QCD

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DISCLAIMER(S)

Purpose(s) of these lectures:

Introduction to QCD

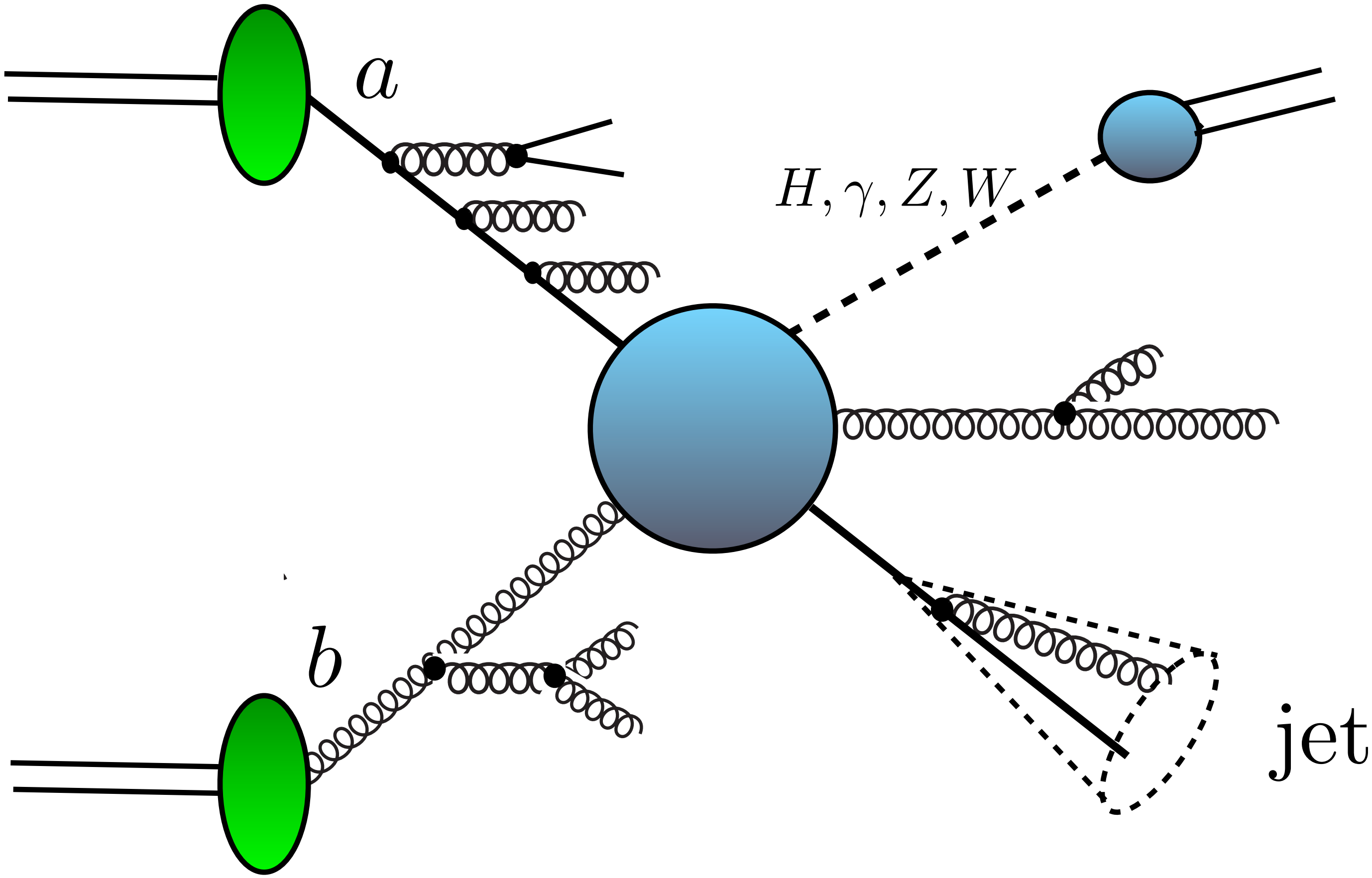
Refresh your knowledge on QCD (another view)

Understand the vocabulary!

New developments in the field (Lectures 3 and 4)

pQCD

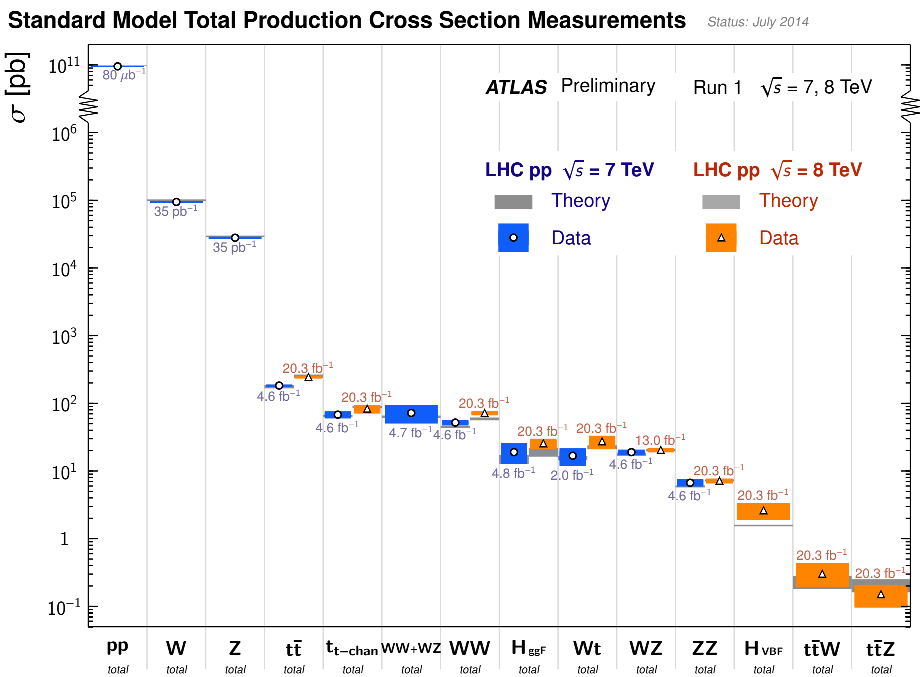
- In the LHC era, QCD is everywhere!

