, CP|\pi\pi\pi\rangle_{\ell=0} = -|\pi\pi\pi\rangle_{\ell=0}$

$\Rightarrow K_S \to \pi\pi, K_L \to \pi\pi\pi$

$$\mathrm{Br}(K_S \to \pi\pi) = 100\%$$
$$\mathrm{Br}(K_L \to \pi\pi) = 0.297\%$$
$$\mathrm{Br}(K_L \to \pi\pi\pi) = 33.9\%$$

- epsilon is small
- CP is not a symmetry
- Longer K_L lifetime accidental

$m_K \approx 490$ MeV $\qquad 3m_\pi \approx 420$ MeV

$$\tau(K_S) = 0.59 \times 10^{-10} \text{ s}$$
$$\tau(K_L) = 5.18 \times 10^{-8} \text{ s}$$

This is no longer the case for heavier mesons.

Perturbation theory (in H_w): connect with underlying fundamentals

see, e.g., Messiah, v2 994-1001

$$M_{ij} = M\delta_{ij} + \langle i|\mathcal{H}|j\rangle + \sum_n{}' \mathrm{PP}\frac{\langle i|\mathcal{H}|n\rangle\langle n|\mathcal{H}|j\rangle}{M - E_n} + \cdots$$

$$\Gamma_{ij} = 2\pi\sum_n{}' \delta(M - E_n)\langle i|\mathcal{H}|n\rangle\langle n|\mathcal{H}|j\rangle + \cdots$$

beware, here: $\langle i|j\rangle = \frac{E}{m}\delta^3(\vec{p} - \vec{p}')$

Time Evolution

$$i\frac{d}{dt}|X_{\substack{H\\L}}\rangle = (M_{\substack{H\\L}} - \tfrac{i}{2}\Gamma_{\substack{H\\L}})|X_{\substack{H\\L}}\rangle \qquad \Rightarrow \qquad |X_{\substack{H\\L}}(t)\rangle = e^{-iM_{\substack{H\\L}}t}e^{-\frac{1}{2}\Gamma_{\substack{H\\L}}t}|X_{\substack{H\\L}}(0)\rangle$$

$|X_{\substack{H\\L}}\rangle$ are eigenvectors: no mixing

But often create X^0 or $\bar{X}^0$. These mix, since they are a combination of X_H and $\bar{X}_L$.

Time evolution:

Invert $\quad |X^0\rangle = \frac{1}{2p}(|X_H\rangle + |X_L\rangle) \qquad |\bar{X}^0\rangle = \frac{1}{2q}(|X_H\rangle - |X_L\rangle)$

$$|X^0(t)\rangle = \frac{1}{2p}\left[e^{-iM_Ht}e^{-\Gamma_Ht}|X_H(0)\rangle + e^{-iM_Lt}e^{-\Gamma_Lt}|X_L(0)\rangle\right]$$

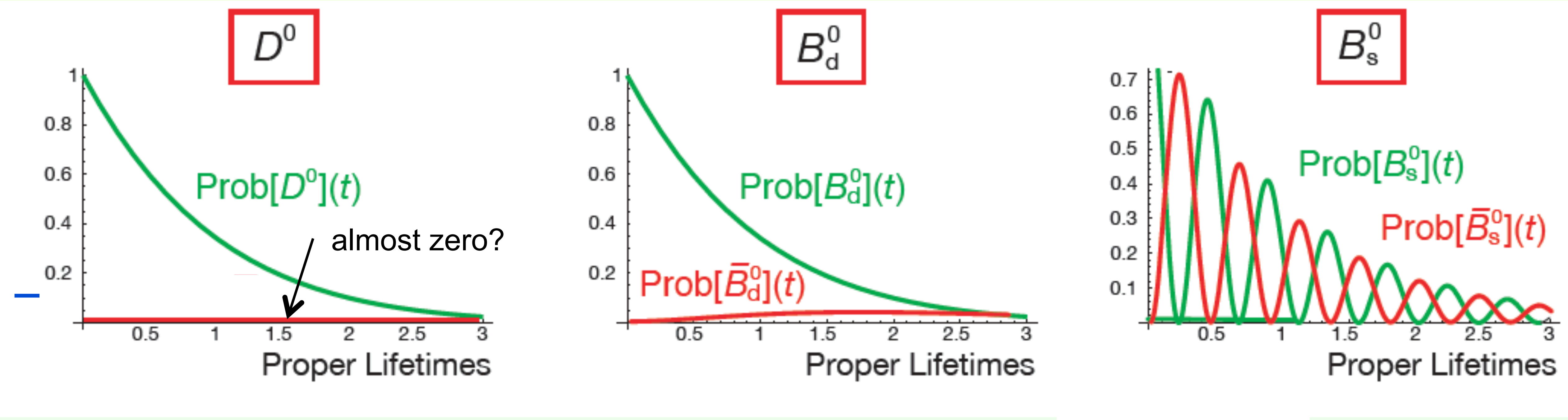
and use $\quad |X(0)_{\substack{H\\L}}\rangle = p|X^0(0)\rangle \pm q|\bar{X}^0(0)\rangle$

$$\boxed{|X^0(t)\rangle = f_+(t)|X^0\rangle + \frac{q}{p}f_-(t)|\bar{X}^0\rangle}$$

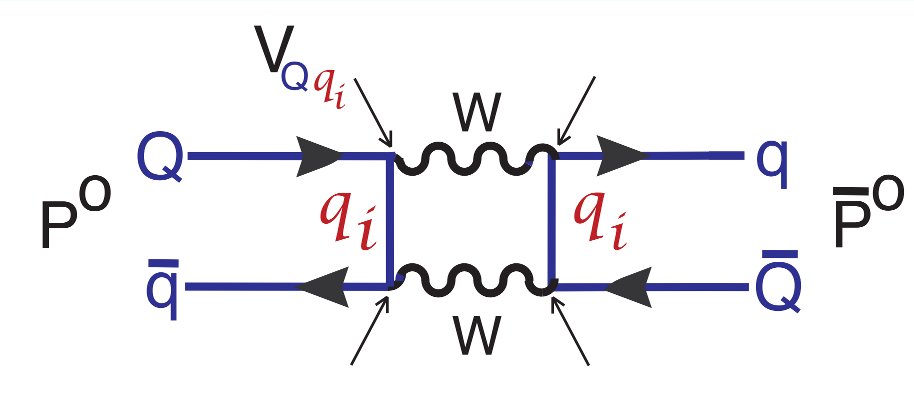
Exercise: $\quad f_\pm(t) = \frac{1}{2}e^{-iM_Lt-\frac{1}{2}\Gamma_Lt}(e^{-i\Delta Mt-\frac{1}{2}\Delta\Gamma t} \pm 1)$

and $\quad |\bar{X}^0(t)\rangle = \frac{p}{q}f_-(t)|X^0\rangle + f_+(t)|\bar{X}^0\rangle$

Mixing: slow/fast?



It's about time we ***connect with SM***! So let's see...



Want this: $$\frac{1+\epsilon}{1-\epsilon} = \frac{p}{q} = 2\frac{M_{12} - \frac{i}{2}\Gamma_{12}}{\Delta M - \frac{i}{2}\Delta\Gamma} = \frac{1}{2}2\frac{\Delta M - \frac{i}{2}\Delta\Gamma}{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}$$

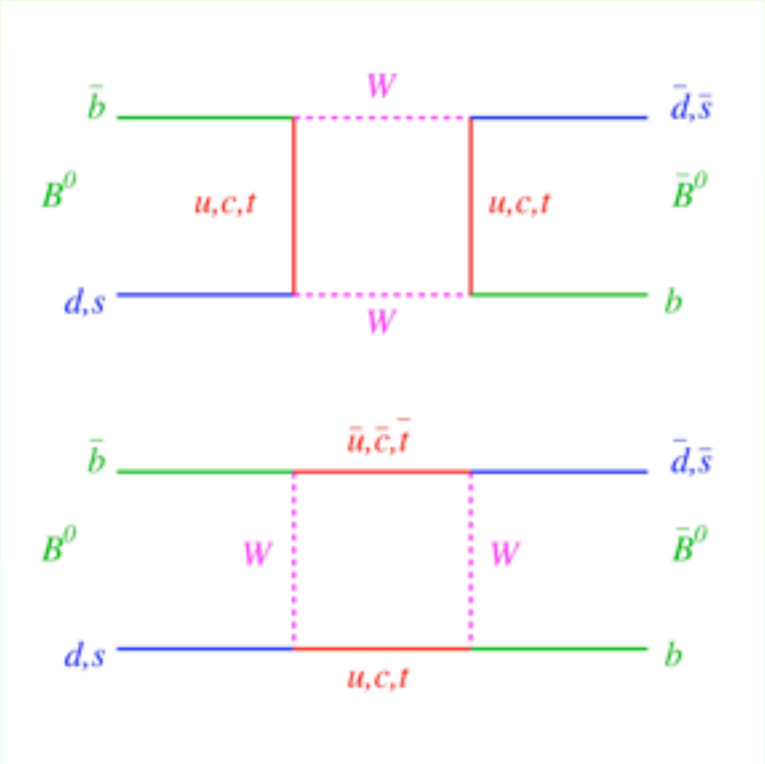
$$(\Delta M)^2 - \frac{1}{4}(\Delta\Gamma)^2 = 4|M_{12}|^2 - |\Gamma_{12}|^2$$

$$\Delta M \Delta\Gamma = 4\mathrm{Re}(M_{12}\Gamma_{12}^*)$$

Use this: $$M_{ij} = M\delta_{ij} + \langle i|\mathcal{H}|j\rangle + \sum_n{}' \mathrm{PP}\frac{\langle i|\mathcal{H}|n\rangle\langle n|\mathcal{H}|j\rangle}{M - E_n} + \cdots$$

$$\Gamma_{ij} = 2\pi\sum_n{}' \delta(M - E_n)\langle i|\mathcal{H}|n\rangle\langle n|\mathcal{H}|j\rangle + \cdots$$

Clear that:



Only M_{12}

t-quark: only M_{12}
u,c-quarks: M_{12} & Γ_{12}

Quickly, for B^0 & B_s-mesons:

- modern-GIM: t-quark dominates
- Neglect Γ_{12}
- Then $\Delta M = 2|M_{12}|$
- Then $p/q = M_{12}/|M_{12}|$
  a pure phase
- No phase in Feynman diagram (no cuts), phase from CKM's only:

$$p/q = (V_{tb}V_{td}^*)^2/|V_{tb}V_{td}^*|^2$$

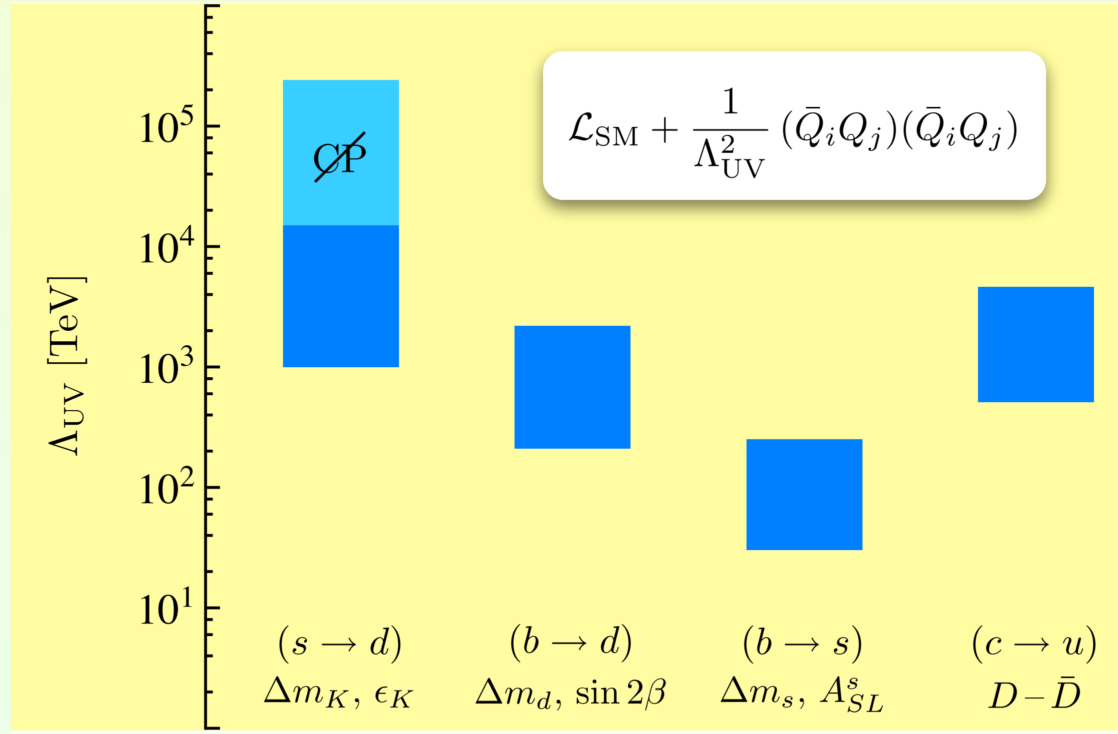
- idem for B_s:

$$p/q = (V_{tb}V_{ts}^*)^2/|V_{tb}V_{ts}^*|^2$$

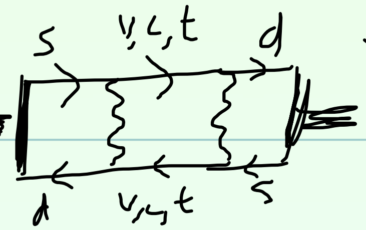
- in SU(3) (Gell-Mann) symmetry limit

$$(\Delta M)_{B_s}/(\Delta M)_{B^0} = |V_{ts}/V_{tb}|^2$$

recall this?



