Introduction

to the EW STANDARD MODEL

CERN School

Ibarra, March 2015

TABLE OF CONTENTS

I. Summary of the Phenomenology

TABLE OF ELEMENTARY PARTICLES		
QUANTA OF RADIATION		
Strong Interactions		Eight gluons
Electromagnetic Interactions		Photon (γ)
Weak Interactions		Bosons W^+ , W^- , Z^0
Gravitational Interactions		Graviton (?)
MATTER PARTICLES		
	Leptons	Quarks
1st Family	ν_e , e^-	u_a , d_a , $a = 1, 2, 3$
2nd Family	ν_μ , μ^-	c_a , s_a , $a = 1, 2, 3$
3rd Family	ν_τ , τ^-	t_a , b_a , $a = 1, 2, 3$
BEH BOSON		

Table : This Table shows our present ideas on the structure of matter. Quarks and gluons do not exist as free particles and the graviton has not yet been observed.

Remarks

- All interactions are produced by the exchange of virtual quanta. For the strong, e.m. and weak interactions they are vector (spin-one) fields, (but ???), while the graviton is assumed to be a tensor, spin-two field.

Remarks

- All interactions are produced by the exchange of virtual quanta. For the strong, e.m. and weak interactions they are vector (spin-one) fields, (but ???), while the graviton is assumed to be a tensor, spin-two field.
- The constituents of matter appear to be all spin one-half particles. They are divided into quarks, which are hadrons, and "leptons" which have no strong interactions.

Remarks

- All interactions are produced by the exchange of virtual quanta. For the strong, e.m. and weak interactions they are vector (spin-one) fields, (but ???), while the graviton is assumed to be a tensor, spin-two field.
- The constituents of matter appear to be all spin one-half particles. They are divided into quarks, which are hadrons, and "leptons" which have no strong interactions.
- Each quark species appears under three forms, often called "colours" (no relation with the ordinary sense of the word).

Remarks

- All interactions are produced by the exchange of virtual quanta. For the strong, e.m. and weak interactions they are vector (spin-one) fields, (but ???), while the graviton is assumed to be a tensor, spin-two field.
- The constituents of matter appear to be all spin one-half particles. They are divided into quarks, which are hadrons, and "leptons" which have no strong interactions.
- Each quark species appears under three forms, often called "colours" (no relation with the ordinary sense of the word).
- Quarks and gluons do not appear as free particles. They form a large number of bound states, the hadrons.

Remarks

- All interactions are produced by the exchange of virtual quanta. For the strong, e.m. and weak interactions they are vector (spin-one) fields, (but ???), while the graviton is assumed to be a tensor, spin-two field.
- The constituents of matter appear to be all spin one-half particles. They are divided into quarks, which are hadrons, and "leptons" which have no strong interactions.
- Each quark species appears under three forms, often called "colours" (no relation with the ordinary sense of the word).
- Quarks and gluons do not appear as free particles. They form a large number of bound states, the hadrons.
- Quarks and leptons seem to fall into three distinct groups, or "families". Why?

Remarks

- All interactions are produced by the exchange of virtual quanta. For the strong, e.m. and weak interactions they are vector (spin-one) fields, (but ???), while the graviton is assumed to be a tensor, spin-two field.
- The constituents of matter appear to be all spin one-half particles. They are divided into quarks, which are hadrons, and "leptons" which have no strong interactions.
- Each quark species appears under three forms, often called "colours" (no relation with the ordinary sense of the word).
- Quarks and gluons do not appear as free particles. They form a large number of bound states, the hadrons.
- Quarks and leptons seem to fall into three distinct groups, or "families". Why?
- The sum of all electric charges inside any family is equal to zero.

Electromagnetic Interactions

$$\mathcal{L}_i \;\sim\; -eA_\mu(x)j^\mu(x)$$

For the matter fields the current is:
(A bit more complicated for the charged vector fields)

$$j^\mu(x) = \sum_i q_i\bar{\psi}_i(x)\gamma^\mu\psi_i(x)$$

- Vector current
- Conservation of P, C, and T
- Absence of more complex terms, such as:

$$j^\mu(x)j_\mu(x), \quad \partial A(x)\bar{\psi}(x)...\psi(x), ...$$

• All these terms, as well as all others we can write, share one common property:

• In a four-dimensional space-time, their canonical dimension is larger than four.

• The resulting quantum field theory is *non-renormalisable*

• For some reason, Nature does not like Non-Renormalisable theories.

Weak Interactions

- Mediated by massive vector bosons

$$\mathcal{L}_i \sim V_\mu(x) j^\mu(x) \quad ; \quad V_\mu \; : \; W_\mu^+, \quad W_\mu^-, \quad Z_\mu^0$$

For the matter part the current is again bi-linear in the fermion fields: $\bar{\psi}...\psi$

- The charged current

  (i) Contains only left-handed fermion fields:

$$j_\mu \sim \bar{\psi}_L \gamma_\mu \psi_L \sim \bar{\psi} \gamma_\mu (1 + \gamma_5) \psi$$

  (ii) It is non-diagonal in the quark flavour space.

- The neutral current

  (i) Contains both left- and right-handed fermion fields:

$$j_\mu \sim C_L \bar{\psi}_L \gamma_\mu \psi_L + C_R \bar{\psi}_R \gamma_\mu \psi_R$$

  (ii) It is diagonal in the quark flavour space.

- Violation of P, C, and T

- With the discovery of the BEH boson we have a new kind of "weak" interaction.

- With the discovery of the BEH boson we have a new kind of "weak" interaction.
- It is the only "non-gauge" interaction in the world.

- With the discovery of the BEH boson we have a new kind of "weak" interaction.
- It is the only "non-gauge" interaction in the world.
- It is mediated by a scalar boson

- With the discovery of the BEH boson we have a new kind of "weak" interaction.
- It is the only "non-gauge" interaction in the world.
- It is mediated by a scalar boson
- In our present understanding, it is as "fundamental" as any one of the others.

SYMMETRIES

- In Physics a Symmetry follows from the assumption that a certain quantity is not measurable.

SYMMETRIES

- In Physics a Symmetry follows from the assumption that a certain quantity is not measurable.
- The equations of motion should not depend on this quantity $\Rightarrow$ Invariance $\Rightarrow$ Conservation law

SYMMETRIES

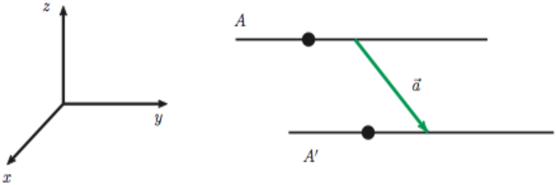
- In Physics a Symmetry follows from the assumption that a certain quantity is not measurable.
- The equations of motion should not depend on this quantity
  $\Rightarrow$ Invariance $\Rightarrow$ Conservation law
- Space-time Symmetries
  Translations, Rotations, Lorentz boosts, Inversions

SYMMETRIES

- In Physics a Symmetry follows from the assumption that a certain quantity is not measurable.
- The equations of motion should not depend on this quantity $\Rightarrow$ Invariance $\Rightarrow$ Conservation law
- Space-time Symmetries
  Translations, Rotations, Lorentz boosts, Inversions
- Internal Symmetries The phase of the wave function, Field redefinitions

Ex. SPACE TRANSLATIONS

$$\vec{x}' = \vec{x} + \vec{a}$$



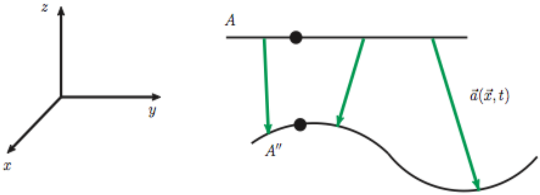
If A is the trajectory of a free particle in the (x,y,z) system, its image, A', is also a possible trajectory of a free particle.

A first abstraction: Local Symmetries

Einstein 1918

Local space translations

$$\vec{x}'' = \vec{x} + \vec{a}(\vec{x}, t)$$



A'' IS NOT a possible trajectory of a free particle.
Are there forces for which A'' is the trajectory?

- **Local space translations**

The question is purely geometrical without any obvious physical meaning, so we expect a mathematical answer with no interest for Physics.

- **Local space translations**

  The question is purely geometrical without any obvious physical meaning, so we expect a mathematical answer with no interest for Physics.

- **Surprise:** The Dynamics which is invariant under local translations is

  GENERAL RELATIVITY

  The resulting force is Gravity
  One of the four fundamental forces.

A second abstraction: Internal Symmetries

- The phase of the wave function in Quantum Mechanics

$$\Psi(x) \to e^{i\theta}\Psi(x)$$

  Leaves the Scrödinger, or the Dirac, equation, as well as the normalisation condition, invariant

A second abstraction: Internal Symmetries

- The phase of the wave function in Quantum Mechanics

$$\Psi(x) \to e^{i\theta}\Psi(x)$$

  Leaves the Scrödinger, or the Dirac, equation, as well as the normalisation condition, invariant

- Isospin Heisenberg 1932

$$N(x) = \begin{pmatrix} p(x) \\ n(x) \end{pmatrix} \to e^{i\vec{\tau}\cdot\vec{\theta}} N(x)$$

A second abstraction: Internal Symmetries

- Heisenberg's iso-space is three dimensional, isomorphic to our physical space.