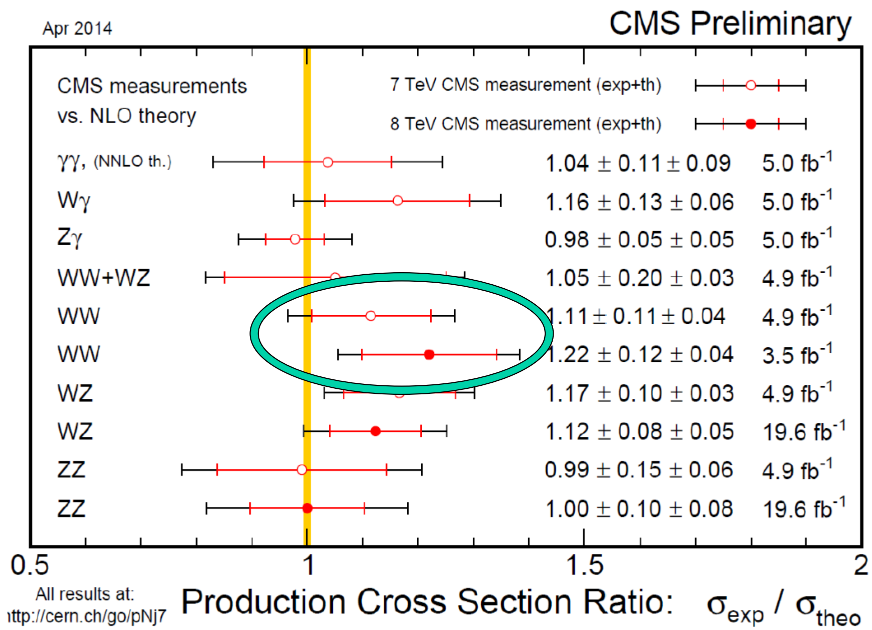


- In these lectures : pQCD as precision QCD for Colliders

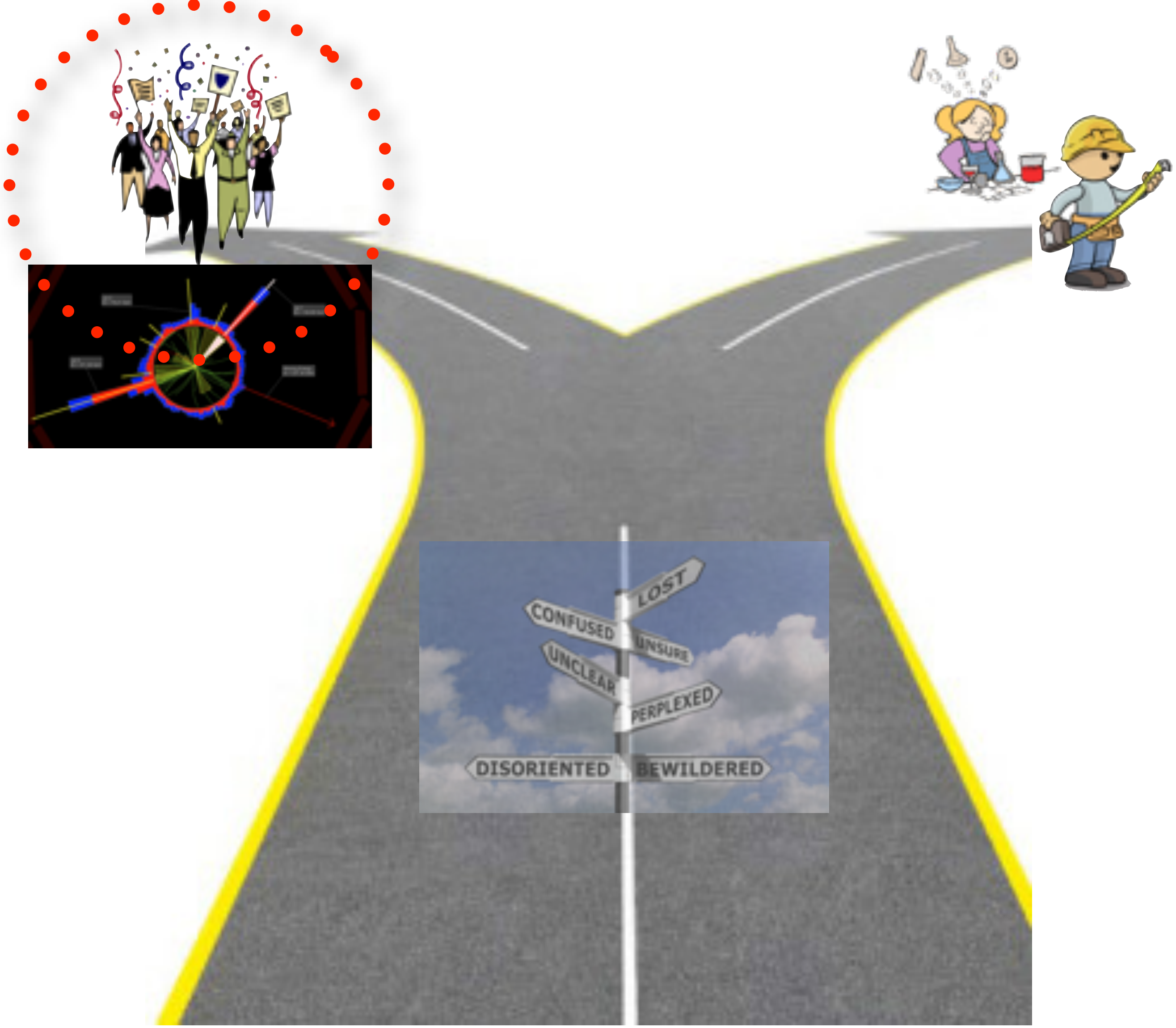
- LHC was incredibly successful at 7 & 8 TeV
- Everything SM like (including Higgs)

LHC cross section measurements



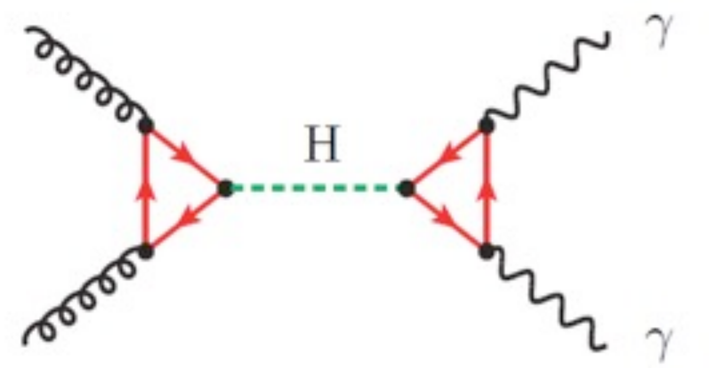


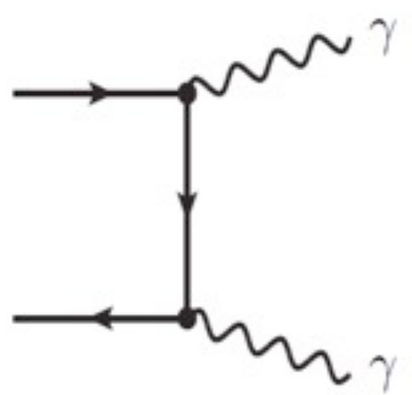
No deviation from Standard Model observed so far.....

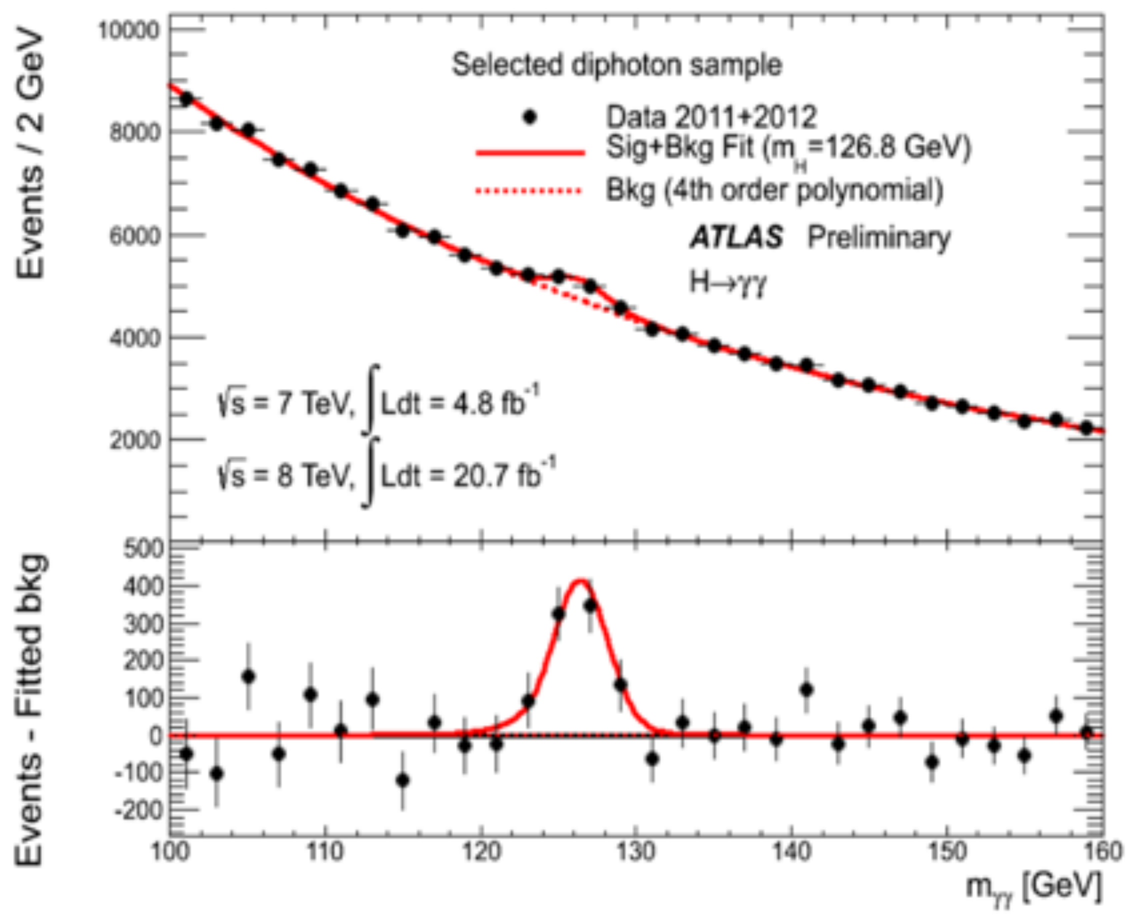


- Next run at 13 TeV ... will find evidence of new physics or not?

discovery ... as for Higgs at LHC



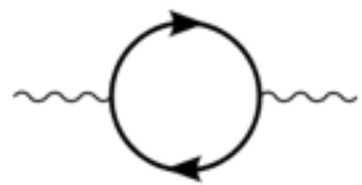




- Observe new particles: **Need good understanding of background**
  - Involve High multiplicities at LHC

▸ Very likely: **New physics might show up in the detail**

- Flavor Physics
- Contribution from new particles at loop level



- Need to be precise on cross-sections and SM parameters

EW vacuum stability $m_H, m_t, \alpha_s, \ldots$

- Explore Higgs sector with precision
- Multiple Gauge boson and HQ production (gauge/couplings to new physics)

Precision is the name of the game

These Lectures
Toolkit for precise TH predictions at the LHC

Outline of the lecture I

- Basics of QCD : Lagrangian and Feynman rules
- QCD at work: beta function and running coupling
- QCD at work in e^+e^-
- Infrared Safety in QCD
- Jets in QCD

Outline of the lecture 2

- Deep Inelastic Scattering
- Parton Model
- Scaling Violations and Evolution
- Factorization
- Parton Distribution Functions

Outline of the lecture 3

- ✤ QCD at Colliders
- ✤ LO calculations : tools and recursions for amplitudes
- ✤ Why higher orders?
- ✤ How to do NLO
- ✤ Automated tools at NLO

Outline of the lecture 4

- NNLO
- Higgs at NNLO and beyond
- Resummation : when fixed order fails
- Parton Showers
- Matching Parton showers and NLO

Some bibliography (and much material on the web)

- QCD and Collider Physics, R.K.Ellis, W.J.Stirling and B.R.Webber , Cambridge University Press Sons (1999)

- Foundations of Quantum Chromodynamics, T. Muta, World Scientific (1998)

- Gauge Theory and Elementary Particle Physics, T. Cheng and L. Li, Oxford Science Publications (1984)