We can now understand it! For example, take $\epsilon_K = \epsilon$

$\mathrm{Im}\, M_{12} \simeq \mathrm{Im}\Big($  $\Big)$

$$\sim \mathrm{Im}\left[\frac{G_F^2 M_W^2}{4\pi^2}\sum_{q=u,c,t} V_{qd}^* V_{qs} \sum_{q'=u,c,t} V_{q'd}^* V_{q's}\, f\left(\frac{m_q}{M_W}, \frac{m_{q'}}{M_W}\right) \langle K^0 | \bar{d}_L \gamma^\mu s_L \bar{d}_L \gamma_\mu s_L | \bar{K}^0\rangle\right]$$

Compare with $$\frac{1}{\Lambda^2}\langle K^0|\bar{d}_L\gamma^\mu s_L \bar{d}_L \gamma_\mu s_L|\bar{K}^0\rangle$$

$$\Lambda^2 \leq \frac{4\pi^2}{G_F^2 M_W^2}\frac{1}{|V_{td}^* V_{ts}|^2} \approx \left[\frac{6}{(10^{-5})(10^2)}\frac{1}{(0.04)(0.004)}\mathrm{GeV}\right]^2 \approx [4\times 10^4 \mathrm{TeV}]^2$$

Exercise: check the other three mixing "bounds."

CPV

CPV in Decay

- Nothing to do with mixing, *per-se*
- Conceptually Simple
- Predictability: difficult
- Later also CPV in *mixing* and decay

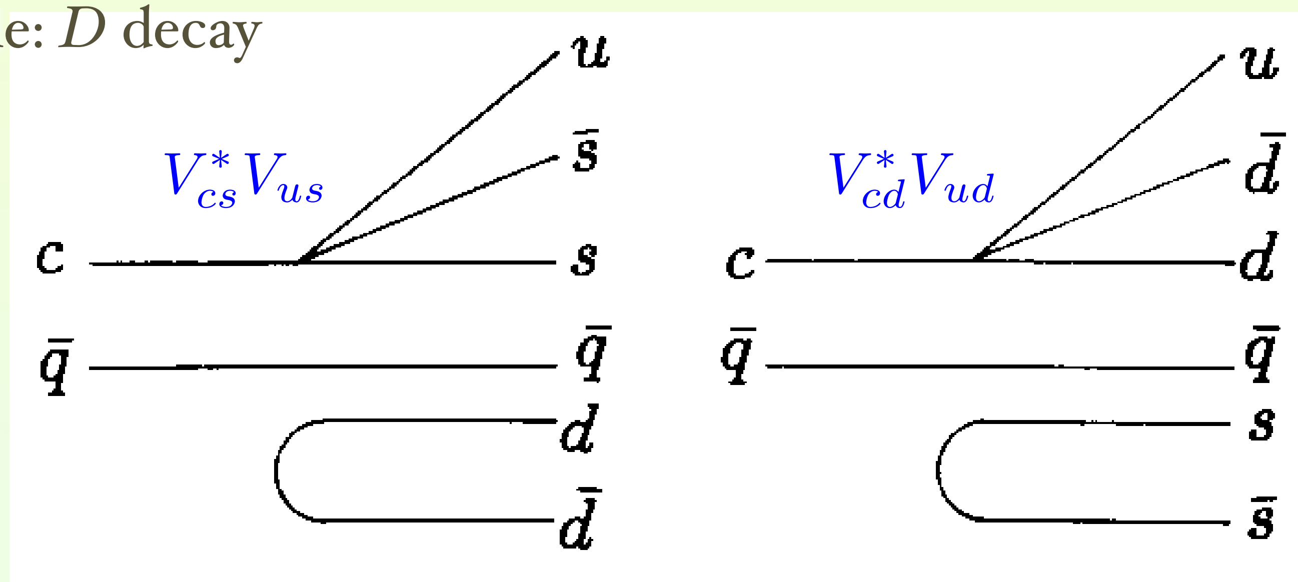
asymmetry $$\mathcal{A} = \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}$$

Γ some rate

$\bar{\Gamma}$ the conjugate of the above
(under something: C, CP, $\theta \to \pi - \theta$)

CP decay-asymmetry $$\mathcal{A} = \frac{|\langle f|X\rangle|^2 - |\langle \bar{f}|\bar{X}\rangle|^2}{|\langle f|X\rangle|^2 + |\langle \bar{f}|\bar{X}\rangle|^2}$$

Example: D decay



$$\langle f|X\rangle = aA + bB$$
$$\langle \bar{f}|\bar{X}\rangle = a^*\bar{A} + b^*\bar{B}$$

$$a = V_{cs}^* V_{us}, \quad b = V_{cd}^* V_{ud}$$

$$A = \langle f|(\bar{u}_L\gamma^\mu s_L)(\bar{s}_L\gamma_\mu c_L)|D\rangle$$
$$B = \langle f|(\bar{u}_L\gamma^\mu d_L)(\bar{d}_L\gamma_\mu c_L)|D\rangle$$

CP invariance of strong interactions:

$$\begin{aligned} A &= \langle f|(\bar{u}_L\gamma^\mu s_L)(\bar{s}_L\gamma_\mu c_L)|D\rangle \\ &= \langle f|(CP)^{-1}(CP)(\bar{u}_L\gamma^\mu s_L)(\bar{s}_L\gamma_\mu c_L)(CP)^{-1}(CP)|D\rangle \\ &= \langle \bar{f}|(\bar{s}_L\gamma^\mu u_L)(\bar{c}_L\gamma_\mu s_L)|\bar{D}\rangle \\ &= \bar{A} \end{aligned}$$

CP decay-asymmetry $\mathcal{A} = \dfrac{|\langle f|X\rangle|^2 - |\langle \bar{f}|\bar{X}\rangle|^2}{|\langle f|X\rangle|^2 + |\langle \bar{f}|\bar{X}\rangle|^2}$ where $\langle f|X\rangle = aA + bB$, $\langle \bar{f}|\bar{X}\rangle = a^*A + b^*B$

$$\Rightarrow \quad \mathcal{A} = \frac{2\mathrm{Im}(a^*b)\mathrm{Im}(A^*B)}{|aA|^2 + |bB|^2 + 2\mathrm{Re}(a^*b)\mathrm{Re}(A^*B)}$$

For direct CPV need both phases! (and knowledge of matrix elements computed with strong interactions):

Note that

$$\mathrm{Im}(a^*b) = \mathrm{Im}((V_{cs}^* V_{us})^* V_{cd}^* V_{ud}) = \mathrm{Im}(V_{cs} V_{cd}^* V_{ud} V_{us}^*) = J$$

as promised ...

$D^{\pm}$ *CP*-violation decay-rate asymmetries

$A_{CP}(\mu^{\pm}\nu) = (8 \pm 8)\%$

$A_{CP}(K_S^0\pi^{\pm}) = (-0.41 \pm 0.09)\%$

$A_{CP}(K^{\mp}2\pi^{\pm}) = (-0.1 \pm 1.0)\%$

$A_{CP}(K^{\mp}\pi^{\pm}\pi^{\pm}\pi^0) = (1.0 \pm 1.3)\%$

$A_{CP}(K_S^0\pi^{\pm}\pi^0) = (0.3 \pm 0.9)\%$

$A_{CP}(K_S^0\pi^{\pm}\pi^+\pi^-) = (0.1 \pm 1.3)\%$

$A_{CP}(\pi^{\pm}\pi^0) = (2.9 \pm 2.9)\%$

$A_{CP}(\pi^{\pm}\eta) = (1.0 \pm 1.5)\%$ (S = 1.4)

$A_{CP}(\pi^{\pm}\eta'(958)) = (-0.5 \pm 1.2)\%$ (S = 1.1)

$A_{CP}(K_S^0 K^{\pm}) = (-0.11 \pm 0.25)\%$

$A_{CP}(K^+K^-\pi^{\pm}) = (0.36 \pm 0.29)\%$

$A_{CP}(K^{\pm}K^{*0}) = (-0.3 \pm 0.4)\%$

$A_{CP}(\phi\pi^{\pm}) = (0.09 \pm 0.19)\%$ (S = 1.2)