A second abstraction: Internal Symmetries

- Heisenberg's iso-space is three dimensional, isomorphic to our physical space.
- With the discovery of new internal symmetries the idea was generalised to multi-dimensional internal spaces.

A second abstraction: Internal Symmetries

- Heisenberg's iso-space is three dimensional, isomorphic to our physical space.
- With the discovery of new internal symmetries the idea was generalised to multi-dimensional internal spaces.
- The space of Physics became an abstract mathematical concept with non-trivial geometrical and topological properties.

A second abstraction: Internal Symmetries

- Heisenberg's iso-space is three dimensional, isomorphic to our physical space.
- With the discovery of new internal symmetries the idea was generalised to multi-dimensional internal spaces.
- The space of Physics became an abstract mathematical concept with non-trivial geometrical and topological properties.
- Only a part of it, the three-dimensional Euclidean space, is directly accessible to our senses.

Local Internal Symmetries

The gravitational forces are not the only ones which have a geometrical origin

- The example of the quantum mechanical phase:

  $\Psi(x) \to e^{i\theta}\Psi(x) \qquad \text{with} \qquad \theta \to \theta(x)$

Local Internal Symmetries

The gravitational forces are not the only ones which have a geometrical origin

- The example of the quantum mechanical phase:

  $\Psi(x) \rightarrow e^{i\theta}\Psi(x)$ with $\theta \rightarrow \theta(x)$

- $\partial_\mu e^{i\theta(x)}\Psi(x) = e^{i\theta(x)}\partial_\mu\Psi(x) + ie^{i\theta(x)}\Psi(x)\partial_\mu\theta(x)$

Local Internal Symmetries

The gravitational forces are not the only ones which have a geometrical origin

- The example of the quantum mechanical phase:

  $\Psi(x) \rightarrow e^{i\theta}\Psi(x)$ with $\theta \rightarrow \theta(x)$

- $\partial_\mu e^{i\theta(x)}\Psi(x) = e^{i\theta(x)}\partial_\mu\Psi(x) + ie^{i\theta(x)}\Psi(x)\partial_\mu\theta(x)$

- Introduce $A_\mu(x)$ such that

  $A_\mu(x) \rightarrow A_\mu(x) - \frac{1}{e}\partial_\mu\theta(x)$

Local Internal Symmetries

The gravitational forces are not the only ones which have a geometrical origin

- The example of the quantum mechanical phase:

  $\Psi(x) \to e^{i\theta}\Psi(x)$ with $\theta \to \theta(x)$

- $\partial_\mu e^{i\theta(x)}\Psi(x) = e^{i\theta(x)}\partial_\mu\Psi(x) + ie^{i\theta(x)}\Psi(x)\partial_\mu\theta(x)$
- Introduce $A_\mu(x)$ such that

  $A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu\theta(x)$

- Then

  $\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu(x)$; $D_\mu e^{i\theta(x)}\Psi(x) = e^{i\theta(x)}D_\mu\Psi(x)$

Local Internal Symmetries

The gravitational forces are not the only ones which have a geometrical origin

- The example of the quantum mechanical phase:

  $\Psi(x) \to e^{i\theta}\Psi(x) \qquad \text{with} \qquad \theta \to \theta(x)$

- $\partial_\mu e^{i\theta(x)}\Psi(x) = e^{i\theta(x)}\partial_\mu\Psi(x) + ie^{i\theta(x)}\Psi(x)\partial_\mu\theta(x)$

- Introduce $A_\mu(x)$ such that

  $A_\mu(x) \to A_\mu(x) - \frac{1}{e}\partial_\mu\theta(x)$

- Then

  $\partial_\mu \to D_\mu = \partial_\mu - ieA_\mu(x) \;\; ; \;\; D_\mu e^{i\theta(x)}\Psi(x) = e^{i\theta(x)}D_\mu\Psi(x)$

- Replacing ∂_μ by D_μ turns any equation which was invariant under the global phase transformation, invariant under the local (gauge) one.

  Fock 1926

Local Internal Symmetries

- The introduction of the covariant derivative:

  The free Scrödinger, or Dirac, equation $\Rightarrow$

  The same equation in the presence of an external electromagnetic field

Local Internal Symmetries

- The introduction of the covariant derivative:

  The free Scrödinger, or Dirac, equation $\Rightarrow$

  The same equation in the presence of an external electromagnetic field

- To obtain the fully interacting theory:

  Add the energy of the new vector field:

  $\sim F_{\mu\nu}^2 = (\partial_\mu A_\nu - \partial_\nu A_\mu)^2$

  The resulting interaction is:

QUANTUM ELECTRODYNAMICS

Think of a field theory formulated on a space-time lattice:

$\Psi(x) \quad \Rightarrow \Psi_i \quad ; \quad \partial\Psi(x) \quad \Rightarrow (\Psi_i - \Psi_{i+1})$

$\Psi(x) \rightarrow e^{i\theta}\Psi(x) \quad \Rightarrow \Psi_i \rightarrow e^{i\theta}\Psi_i$

- For global transformations, *i.e.* constant θ:

  $\bar{\Psi}_i\Psi_i$ as well as $\bar{\Psi}_i\Psi_{i+1}$ remain invariant

Think of a field theory formulated on a space-time lattice:

$\Psi(x) \quad \Rightarrow \Psi_i \quad ; \quad \partial\Psi(x) \quad \Rightarrow (\Psi_i - \Psi_{i+1})$

$\Psi(x) \to e^{i\theta}\Psi(x) \quad \Rightarrow \Psi_i \to e^{i\theta}\Psi_i$

- For global transformations, *i.e.* constant θ:

  $\bar{\Psi}_i\Psi_i$ as well as $\bar{\Psi}_i\Psi_{i+1}$ remain invariant
- For gauge transformations, *i.e.* $\theta(x) \Rightarrow \theta_i$, the term:

  $\bar{\Psi}_i\Psi_{i+1}$ transforms into $e^{-i\theta_i}\bar{\Psi}_i\Psi_{i+1}e^{i\theta_{i+1}}$

Think of a field theory formulated on a space-time lattice:

$\Psi(x) \quad \Rightarrow \Psi_i \quad ; \quad \partial\Psi(x) \quad \Rightarrow (\Psi_i - \Psi_{i+1})$

$\Psi(x) \rightarrow e^{i\theta}\Psi(x) \quad \Rightarrow \Psi_i \rightarrow e^{i\theta}\Psi_i$

- For global transformations, *i.e.* constant θ:

  $\bar{\Psi}_i\Psi_i$ as well as $\bar{\Psi}_i\Psi_{i+1}$ remain invariant

- For gauge transformations, *i.e.* $\theta(x) \Rightarrow \theta_i$, the term:

  $\bar{\Psi}_i\Psi_{i+1}$ transforms into $e^{-i\theta_i}\bar{\Psi}_i\Psi_{i+1}e^{i\theta_{i+1}}$

- We need a field to *connect* the points i and $i+1$.

  $U_{i,i+1}$ which transforms as $U_{i,i+1} \rightarrow e^{i\theta_i}U_{i,i+1}e^{-i\theta_{i+1}}$

  The term $\bar{\Psi}_i U_{i,i+1}\Psi_{i+1}$ is now invariant. In the continuum limit the field U becomes the gauge potential A

Think of a field theory formulated on a space-time lattice:

$\Psi(x) \quad \Rightarrow \Psi_i \quad ; \quad \partial\Psi(x) \quad \Rightarrow (\Psi_i - \Psi_{i+1})$

$\Psi(x) \to e^{i\theta}\Psi(x) \quad \Rightarrow \Psi_i \to e^{i\theta}\Psi_i$

- For global transformations, *i.e.* constant θ:

  $\bar{\Psi}_i\Psi_i$ as well as $\bar{\Psi}_i\Psi_{i+1}$ remain invariant
- For gauge transformations, *i.e.* $\theta(x) \Rightarrow \theta_i$, the term:

  $\bar{\Psi}_i\Psi_{i+1}$ transforms into $e^{-i\theta_i}\bar{\Psi}_i\Psi_{i+1}e^{i\theta_{i+1}}$
- We need a field to *connect* the points i and $i+1$.

  $U_{i,i+1}$ which transforms as $U_{i,i+1} \to e^{i\theta_i}U_{i,i+1}e^{-i\theta_{i+1}}$

  The term $\bar{\Psi}_i U_{i,i+1}\Psi_{i+1}$ is now invariant. In the continuum limit the field U becomes the gauge potential A
- The matter fields live on the lattice points.