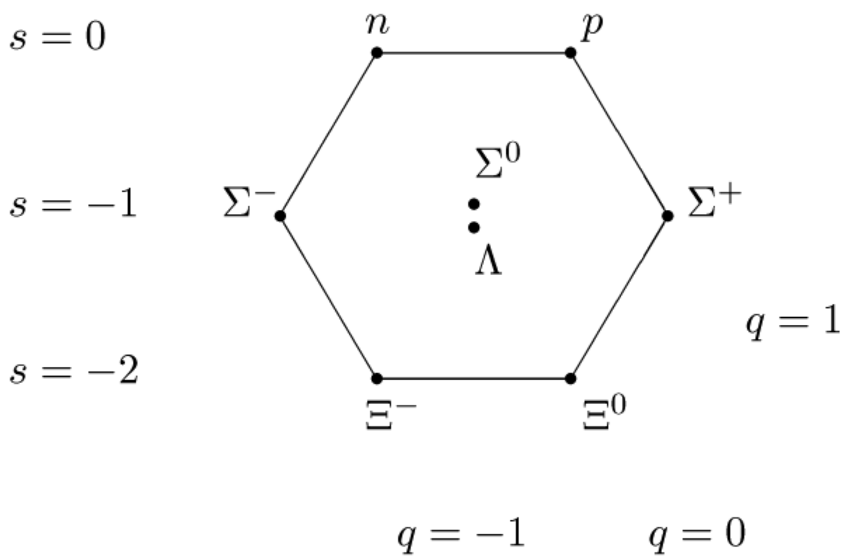
- The theory of quark and gluon interactions, F.J. Ynduráin, Springer-Verlag (1999)

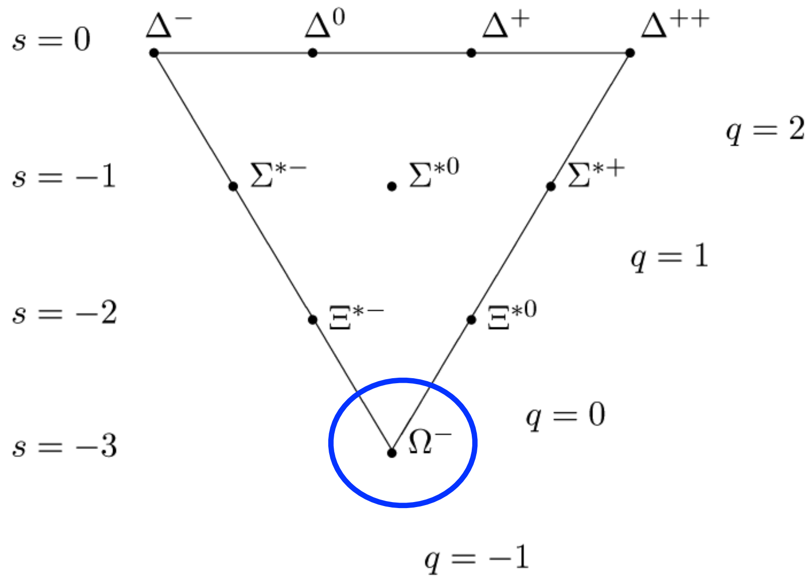
- Collider Physics, V. Barger and R. Phillips, Addison-Wesley (1996)

- Quantum Chromodynamics: High Energy Experiments and Theory, G. Dissertori, I. Knowles and M. Schmelling, International Series of Monograph on Physics (2009)

The eightfold way (1961) Gell-Mann and Ne'eman

Everything starts by organizing hadron spectrum to show some pattern of symmetry (such as Mendeleev did for atoms in periodic table)



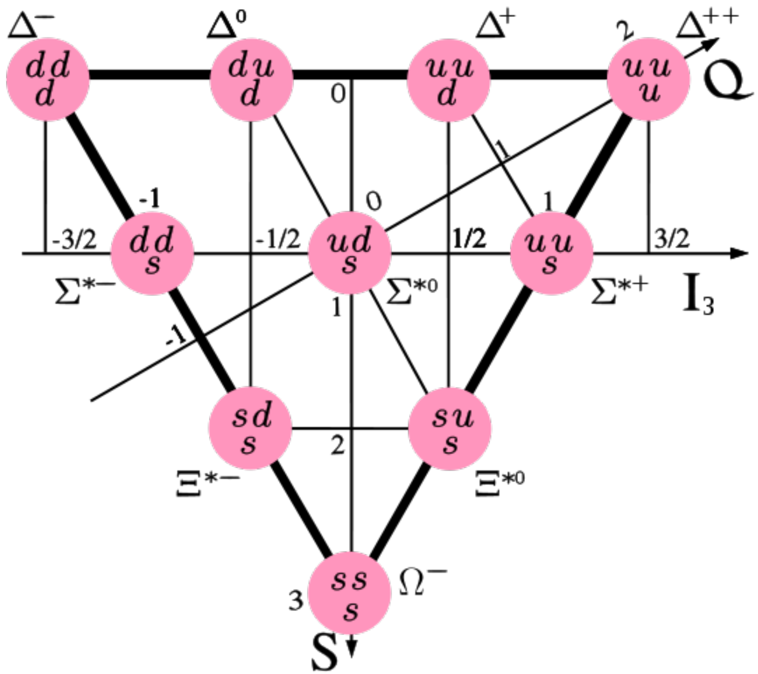


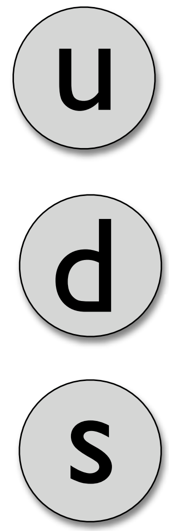
One still missing by that time, but predicted following pattern

Then one asks ... what is the reason for this pattern?

Quarks (1964)

Gell-Mann and Zweig propose the existence of elementary (spin 1/2) particles named quarks : with 3 of them (plus antiquarks) can explain the composition of all known hadrons

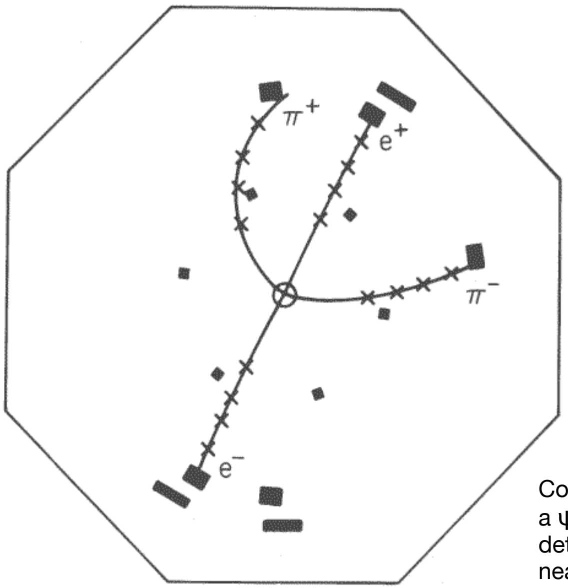




Bound states are only made by 3 quarks (baryon) or by a quark+antiquark (meson). No other structure observed.

Baryon qqq

Meson $q\bar{q}$



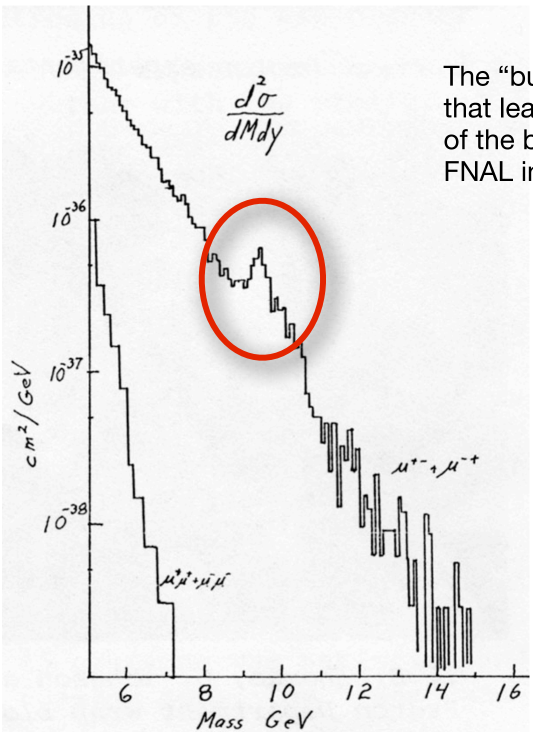
Computer reconstruction of a ψ′ decay in the Mark I detector at SLAC, making a near-perfect image of the Greek letter ψ

c

$m_c \approx 1.1 - 1.3 GeV$

$$J/\Psi = (c\bar{c})$$

(1974) Discovered at SLAC and Brookhaven. Expected due to strong theoretical arguments (GIM mechanism)