$A_{CP}(K^{\pm}K_0^*(1430)^0) = (8^{+7}_{-6})\%$

$A_{CP}(K^{\pm}K_2^*(1430)^0) = (43^{+20}_{-26})\%$

$A_{CP}(K^{\pm}K_0^*(800)) = (-12^{+18}_{-13})\%$

$A_{CP}(a_0(1450)^0\pi^{\pm}) = (-19^{+14}_{-16})\%$

$A_{CP}(\phi(1680)\pi^{\pm}) = (-9 \pm 26)\%$

$A_{CP}(\pi^+\pi^-\pi^{\pm}) = (-2 \pm 4)\%$

$A_{CP}(K_S^0 K^{\pm}\pi^+\pi^-) = (-4 \pm 7)\%$

$A_{CP}(K^{\pm}\pi^0) = (-4 \pm 11)\%$

$B^{\pm}$

***CP* violation**

$A_{CP}(B^+ \to J/\psi(1S) K^+) = 0.003 \pm 0.006$ (S = 1.8)
$A_{CP}(B^+ \to J/\psi(1S) \pi^+) = (0.1 \pm 2.8) \times 10^{-2}$ (S = 1.2)
$A_{CP}(B^+ \to J/\psi \rho^+) = -0.11 \pm 0.14$
$A_{CP}(B^+ \to J/\psi K^*(892)^+) = -0.048 \pm 0.033$
$A_{CP}(B^+ \to \eta_c K^+) = -0.02 \pm 0.10$ (S = 2.0)
$A_{CP}(B^+ \to \psi(2S) \pi^+) = 0.03 \pm 0.06$
$A_{CP}(B^+ \to \psi(2S) K^+) = -0.024 \pm 0.023$
$A_{CP}(B^+ \to \psi(2S) K^*(892)^+) = 0.08 \pm 0.21$
$A_{CP}(B^+ \to \chi_{c1}(1P) \pi^+) = 0.07 \pm 0.18$
$A_{CP}(B^+ \to \chi_{c0} K^+) = -0.20 \pm 0.18$ (S = 1.5)
$A_{CP}(B^+ \to \chi_{c1} K^+) = -0.009 \pm 0.033$
$A_{CP}(B^+ \to \chi_{c1} K^*(892)^+) = 0.5 \pm 0.5$
$A_{CP}(B^+ \to \overline{D}^0 \pi^+) = -0.007 \pm 0.007$
$A_{CP}(B^+ \to D_{CP(+1)} \pi^+) = 0.035 \pm 0.024$
$A_{CP}(B^+ \to D_{CP(-1)} \pi^+) = 0.017 \pm 0.026$
$A_{CP}([K^\mp \pi^\pm \pi^+ \pi^-]_D \pi^+) = 0.13 \pm 0.10$
$A_{CP}(B^+ \to \overline{D}^0 K^+) = 0.01 \pm 0.05$ (S = 2.1)
$A_{CP}([K^\mp \pi^\pm \pi^+ \pi^-]_D K^+) = -0.42 \pm 0.22$
$r_B(B^+ \to D^0 K^+) = 0.096 \pm 0.008$
$\delta_B(B^+ \to D^0 K^+) = (115 \pm 13)^\circ$
$r_B(B^+ \to \overline{D}^0 K^{*+}) = 0.17 \pm 0.11$ (S = 2.3)
$\delta_B(B^+ \to D^0 K^{*+}) = (155 \pm 70)^\circ$ (S = 2.0)
$A_{CP}(B^+ \to [K^- \pi^+]_D K^+) = -0.58 \pm 0.21$
$A_{CP}(B^+ \to [K^- \pi^+ \pi^0]_D K^+) = 0.41 \pm 0.30$
$A_{CP}(B^+ \to [K^- \pi^+]_{\overline{D}} K^*(892)^+) = -0.3 \pm 0.5$
$A_{CP}(B^+ \to [K^- \pi^+]_D \pi^+) = 0.00 \pm 0.09$
$A_{CP}(B^+ \to [K^- \pi^+ \pi^0]_D \pi^+) = 0.16 \pm 0.27$
$A_{CP}(B^+ \to [K^- \pi^+]_{(D\pi)} \pi^+) = -0.09 \pm 0.27$
$A_{CP}(B^+ \to [K^- \pi^+]_{(D\gamma)} \pi^+) = -0.7 \pm 0.6$
$A_{CP}(B^+ \to [K^- \pi^+]_{(D\pi)} K^+) = 0.8 \pm 0.4$
$A_{CP}(B^+ \to [K^- \pi^+]_{(D\gamma)} K^+) = 0.4 \pm 1.0$
$A_{CP}(B^+ \to [\pi^+ \pi^- \pi^0]_D K^+) = -0.02 \pm 0.15$
$\boldsymbol{A_{CP}(B^+ \to D_{CP(+1)} K^+)} = 0.170 \pm 0.033$ (S = 1.2)
$A_{ADS}(B^+ \to D K^+) = -0.52 \pm 0.15$
$A_{ADS}(B^+ \to D \pi^+) = 0.14 \pm 0.06$
$A_{CP}(B^+ \to D_{CP(-1)} K^+) = -0.10 \pm 0.07$
$A_{CP}(B^+ \to \overline{D}^{*0} \pi^+) = -0.014 \pm 0.015$
$A_{CP}(B^+ \to (D^*_{CP(+1)})^0 \pi^+) = -0.02 \pm 0.05$
$A_{CP}(B^+ \to (D^*_{CP(-1)})^0 \pi^+) = -0.09 \pm 0.05$
$A_{CP}(B^+ \to D^{*0} K^+) = -0.07 \pm 0.04$
$r^*_B(B^+ \to D^{*0} K^+) = 0.114^{+0.023}_{-0.040}$ (S = 1.2)
$\delta^*_B(B^+ \to D^{*0} K^+) = (310^{+22}_{-28})^\circ$ (S = 1.3)
$A_{CP}(B^+ \to D^{*0}_{CP(+1)} K^+) = -0.12 \pm 0.08$
$A_{CP}(B^+ \to D^*_{CP(-1)} K^+) = 0.07 \pm 0.10$
$A_{CP}(B^+ \to D_{CP(+1)} K^*(892)^+) = 0.09 \pm 0.14$
$A_{CP}(B^+ \to D_{CP(-1)} K^*(892)^+) = -0.23 \pm 0.22$
$A_{CP}(B^+ \to D^+_s \phi) = 0.0 \pm 0.4$
$A_{CP}(B^+ \to D^{*+} \overline{D}^{*0}) = -0.15 \pm 0.11$
$A_{CP}(B^+ \to D^{*+} \overline{D}^0) = -0.06 \pm 0.13$
$A_{CP}(B^+ \to D^+ \overline{D}^{*0}) = 0.13 \pm 0.18$
$A_{CP}(B^+ \to D^+ \overline{D}^0) = -0.03 \pm 0.07$
$A_{CP}(B^+ \to K^0_S \pi^+) = -0.017 \pm 0.016$
$A_{CP}(B^+ \to K^+ \pi^0) = 0.037 \pm 0.021$
$A_{CP}(B^+ \to \eta' K^+) = 0.013 \pm 0.017$
$A_{CP}(B^+ \to \eta' K^*(892)^+) = -0.26 \pm 0.27$
$A_{CP}(B^+ \to \eta' K^*_0(1430)^+) = 0.06 \pm 0.20$
$A_{CP}(B^+ \to \eta' K^*_2(1430)^+) = 0.15 \pm 0.13$
$\boldsymbol{A_{CP}(B^+ \to \eta K^+)} = -0.37 \pm 0.08$
$A_{CP}(B^+ \to \eta K^*(892)^+) = 0.02 \pm 0.06$
$A_{CP}(B^+ \to \eta K^*_0(1430)^+) = 0.05 \pm 0.13$
$A_{CP}(B^+ \to \eta K^*_2(1430)^+) = -0.45 \pm 0.30$
$A_{CP}(B^+ \to \omega K^+) = 0.02 \pm 0.05$
$A_{CP}(B^+ \to \omega K^{*+}) = 0.29 \pm 0.35$
$A_{CP}(B^+ \to \omega (K\pi)^{*+}_0) = -0.10 \pm 0.09$
$A_{CP}(B^+ \to \omega K^*_2(1430)^+) = 0.14 \pm 0.15$
$A_{CP}(B^+ \to K^{*0} \pi^+) = -0.04 \pm 0.09$ (S = 2.1)
$A_{CP}(B^+ \to K^*(892)^+ \pi^0) = -0.06 \pm 0.24$
$\boldsymbol{A_{CP}(B^+ \to K^+ \pi^- \pi^+)} = 0.033 \pm 0.010$
$A_{CP}(B^+ \to K^+ K^- K^+ \text{ nonresonant}) = 0.06 \pm 0.05$
$A_{CP}(B^+ \to f(980)^0 K^+) = -0.08 \pm 0.09$
$\boldsymbol{A_{CP}(B^+ \to f_2(1270) K^+)} = -0.68^{+0.19}_{-0.17}$

$A_{CP}(B^+ \to f_0(1500) K^+) = 0.28 \pm 0.30$
$A_{CP}(B^+ \to f_2'(1525)^0 K^+) = -0.08^{+0.05}_{-0.04}$
$\boldsymbol{A_{CP}(B^+ \to \rho^0 K^+)} = 0.37 \pm 0.10$
$A_{CP}(B^+ \to K_0^*(1430)^0 \pi^+) = 0.055 \pm 0.033$
$A_{CP}(B^+ \to K_2^*(1430)^0 \pi^+) = 0.05^{+0.29}_{-0.24}$
$A_{CP}(B^+ \to K^+ \pi^0 \pi^0) = -0.06 \pm 0.07$
$A_{CP}(B^+ \to K^0 \rho^+) = -0.12 \pm 0.17$
$A_{CP}(B^+ \to K^{*+} \pi^+ \pi^-) = 0.07 \pm 0.08$
$A_{CP}(B^+ \to \rho^0 K^*(892)^+) = 0.31 \pm 0.13$
$A_{CP}(B^+ \to K^*(892)^+ f_0(980)) = -0.15 \pm 0.12$
$A_{CP}(B^+ \to a_1^+ K^0) = 0.12 \pm 0.11$
$A_{CP}(B^+ \to b_1^+ K^0) = -0.03 \pm 0.15$
$A_{CP}(B^+ \to K^*(892)^0 \rho^+) = -0.01 \pm 0.16$
$A_{CP}(B^+ \to b_1^0 K^+) = -0.46 \pm 0.20$
$A_{CP}(B^+ \to \overline{K}^0 K^+) = 0.04 \pm 0.14$
$A_{CP}(B^+ \to K_S^0 K^+) = -0.21 \pm 0.14$
$A_{CP}(B^+ \to K^+ K_S^0 K_S^0) = 0.04^{+0.04}_{-0.05}$
$A_{CP}(B^+ \to K^+ K^- \pi^+) = -0.12 \pm 0.05$ (S = 1.2)
$\boldsymbol{A_{CP}(B^+ \to K^+ K^- K^+)} = -0.036 \pm 0.012$ (S = 1.1)
$A_{CP}(B^+ \to \phi K^+) = 0.04 \pm 0.04$ (S = 2.1)
$A_{CP}(B^+ \to X_0(1550) K^+) = -0.04 \pm 0.07$
$A_{CP}(B^+ \to K^{*+} K^+ K^-) = 0.11 \pm 0.09$
$A_{CP}(B^+ \to \phi K^*(892)^+) = -0.01 \pm 0.08$
$A_{CP}(B^+ \to \phi (K\pi)_0^{*+}) = 0.04 \pm 0.16$
$A_{CP}(B^+ \to \phi K_1(1270)^+) = 0.15 \pm 0.20$
$A_{CP}(B^+ \to \phi K_2^*(1430)^+) = -0.23 \pm 0.20$
$A_{CP}(B^+ \to K^+ \phi \phi) = -0.10 \pm 0.08$
$A_{CP}(B^+ \to K^+ [\phi \phi]_{\eta_c}) = 0.09 \pm 0.10$
$A_{CP}(B^+ \to K^*(892)^+ \gamma) = 0.018 \pm 0.029$
$A_{CP}(B^+ \to \eta K^+ \gamma) = -0.12 \pm 0.07$
$A_{CP}(B^+ \to \phi K^+ \gamma) = -0.13 \pm 0.11$ (S = 1.1)
$A_{CP}(B^+ \to \rho^+ \gamma) = -0.11 \pm 0.33$
$A_{CP}(B^+ \to \pi^+ \pi^0) = 0.03 \pm 0.04$
$\boldsymbol{A_{CP}(B^+ \to \pi^+ \pi^- \pi^+)} = 0.105 \pm 0.029$ (S = 1.3)
$A_{CP}(B^+ \to \rho^0 \pi^+) = 0.18^{+0.09}_{-0.17}$
$A_{CP}(B^+ \to f_2(1270) \pi^+) = 0.41 \pm 0.30$
$A_{CP}(B^+ \to \rho^0(1450) \pi^+) = -0.1^{+0.4}_{-0.5}$
$\boldsymbol{A_{CP}(B^+ \to f_0(1370) \pi^+)} = 0.72 \pm 0.22$
$A_{CP}(B^+ \to \pi^+ \pi^- \pi^+ \text{ nonresonant}) = -0.14^{+0.23}_{-0.16}$
$A_{CP}(B^+ \to \rho^+ \pi^0) = 0.02 \pm 0.11$
$A_{CP}(B^+ \to \rho^+ \rho^0) = -0.05 \pm 0.05$
$A_{CP}(B^+ \to \omega \pi^+) = -0.04 \pm 0.06$
$A_{CP}(B^+ \to \omega \rho^+) = -0.20 \pm 0.09$
$A_{CP}(B^+ \to \eta \pi^+) = -0.14 \pm 0.07$ (S = 1.4)
$A_{CP}(B^+ \to \eta \rho^+) = 0.11 \pm 0.11$
$A_{CP}(B^+ \to \eta' \pi^+) = 0.06 \pm 0.16$
$A_{CP}(B^+ \to \eta' \rho^+) = 0.26 \pm 0.17$
$A_{CP}(B^+ \to b_1^0 \pi^+) = 0.05 \pm 0.16$
$A_{CP}(B^+ \to p \overline{p} \pi^+) = 0.00 \pm 0.04$
$A_{CP}(B^+ \to p \overline{p} K^+) = -0.08 \pm 0.04$ (S = 1.1)
$A_{CP}(B^+ \to p \overline{p} K^*(892)^+) = 0.21 \pm 0.16$ (S = 1.4)
$A_{CP}(B^+ \to p \overline{\Lambda} \gamma) = 0.17 \pm 0.17$
$A_{CP}(B^+ \to p \overline{\Lambda} \pi^0) = 0.01 \pm 0.17$
$A_{CP}(B^+ \to K^+ \ell^+ \ell^-) = -0.02 \pm 0.08$
$A_{CP}(B^+ \to K^+ e^+ e^-) = 0.14 \pm 0.14$
$A_{CP}(B^+ \to K^+ \mu^+ \mu^-) = -0.003 \pm 0.033$
$A_{CP}(B^+ \to K^{*+} \ell^+ \ell^-) = -0.09 \pm 0.14$
$A_{CP}(B^+ \to K^* e^+ e^-) = -0.14 \pm 0.23$
$A_{CP}(B^+ \to K^* \mu^+ \mu^-) = -0.12 \pm 0.24$
$\boldsymbol{\gamma(B^+ \to D^{(*)0} K^{(*)+})} = (73^{+7}_{-9})^\circ$

CPV in mixing

Kaons first (will come back to heavier mesons)

Physical approximations:

If CP were conserved $\epsilon = 0, \mathrm{Im}M_{12} = 0, \mathrm{Im}\Gamma_{12} = 0$

and we would have $\Delta M = 2\mathrm{Re}M_{12}, \Delta\Gamma = 2\mathrm{Re}\Gamma_{12}$

CPV is small: assume $\mathrm{Im}M_{12} \ll \mathrm{Re}M_{12}, \quad \mathrm{Im}\Gamma_{12} \ll \mathrm{Re}\Gamma_{12}$

$$\epsilon \approx i\frac{\mathrm{Im}M_{12} - \frac{i}{2}\mathrm{Im}\Gamma_{12}}{\Delta M - \frac{i}{2}\Delta\Gamma}$$

We'll see $\mathrm{Im}\Gamma_{12} \ll \mathrm{Im}M_{12}$ Empirically $\Delta\Gamma \approx -2\Delta M$ $\Rightarrow$ $\epsilon \approx e^{i\pi/4}\frac{\mathrm{Im}M_{12}}{\sqrt{2}\Delta M}$