Think of a field theory formulated on a space-time lattice:

$\Psi(x) \quad \Rightarrow \Psi_i \quad ; \quad \partial\Psi(x) \quad \Rightarrow (\Psi_i - \Psi_{i+1})$

$\Psi(x) \to e^{i\theta}\Psi(x) \quad \Rightarrow \Psi_i \to e^{i\theta}\Psi_i$

- For global transformations, *i.e.* constant θ:

  $\bar{\Psi}_i\Psi_i$ as well as $\bar{\Psi}_i\Psi_{i+1}$ remain invariant

- For gauge transformations, *i.e.* $\theta(x) \Rightarrow \theta_i$, the term:

  $\bar{\Psi}_i\Psi_{i+1}$ transforms into $e^{-i\theta_i}\bar{\Psi}_i\Psi_{i+1}e^{i\theta_{i+1}}$

- We need a field to *connect* the points i and $i+1$.

  $U_{i,i+1}$ which transforms as $U_{i,i+1} \to e^{i\theta_i}U_{i,i+1}e^{-i\theta_{i+1}}$

  The term $\bar{\Psi}_i U_{i,i+1}\Psi_{i+1}$ is now invariant. In the continuum limit the field U becomes the gauge potential A

- The matter fields live on the lattice points.

- The gauge potentials live on the oriented lattice links

Non-Abelian, Local, Internal Symmetries

(Klein 1937, Pauli 1953, Yang and Mills 1954)

- $\Psi = \begin{pmatrix} \psi^1 \\ \vdots \\ \psi^r \end{pmatrix} \quad ; \quad \Psi(x) \to U(\omega)\Psi(x) \quad ; \quad \omega \in G$

Non-Abelian, Local, Internal Symmetries

(Klein 1937, Pauli 1953, Yang and Mills 1954)

- $\Psi = \begin{pmatrix} \psi^1 \\ \vdots \\ \psi^r \end{pmatrix} \quad ; \quad \Psi(x) \to U(\omega)\Psi(x) \quad ; \quad \omega \in G$
- For infinitesimal transformations:

  $\Psi(x) \to e^{i\Theta}\Psi \quad ; \quad \Theta = \sum_{a=1}^{m} \theta^a T^a \quad ; \quad \left[T^a, T^b\right] = if^{abc} T^c$

Non-Abelian, Local, Internal Symmetries

(Klein 1937, Pauli 1953, Yang and Mills 1954)

- $\Psi = \begin{pmatrix} \psi^1 \\ \vdots \\ \psi^r \end{pmatrix}$; $\Psi(x) \to U(\omega)\Psi(x)$; $\omega \in G$
- For infinitesimal transformations:

  $\Psi(x) \to e^{i\Theta}\Psi$; $\Theta = \sum_{a=1}^{m} \theta^a T^a$; $\left[T^a, T^b\right] = if^{abc}T^c$
- $\mathcal{L}(\Psi, \partial\Psi)$ is assumed to be invariant under these global transformations

Non-Abelian, Local, Internal Symmetries

(Klein 1937, Pauli 1953, Yang and Mills 1954)

- $\Psi = \begin{pmatrix} \psi^1 \\ \vdots \\ \psi^r \end{pmatrix}$; $\Psi(x) \to U(\omega)\Psi(x)$; $\omega \in G$
- For infinitesimal transformations:

  $\Psi(x) \to e^{i\Theta}\Psi$; $\Theta = \sum_{a=1}^{m} \theta^a T^a$; $\left[T^a, T^b\right] = if^{abc} T^c$
- $\mathcal{L}(\Psi, \partial\Psi)$ is assumed to be invariant under these global transformations
- As was done for the Abelian case, we wish to find a new $\mathcal{L}$, invariant under the corresponding gauge transformations in which the θ^a's are arbitrary functions of x.

Non-Abelian, Local, Internal Symmetries

(Klein 1937, Pauli 1953, Yang and Mills 1954)

- $\Psi = \begin{pmatrix} \psi^1 \\ \vdots \\ \psi^r \end{pmatrix} \quad ; \quad \Psi(x) \to U(\omega)\Psi(x) \quad ; \quad \omega \in G$
- For infinitesimal transformations:

  $\Psi(x) \to e^{i\Theta}\Psi \quad ; \quad \Theta = \sum_{a=1}^{m} \theta^a T^a \quad ; \quad \left[T^a, T^b\right] = if^{abc} T^c$
- $\mathcal{L}(\Psi, \partial\Psi)$ is assumed to be invariant under these global transformations
- As was done for the Abelian case, we wish to find a new $\mathcal{L}$, invariant under the corresponding gauge transformations in which the θ^a's are arbitrary functions of x.
- In a qualitative sense, we look for a theory invariant under G^∞

Non-Abelian, Local, Internal Symmetries

- We need a gauge field, the analogue of the electromagnetic field, to transport the information from one point to the next. Since we can perform m independent transformations, the number of generators in the Lie algebra of G, we need m gauge fields $A_\mu^a(x)$, $a = 1, ..., m$. It is easy to show that they belong to the adjoint representation of G.

  $\mathcal{A}_\mu(x) = \sum_{a=1}^m A_\mu^a(x) T^a$

Non-Abelian, Local, Internal Symmetries

- We need a gauge field, the analogue of the electromagnetic field, to transport the information from one point to the next. Since we can perform m independent transformations, the number of generators in the Lie algebra of G, we need m gauge fields $A_\mu^a(x)$, $a = 1, ..., m$. It is easy to show that they belong to the adjoint representation of G.

  $\mathcal{A}_\mu(x) = \sum_{a=1}^{m} A_\mu^a(x) T^a$
- We construct the covariant derivative: $\mathcal{D}_\mu = \partial_\mu + ig\mathcal{A}_\mu$

Non-Abelian, Local, Internal Symmetries

- We need a gauge field, the analogue of the electromagnetic field, to transport the information from one point to the next. Since we can perform m independent transformations, the number of generators in the Lie algebra of G, we need m gauge fields $A_\mu^a(x)$, $a = 1, ..., m$. It is easy to show that they belong to the adjoint representation of G.

  $\mathcal{A}_\mu(x) = \sum_{a=1}^m A_\mu^a(x) T^a$
- We construct the covariant derivative: $\mathcal{D}_\mu = \partial_\mu + ig\mathcal{A}_\mu$
- It satisfies $\mathcal{D}_\mu e^{i\Theta(x)}\Psi(x) = e^{i\Theta(x)}\mathcal{D}_\mu\Psi(x)$

  provided $\mathcal{A}_\mu(x) \to e^{i\Theta(x)}\mathcal{A}_\mu(x)e^{-i\Theta(x)} + \frac{i}{g}\left(\partial_\mu e^{i\Theta(x)}\right)e^{-i\Theta(x)}$

Non-Abelian, Local, Internal Symmetries

- We need a gauge field, the analogue of the electromagnetic field, to transport the information from one point to the next. Since we can perform m independent transformations, the number of generators in the Lie algebra of G, we need m gauge fields $A_\mu^a(x)$, $a = 1, ..., m$. It is easy to show that they belong to the adjoint representation of G.

  $\mathcal{A}_\mu(x) = \sum_{a=1}^m A_\mu^a(x) T^a$
- We construct the covariant derivative: $\mathcal{D}_\mu = \partial_\mu + ig\mathcal{A}_\mu$
- It satisfies $\mathcal{D}_\mu e^{i\Theta(x)}\Psi(x) = e^{i\Theta(x)}\mathcal{D}_\mu\Psi(x)$

  provided $\mathcal{A}_\mu(x) \to e^{i\Theta(x)}\mathcal{A}_\mu(x)e^{-i\Theta(x)} + \frac{i}{g}\left(\partial_\mu e^{i\Theta(x)}\right) e^{-i\Theta(x)}$
- If $\mathcal{L}(\Psi, \partial\Psi)$ is invariant under the global G

  $\mathcal{L}(\Psi, \mathcal{D}\Psi)$ is invariant under the gauge G

Non-Abelian, Local, Internal Symmetries

- $\mathcal{L}(\Psi, \mathcal{D}\Psi)$ describes the interaction of the fields Ψ in an external Yang-Mills field

Non-Abelian, Local, Internal Symmetries

- $\mathcal{L}(\Psi, \mathcal{D}\Psi)$ describes the interaction of the fields Ψ in an external Yang-Mills field
- In order to obtain the fully interacting theory, we must include the degrees of freedom of the gauge fields by adding to the Lagrangian density a gauge invariant kinetic term.

  $$\mathcal{L}_{inv} = -\frac{1}{2} Tr\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu} + \mathcal{L}(\Psi, \mathcal{D}\Psi)$$

  $$\mathcal{G}_{\mu\nu} = \partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu - ig\left[\mathcal{A}_\mu, \mathcal{A}_\nu\right]$$

  $$\mathcal{G}_{\mu\nu}(x) \to e^{i\theta^a(x)t^a} \; \mathcal{G}_{\mu\nu}(x) \; e^{-i\theta^a(x)t^a}$$

Non-Abelian, Local, Internal Symmetries

Remarks:

- No terms proportional to $A_\mu A^\mu$. $\Rightarrow$ the gauge fields describe massless particles. Useless for Physics??

Non-Abelian, Local, Internal Symmetries

Remarks:

- No terms proportional to $A_\mu A^\mu$. $\Rightarrow$ the gauge fields describe massless particles. Useless for Physics??
- Yang-Mills fields alone are coupled. The em field is not.