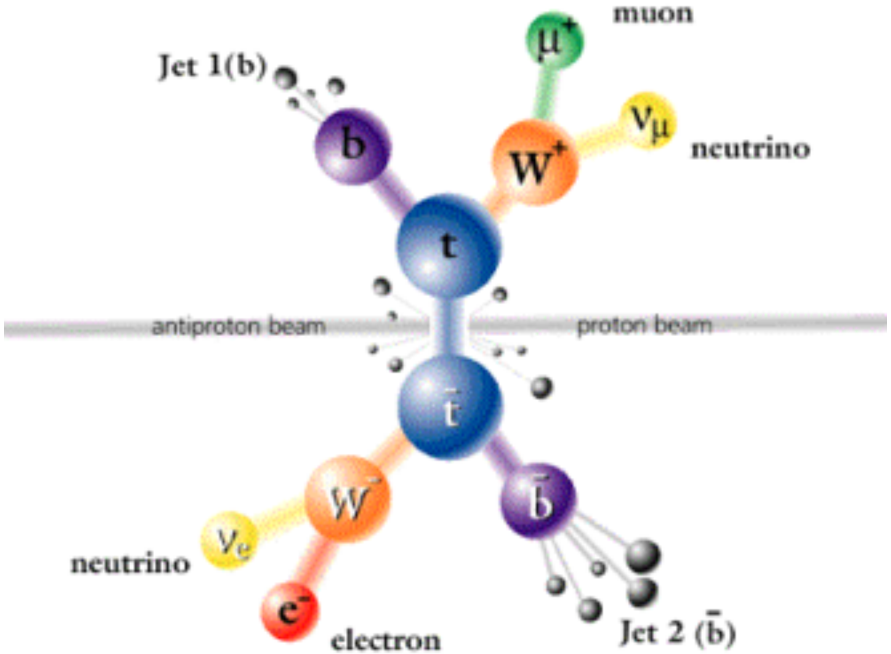
The “bump” at 9.5 GeV that lead to the discovery of the bottom quark at FNAL in 1977

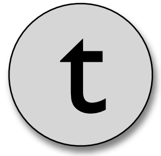
b

$m_b \approx 4.0 - 4.4 GeV$

$$\Upsilon = (b\bar{b})$$

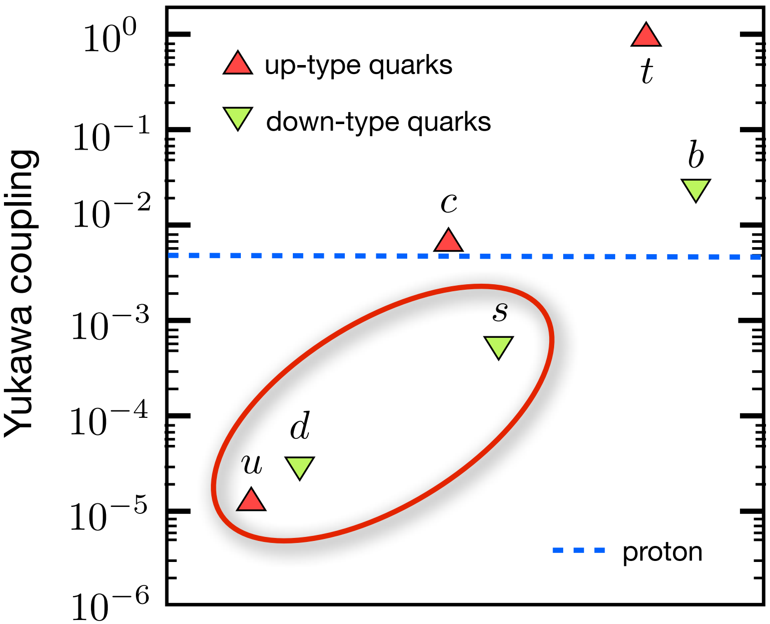
(1977) Discovered at Fermilab (E288) 3rd family of quarks needed to account for CP violation





$m_t \approx 171 GeV$

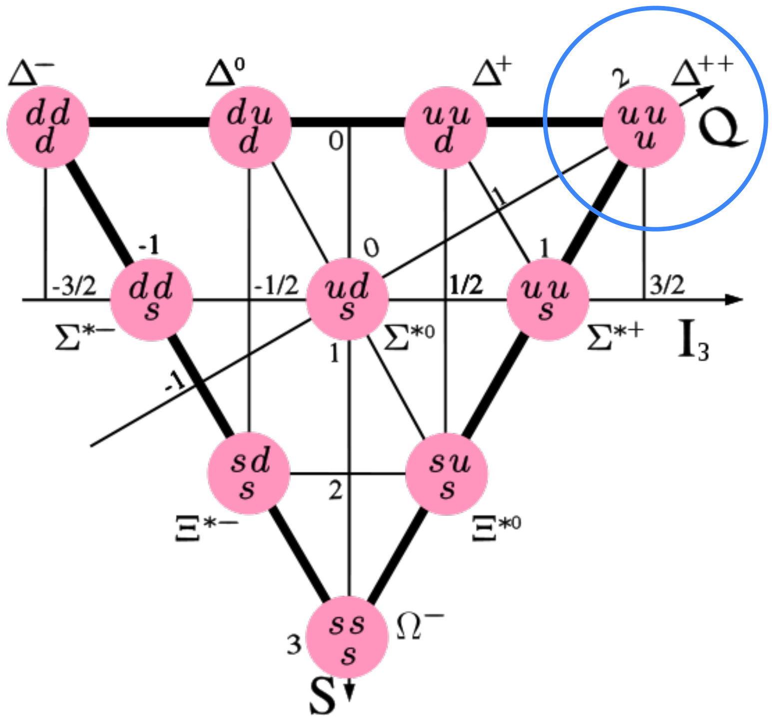
(1995) Discovered at Tevatron
EW precision measurements predicted mass with accuracy



Several orders of magnitude in masses

quark	charge	mass (approx.)
u	2/3	~4 MeV
d	-1/3	~ 7 MeV
c	2/3	~ 1.3 GeV
s	-1/3	~150 MeV
t	2/3	~171 GeV
b	-1/3	~4.4 GeV

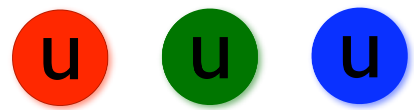
Spin-statistics issue



$$\Delta^{++} = u\uparrow u\uparrow u\uparrow$$

Wave function (flavor+spin) completely symmetric : forbidden by Pauli exclusion principle

Introduce new additional quantum number : color



$$\Delta^{++} = \epsilon_{ijk}\; u_i\uparrow u_j\uparrow u_k\uparrow$$

wave function becomes antisymmetric

Will see that experiment directly confirms 3 colors

$$\frac{\sigma(e^+e^- \to hadrons)}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

Upgrade color to "charge of the strong interactions"

So strong that only hadrons observed in nature are those combinations of quarks that result in color singlets!

Only Baryon qqq, Meson $q\bar{q}$ → states results in color singlets

3 colors explain observed spectrum of hadrons!

$SU(3)_{color}$ is an exact symmetry of nature

$$\psi^i_f$$

i: color, f: flavor

QCD: non-abelian gauge theory under SU(3)

Simple recipe: take free Lagrangian for fermions

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$$

Force it to be invariant under non-abelian local transformation

$$\psi(x) \rightarrow e^{i\alpha_a(x)T_a}\psi(x)$$

with 8 generators obeying $\quad [T^a, T^b] \;=\; i f^{abc} T^c$

$t^A = \frac{1}{2}\lambda^A$ 3x3 Gell-Mann matrices (1 representation)

$$\lambda^1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; \lambda^2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; \lambda^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \; \lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix},$$

$$\lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \; \lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \; \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \; \lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

Original Lagrangian not invariant due to derivative of $\alpha_a(x)$

To correct for that change derivative to covariant derivative adding extra spin-1 fields (one per generator)

$$(D_\mu)_{ij} = \delta_{ij}\partial_\mu - i g_s T^a_{ij} A^a_\mu$$

D transforms as the quark field

Add all gauge invariants! (*F* is not invariant in non-abelian theories, but..)

$$F^{(a)}_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_s f_{abc} A^b_\mu A^c_\nu$$

→ QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^{(a)}_{\mu\nu} F^{(a)\mu\nu} + \sum_q \bar{\psi}^q_i \left(i\gamma^\mu (D_\mu)_{ij} - m_q \delta_{ij} \right) \psi^q_j$$

(+ gauge fixing terms and eventually ghosts)

one single coupling constant $\alpha_S \equiv \frac{g_s^2}{4\pi}$

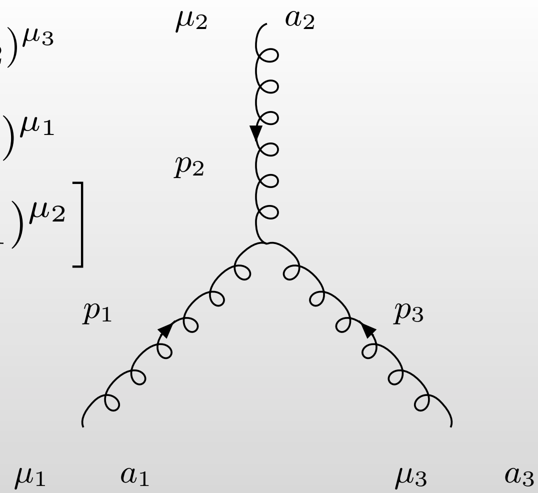
no mass term for gluon (gauge invariance) $\cancel{m^2 A_\mu A^\mu}$