Example: Conceptually clean measurement, semileptonic charge-asymmetry

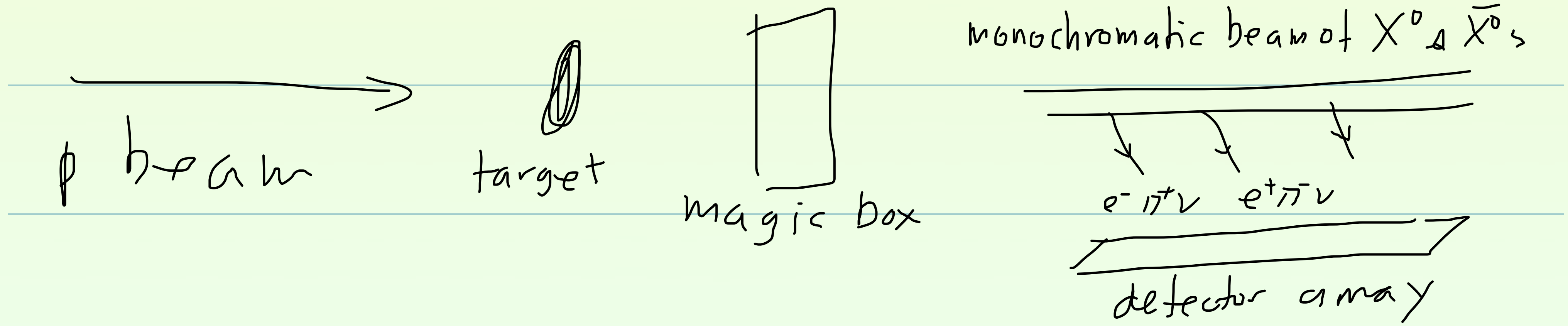
$$\delta \equiv \frac{\Gamma(K_L \to \pi^- e^+ \nu) - \Gamma(K_L \to \pi^+ e^- \nu)}{\Gamma(K_L \to \pi^- e^+ \nu) + \Gamma(K_L \to \pi^+ e^- \nu)} = \frac{|1+\epsilon|^2 - |1-\epsilon|^2}{|1+\epsilon|^2 + |1-\epsilon|^2} \simeq 2\,\mathrm{Re}\,\epsilon$$

$\delta_{exp} = 0.330 \pm 0.012\%$ gives $\mathrm{Re}\,\epsilon = 1.65 \times 10^{-3}$

Example: Time dependent charge-asymmetry in semileptonic X decay ("X_{l3} decay")

Like δ above but now $\delta(t)$

setup



Assume beam has N_{X^0} and $N_{\bar{X}^0}$ of X^0 and $\bar{X}^0$

$$\delta(t) = \frac{N^+ - N^-}{N^+ + N^-}$$ where t is from distance from target/magic box

$$\delta(t) = \frac{N_{X^0}[\Gamma(X^0(t) \to \pi^- e^+ \nu) - \Gamma(X^0(t) \to \pi^+ e^- \nu)] + N_{\bar{X}^0}[\Gamma(\bar{X}^0(t) \to \pi^- e^+ \nu) - \Gamma(\bar{X}^0(t) \to \pi^+ e^- \nu)]}{\text{same but with } + + ++ \text{ signs}}$$

yeach! real life is complicated...

Exercises

Exercise 2.5.2-2: Use $\Gamma(K^0(t) \to \pi^- e^+ \nu) \propto |\langle \pi^- e^+ \nu | H_W | K^0(t) \rangle|^2$ and the assumptions that

(i) $\langle \pi^- e^+ \nu | H_W | \overline{K}^0(t) \rangle = 0 = \langle \pi^+ e^- \nu | H_W | K^0(t) \rangle$

(ii) $\langle \pi^- e^+ \nu | H_W | K^0(t) \rangle = \langle \pi^+ e^- \nu | H_W | \overline{K}^0(t) \rangle$

to show that

$$\delta(t) = \frac{(N_{K^0} - N_{\overline{K}^0}) \left[|f_+(t)|^2 - |f_-(t)|^2 \frac{1}{2} \left(\left| \frac{q}{p} \right|^2 + \left| \frac{p}{q} \right|^2 \right) \right] + \frac{1}{2}(N_{K^0} + N_{\overline{K}^0}) |f_-(t)|^2 \left(\left| \frac{p}{q} \right|^2 - \left| \frac{q}{p} \right|^2 \right)}{(N_{K^0} + N_{\overline{K}^0}) \left[|f_+(t)|^2 + |f_-(t)|^2 \frac{1}{2} \left(\left| \frac{q}{p} \right|^2 + \left| \frac{p}{q} \right|^2 \right) \right] - \frac{1}{2}(N_{K^0} - N_{\overline{K}^0}) |f_-(t)|^2 \left(\left| \frac{p}{q} \right|^2 - \left| \frac{q}{p} \right|^2 \right)}$$

Justify assumptions (i) and (ii).

KAONS: $p/q = (1+\epsilon)/(1-\epsilon)$ $\quad a \equiv (N_{K^0} - N_{\bar{K}^0})/(N_{K^0} + N_{\bar{K}^0})$ $\quad \Delta\Gamma \approx -\Gamma_S$

$$\begin{aligned} \delta(t) &= \frac{a\left[|f_+(t)|^2 - |f_-(t)|^2 \right] + 4\mathrm{Re}(\epsilon)|f_-(t)|^2}{\left[|f_+(t)|^2 + |f_-(t)|^2 \right] - 4a\mathrm{Re}(\epsilon)|f_-(t)|^2} \\ &\approx \frac{2ae^{-\frac{1}{2}\Gamma_S t}\cos(\Delta M t) + \left(1 + e^{-\Gamma_S t} - 2e^{-\frac{1}{2}\Gamma_S t}\cos(\Delta M t)\right) 2\left(1 + \frac{a}{2}\right)\mathrm{Re}(\epsilon)}{1 + e^{-\Gamma_S t}} \end{aligned}$$

(2.16

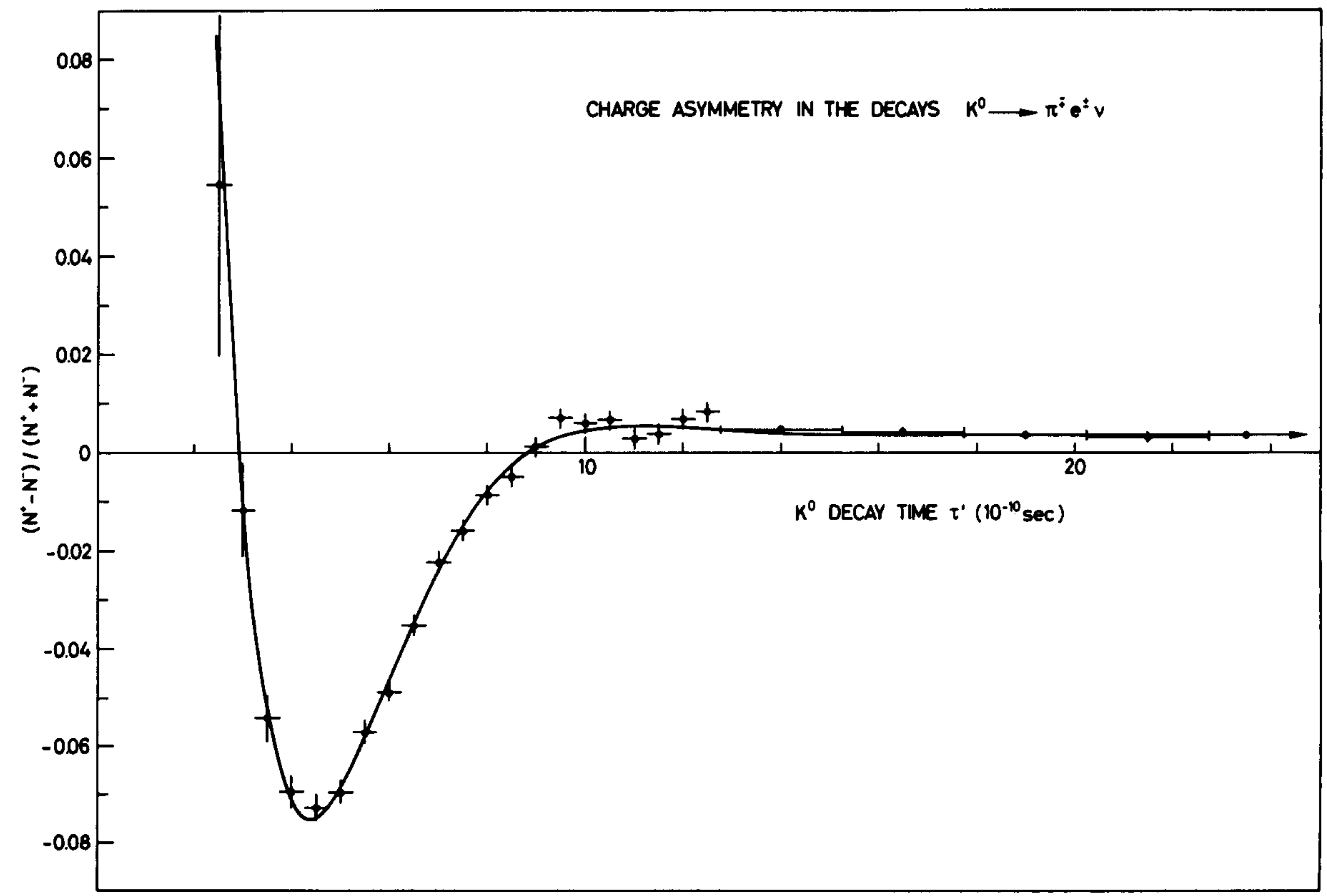


Fig. 1. The charge asymmetry as a function of the reconstructed decay time τ' for the K_{e3} decays. The experimental data are compared to the best fit as indicated by the solid line.

muons:

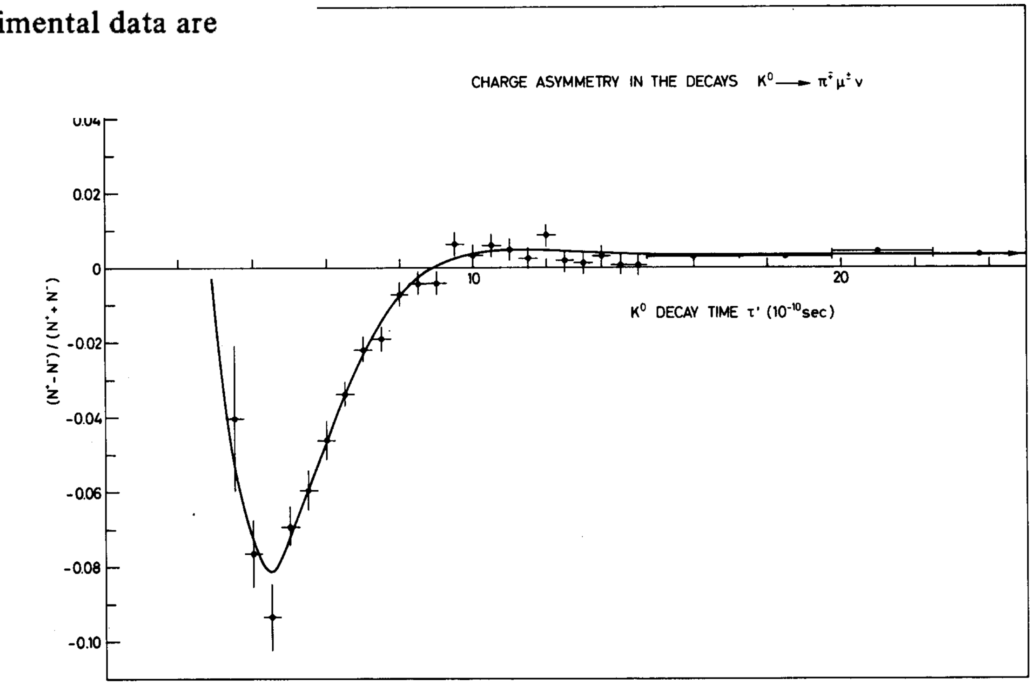


Fig. 2. The charge asymmetry as a function of the reconstructed decay time τ' for the $K_{\mu 3}$ decays. The experimental data are compared to the best fit as indicated by the solid line.

S. Gjesdal, et al, Phys.Lett. B52 (1974) 113

The solid curve is a fit to the formula of previous slide from which the parameters Γ_S, ΔM, a and $\text{Re}(\epsilon)$ are extracted.