Non-Abelian, Local, Internal Symmetries

Remarks:

- No terms proportional to $A_\mu A^\mu$. $\Rightarrow$ the gauge fields describe massless particles. Useless for Physics??
- Yang-Mills fields alone are coupled. The em field is not.
- The coupling constant g appears in the covariant derivative of the fields Ψ. They are all coupled with the same strength.

Non-Abelian, Local, Internal Symmetries

Remarks:

- No terms proportional to $A_\mu A^\mu$. $\Rightarrow$ the gauge fields describe massless particles. Useless for Physics??
- Yang-Mills fields alone are coupled. The em field is not.
- The coupling constant g appears in the covariant derivative of the fields Ψ. They are all coupled with the same strength.
- Extension to $G = G_1 \times ... \times G_n$ semi-simple. $\Rightarrow$ n independent coupling constants.
  Gell-Mann and Glashow, 1960

Non-Abelian, Local, Internal Symmetries

Remarks:

- No terms proportional to $A_\mu A^\mu$. $\Rightarrow$ the gauge fields describe massless particles. Useless for Physics??
- Yang-Mills fields alone are coupled. The em field is not.
- The coupling constant g appears in the covariant derivative of the fields Ψ. They are all coupled with the same strength.
- Extension to $G = G_1 \times ... \times G_n$ semi-simple. $\Rightarrow$ n independent coupling constants.
  Gell-Mann and Glashow, 1960
- If one of the factors is Abelian $\Rightarrow$ no charge quantisation.

Spontaneous Symmetry Breaking (SSB)

- An infinite system may exhibit the phenomenon of phase transitions. It often implies a reduction in the symmetry of the ground state.

Spontaneous Symmetry Breaking (SSB)

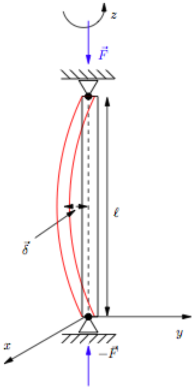
- An infinite system may exhibit the phenomenon of phase transitions. It often implies a reduction in the symmetry of the ground state.
- For a field theory, in many cases, we encounter at least two phases:

  (i) *The unbroken*, or, *the Wigner phase*: A symmetry is manifest in the spectrum of the theory whose excitations form irreducible representations of the symmetry group. For a gauge theory the vector gauge bosons are massless and belong to the adjoint representation.

  (ii) *The spontaneously broken phase*: Part of the symmetry is hidden from the spectrum. For a gauge theory, some of the gauge bosons become massive.

SSB: Global Symmetries

An example from Classical Mechanics



$$IE\frac{d^4X}{dz^4} + F\frac{d^2X}{dz^2} = 0 \quad ; \quad IE\frac{d^4Y}{dz^4} + F\frac{d^2Y}{dz^2} = 0$$

$X = X'' = Y = Y'' = 0$ for $z = 0$ and $z = l$

SSB: Global Symmetries

An example from Classical Mechanics

- A symmetric solution always exists: $X = Y = 0$

SSB: Global Symmetries

An example from Classical Mechanics

- A symmetric solution always exists: $X = Y = 0$
- For $F \geq F_{cr} = \frac{\pi^2 EI}{l^2}$ asymmetric solutions appear:

  $X = C\sin kz \quad ; \quad kl = n\pi \quad ; \quad n = 1, \ldots \quad ; \quad k^2 = F/EI$

  They correspond to lower energy.

SSB: Global Symmetries

An example from Classical Mechanics

- A symmetric solution always exists: $X = Y = 0$
- For $F \geq F_{cr} = \frac{\pi^2 EI}{l^2}$ asymmetric solutions appear:

  $X = C \sin kz \quad ; \quad kl = n\pi \quad ; \quad n = 1, \ldots \quad ; \quad k^2 = F/EI$

  They correspond to lower energy.
- What happened to the original symmetry?

SSB: Global Symmetries

An example from Classical Mechanics

- A symmetric solution always exists: $X = Y = 0$
- For $F \geq F_{cr} = \frac{\pi^2 EI}{l^2}$ asymmetric solutions appear:

  $X = C \sin kz \quad ; \quad kl = n\pi \quad ; \quad n = 1, \ldots \quad ; \quad k^2 = F/EI$

  They correspond to lower energy.
- What happened to the original symmetry?
- The ground state is degenerate.

SSB: Global Symmetries

An example from Classical Mechanics

- A symmetric solution always exists: $X = Y = 0$
- For $F \geq F_{cr} = \frac{\pi^2 EI}{l^2}$ asymmetric solutions appear:

  $X = C \sin kz \quad ; \quad kl = n\pi \quad ; \quad n = 1, \ldots \quad ; \quad k^2 = F/EI$

  They correspond to lower energy.
- What happened to the original symmetry?
- The ground state is degenerate.
- The state is characterised by the two-component vector $\vec{\delta} = |\vec{\delta}| e^{i\theta}$

  The modulus does have a physical meaning, the phase does not.

SSB: Global Symmetries

An example from Quantum Mechanics

The Heisenberg ferromagnet

$$H = -J_{ij}\vec{S}_i \cdot \vec{S}_j$$

Symmetry breaking $O(3) \to O(2)$

SSB: Global Symmetries

A field theory example

- $\mathcal{L}_1 \;=\; (\partial_\mu \phi)(\partial^\mu \phi^*) - M^2 \phi\phi^* - \lambda(\phi\phi^*)^2$

Invariant under $U(1)$ global transformations: $\phi(x) \;\rightarrow\; e^{i\theta}\phi(x)$

- The Hamiltonian is given by:

$$\mathcal{H}_1 \;=\; (\partial_0 \phi)(\partial_0 \phi^*) + (\partial_i \phi)(\partial_i \phi^*) + V(\phi)$$

$$V(\phi) = M^2 \phi\phi^* + \lambda(\phi\phi^*)^2$$

- The symmetric solution is $\phi(x) = 0$.

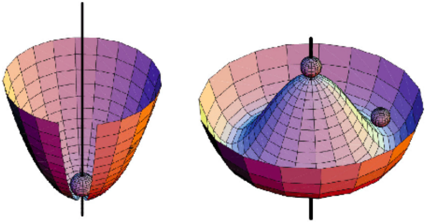
- The minimum energy configuration corresponds to:

$\phi(x) =$ constant $= \phi$ such that $V(\phi)$ is minimum, solution of:

$V' = 0$

SSB: Global Symmetries

A field theory example



- The potential $V(\phi)$ with $\lambda > 0$ and $M^2 \geq 0$ (left).

The only solution is the symmetric one $\phi = 0$.

- The potential $V(\phi)$ with $\lambda > 0$ and $M^2 < 0$ (right).

$\phi = 0$ is a local maximum. An entire circle of minima at the complex ϕ-plane with radius $v = (-M^2/2\lambda)^{1/2}$. Any point on it corresponds to a spontaneous breaking of the $U(1)$ symmetry.

SSB: Global Symmetries

A field theory example

- Conclusion: $M^2 = 0$ is a critical point.

  For $M^2 > 0$ the symmetric solution is stable.

  For $M^2 < 0$ spontaneous symmetry breaking occurs.

SSB: Global Symmetries

A field theory example

- Conclusion: $M^2 = 0$ is a critical point.

  For $M^2 > 0$ the symmetric solution is stable.

  For $M^2 < 0$ spontaneous symmetry breaking occurs.

- In order to reach the stable solution we translate the field ϕ.

  $\phi(x) = \frac{1}{\sqrt{2}} \left[v + \psi(x) + i\chi(x) \right]$

$$\begin{aligned} \mathcal{L}_1(\phi) \ \rightarrow \ \mathcal{L}_2(\psi, \chi) = \frac{1}{2}(\partial_\mu \psi)^2 + \frac{1}{2}(\partial_\mu \chi)^2 - \frac{1}{2}(2\lambda v^2)\psi^2 \\ - \lambda v \psi(\psi^2 + \chi^2) - \frac{\lambda}{4}(\psi^2 + \chi^2)^2 \end{aligned}$$

SSB: Global Symmetries

A field theory example

- Conclusion: $M^2 = 0$ is a critical point.

  For $M^2 > 0$ the symmetric solution is stable.

  For $M^2 < 0$ spontaneous symmetry breaking occurs.

- In order to reach the stable solution we translate the field ϕ.

  $\phi(x) = \frac{1}{\sqrt{2}}\left[v + \psi(x) + i\chi(x)\right]$

$$\mathcal{L}_1(\phi) \;\rightarrow\; \mathcal{L}_2(\psi,\chi) = \frac{1}{2}(\partial_\mu\psi)^2 + \frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2}(2\lambda v^2)\psi^2$$
$$- \lambda v\psi(\psi^2 + \chi^2) - \frac{\lambda}{4}(\psi^2 + \chi^2)^2$$

- χ is massless (*Goldstone mode*).

SSB: Global Symmetries

A field theory example

- $\mathcal{L}_2$ is still invariant.

  $\delta\psi = -\theta\chi \quad ; \quad \delta\chi = \theta\psi + \theta v$

  We still have a conserved current:

  $j_\mu \sim \psi\partial_\mu\chi - \chi\partial_\mu\psi + v\partial_\mu\chi$

  $\partial^\mu j_\mu(x) = 0$

  It is the minimum energy configuration which is not invariant.