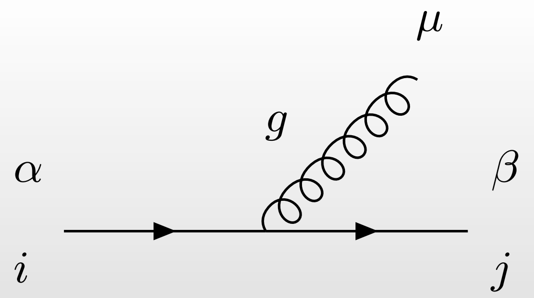
$$\mathcal{L}_{free} + \mathcal{L}_{int}$$

$$\mathcal{L}_{int} = g\sum_{f=1}^{N_f} \bar{\psi}^i_f \gamma^\mu t^a_{ij} A^a_\mu \psi^i_f \quad q\bar{q}g \text{ vertex}$$
$$- g\, f^{abc} \partial^\mu A^a_\nu A^b_\mu A^{\nu c} \quad ggg \text{ vertex}$$
$$- \frac{1}{4} g^2 f^{abc} f^{ade} A^b_\mu A^c_\nu A^{\mu d} A^{\nu e} \quad gggg \text{ vertex}$$

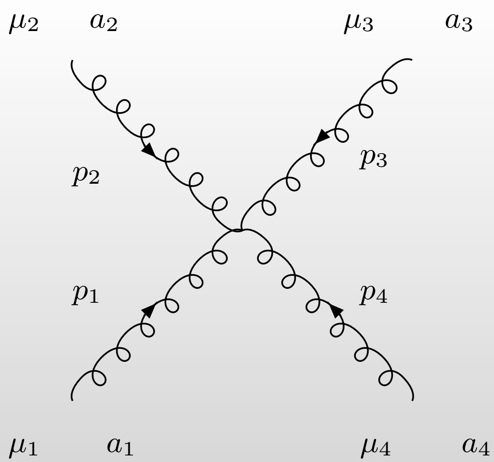
Feynman rules

$$-g f^{a_1 a_2 a_3} \Big[g^{\mu_1 \mu_2} (p_1 - p_2)^{\mu_3} + g^{\mu_2 \mu_3} (p_2 - p_3)^{\mu_1} + g^{\mu_3 \mu_1} (p_3 - p_1)^{\mu_2} \Big]$$



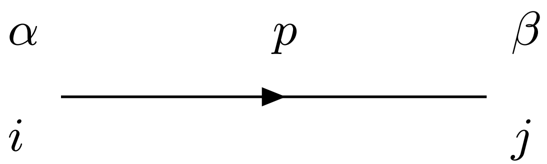


$$-ig(t^a)_{ij}(\gamma^\mu)_{\alpha\beta}$$



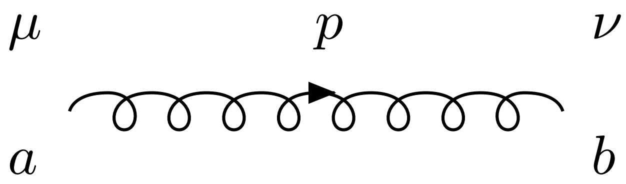
$$-ig^2 \Big[f^{ba_1 a_2} f^{ba_3 a_4} (g^{\mu_1 \mu_3} g^{\mu_2 \mu_4} - g^{\mu_1 \mu_4} g^{\mu_2 \mu_3}) + (2 \leftrightarrow 3) + (2 \leftrightarrow 4) \Big]$$

Propagators



$$\frac{i(\not{p} + m)_{\alpha\beta}}{p^2 - m^2 + i\epsilon}\ \delta^{ij}$$

Quark



$$\frac{i}{p^2 + i\epsilon}\ d^{\mu\nu}(p)\ \delta^{ab}$$

Gluon

spin polarization tensor

$$d^{\mu\nu}(p) = \sum_{\lambda} \varepsilon^{\mu}_{(\lambda)}(p)\varepsilon^{\nu\,*}_{(\lambda)}(p)$$

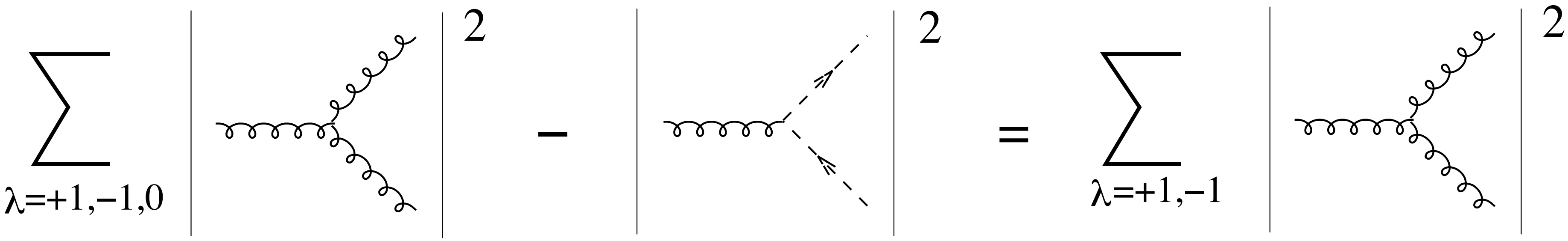
Explicit expression depends on gauge

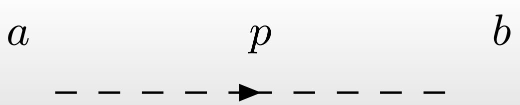
propagation of physical and unphysical polarizations

$$d^{\mu\nu}(p) = \begin{cases} -g^{\mu\nu} + (1-\alpha)\dfrac{p^{\mu}p^{\nu}}{p^2 + i\epsilon} & \text{covariant gauges} \\[2ex] -g^{\mu\nu} + \dfrac{p^{\mu}n^{\nu} + p^{\nu}n^{\mu}}{p\cdot n} - n^2\,\dfrac{p^{\mu}\,p^{\nu}}{(p\cdot n)^2} & \text{axial gauges} \end{cases}$$

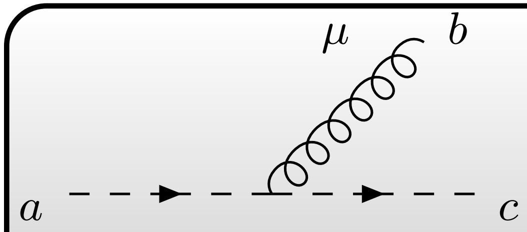
propagation of physical (transverse) polarizations only

In covariant gauges Lorentz invariance is manifest but ghosts must be included to cancel effect of unphysical polarizations in propagator





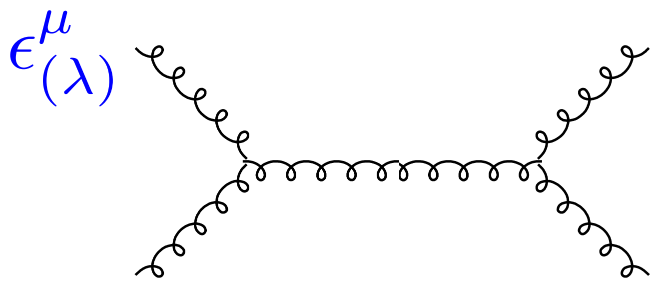
$$\frac{i}{p^2 + i\epsilon}\delta^{ab}$$



$$g f^{abc} p^{\mu}$$

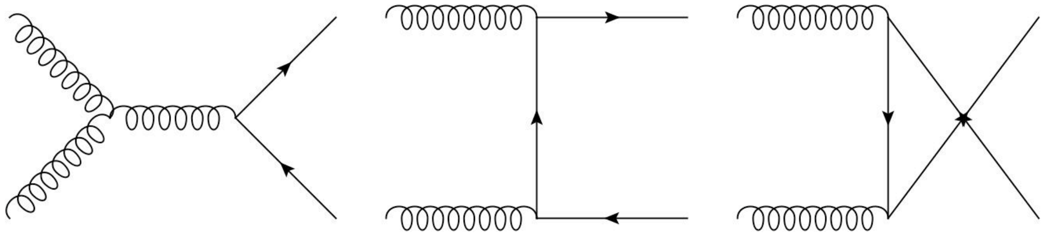
Similar trick can be used to simplify calculations when gluon (initial of final state) polarization enters in any amplitude[2]

$$\sum_{\lambda} \varepsilon^{\mu}_{(\lambda)}(p)\varepsilon^{\nu\,*}_{(\lambda)}(p)$$



Example gg → qq

do it!

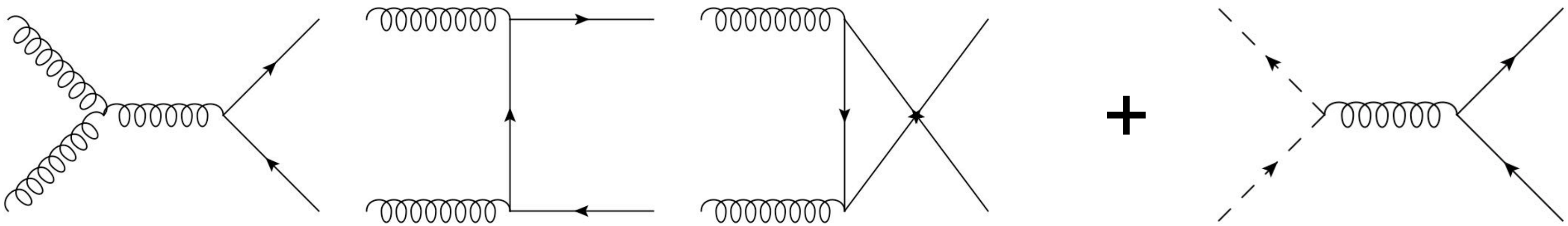


In QED it is OK to use $$\sum_{pol} \epsilon_i^{\mu} \epsilon_i^{*\nu} = -g_{\mu\nu}$$