This is B^0 (in hadronic decays)

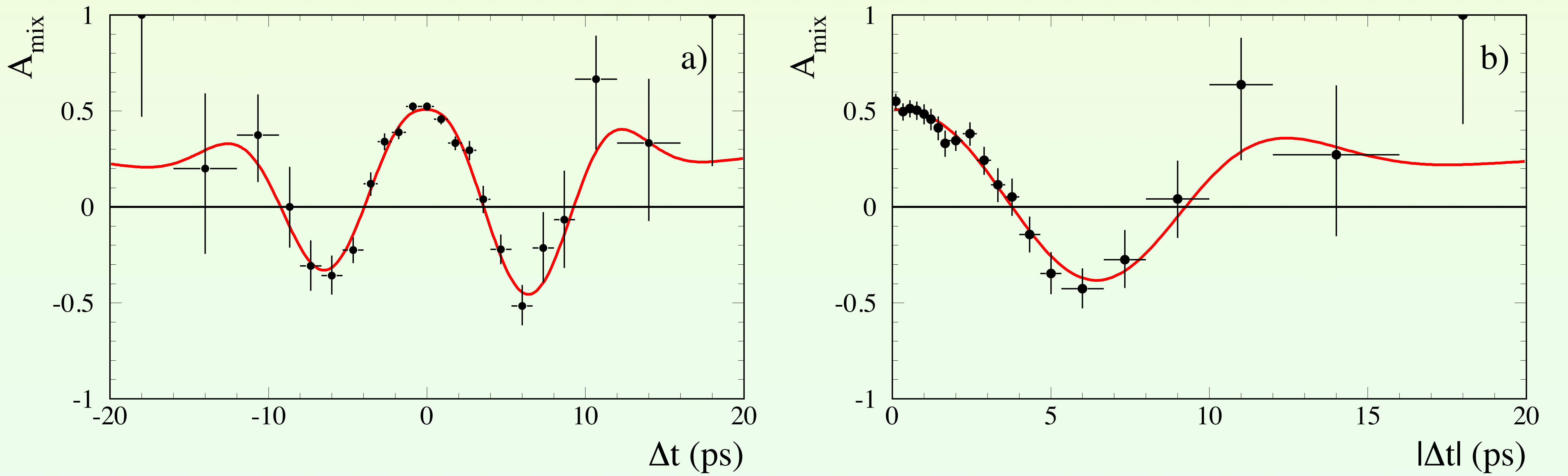
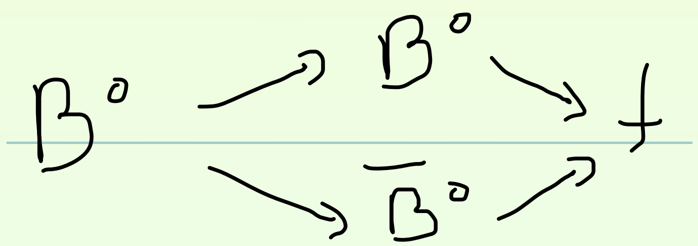


FIG. 25: Time-dependent asymmetry $\mathcal{A}(\Delta t)$ between unmixed and mixed events for hadronic B candidates with $m_{\rm ES} > 5.27\,{\rm GeV}/c^2$, a) as a function of Δt; and b) folded as a function of $|\Delta t|$. The asymmetry in a) is due to the fitted bias in the Δt resolution function.

Babar, arXiv.org > hep-ex > arXiv:hep-ex/0201020

CP Asymmetries in Interference Mixing-Decay



Mixing gives two paths to same final state.
If final state is a CP eigenstate this can test for CPV in the two decays.

This we know: $\Gamma(X^0(t) \to f) = |f_+(t)\langle f|H_w|X^0\rangle + \frac{q}{p} f_-(t)\langle f|H_w|\bar{X}^0\rangle|^2$

This defines shorthand: $\equiv |f_+(t)A_f + \frac{q}{p} f_-(t)\bar{A}_f|^2$

idem $\Gamma(\bar{X}^0(t) \to \bar{f}) = |\frac{p}{q} f_-(t)A_{\bar{f}} + f_+(t)\bar{A}_{\bar{f}}|^2$

Time-dependent asymmetry

$$\mathcal{A}(t) = \frac{\Gamma(\bar{X}^0(t) \to \bar{f}) - \Gamma(X^0(t) \to f)}{\Gamma(\bar{X}^0(t) \to \bar{f}) + \Gamma(X^0(t) \to f)}$$

(similar to $\delta(t)$)

1.Semileptonic (much like $\delta(t)$): $f = e^- + any$

$\bar{b} \to \bar{c} e^+ \nu \quad \Rightarrow \quad X^0 \to e^+ + \text{any}$

$b \to c e^- \bar{\nu} \quad \Rightarrow \quad \bar{X}^0 \to e^- + \text{any}$

Then $\quad A_f = 0$, $\quad \bar{A}_{\bar{f}} = 0$

$$\Gamma(X^0(t) \to f) = |\tfrac{q}{p} f_-(t) \bar{A}_f|^2$$
$$\Gamma(\bar{X}^0(t) \to \bar{f}) = |\tfrac{p}{q} f_-(t) A_{\bar{f}}|^2$$

$$\mathcal{A}_{\text{SL}}(t) = \frac{\left|\frac{p}{q}\right|^2 - \left|\frac{q}{p}\right|^2}{\left|\frac{p}{q}\right|^2 + \left|\frac{q}{p}\right|^2}$$

- Directly probes $|q/p|$
- Time dependence? time independent
- We already saw that in SM this is expected to vanish to good approximation (if $\Gamma_{12} = 0$)
- We did not try to improve on our approximation nor estimate deviations; guesstimate

B_d: $\mathcal{A}^d_{\text{SL}} = \mathcal{O}\left[(m_c^2/m_t^2)\sin\beta\right] \lesssim 0.001\ .$ $\quad B_s$: $\mathcal{A}^s_{\text{SL}} = \mathcal{O}\left[(m_c^2/m_t^2)\sin\beta_s\right] \lesssim 10^{-4}.$

- Experiment

$$\mathcal{A}^d_{\text{SL}} = (+0.7 \pm 2.7) \times 10^{-3} \quad \Longrightarrow \quad |q/p| = 0.9997 \pm 0.0013\ .$$

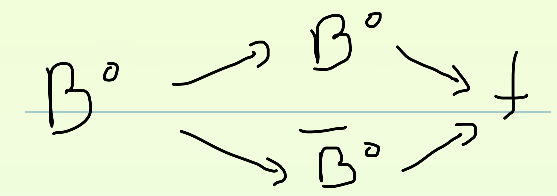
$$\mathcal{A}^s_{\text{SL}} = (-17.1 \pm 5.5) \times 10^{-3} \quad \Longrightarrow \quad |q/p| = 1.0086 \pm 0.0028\ .$$

In what follows take $\left|\frac{p}{q}\right| = 1$ and it makes sense to use $\Delta\Gamma \approx 0$

Simplification:

$$f_\pm(t) = e^{-iMt} e^{-\Gamma t} \begin{cases} \cos(\frac{1}{2}\Delta M t) \\ -i\sin(\frac{1}{2}\Delta M t) \end{cases}$$

2. CPV in interference between a decay with mixing and a decay without mixing



No distinction between final states $A_{\bar{f}} = A_f$ $\qquad \bar{A}_{\bar{f}} = \bar{A}_f$

$$\mathcal{A}_{f_{CP}} = \frac{|\frac{p}{q} f_-(t) A_f + f_+(t)\bar{A}_f|^2 - |f_+(t)A_f + \frac{q}{p} f_-(t)\bar{A}_f|^2}{|\frac{p}{q} f_-(t) A_f + f_+(t)\bar{A}_f|^2 + |f_+(t)A_f + \frac{q}{p} f_-(t)\bar{A}_f|^2}$$

Divide by $|A|^2$, use $|p/q| = 1$ and define

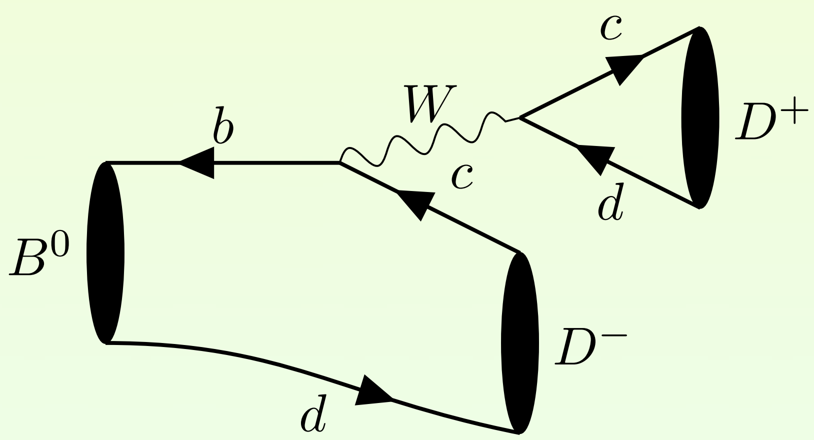
$$\lambda_f = \frac{q}{p}\frac{\bar{A}_f}{A_f}$$

$$= \frac{|f_-(t) + f_+(t)\lambda_f|^2 - |f_+(t) + f_-(t)\lambda_f|^2}{|f_-(t) + f_+(t)\lambda_f|^2 - |f_+(t) + f_-(t)\lambda_f|^2}$$

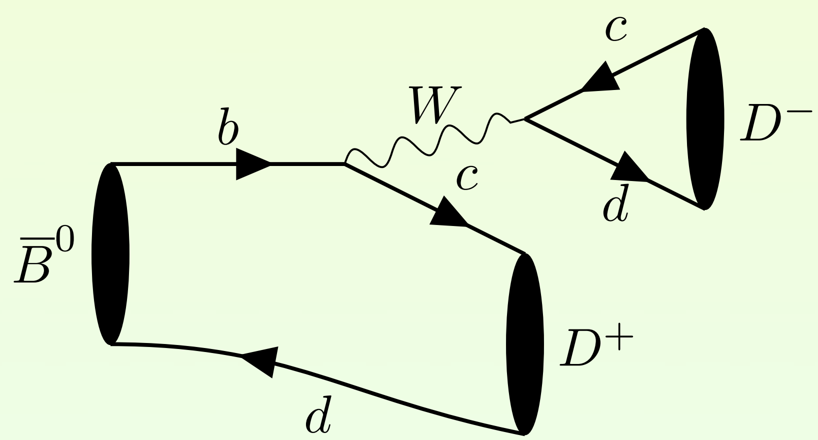
$$= -\frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2}\cos(\Delta M t) + \frac{2\mathrm{Im}\lambda_f}{1 + |\lambda_f|^2}\sin(\Delta M t)$$

$$\equiv -C_f \cos(\Delta M t) + S_f \sin(\Delta M t)$$

Example: $f = D^+ D^-$



$A_{D^+D^-} \propto V_{cb}^* V_{cd}$



$\bar{A}_{D^+D^-} \propto V_{cb} V_{cd}^*$

$$\frac{\bar{A}_f}{A_f} = \frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}}$$

We have already seen that

$$\frac{p}{q} = \frac{2M_{12}}{\Delta M} = \frac{\Delta M}{2M_{12}^*} = \frac{M_{12}}{|M_{12}|} = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}$$

Putting these together $|\lambda_{D^+D^-}| = 1$

and

$$S_{D^+D^-} = \mathrm{Im}\,(\lambda_{D^+D^-}) = \mathrm{Im}\left(\frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}} \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\right) = \mathrm{Im}(e^{2i\beta}) = \sin(2\beta)$$

This is pure KM phase! No hadronic uncertainties.

Just as in direct CPV:

$$A_f = aT + bP$$
$$\bar{A}_f = a^*T + b^*P$$

a, b = CKMs
T, P = M.E.s
("tree" and "penguin")

Suppose $|P| = 0 \quad \Rightarrow \quad \lambda_f = \frac{q}{p}\frac{a^*}{a}$

That's just CKM's. No dependence on unknown M.E.s !