SSB: Global Symmetries

A field theory example

- $\mathcal{L}_2$ is still invariant.

  $\delta\psi = -\theta\chi \quad ; \quad \delta\chi = \theta\psi + \theta v$

  We still have a conserved current:

  $j_\mu \sim \psi\partial_\mu\chi - \chi\partial_\mu\psi + v\partial_\mu\chi$

  $\partial^\mu j_\mu(x) = 0$

  It is the minimum energy configuration which is not invariant.
- *Goldstone Theorem:* Spontaneous breaking of a continuous symmetry $\Rightarrow$ A massless particle
  (*Needs Lorentz invariance and positivity*)

SSB: Gauge Symmetries: I. Abelian

- Consider the gauge theory extension of the previous model:

  $$\mathcal{L}_1 = -\frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu + ieA_\mu)\phi|^2 - M^2\phi\phi^* - \lambda(\phi\phi^*)^2$$

  $\mathcal{L}_1$ is invariant under the gauge transformation:

  $$\phi(x) \quad \rightarrow \quad e^{i\theta(x)}\phi(x) \quad ; \quad A_\mu \quad \rightarrow \quad A_\mu - \frac{1}{e}\partial_\mu\theta(x)$$

SSB: Gauge Symmetries: I. Abelian

- Consider the gauge theory extension of the previous model:

  $\mathcal{L}_1 = -\frac{1}{4}F_{\mu\nu}^2 + |(\partial_\mu + ieA_\mu)\phi|^2 - M^2\phi\phi^* - \lambda(\phi\phi^*)^2$

  $\mathcal{L}_1$ is invariant under the gauge transformation:

  $\phi(x) \quad \rightarrow \quad e^{i\theta(x)}\phi(x) \quad ; \quad A_\mu \quad \rightarrow \quad A_\mu - \frac{1}{e}\partial_\mu\theta(x)$

- Same analysis for $\lambda > 0$ and $M^2 < 0$ yields:

$$\mathcal{L}_1 \;\rightarrow\; \mathcal{L}_2 = -\frac{1}{4}F_{\mu\nu}^2 + \frac{e^2v^2}{2}A_\mu^2 + evA_\mu\partial^\mu\chi + \frac{1}{2}(\partial_\mu\psi)^2 + \frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2}(2\lambda v^2)\psi^2 \quad + \ldots$$

SSB: Gauge Symmetries: I. Abelian

- $\mathcal{L}_2$ is invariant under the gauge transformation:

$$
\begin{aligned}
\psi(x) &\rightarrow cos\theta(x)[\psi(x) + v] - sin\theta(x)\chi(x) - v \\
\chi(x) &\rightarrow cos\theta(x)\chi(x) + sin\theta(x)[\psi(x) + v] \\
A_\mu &\rightarrow A_\mu - \frac{1}{e}\partial_\mu\theta(x)
\end{aligned}
$$

SSB: Gauge Symmetries: I. Abelian

- $\mathcal{L}_2$ is invariant under the gauge transformation:

$$
\begin{aligned}
\psi(x) &\rightarrow cos\theta(x)[\psi(x)+v] - sin\theta(x)\chi(x) - v \\
\chi(x) &\rightarrow cos\theta(x)\chi(x) + sin\theta(x)[\psi(x)+v] \\
A_\mu &\rightarrow A_\mu - \frac{1}{e}\partial_\mu\theta(x)
\end{aligned}
$$

- $\mathcal{L}_2$ contains a term proportional to A^2. A massive photon??

SSB: Gauge Symmetries: I. Abelian

- $\mathcal{L}_2$ is invariant under the gauge transformation:

$$\begin{aligned}\psi(x) &\rightarrow cos\theta(x)[\psi(x)+v]-sin\theta(x)\chi(x)-v\\ \chi(x) &\rightarrow cos\theta(x)\chi(x)+sin\theta(x)[\psi(x)+v]\\ A_\mu &\rightarrow A_\mu-\frac{1}{e}\partial_\mu\theta(x)\end{aligned}$$

- $\mathcal{L}_2$ contains a term proportional to A^2. A massive photon??
- Degrees of freedom:

  $\mathcal{L}_1$: 2+2=4

  $\mathcal{L}_2$: 2+3=5 ??

  Notice the term $evA_\mu\partial^\mu\chi$

SSB: Gauge Symmetries: I. Abelian

In order to make this counting easier, let us choose a different parametrisation:

$\phi(x) = \frac{1}{\sqrt{2}}[v + \rho(x)]e^{i\zeta(x)/v}$; $A_\mu(x) = B_\mu(x) - \frac{1}{ev}\partial_\mu\zeta(x)$

A gauge transformation: $\zeta(x) \to \zeta(x) + v\theta(x)$

$$
\begin{aligned}
\mathcal{L}_1 \;\to\; \mathcal{L}_3 &= -\frac{1}{4}B_{\mu\nu}^2 + \frac{e^2 v^2}{2}B_\mu^2 + \frac{1}{2}(\partial_\mu\rho)^2 - \frac{1}{2}(2\lambda v^2)\rho^2 \\
&\quad - \frac{\lambda}{4}\rho^4 + \frac{1}{2}e^2 B_\mu^2(2v\rho + \rho^2) \\
B_{\mu\nu} &= \partial_\mu B_\nu \partial_\nu B_\mu
\end{aligned}
$$

The ζ field has disappeared!!

SSB: Gauge Symmetries: I. Abelian

- $\mathcal{L}_3$ describes the interaction of:
  - A massive spin-one field : $B_\mu(x) \quad \rightarrow \quad$ 3 degrees of freedom
  - A massive, real, scalar field : $\rho(x) \quad \rightarrow \quad$ 1 degree of freedom

  There is no more any gauge invariance left. No massless particles. The ζ degree of freedom gave rise to the longitudinal component of the vector boson.

SSB: Gauge Symmetries: I. Abelian

- $\mathcal{L}_3$ describes the interaction of:
  • A massive spin-one field : $B_\mu(x) \quad \rightarrow \quad$ 3 degrees of freedom
  • A massive, real, scalar field : $\rho(x) \quad \rightarrow \quad$ 1 degree of freedom

  There is no more any gauge invariance left. No massless particles. The ζ degree of freedom gave rise to the longitudinal component of the vector boson.

- **Conclusion:** We obtained three different Lagrangian densities: $\mathcal{L}_1$, $\mathcal{L}_2$, $\mathcal{L}_3$.

SSB: Gauge Symmetries: I. Abelian

- $\mathcal{L}_3$ describes the interaction of:
  • A massive spin-one field : $B_\mu(x) \quad \rightarrow \quad$ 3 degrees of freedom
  • A massive, real, scalar field : $\rho(x) \quad \rightarrow \quad$ 1 degree of freedom

  There is no more any gauge invariance left. No massless particles. The ζ degree of freedom gave rise to the longitudinal component of the vector boson.

- **Conclusion:** We obtained three different Lagrangian densities: $\mathcal{L}_1$, $\mathcal{L}_2$, $\mathcal{L}_3$.
- They all describe the same Physics.

SSB: Gauge Symmetries: I. Abelian

- $\mathcal{L}_3$ describes the interaction of:
  - A massive spin-one field : $B_\mu(x) \quad \rightarrow \quad$ 3 degrees of freedom
  - A massive, real, scalar field : $\rho(x) \quad \rightarrow \quad$ 1 degree of freedom

  There is no more any gauge invariance left. No massless particles. The ζ degree of freedom gave rise to the longitudinal component of the vector boson.

- **Conclusion:** We obtained three different Lagrangian densities: $\mathcal{L}_1$, $\mathcal{L}_2$, $\mathcal{L}_3$.
- They all describe the same Physics.
- But not necessarily in perturbation theory!

SSB: Gauge Symmetries: I. Abelian

- $\mathcal{L}_1$:

  • It is gauge invariant.

  • In the quadratic part it has an imaginary mass $\rightarrow$ not suitable for perturbation expansion.

SSB: Gauge Symmetries: I. Abelian

- $\mathcal{L}_1$:

  • It is gauge invariant.

  • In the quadratic part it has an imaginary mass $\rightarrow$ not suitable for perturbation expansion.

- $\mathcal{L}_2$:

  • It is gauge invariant.

  • It has redundant degrees of freedom.

  • It has a correct quadratic part for a perturbation expansion.

SSB: Gauge Symmetries: I. Abelian

- $\mathcal{L}_1$:

  • It is gauge invariant.

  • In the quadratic part it has an imaginary mass $\rightarrow$ not suitable for perturbation expansion.

- $\mathcal{L}_2$:

  • It is gauge invariant.

  • It has redundant degrees of freedom.

  • It has a correct quadratic part for a perturbation expansion.

- $\mathcal{L}_3$:

  • No gauge invariance. Only physical degrees of freedom.

  • Non-renormalisable by power counting.

  • It can be obtained from $\mathcal{L}_2$ by a suitable choice of gauge.