But in QCD one needs to use physical polarizations $$\sum_{phys\,pol} \epsilon_i^{\mu} \epsilon_i^{*\nu} = -g_{\mu\nu} + \frac{k_\mu \bar{k}_\nu + k_\nu \bar{k}_\mu}{k \cdot \bar{k}}$$

$\bar{k}$ is a light-like vector,

Alternatively on could add ghosts in the initial state and use again $$\sum_{pol} \epsilon_i^{\mu} \epsilon_i^{*\nu} = -g_{\mu\nu}$$



Color algebra

Conventional normalization

$$Tr(t^a t^b) = T_R \delta_{ab} \quad T_R = 1/2$$

Fundamental representation 3

$$(t^a t^a)_{il} = C_F \delta_{il} \qquad C_F = \frac{N_c^2 - 1}{2N_c}$$

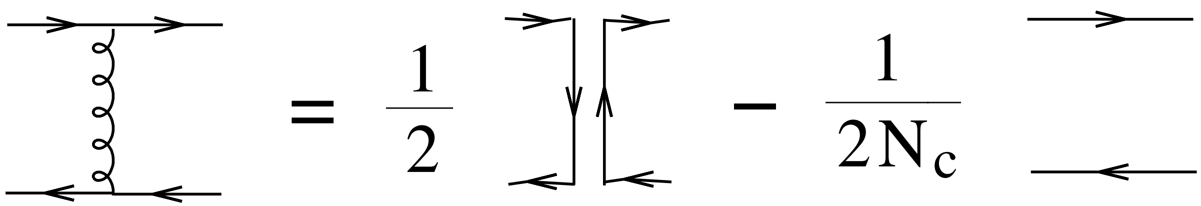
i,j,.. quark
a,b,.. gluon

Adjoint representation 8

$$f^{adc} f^{bdc} = C_A \delta^{ab} \qquad C_A = N_c$$

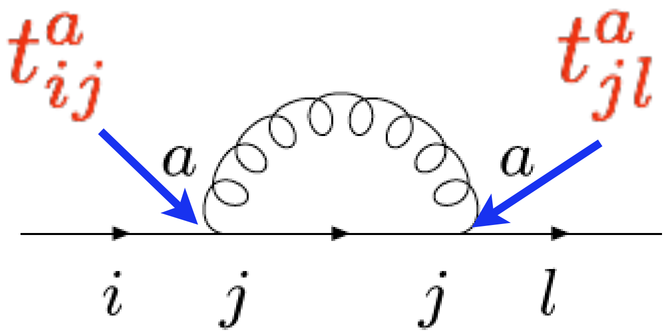
Very useful Fierz identity

$$(t^a)^i_k (t^a)^l_j = \frac{1}{2} \delta^i_j \delta^l_k - \frac{1}{2N_c} \delta^i_k \delta^l_j$$



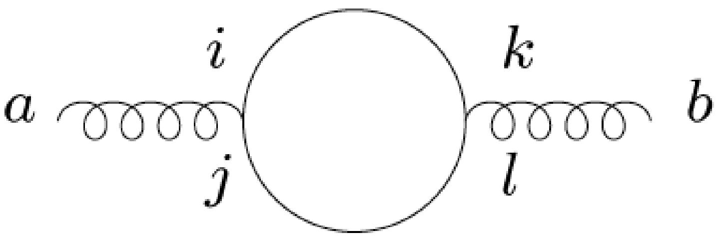
Most relevant color structures

Compute those!



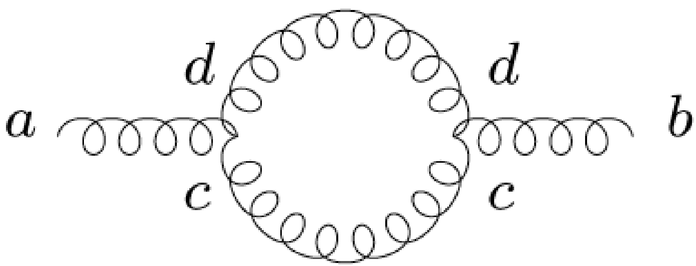
$$t^a_{ij} t^a_{jl} = C_F \, \delta_{il}$$

quark → gluon



$$Tr(t^a t^b) = T_R \delta_{ab}$$

gluon → quark



$$f^{adc} f^{bdc} = C_A \, \delta^{ab}$$

gluon → gluon

QCD at work

- QCD can not be solved exactly: use perturbation theory

$$\sigma = \sigma^{(0)} + \alpha_s(\mu)\,\sigma^{(1)} + \alpha_s^2(\mu)\,\sigma^{(2)} + \ldots$$

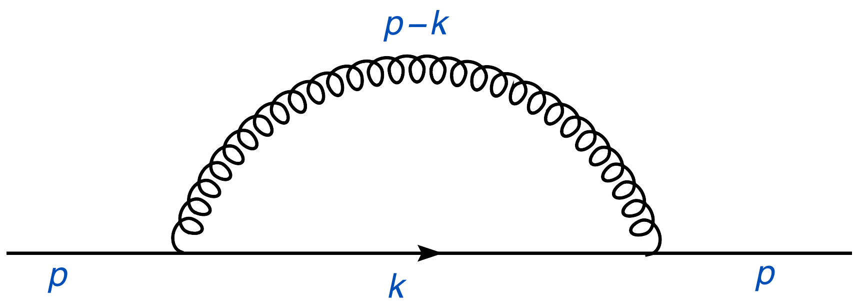
- Coupling constant "large" : many orders needed for precision

- Several problems appear in the calculation of perturbative corrections

Ultraviolet (UV) and InfraRed (IR) divergences

QFT has problems with loops: ultraviolet divergences

originate from integration over very large momentum



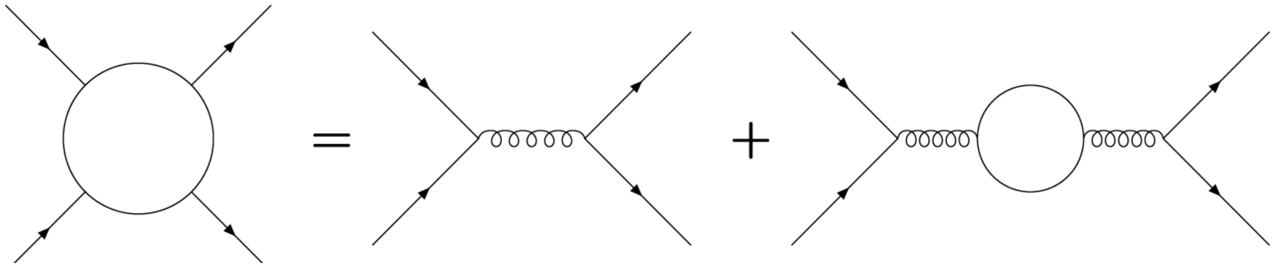
$$\sim g^2 \int_{p^2}^{\infty} d^4k \, \frac{1}{k^2} \frac{1}{(p-k)^2} \to \infty$$

A manifestation that QFT **FAIL** at very large energies!

To be able to use QFT, search for a procedure to isolate the "large" energy regime were it fails ➡ renormalization

1. Regularize the divergency
2. "Absorb" it by redefinition of "bare" (g, m, A, ψ) parameters in Lagrangian (thanks to gauge symmetry!)

Example



Regularization Λ_{cut} $\quad \sim \alpha_B \Big\{ 1 \quad + \quad \alpha_B \beta_0 \int_{p^2}^{\Lambda_{cut}^2} \frac{d^4k}{(k^2)^2} + \mathcal{O}(\alpha_B^2) \Big\}$

Renormalization scale μ $\quad \sim \alpha_B \Big\{ 1 \quad + \quad \alpha_B \beta_0 (\log \frac{\Lambda_{cut}^2}{\mu^2} + \log \frac{\mu^2}{p^2}) + \mathcal{O}(\alpha_B{}^2) \Big\}$

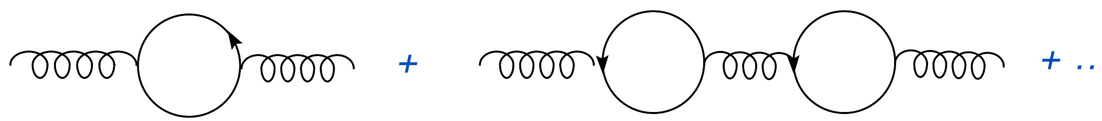
Renormalization $\quad = \alpha(\mu^2) \Big\{ 1 \quad + \quad \beta_0 \alpha(\mu^2) \log \frac{\mu^2}{p^2} + \mathcal{O}(\alpha_B{}^2) \Big\}$

$$\alpha(\mu^2) \equiv \alpha_B \Big(1 + \beta_0 \alpha_B \log \frac{\Lambda_{cut}^2}{\mu^2} + \mathcal{O}(\alpha_B^2) \Big)$$

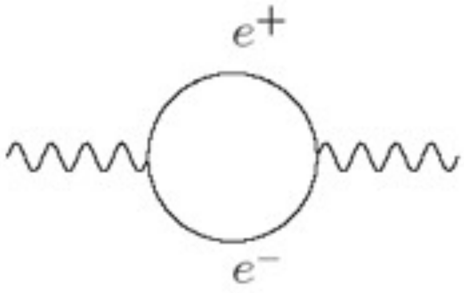
Renormalized (running) coupling constant : μ dependent

RGE → $\frac{d\alpha_s(\mu^2)}{d \log \mu^2} = -\beta(\alpha_s)$ $\qquad \beta(\alpha_s) = \beta_0 \alpha_s^2 + \ldots$