For pointing this out I. Bigi and A. Sanda received the Sakurai Prize 2004

... and a race to build *B*-factories was on!
(well, with the added idea of asymmetric colliders)

$B \to J/\psi K_S$

$$\frac{\bar{A}_{\psi K_S}}{A_{\psi K_S}} = -\frac{(V_{cb}V_{cs}^*)T + (V_{ub}V_{us}^*)P}{(V_{cb}^*V_{cs})T + (V_{ub}^*V_{us})P} \times \frac{V_{cd}^*V_{cs}}{V_{cd}V_{cs}^*}$$

Neglecting P:

$$\lambda_{\psi K_S} = -e^{-2i\beta} \qquad S_{\psi K_S} = \sin(2\beta), \quad C_{\psi K_S} = 0$$

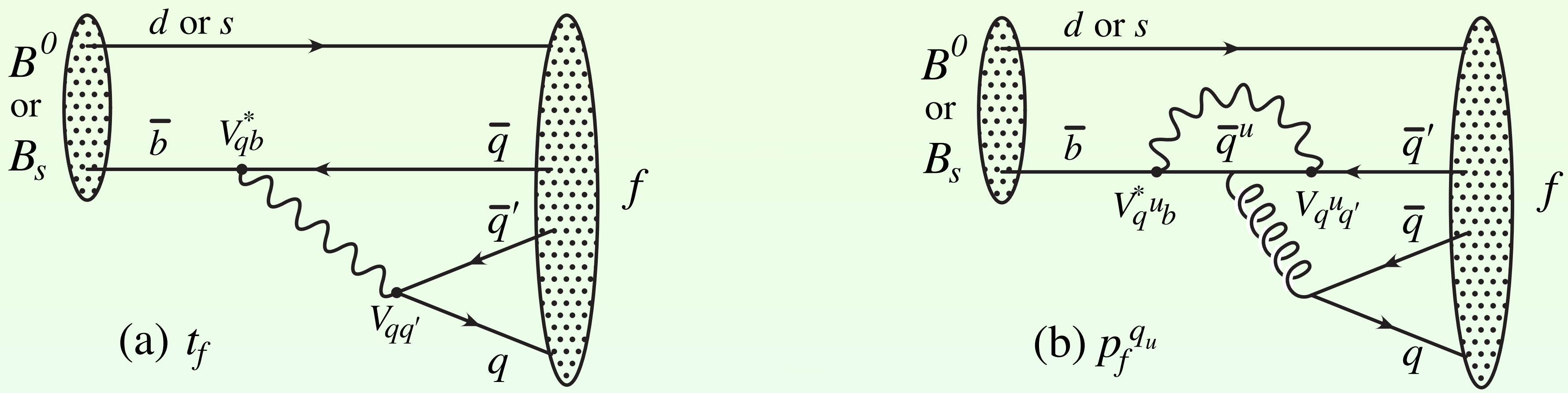
PDG: $S_{\psi K} = +0.682 \pm 0.019, \quad C_{\psi K} = (0.5 \pm 2.0) \times 10^{-2}$

$B \to \pi\pi$

$$\frac{\bar{A}_{\pi\pi}}{A_{\pi\pi}} = -\frac{(V_{ub}V_{ud}^*)T + (V_{tb}V_{td}^*)P}{(V_{ub}^*V_{ud})T + (V_{tb}^*V_{td})P}$$

PDG: $C_{\pi^+\pi^-} = -0.31 \pm 0.05$

Real life, $|P| \neq 0$ (for this observation, I got no prize; Phys.Lett. B229 (1989) 280)



$\bar b \to \bar q q \bar q'$	$B^0 \to f$	$B_s \to f$	CKM dependence of A_f	Suppression
$\bar b \to \bar c c \bar s$	ψK_S	$\psi\phi$	$(V_{cb}^* V_{cs})T + (V_{ub}^* V_{us})P^u$	loop $\times\ \lambda^2$
$\bar b \to \bar s s \bar s$	ϕK_S	$\phi\phi$	$(V_{cb}^* V_{cs})P^c + (V_{ub}^* V_{us})P^u$	λ^2
$\bar b \to \bar u u \bar s$	$\pi^0 K_S$	K^+K^-	$(V_{cb}^* V_{cs})P^c + (V_{ub}^* V_{us})T$	λ^2/loop
$\bar b \to \bar c c \bar d$	D^+D^-	ψK_S	$(V_{cb}^* V_{cd})T + (V_{tb}^* V_{td})P^t$	loop
$\bar b \to \bar s s \bar d$	$K_S K_S$	ϕK_S	$(V_{tb}^* V_{td})P^t + (V_{cb}^* V_{cd})P^c$	$\lesssim 1$
$\bar b \to \bar u u \bar d$	$\pi^+\pi^-$	$\rho^0 K_S$	$(V_{ub}^* V_{ud})T + (V_{tb}^* V_{td})P^t$	loop

$C_{D^{*-}D^{+}}\ (B^0 \rightarrow\ D^*(2010)^- D^+) = -0.01 \pm 0.11$

$\boldsymbol{S_{D^{*-}D^{+}}\ (B^0 \rightarrow\ D^*(2010)^- D^+)} = -0.72 \pm 0.15$

$C_{D^{*+}D^{-}}\ (B^0 \rightarrow\ D^*(2010)^+ D^-) = 0.00 \pm 0.13 \quad (S = 1.3)$

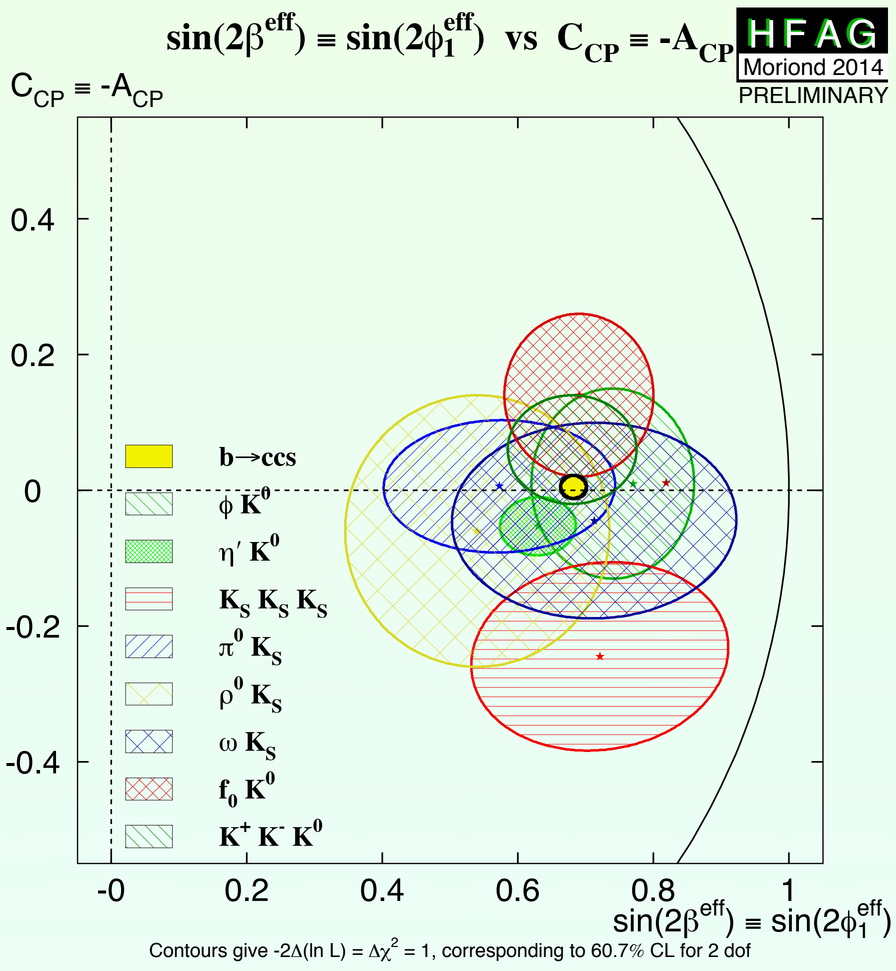
$\boldsymbol{S_{D^{*+}D^{-}}\ (B^0 \rightarrow\ D^*(2010)^+ D^-)} = -0.73 \pm 0.14$

$C_{D^{*+}D^{*-}}\ (B^0 \rightarrow\ D^{*+} D^{*-}) = 0.01 \pm 0.09 \quad (S = 1.6)$

$\boldsymbol{S_{D^{*+}D^{*-}}\ (B^0 \rightarrow\ D^{*+} D^{*-})} = -0.59 \pm 0.14 \quad (S = 1.8)$

$C_+\ (B^0 \rightarrow\ D^{*+} D^{*-}) = 0.00 \pm 0.10 \quad (S = 1.6)$

$\boldsymbol{S_+\ (B^0 \rightarrow\ D^{*+} D^{*-})} = -0.73 \pm 0.09$



$C_-\ (B^0 \rightarrow\ D^{*+} D^{*-}) = 0.19 \pm 0.31$

$S_-\ (B^0 \rightarrow\ D^{*+} D^{*-}) = 0.1 \pm 1.6 \quad (S = 3.5)$

$C\ (B^0 \rightarrow\ D^*(2010)^+ D^*(2010)^- K^0_S) = 0.01 \pm 0.29$

$S\ (B^0 \rightarrow\ D^*(2010)^+ D^*(2010)^- K^0_S) = 0.1 \pm 0.4$

$C_{D^+D^-}\ (B^0 \rightarrow\ D^+ D^-) = -0.46 \pm 0.21 \quad (S = 1.8)$

$\boldsymbol{S_{D^+D^-}\ (B^0 \rightarrow\ D^+ D^-)} = -0.99^{+0.17}_{-0.14}$

$C_{J/\psi(1S)\pi^0}\ (B^0 \rightarrow\ J/\psi(1S)\pi^0) = -0.13 \pm 0.13$

$\boldsymbol{S_{J/\psi(1S)\pi^0}\ (B^0 \rightarrow\ J/\psi(1S)\pi^0)} = -0.94 \pm 0.29 \quad (S = 1.9)$

$C_{D^{(*)}_{CP}h^0}\ (B^0 \rightarrow\ D^{(*)}_{CP} h^0) = -0.23 \pm 0.16$

$S_{D^{(*)}_{CP}h^0}\ (B^0 \rightarrow\ D^{(*)}_{CP} h^0) = -0.56 \pm 0.24$

$C_{K^0\pi^0}\ (B^0 \rightarrow\ K^0\pi^0) = 0.00 \pm 0.13 \quad (S = 1.4)$

$\boldsymbol{S_{K^0\pi^0}\ (B^0 \rightarrow\ K^0\pi^0)} = 0.58 \pm 0.17$

$C_{\eta'(958)K^0_S}\ (B^0 \rightarrow\ \eta'(958) K^0_S) = -0.04 \pm 0.20 \quad (S = 2.5)$

$S_{\eta'(958)K^0_S}\ (B^0 \rightarrow\ \eta'(958) K^0_S) = 0.43 \pm 0.17 \quad (S = 1.5)$

$C_{\eta' K^0}\ (B^0 \rightarrow\ \eta' K^0) = -0.05 \pm 0.05$

$\boldsymbol{S_{\eta' K^0}\ (B^0 \rightarrow\ \eta' K^0)} = 0.60 \pm 0.07$

$C_{\omega K^0_S}\ (B^0 \rightarrow\ \omega K^0_S) = -0.30 \pm 0.28 \quad (S = 1.6)$

$S_{\omega K^0_S}\ (B^0 \rightarrow\ \omega K^0_S) = 0.43 \pm 0.24$

$C\ (B^0 \rightarrow\ K^0_S \pi^0 \pi^0) = 0.2 \pm 0.5$

$S\ (B^0 \rightarrow\ K^0_S \pi^0 \pi^0) = 0.7 \pm 0.7$

$C_{\rho^0 K^0_S}\ (B^0 \rightarrow\ \rho^0 K^0_S) = -0.04 \pm 0.20$

$S_{\rho^0 K^0_S}\ (B^0 \rightarrow\ \rho^0 K^0_S) = 0.50^{+0.17}_{-0.21}$

$C_{f_0 K^0_S}\ (B^0 \rightarrow\ f_0(980) K^0_S) = 0.29 \pm 0.20$

$\boldsymbol{S_{f_0 K^0_S}\ (B^0 \rightarrow\ f_0(980) K^0_S)} = -0.50 \pm 0.16$

$S_{f_2 K^0_S}\ (B^0 \rightarrow\ f_2(1270) K^0_S) = -0.5 \pm 0.5$

$C_{f_2 K^0_S}\ (B^0 \rightarrow\ f_2(1270) K^0_S) = 0.3 \pm 0.4$

$S_{f_x K^0_S}\ (B^0 \rightarrow\ f_x(1300) K^0_S) = -0.2 \pm 0.5$

$C_{f_x K^0_S}\ (B^0 \rightarrow\ f_x(1300) K^0_S) = 0.13 \pm 0.35$

$S_{K^0\pi^+\pi^-}\ (B^0 \rightarrow\ K^0\pi^+\pi^-\ \text{nonresonant}) = -0.01 \pm 0.33$

$C_{K^0\pi^+\pi^-}\ (B^0 \rightarrow\ K^0\pi^+\pi^-\ \text{nonresonant}) = 0.01 \pm 0.26$