SSB: Gauge Symmetries: II. Non Abelian

• Assume a gauge Lie-group G with m generators $\rightarrow$ m massless gauge bosons.

• Add a multiplet of scalar fields ϕ_i belonging to an n-dim. repr.

$$\mathcal{L} = -\frac{1}{4} Tr(F_{\mu\nu}F^{\mu\nu}) + (D_\mu \Phi)^\dagger D^\mu \Phi - V(\Phi)$$

$$D_\mu \phi_i = \partial_\mu \phi_i - ig^{(a)} T^a_{ij} A^a_\mu \phi_j$$

• Choose the parameters in V such that the minimum is not at $\Phi = 0$ but rather at $\Phi = v$.

• The m generators of G can be separated into two classes: h generators which annihilate v and form the Lie algebra of a subgroup H and the $m - h$ others represented, in the representation of Φ, by matrices T^a, such that $T^a v \neq 0$.

• Any vector in the orbit of v, *i.e.* of the form $e^{iw^a T^a} v$, is an equivalent minimum of the potential.

SSB: Gauge Symmetries: II. Non Abelian

- $\Phi \to \Phi + v$

- Decompose Φ into components along the orbit of v and orthogonal to it:

$$\Phi(x) = i \sum_{a=1}^{m-h} \frac{\chi^a(x) T^a v}{|T^a v|} + \sum_{b=1}^{n-m+h} \psi^b(x) u^b + v$$

The u^b's are orthogonal to all the $T^a v$'s. The corresponding generators span the coset space G/H.

- The fields χ^a will give the longitudinal components of the $m-h$ gauge bosons.

- The fields ψ^b will remain physical.

- There is always at least one field ψ.

SSB: Gauge Symmetries. Conclusions:

The Brout-Englert-Higgs Mechanism

• The vector bosons corresponding to spontaneously broken generators of a gauge group become massive.

• The corresponding Goldstone bosons decouple and disappear from the physical spectrum.

• Their degrees of freedom become the longitudinal components of the vector bosons.

• Gauge bosons corresponding to unbroken generators remain massless.

• There is always at least one physical, massive, scalar particle.

Model Building: A five step programme

1) Choose a gauge group G.

2) Choose the fields of the "elementary" particles and assign them to representations of G. Include scalar fields to allow for the BEH mechanism.

3) Write the most general renormalisable Lagrangian invariant under G. At this stage gauge invariance is still exact and all gauge vector bosons are massless.

4) Choose the parameters of the scalar potential so that spontaneous symmetry breaking occurs.

5) Translate the scalars and rewrite the Lagrangian in terms of the translated fields. Choose a suitable gauge and quantise the theory.

A remark: Gauge theories provide only the general framework, not a detailed model. The latter will depend on the particular choices made in steps 1) and 2).

The EW Standard Model

A. The lepton world

- Step 1. We have four vector bosons: W^+, W^-, Z^0 and $\gamma \Rightarrow$
  We need a group with four generators. $\Rightarrow$
  $G = U(2) \sim SU(2) \times U(1)$

  Quantum number assignment: $Q = T_3 + \frac{1}{2} Y$

The EW Standard Model

A. The lepton world

- Step 1. We have four vector bosons: W^+, W^-, Z^0 and $\gamma \Rightarrow$ We need a group with four generators. $\Rightarrow$ $G = U(2) \sim SU(2) \times U(1)$

  Quantum number assignment: $Q = T_3 + \frac{1}{2}Y$

- Step 2. Three families $\Rightarrow$ Simplest solution: Three copies

$$\Psi_L^i(x) = \frac{1}{2}(1+\gamma_5)\left(\begin{array}{c} \nu_i(x) \\ \ell_i^-(x) \end{array}\right) \quad ; \quad i = 1,2,3$$

$$\nu_{iR}(x) = \frac{1}{2}(1-\gamma_5)\nu_i(x) \ \ (?) \quad ; \quad \ell_{iR}^-(x) = \frac{1}{2}(1-\gamma_5)\ell_i^-(x)$$

$$\Psi_L^i(x) \to e^{i\vec{\tau}\vec{\theta}(x)}\Psi_L^i(x) \quad ; \quad R_i(x) \to R_i(x)$$

$$Y(\Psi_L^i) = -1 \quad ; \quad Y(R_i) = -2$$

The EW Standard Model

The scalar field choice:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad ; \quad \Phi(x) \to e^{i\vec{\tau}\vec{\theta}(x)}\Phi(x) \quad ; \quad Y(\Phi) = 1$$

- Step 3. Assume conservation of lepton numbers:

$$\begin{aligned} \mathcal{L} &= -\frac{1}{4}\vec{W}_{\mu\nu}\cdot\vec{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + |D_\mu\Phi|^2 - V(\Phi) \\ &+ \sum_{i=1}^{3}\left[\bar{\Psi}_L^i i\not{D}\Psi_L^i + \bar{R}_i i\not{D}R_i - G_i(\bar{\Psi}_L^i R_i\Phi + h.c.)\right] \end{aligned}$$

$$\vec{W}_{\mu\nu} = \partial_\mu\vec{W}_\nu - \partial_\nu\vec{W}_\mu + g\vec{W}_\mu\times\vec{W}_\nu \qquad B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$D_\mu\Psi_L^i = \left(\partial_\mu - ig\tfrac{\vec{\tau}}{2}\cdot\vec{W}_\mu + i\tfrac{g'}{2}B_\mu\right)\Psi_L^i \quad ; \quad D_\mu R_i = (\partial_\mu + ig'B_\mu)R_i$$

$$D_\mu\Phi = \left(\partial_\mu - ig\tfrac{\vec{\tau}}{2}\cdot\vec{W}_\mu - i\tfrac{g'}{2}B_\mu\right)\Phi$$

The EW Standard Model

$$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$$

Gauge bosons and leptons are massless

- Step 4. We choose $\mu^2 < 0 \to v^2 = -\mu^2/\lambda$

  We put the breaking along the real part of ϕ^0

The EW Standard Model

$V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2$

Gauge bosons and leptons are massless

- Step 4. We choose $\mu^2 < 0 \rightarrow v^2 = -\mu^2/\lambda$

  We put the breaking along the real part of ϕ^0

- Step 5. Translate the scalar field:

$$\Phi \rightarrow \Phi + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad v^2 = -\frac{\mu^2}{\lambda}$$

  This transforms the Lagrangian and generates new terms. Some of them:

- Fermion mass terms:

$$m_e = \frac{1}{\sqrt{2}} G_e v \qquad m_\mu = \frac{1}{\sqrt{2}} G_\mu v \qquad m_\tau = \frac{1}{\sqrt{2}} G_\tau v$$

- Fermion mass terms:

$$m_e = \frac{1}{\sqrt{2}} G_e v \quad m_\mu = \frac{1}{\sqrt{2}} G_\mu v \quad m_\tau = \frac{1}{\sqrt{2}} G_\tau v$$

- Gauge boson mass terms:

$$\frac{1}{8} v^2 [g^2 (W^1_\mu W^{1\mu} + W^2_\mu W^{2\mu}) + (g' B_\mu - g W^3_\mu)^2]$$

$$W^\pm_\mu = \frac{W^1_\mu \mp i W^2_\mu}{\sqrt{2}} \quad ; \quad m_W = \frac{vg}{2}$$

After diagonalisation, we obtain the neutral bosons:

$$Z_\mu = sin\theta_W B_\mu - cos\theta_W W^3_\mu \quad ; \quad m_Z = \frac{v(g^2 + g'^2)^{1/2}}{2} = \frac{m_W}{cos\theta_W}$$

$$A_\mu = cos\theta_W B_\mu + sin\theta_W W^3_\mu \quad ; \quad m_A = 0$$

$$g'/g = tan\theta_W$$

- Fermion mass terms:

$$m_e = \frac{1}{\sqrt{2}} G_e v \quad m_\mu = \frac{1}{\sqrt{2}} G_\mu v \quad m_\tau = \frac{1}{\sqrt{2}} G_\tau v$$

- Gauge boson mass terms:

$$\frac{1}{8} v^2 [g^2 (W^1_\mu W^{1\mu} + W^2_\mu W^{2\mu}) + (g' B_\mu - g W^3_\mu)^2]$$

$$W^\pm_\mu = \frac{W^1_\mu \mp i W^2_\mu}{\sqrt{2}} \quad ; \quad m_W = \frac{vg}{2}$$

After diagonalisation, we obtain the neutral bosons:

$$Z_\mu = sin\theta_W B_\mu - cos\theta_W W^3_\mu \quad ; \quad m_Z = \frac{v(g^2 + g'^2)^{1/2}}{2} = \frac{m_W}{cos\theta_W}$$

$$A_\mu = cos\theta_W B_\mu + sin\theta_W W^3_\mu \quad ; \quad m_A = 0$$

$$g'/g = tan\theta_W$$

- Physical higgs mass: $m_h = \sqrt{-2\mu^2} = \sqrt{2\lambda v^2}$

Extension to hadrons

• The lepton-hadron universality suggests to use also doublets for the left-handed quarks and singlets for the right-handed ones.