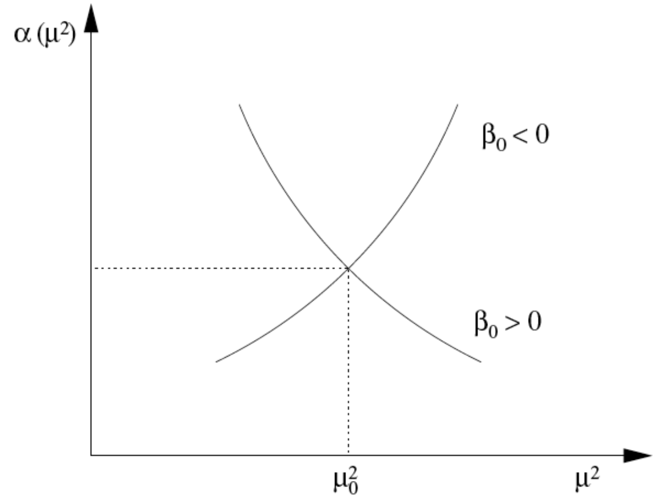
All order sum of logs



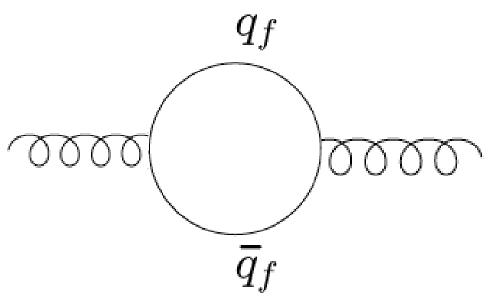
QED



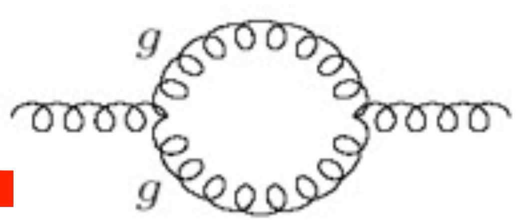
$\beta_0 = -\frac{1}{3\pi} < 0$



QCD



$\beta_0^{quark} = -\frac{1}{3\pi} T_R N_F < 0$



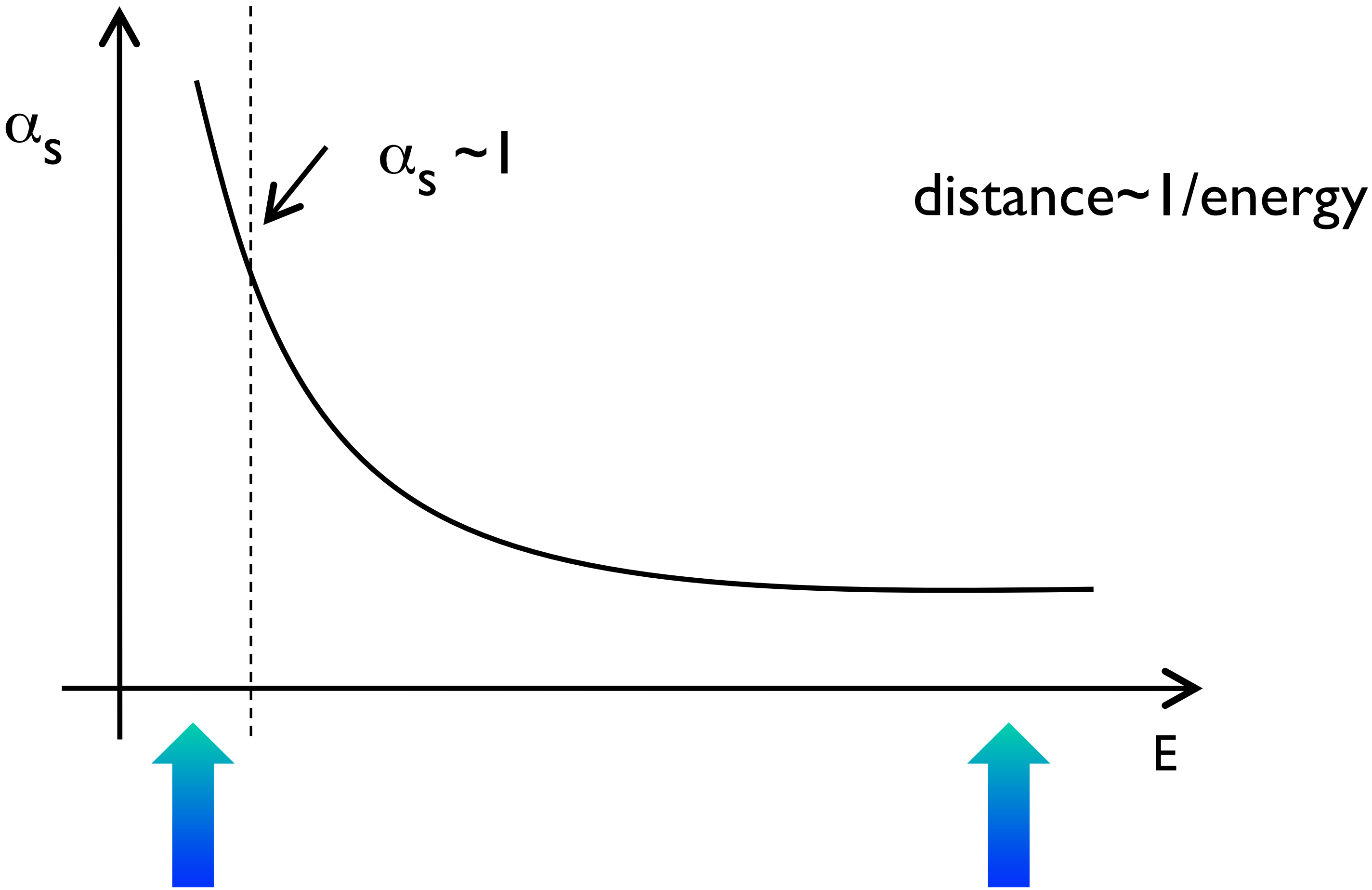
$\beta_0^{gluon} = \frac{11}{12\pi} C_A$

Gross, Wilczek, Politzer

$$\text{QCD} \quad \beta_0 = \frac{11C_A - 2n_F}{12\pi} > 0 \qquad (n_F < 16)$$

Coupling constant DEcreases with energy

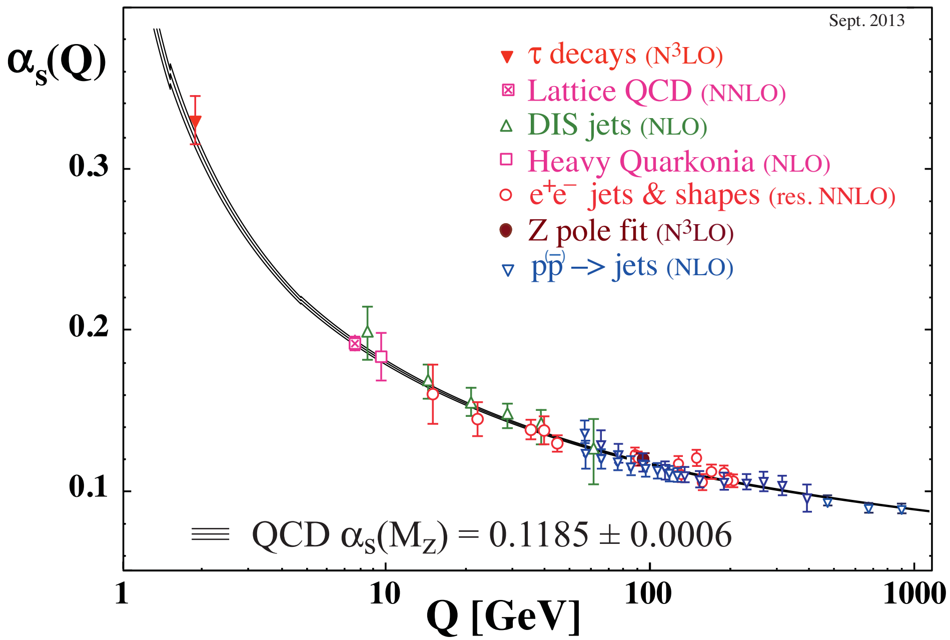
The two faces of QCD



confinement
large distances ~1 fermi

asymptotic freedom
short distances

Quarks do not show up as “free particles”

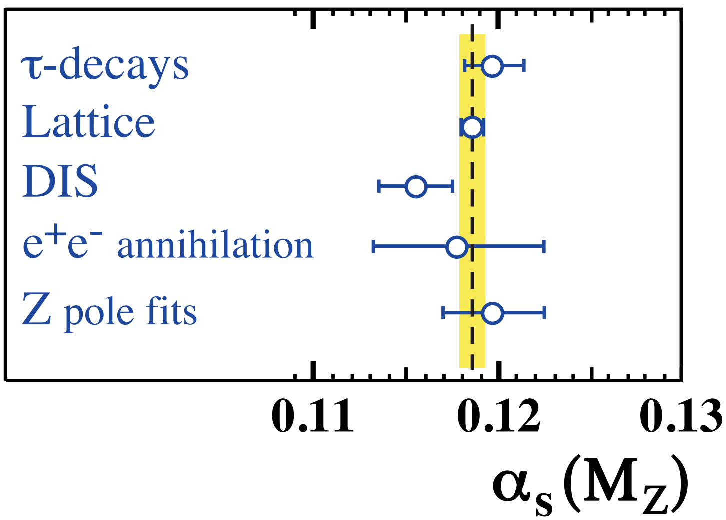


asymptotic freedom

It is a prediction of perturbation theory and allows to use it at high energies

confinement

Perturbation theory breaks down: no rigorous proof yet ...