$C_{K^0_S K^0_S}\ (B^0 \rightarrow\ K^0_S K^0_S) = 0.0 \pm 0.4 \quad (S = 1.4)$

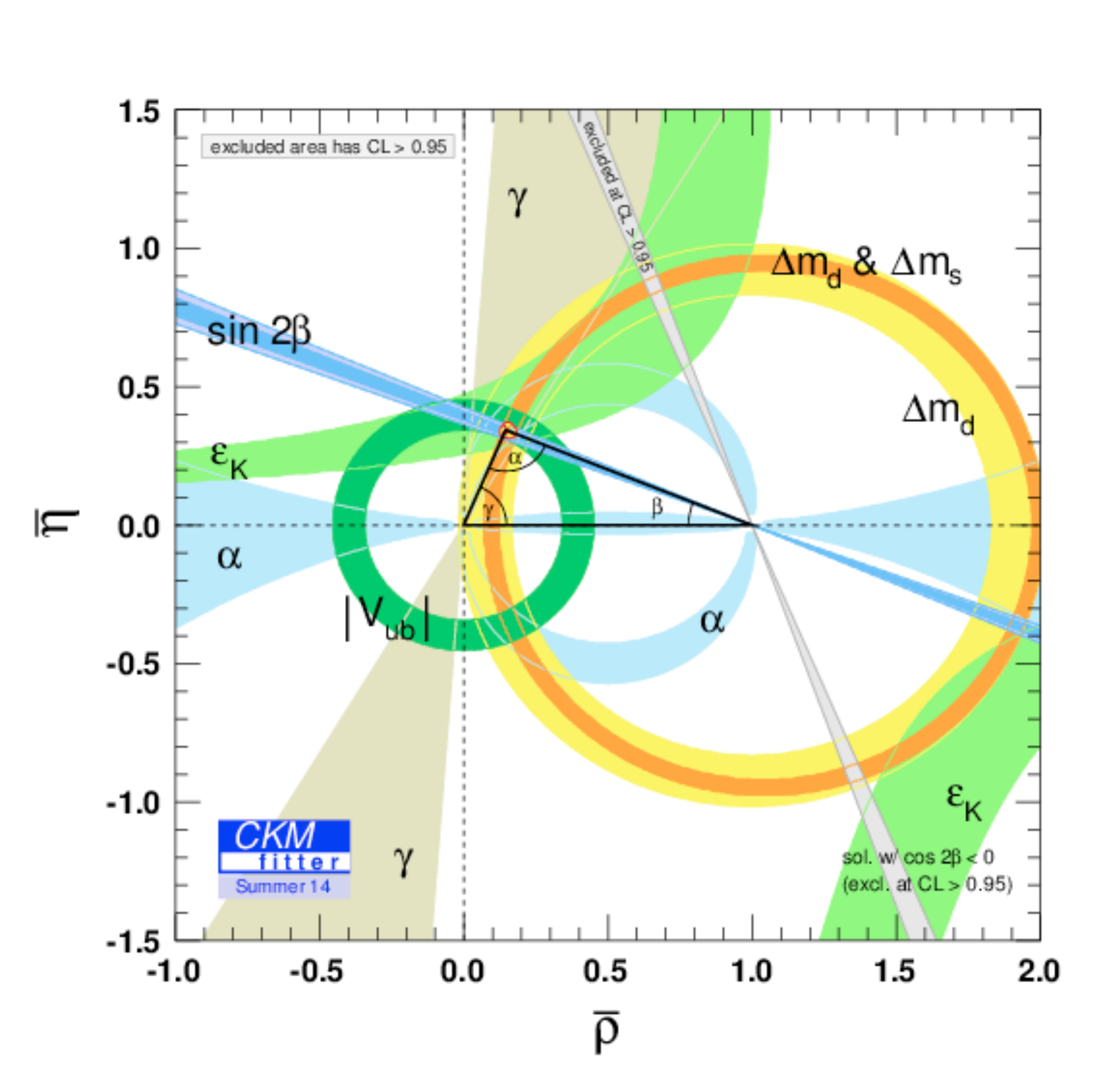
$S_{K^0_S K^0_S}\ (B^0 \rightarrow\ K^0_S K^0_S) = -0.8 \pm 0.5$

$C_{K^+K^-K^0_S}\ (B^0 \rightarrow\ K^+ K^- K^0_S\ \text{nonresonant}) = 0.06 \pm 0.08$

+ two more pages

EPILOGUE

State of the art:



Exercise: you should be able to understand these shapes

The End

Frogatt & Nielsen, NPB147, 277 ('79)

Hierarchies from small order parameter(s).

IDEA:

(i) Small parameter $\epsilon = \frac{\langle\phi\rangle}{M} \ll 1$

(ii) Symmetry group G prevents mass terms $\bar{\psi}_{Li}\psi_{Rj}$

(iii) Terms $\left(\frac{\phi}{M}\right)^{\Delta_{ij}} \bar{\psi}_{Li}\psi_{Rj}$ allowed by G

(iv) $\langle\phi\rangle \neq 0$ breaks G spontaneously

(v) Different charges under G for different $\psi_{L/R\,i}$

Simplest if $G = U(1)$, with $Q(\phi) = 1$, $Q(\psi_{Li}) = c + b_i$ $Q(\psi_{Ri}) = c - a_i$

Then $Q(\bar{\psi}_{Li}\psi_{Rj}) = -(a_i + b_i)$. If $a_i + b_i > 0$ $\left(\frac{\phi}{M}\right)^{\Delta_{ij}} \bar{\psi}_{Li}\psi_{Rj}$ $\Delta_{ij} = a_i + b_j$

else $\left(\frac{\phi^*}{M}\right)^{\Delta_{ij}} \bar{\psi}_{Li}\psi_{Rj}$

Take $M_{ij} = g_{ij}\,\epsilon^{a_i+b_j}$ $a_i>0$, $b_j\geq 0$

and order them $a_1\leq a_2\leq a_3$, $b_1\leq b_2\leq b_3$

Anarchy (democracy? Ask the Greeks) $G=(g_{ij})$ = all entries of same order

What can we say about masses/mixing ?

(i) product of largest n eigenvalues $= \prod_{i=1}^{n}\lambda_i \approx (\det G^{(n)})\,\epsilon^{K_n}$ $G^{(n)}$ = top $n\times n$ block in G, $K_n=\sum_{i=1}^{n}(a_i+b_i)$

(ii) n-th largest mass $m_n = \dfrac{\prod_1^n \lambda_i}{\prod_1^{n-1}\lambda_i} = \dfrac{(\det G^{(n)})}{(\det G^{(n-1)})}\,\epsilon^{a_n+b_n} \Rightarrow \dfrac{m_i}{m_j} = \epsilon^{a_i-a_j+b_i-b_j}$

(iii) CKM? First need two mass matrices, so let's introduce

$\bar{u}_R M_u u_L + \bar{d}_R M_d d_L + h.c.$

Assume each is of the form above

CKM: recall if $U_{U_R}^\dagger M_U U_{U_L}$ and $U_{d_R}^\dagger M_d U_{d_L}$ are diagonal then $V = U_{U_L}^\dagger U_{d_L}$

$U_{U_L}^\dagger M_U^\dagger M_U U_{U_L}$ and $U_{d_L}^\dagger M_d^\dagger M_d U_{d_L}$ diagonal

$\Rightarrow$ columns of U are (normalized) eigenvectors of $M^\dagger M$.

$$(M^\dagger M)_{ij} = M_{ki}^* M_{kj} = \sum_k g_{ki}^* g_{kj} \epsilon^{2a_k + b_i + b_j} = g_{1i}^* g_{1j} \epsilon^{2a_1} \epsilon^{b_i + b_j} + \cdots$$

- For largest eigenvalue (m_b^2 or m_t^2) eigenvector is

$$\sum_j \left[(g_{1i}^* \epsilon^{b_i})(g_{1j} \epsilon^{b_j})\right](g_{1j}^* \epsilon^{b_j}) = \left(\sum_j |g_{1j} \epsilon^{b_j}|^2\right) g_{1i}^* \epsilon^{b_i}$$

Normalize: $\text{norm}^2 = \sum_i |g_{1i}^* \epsilon^{b_i}|^2 \simeq |g_{11}|^2 \epsilon^{2b_1} \quad \Rightarrow \quad U_{i1} = \dfrac{g_{1i}^*}{g_{11}^*} \epsilon^{b_i - b_1}$

(used freedom to choose overall phase)

- For second largest,

for $i \geq 2$: $U_{i2}^* = \dfrac{g_{11} g_{2i} - g_{21} g_{1i}}{g_{11} g_{22} - g_{12} g_{21}} \epsilon^{b_i - b_2}$, and $U_{12}^* = - U_{21}$

Exercise: Use perturbation theory (as in your QM courses) to show this. Keep in mind this is only leading order in ϵ.

- In general one may show $U_{ij} = u_{ij}\,\epsilon^{|b_i - b_j|}$

  with u_{ij} order 1 complex.

- CKM: $(U_{u_L})_{ij} = u_{ij}\,\epsilon^{|b_i-b_j|}$ $\quad (U_{d_L}) = d_{ij}\,\epsilon^{|b_i-b_j|}$

  Same b's: u_L and d_L are members of doublet q_L and G must commute with $SU(2)_W \times U(1)_Y$.

$$V_{ij} = (U_{u_L}^\dagger U_{d_L})_{ij} = \sum_k u^*_{ki}\, d_{kj}\, \epsilon^{|b_i-b_k|}\,\epsilon^{|b_k-b_j|}$$

Largest term when $k = i$ or j

$$V_{ij} = v_{ij}\,\epsilon^{|b_i-b_j|}$$

Note this gives $V_{ij} = \delta_{ij} + O(\epsilon^{\text{power}})$

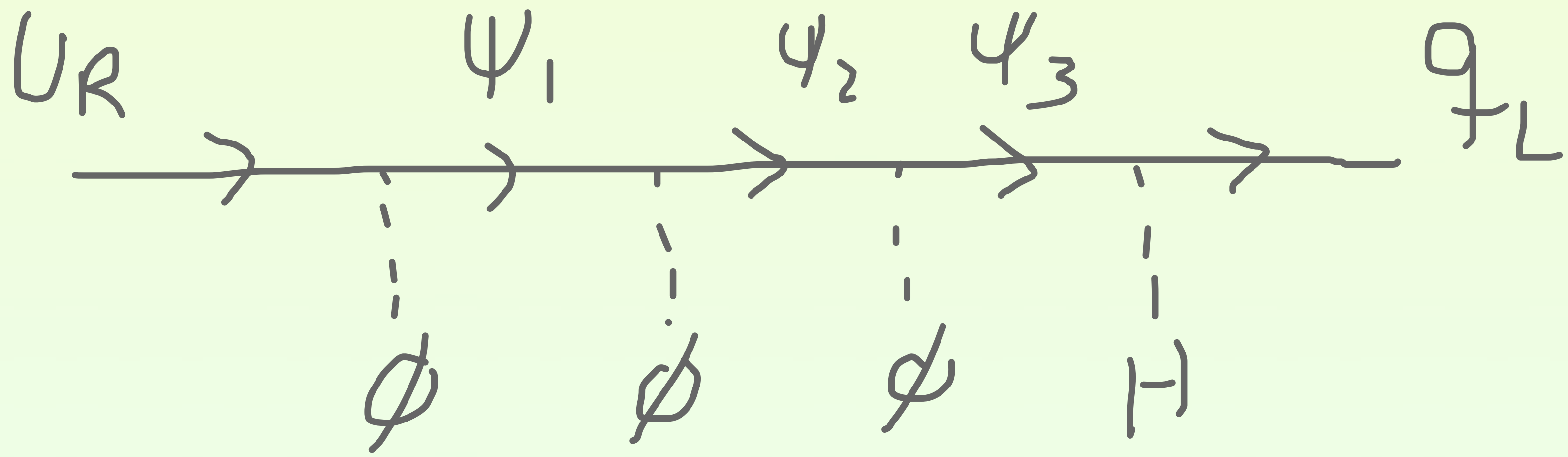
Also, some relation (vague) to mass ratios (as in Fritsch):

$$\frac{m_{u_i}}{m_{u_j}} \sim \epsilon^{a_i^u - a_j^u + b_i - b_j} \qquad \frac{m_{d_i}}{m_{d_j}} \sim \epsilon^{a_i^d - a_j^d + b_i - b_j}$$

So $V_{ij} \sim \left(\frac{m_{u_i}}{m_{u_j}}\right)^{c^u_{ij}} \sim \left(\frac{m_{d_i}}{m_{d_j}}\right)^{c^d_{ij}}$ $\quad c_{ij} = \left(1 + \frac{a_i - a_j}{b_i - b_j}\right)^{-1}$ $\Bigg\|$ e.g., if $\Delta a^d \simeq \Delta b$ $\quad c^d_{ij} = \frac{1}{2}$

$\Rightarrow \sin\theta_c = \sqrt{\frac{m_d}{m_s}}$

Model?



ψ_I are vector-like: have masses $\mathcal{L}_{kin} = \sum_I \bar{\psi}_I (i\not{\partial} - M_I)\psi_I$

For large M this gives $\mathcal{L}_{eff,interaction} = \left(\frac{\phi}{M_1}\right)\left(\frac{\phi}{M_2}\right)\left(\frac{\phi}{M_3}\right) H \bar{q}_L u_R$

(times some $\mathcal{O}(1)$ couplings).

So we also need

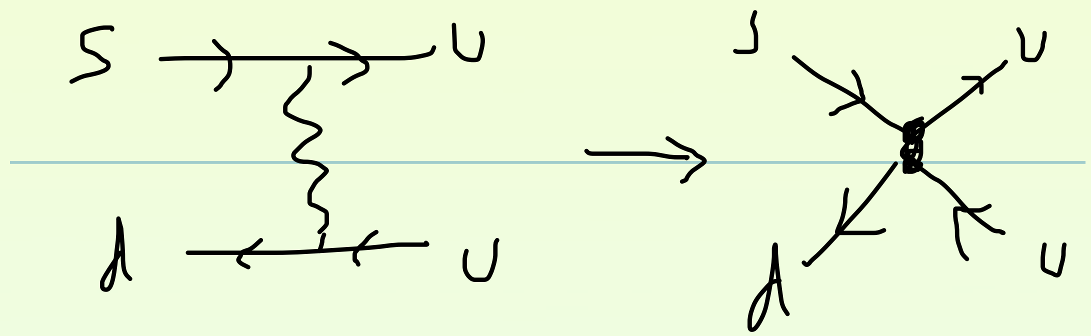
$$\mathcal{L} = \cdots \quad H\bar{q}_L\psi_3 , \quad \phi\bar{\psi}_3\psi_2 , \quad \cdots \quad \phi\bar{\psi}_1 u_R$$

$\bar{q}_L$: $-(c+b)$, ψ_3: $c+b$; ϕ: 1, $\bar{\psi}_3$: $-(c+b)$, ψ_2: $c+b-1$; ϕ: 1, $\bar{\psi}_1$: $-c+a-1$, u_R: $c-a$

In this example $c+b-2 = c-a+1 \Rightarrow b+a=3$

Clearly easy to construct models. Use freedom in c, b for anomaly cancelation.