• New features: Individual quantum numbers are not separately conserved. All quarks have non-vanishing masses.

• A naïve assignment:

$$Q_L^i(x) = \frac{1}{2}(1+\gamma_5)\begin{pmatrix} U^i(x) \\ D^i(x) \end{pmatrix} \quad ; \quad U_R^i(x) \quad ; \quad D_R^i(x)$$

$U^i = u, c, t \quad ; \quad D^i = d, s, b \quad i = 1, 2, 3$

$Y(Q_L^i) = \frac{1}{3} \quad ; \quad Y(U_R^i) = \frac{4}{3} \quad ; \quad Y(D_R^i) = -\frac{2}{3}$

• The presence of the second right-hand singlet implies a second Yukawa term:

$$\mathcal{L}_{Yuk} = G_d(\bar{Q}_L D_R \Phi + h.c.) + G_u(\bar{Q}_L U_R \tilde{\Phi} + h.c.)$$

Extension to hadrons

• Had we only one family, this would have been the end of the story! BUT...

• The correct Yukawa term is:

$$\mathcal{L}_{Yuk} = \sum_{i,j}\left[(\bar{Q}^i_L G^{ij}_d D^j_R \Phi + h.c.)\right] + \sum_i \left[G^i_u(\bar{Q}^i_L U^i_R \tilde{\Phi} + h.c.)\right]$$

• After translation of the scalar field:
Masses for the up quarks $m_u = G^1_u v$, $m_c = G^2_u v$ and $m_t = G^3_u v$.
A mass matrix for the down quarks $G^{ij}_d v$.

• We prefer to work in a field space with diagonal masses:
$\tilde{D}^i = U^{ij} D^j$ such that $U^\dagger G_d U =$diag(m_d, m_s, m_b).

• For two families, with θ=The Cabibbo angle:

$$C = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix}$$

Extension to hadrons

- For three families:

$$KM = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_3 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}$$

The Standard Model: The full Lagrangian

$$\begin{aligned}
\mathcal{L} &= -\frac{1}{4}\vec{W}_{\mu\nu}\cdot\vec{W}^{\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + |D_\mu\Phi|^2 - V(\Phi) \\
&+ \sum_{i=1}^{3}\left[\bar{\Psi}_L^i i\not{D}\Psi_L^i + \bar{R}_i i\not{D}R_i - G_i(\bar{\Psi}_L^i R_i\Phi + h.c.)\right. \\
&+ \left.\bar{Q}_L^i i\not{D}Q_L^i + \bar{U}_R^i i\not{D}U_R^i + \bar{D}_R^i i\not{D}D_R^i + G_u^i(\bar{Q}_L^i U_R^i\tilde{\Phi} + h.c.)\right] \\
&+ \sum_{i,j=1}^{3}\left[(\bar{Q}_L^i G_d^{ij} D_R^j\Phi + h.c.)\right]
\end{aligned}$$

$$D_\mu Q_L^i = \left(\partial_\mu - ig\frac{\vec{\tau}}{2}\cdot\vec{W}_\mu - i\frac{g'}{6}B_\mu\right)Q_L^i$$

$$D_\mu U_R^i = \left(\partial_\mu - i\frac{2g'}{3}B_\mu\right)U_R^i$$

$$D_\mu D_R^i = \left(\partial_\mu + i\frac{g'}{3}B_\mu\right)D_R^i$$

The Standard Model: Arbitrary parameters

- The two gauge coupling constants g and g'.
- The two parameters of the scalar potential λ and μ^2.
- Three Yukawa coupling constants for the three lepton families, $G_{e,\mu,\tau}$. ($m_\nu = 0$).
- Six Yukawa coupling constants for the three quark families, $G_u^{u,c,t}$, and $G_d^{d,s,b}$.
- Four parameters of the KM matrix, the three angles and the phase δ.
- All but two come from the scalar fields.

The Standard Model: The couplings

- The gauge boson-fermion couplings.
  - The photon couplings

$$\frac{gg'}{(g^2+g'^2)^{1/2}}\left[\bar{e}\gamma^\mu e+\sum_{a=1}^{3}\left(\frac{2}{3}\bar{u}^a\gamma^\mu u^a-\frac{1}{3}\bar{d}^a\gamma^\mu d^a\right)\quad+...\right]A_\mu$$

$$e=\frac{gg'}{(g^2+g'^2)^{1/2}}=gsin\theta_W=g'cos\theta_W$$

  - The charged W couplings

$$\frac{g}{2\sqrt{2}}\left(\bar{\nu}_e\gamma^\mu(1+\gamma_5)e+\sum_{a=1}^{3}\bar{u}^a\gamma^\mu(1+\gamma_5)d^a_{KM}\quad+...\right)W^+_\mu\ +h.c.$$

$$\frac{G}{\sqrt{2}}=\frac{g^2}{8m_W^2}=\frac{1}{2v^2}$$

The Standard Model: The couplings

- The Z^0 couplings

$$-\frac{e}{2}\frac{1}{sin\theta_W cos\theta_W}\left[\bar{\nu}_L\gamma^\mu\nu_L + (sin^2\theta_W - cos^2\theta_W)\bar{e}_L\gamma^\mu e_L \right.$$
$$\left. +2sin^2\theta_W\bar{e}_R\gamma^\mu e_R \; + \; ...\right] Z_\mu$$

$$\frac{e}{2}\sum_{a=1}^{3}\left[(\frac{1}{3}tan\theta_W - cot\theta_W)\bar{u}_L^a\gamma^\mu u_L^a + (\frac{1}{3}tan\theta_W + cot\theta_W)\bar{d}_L^a\gamma^\mu d_L^a\right.$$
$$\left. +\frac{2}{3}tan\theta_W(2\bar{u}_R^a\gamma^\mu u_R^a - \bar{d}_R^a\gamma^\mu d_R^a) \; + \; ...\right] Z_\mu$$

Remarks :
(i) The neutral current is diagonal in flavour space.
(ii) The axial part is $\sim [\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d]$

The Standard Model: The couplings

- The gauge boson self-couplings

  • $-\frac{1}{4}\vec{W}_{\mu\nu}\cdot\vec{W}^{\mu\nu} \quad \Rightarrow$

$$
\begin{aligned}
& - ig(sin\theta_W A^\mu - cos\theta_W Z^\mu)(W^{\nu -}W^+_{\mu\nu} - W^{\nu +}W^-_{\mu\nu}) \\
& - ig(sin\theta_W F^{\mu\nu} - cos\theta_W Z^{\mu\nu})W^-_\mu W^+_\nu \\
& - g^2(sin\theta_W A^\mu - cos\theta_W Z^\mu)^2 W^+_\nu W^{\nu -} \\
& + g^2(sin\theta_W A^\mu - cos\theta_W Z^\mu)(sin\theta_W A^\nu - cos\theta_W Z^\nu)W^+_\mu W^-_\nu \\
& - \frac{g^2}{2}(W^+_\mu W^{\mu -})^2 + \frac{g^2}{2}(W^+_\mu W^-_\nu)^2
\end{aligned}
$$

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu,$$

  • For a charged, massive W, the magnetic moment μ and the quadrupole moment Q are given by:

$$\mu = \frac{(1+\kappa)e}{2m_W} \qquad Q = -\frac{e\kappa}{m_W^2}$$

  • An $SU(2)$ prediction: $\kappa = 1$

The Standard Model: The couplings

- The higgs-fermion couplings

  Proportional to the fermion mass!

The Standard Model: The couplings

- The higgs-fermion couplings

  Proportional to the fermion mass!

- The higgs-gauge boson couplings

  $\frac{1}{4}(v+\phi)^2\left[g^2 W_\mu^+ W^{-\mu} + (g^2+g'^2) Z_\mu Z^\mu\right]$

The Standard Model: The couplings

- The higgs-fermion couplings

  Proportional to the fermion mass!

- The higgs-gauge boson couplings

  $\frac{1}{4}(v+\phi)^2\left[g^2 W_\mu^+ W^{-\mu} + (g^2+g'^2) Z_\mu Z^\mu\right]$

- The higgs self coupling

  Given by $\lambda = G m_h^2/\sqrt{2}$. Not easy to measure.

The Standard Model: The couplings

- The higgs-fermion couplings

  Proportional to the fermion mass!

- The higgs-gauge boson couplings

  $\frac{1}{4}(v+\phi)^2\left[g^2 W^+_\mu W^{-\mu} + (g^2+g'^2) Z_\mu Z^\mu\right]$

- The higgs self coupling

  Given by $\lambda = G m_h^2/\sqrt{2}$. Not easy to measure.

- **The five-step programme is complete**

The Standard Model and experiment

The Standard Model and experiment

- The Standard Model has 17 arbitrary parameters.

  They are related to masses and coupling constants and should be determined experimentally.

  All have been measured.

The Standard Model and experiment

- The Standard Model has 17 arbitrary parameters.

  They are related to masses and coupling constants and should be determined experimentally.