Fermi Theory



$$\left(-\frac{ig_2}{\sqrt{2}}V_{ud}^*\bar{d}\gamma^\mu P_L u\right)\left(-i\frac{g_{\mu\nu}-q_\mu q_\nu/M_W^2}{q^2-M_W^2}\right)\left(-\frac{ig_2}{\sqrt{2}}V_{us}\bar{u}\gamma^\nu P_L s\right) \;\to -\frac{ig_2^2}{2M_W^2}V_{ud}^*V_{us}\,\bar{u}\gamma^\mu P_L s\,\bar{d}\gamma_\mu P_L u$$

So you can use this $\quad \mathcal{H}_{\text{eff}}^{\Delta S=1} = \frac{g_2^2}{2M_W^2}V_{ud}^*V_{us}\,\bar{u}_L\gamma^\mu s_L\,\bar{d}_L\gamma_\mu u_L$

in

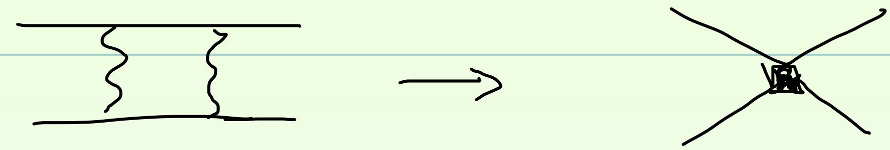
$$M_{12} = M\delta_{12} + \langle 1|\mathcal{H}|2\rangle + \sum_n{}' \mathrm{PP}\frac{\langle 1|\mathcal{H}|n\rangle\langle n|\mathcal{H}|2\rangle}{M-E_n} + \cdots$$

The intermediate states are pions, rho-mesons, ... “long-distance contributions”

Graphs with *u* replaced by *c*, *t*...

(i) It seems difficult to evaluate $\sum PP\cdots$

(ii) We have used a very effective approximation $m_K \ll M_W$, why not $m_K \ll m_t$ or even $m_K \ll m_c$?



"short distance contributions"

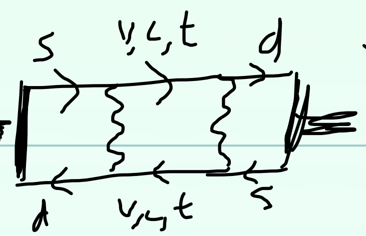
Sweet: use 1st order $M_{12} = \langle 1|\mathcal{H}|2\rangle + \cdots$

short distance: difficult

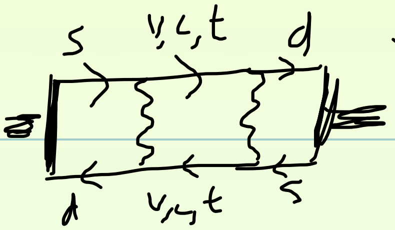
long distance: way more difficult

Im(M_{12}) is CPV $\Rightarrow$ non-zero requires *c, t* quarks $\Rightarrow$ short distance $\Rightarrow$ doable

Do this, leave Re for lattice; see, *e.g.*, 1212.5931. Use, for numerics, $\mathrm{Re}(M_{12}) = \frac{1}{2}\Delta M$ from data

$\mathrm{Im}\, M_{12} = \mathrm{Im}\Big($ 

$\Big) \sim \mathrm{Im}\left[\frac{G_F^2 M_W^2}{4\pi^2}\sum_{q=u,c,t} V^*_{qd}V_{qs}\sum_{q'=u,c,t} V^*_{q'd}V_{q's}\, f\left(\frac{m_q}{M_W},\frac{m_{q'}}{M_W}\right)\langle K^0|\bar d_L\gamma^\mu s_L\, \bar d_L\gamma_\mu s_L|\bar K^0\rangle\right]$

$\mathrm{Im}M_{12} = \mathrm{Im}\Big($ 

$\Big) \sim \mathrm{Im}\left[\frac{G_F^2 M_W^2}{4\pi^2}\sum_{q=u,c,t} V_{qd}^* V_{qs} \sum_{q'=u,c,t} V_{q'd}^* V_{q's}\, f\left(\frac{m_q}{M_W}, \frac{m_{q'}}{M_W}\right) \langle K^0|\bar{d}_L\gamma^\mu s_L\, \bar{d}_L\gamma_\mu s_L|\bar{K}^0\rangle\right]$

- *f(x,y)*: can compute, Feynman diagrams
- double GIM!
- non-zero Im-part form CKM's only
  Exercise: show the matrix element is real (use CP of strong interactions)
- std parametrization: V_{ud} and V_{us} real need at least one *c* or *t*-quark
- EFT not valid with 1 or 2 *u*-quarks, but these very suppressed (EFT explanation is cleanest, but for now think GIM again)
- Left with *c,t* contributions. But

$$\sum V_{qd}V_{qs}^* = 0 \quad \text{and} \quad \mathrm{Im}V_{ud}V_{us}^* = 0 \qquad\qquad \mathrm{Im}V_{cd}V_{cs}^* = -\mathrm{Im}V_{td}V_{ts}^* = A^2\lambda^5\eta$$

- Last we need M.E. We parametrize our ignorance using the "vacuum insertion approximation:

$$\langle K^0|\bar{d}_L\gamma^\mu s_L\, \bar{d}_L\gamma_\mu s_L|\bar{K}^0\rangle = \tfrac{2}{3} f_K^2 m_K^2 B_K$$

where $B_K = 1$ in vacuum insertion approx.

Exercise: Use

$$\langle K^0|\bar{d}_L\gamma^\mu s_L\, \bar{d}_L\gamma_\mu s_L|\bar{K}^0\rangle \to \langle K^0|\bar{d}_L\gamma^\mu s_L|0\rangle\langle 0|\bar{d}_L\gamma_\mu s_L|\bar{K}^0\rangle + \langle K^0|\bar{d}_L^a\gamma^\mu s_{Lb}|0\rangle\langle 0|\bar{d}_L^b\gamma_\mu s_{La}|\bar{K}^0\rangle$$

and

$$\langle 0|\bar{d}_L\gamma_\mu s_L|\bar{K}^0\rangle = \tfrac{1}{2} p_\mu f_K$$

to show $B_K = 1$ in vacuum insertion approx. Note: here we are using the relativistic normalization of states

Ready to put it all together?

$$\mathrm{Im} M_{12} = -2A^2\lambda^5\eta\, \tfrac{2}{3} B_K \frac{G_F^2 m_K^2 f_K^2}{4\pi^2}\left[A^2\lambda^5(1-\rho)f(m_t) - \lambda f(m_c) + \lambda f(m_c, m_t)\right]\frac{1}{2m_K}$$

where, using $x_i = \dfrac{m_i^2}{M_W^2}$

$$f(m_c, m_t) = x_c\left[\ln\frac{x_c}{x_t} - \frac{3x_t}{4(1-x_t)} - \frac{3x_t^2 \ln x_t}{4(1-x_t)^2}\right] \qquad \text{used} \quad x_c \ll 1$$

$$f(m) = \frac{4x - 11x^2 + x^3}{4(1-x)^2} - \frac{3x^3 \ln x}{2(1-x)^3}$$

Finally

$$\epsilon_K = e^{i\pi/4} C_\epsilon\, A^2\lambda^5\eta\,\left[A^2\lambda^5(1-\rho)f(m_t) - \lambda(f(m_c) - f(m_c, m_t))\right]$$

$$C_\epsilon = \frac{G_F^2 f_K^2 m_K M_W^2 B_K}{6\sqrt{2}\pi^2 \Delta m_K} \approx 3\times 10^4 B_K$$

Instead of detailed numerics, let's check order of magnitude: $A^2\lambda^5 \sim (0.2)^5 \sim 3\times 10^{-4}$

$$A^2\lambda^5(1-\rho)f(m_t) \sim (0.2)^5 \sim 3\times 10^{-4} \qquad \Rightarrow \qquad \epsilon \sim 3\times 10^{-3}$$

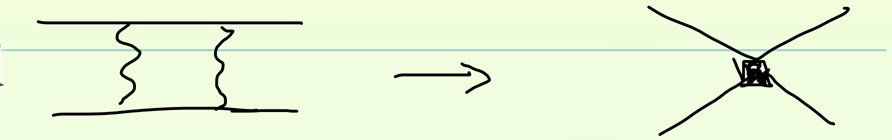
$$\lambda f(m_c) \sim \lambda f(m_c, m_t) \sim (0.2)\left(\frac{1.5}{80}\right)^2 \sim \times 10^{-4} \qquad \Rightarrow \qquad \epsilon \sim \times 10^{-3}$$

All give contributions of the right order of magnitude!
This is a great success of the SM!!! (how many exclamations marks do we need?)

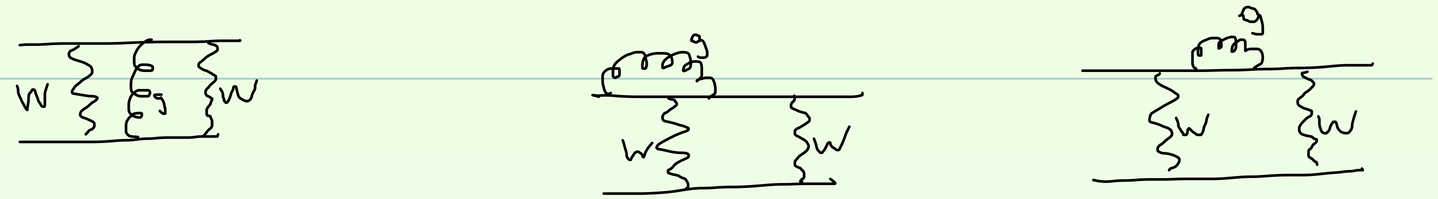
Exercise: (i) pretend you can compute $\mathrm{Re}\, M_{12}$ by computing Feynman diagram and using $\mathcal{H}_W = \frac{1}{4\sqrt{2}} G_F^2 M_W^2 (\cdots)(\bar{s}d)(\bar{s}d)$, so as to ignore $\sum_n{}' PP$. Estimate ΔM. Compare with experimental value.

(ii) What if you ignore c, t quarks? (so no GIM). (iii) Ignore t-quark. How large does m_c have to be to account for ΔM? This is how m_c was predicted and GIM discovered.

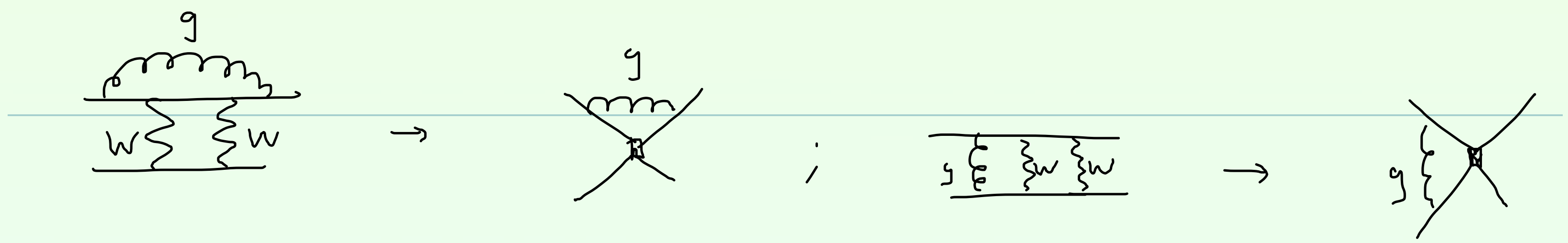
Before we move in, there is a sticky point...

We have replaced  and expanded in powers of $1/M_W$

while pretending we have kept strong interactions exact. But these are QCD, we know. And what about



Graphs with gluons connecting external legs accounted for:



EFT organizes the computation, factorizing

long distance contributions (that go into M.E.)	×	short distance contributions (computable)

and allows RGE to resum logs, eg, $\sim \sum_n (\frac{\alpha_s}{\pi} \ln \frac{M}{\mu})^n$, systematically,

$$\epsilon_K = e^{i\pi/4} C_\epsilon \, A^2 \lambda^5 \eta \, \left[\eta_2 A^2 \lambda^5 (1-\rho) f(m_t) - \lambda(\eta_1 f(m_c) - \eta_3 f(m_c, m_t)) \right]$$

$$\eta_{1,2,3} \approx 0.7, 0.6, 0.4$$