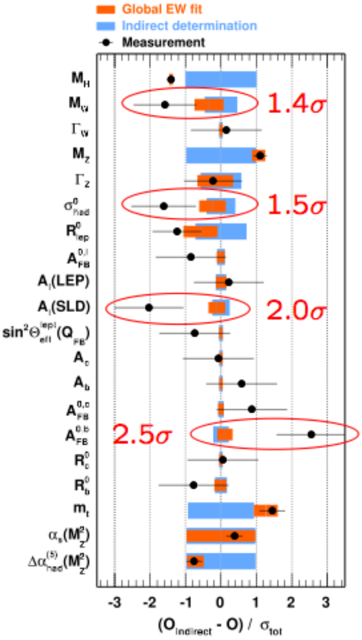
  All have been measured.
- The Model gives a large number of predictions.

The Standard Model and experiment

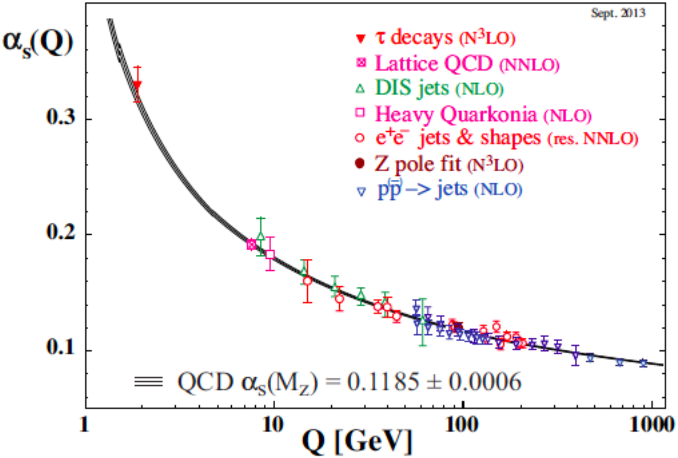
- The Standard Model has 17 arbitrary parameters.

  They are related to masses and coupling constants and should be determined experimentally.

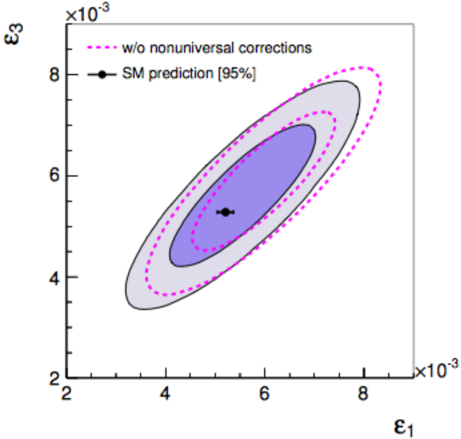
  All have been measured.
- The Model gives a large number of predictions.
- **THE STANDARD MODEL HAS BEEN ENORMOUSLY SUCCESSFUL**



Global EW fit
Indirect determination
Measurement
M_H
M_W
Γ_W
M_Z
Γ_Z
σ⁰_had
R⁰_lep
A_FB^0,l
A_l(LEP)
A_l(SLD)
sin²Θ_eff^lept(Q_FB)
A_c
A_b
A_FB^0,c
A_FB^0,b
R⁰_c
R⁰_b
m_t
α_s(M²_Z)
Δα_had^(5)(M²_Z)
1.4σ
1.5σ
2.0σ
2.5σ
-3
-2
-1
0
1
2
3
(O_indirect - O) / σ_tot



Sept. 2013
$\alpha_s(Q)$
τ decays (N^3LO)
Lattice QCD (NNLO)
DIS jets (NLO)
Heavy Quarkonia (NLO)
e^+e^- jets & shapes (res. NNLO)
Z pole fit (N^3LO)
$p\bar{p}$ –> jets (NLO)
0.3
0.2
0.1
QCD $\alpha_s(M_Z) = 0.1185 \pm 0.0006$
1
10
100
1000
Q [GeV]



ε_3
$\times 10^{-3}$
w/o nonuniversal corrections
SM prediction [95%]
8
6
4
2
2
4
6
8
$\times 10^{-3}$
ε_1

$$\epsilon_1 = \frac{3G_F m_t^2}{8\sqrt{2}\pi^2} - \frac{3G_F m_W^2}{4\sqrt{2}\pi^2} \tan^2\theta_W \ln\frac{m_H}{m_Z} + \ldots \quad (1)$$

$$\epsilon_3 = \frac{G_F m_W^2}{12\sqrt{2}\pi^2} \ln\frac{m_H}{m_Z} - \frac{G_F m_W^2}{6\sqrt{2}\pi^2} \ln\frac{m_t}{m_Z} + \ldots \quad (2)$$

The Standard Model and experiment

The precision of the measurements often led to successful predictions of new Physics.

The Standard Model and experiment

The precision of the measurements often led to successful predictions of new Physics.

- The discovery of weak neutral currents by Gargamelle in 1972

  $$\nu_\mu + e^- \to \nu_\mu + e^- \quad ; \quad \nu_\mu + N \to \nu_\mu + X$$

  Both, their strength and their properties were predicted by the Model.

The Standard Model and experiment

The precision of the measurements often led to successful predictions of new Physics.

- The discovery of weak neutral currents by Gargamelle in 1972

  $\nu_\mu + e^- \to \nu_\mu + e^- \quad ; \quad \nu_\mu + N \to \nu_\mu + X$

  Both, their strength and their properties were predicted by the Model.

- The discovery of charmed particles at SLAC in 1974

  Their presence was essential to ensure the absence of strangeness changing neutral currents, ex. $K^0 \to \mu^+ + \mu^-$

  Their characteristic property is to decay predominantly in strange particles.

The Standard Model and experiment

The precision of the measurements often led to successful predictions of new Physics.

- The discovery of weak neutral currents by Gargamelle in 1972

  $$\nu_\mu + e^- \to \nu_\mu + e^- \quad ; \quad \nu_\mu + N \to \nu_\mu + X$$

  Both, their strength and their properties were predicted by the Model.

- The discovery of charmed particles at SLAC in 1974

  Their presence was essential to ensure the absence of strangeness changing neutral currents, ex. $K^0 \to \mu^+ + \mu^-$

  Their characteristic property is to decay predominantly in strange particles.

- A necessary condition for the consistency of the Model is that $\sum_i Q_i = 0$ inside each family.

  When the τ lepton was discovered the b and t quarks were predicted with the right electric charges.

The Standard Model and experiment

- The discovery of the W and Z bosons at CERN in 1983

  The characteristic relation of the Standard Model with an isodoublet BEH mechanism $m_Z = m_W / cos\theta_W$ is checked with very high accuracy (including radiative corrections).

The Standard Model and experiment

- The discovery of the W and Z bosons at CERN in 1983

  The characteristic relation of the Standard Model with an isodoublet BEH mechanism $m_Z = m_W / cos\theta_W$ is checked with very high accuracy (including radiative corrections).

- The t-quark was *seen* at LEP through its effects in radiative corrections before its actual discovery at Fermilab.

The Standard Model and experiment

- The discovery of the W and Z bosons at CERN in 1983

  The characteristic relation of the Standard Model with an isodoublet BEH mechanism $m_Z = m_W / cos\theta_W$ is checked with very high accuracy (including radiative corrections).
- The t-quark was *seen* at LEP through its effects in radiative corrections before its actual discovery at Fermilab.
- The final touch: the discovery of the BEH scalar at CERN

The Standard Model and experiment

- The discovery of the W and Z bosons at CERN in 1983

  The characteristic relation of the Standard Model with an isodoublet BEH mechanism $m_Z = m_W/cos\theta_W$ is checked with very high accuracy (including radiative corrections).
- The t-quark was *seen* at LEP through its effects in radiative corrections before its actual discovery at Fermilab.
- The final touch: the discovery of the BEH scalar at CERN
- **THE STANDARD MODEL HAS BEEN ENORMOUSLY SUCCESSFUL**

The Standard Model and experiment

- The discovery of the W and Z bosons at CERN in 1983

  The characteristic relation of the Standard Model with an isodoublet BEH mechanism $m_Z = m_W / cos\theta_W$ is checked with very high accuracy (including radiative corrections).

- The t-quark was *seen* at LEP through its effects in radiative corrections before its actual discovery at Fermilab.
- The final touch: the discovery of the BEH scalar at CERN
- **THE STANDARD MODEL HAS BEEN ENORMOUSLY SUCCESSFUL**
- **RENORMALISED PERTURBATION THEORY HAS BEEN ENORMOUSLY SUCCESSFUL**

The Standard Model and experiment

- The discovery of the W and Z bosons at CERN in 1983

  The characteristic relation of the Standard Model with an isodoublet BEH mechanism $m_Z = m_W / cos\theta_W$ is checked with very high accuracy (including radiative corrections).
- The t-quark was *seen* at LEP through its effects in radiative corrections before its actual discovery at Fermilab.
- The final touch: the discovery of the BEH scalar at CERN
- **THE STANDARD MODEL HAS BEEN ENORMOUSLY SUCCESSFUL**
- **RENORMALISED PERTURBATION THEORY HAS BEEN ENORMOUSLY SUCCESSFUL**
- **WHY?**

The Standard Model and experiment

BASED ON THIS SUCCESS, NEW PHYSICS IS PREDICTED FOR LHC