World Average

$$\alpha_s(M_Z^2) = 0.1185 \pm 0.0006$$

Dominated by Lattice 

RGE $\frac{d\alpha_s(\mu^2)}{d\log\mu^2} = -\beta(\alpha_s)$ at leading order (LO)

$$\frac{d\alpha_s(\mu^2)}{d\log\mu^2} = -\beta_0\alpha_s(\mu^2) \quad \Rightarrow \quad \alpha_s(\mu^2) = \frac{\alpha_s(\mu_0^2)}{1+\beta_0\alpha_s(\mu_0^2)\log\frac{\mu^2}{\mu_0^2}}$$

This expression allows to compute coupling at any scale by knowing it at a reference value, e.g. $\mu_0 = M_Z$

But it is convenient to introduce the fundamental parameter of QCD

$$\Lambda_{QCD}$$

Such as

$$\Lambda_{QCD} = \mu_0 \exp\left[-\frac{1}{2\beta_0\alpha_s(\mu_0^2)}\right] \quad \Rightarrow \quad \alpha_s(\mu^2) = \frac{1}{\beta_0\log\frac{\mu^2}{\Lambda_{QCD}^2}}$$

- Scale at which coupling becomes large
- Scale that control hadron masses

$$\Lambda_{QCD} \sim 200\,\mathrm{MeV}$$

In real life:

- Dimensional regularization 4 → D dimensions,
- "divergences" appear as 1/(D-4) poles
- Finite terms can be subtracted: renormalization scheme

Next-to-Next-to-Leading Order (NNLO) in $\overline{\text{MS}}$ scheme

$\overline{MS}$ scheme. Subtract → $\frac{2}{4-D} + \ln(4\pi) - \gamma_E$

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda^2)}\left[1 - \frac{2\beta_1}{\beta_0^2}\frac{\ln\left[\ln(\mu^2/\Lambda^2)\right]}{\ln(\mu^2/\Lambda^2)} + \frac{4\beta_1^2}{\beta_0^4 \ln^2(\mu^2/\Lambda^2)}\right.$$
$$\left.\times\left(\left(\ln\left[\ln(\mu^2/\Lambda^2)\right] - \frac{1}{2}\right)^2 + \frac{\beta_2\beta_0}{8\beta_1^2} - \frac{5}{4}\right)\right].$$

$$\beta_0 = \frac{11}{3}C_A - \frac{4}{3}T_F n_f$$

$$\beta_1 = \frac{34}{3}C_A^2 - 4C_F T_F n_f - \frac{20}{3}C_A T_F n_f$$

$$\begin{aligned}\beta_2 = &\frac{2857}{54}C_A^3 + 2C_F^2 T_F n_f - \frac{205}{9}C_F C_A T_F n_f \\ &-\frac{1415}{27}C_A^2 T_F n_f + \frac{44}{9}C_F T_F^2 n_f^2 + \frac{158}{27}C_A T_F^2 n_f^2\end{aligned}$$

$$\begin{aligned}\beta_3 = &C_A^4\left(\frac{150653}{486} - \frac{44}{9}\zeta_3\right) + C_A^3 T_F n_f\left(-\frac{39143}{81} + \frac{136}{3}\zeta_3\right) \\ &+C_A^2 C_F T_F n_f\left(\frac{7073}{243} - \frac{656}{9}\zeta_3\right) + C_A C_F^2 T_F n_f\left(-\frac{4204}{27} + \frac{352}{9}\zeta_3\right) \\ &+46 C_F^3 T_F n_f + C_A^2 T_F^2 n_f^2\left(\frac{7930}{81} + \frac{224}{9}\zeta_3\right) + C_F^2 T_F^2 n_f^2\left(\frac{1352}{27} - \frac{704}{9}\zeta_3\right) \\ &+C_A C_F T_F^2 n_f^2\left(\frac{17152}{243} + \frac{448}{9}\zeta_3\right) + \frac{424}{243}C_A T_F^3 n_f^3 + \frac{1232}{243}C_F T_F^3 n_f^3 \\ &+\frac{d_A^{abcd}d_A^{abcd}}{N_A}\left(-\frac{80}{9} + \frac{704}{3}\zeta_3\right) + n_f\frac{d_F^{abcd}d_A^{abcd}}{N_A}\left(\frac{512}{9} - \frac{1664}{3}\zeta_3\right) \\ &+n_f^2\frac{d_F^{abcd}d_F^{abcd}}{N_A}\left(-\frac{704}{9} + \frac{512}{3}\zeta_3\right)\end{aligned}$$

QCD at work

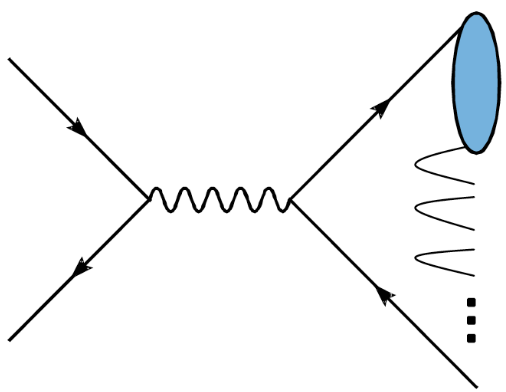
Observable computed as an expansion in strong coupling constant

$$\sigma = \sigma^{(0)} + \alpha_s(\mu)\,\sigma^{(1)} + \alpha_s^2(\mu)\,\sigma^{(2)} + \ldots$$

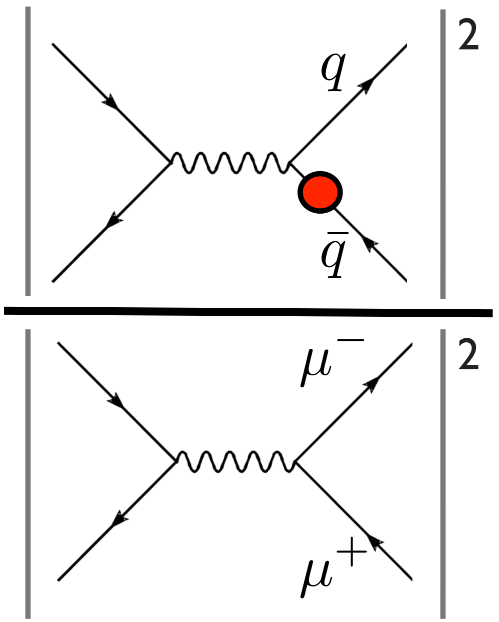
Example: $e^+e^- \to \text{hadrons}$

We can not compute "hadrons" but can assume that once there are partons in the final state they will form hadrons. If we neglect some hadronization effects then "hadrons ~ partons"

LO: $\sigma(e^+e^- \to \text{hadrons}) \approx \sigma(e^+e^- \to \text{quarks})$



$$R_{\text{had}} \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

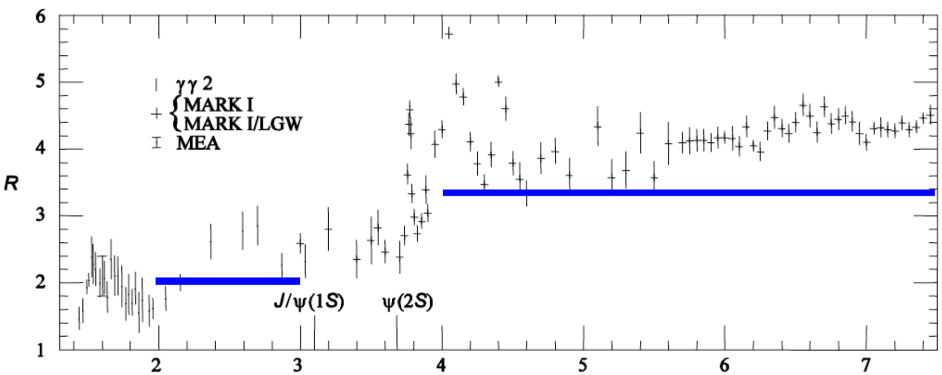


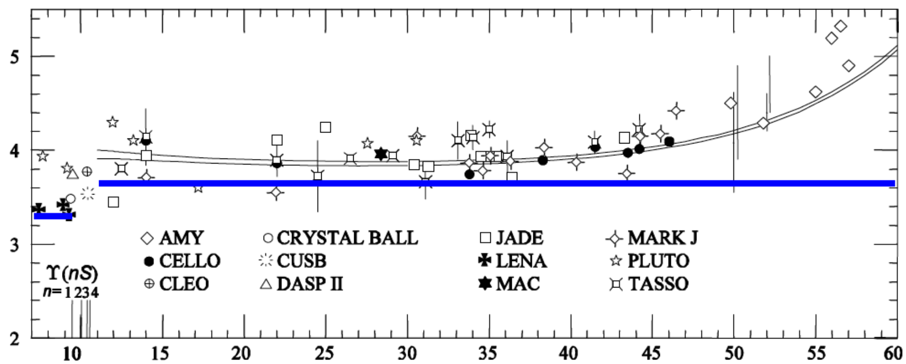
R is Sensitive to number of colors!

$$R_{\text{had}} \equiv \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_q e_q^2 N_c$$

- ☑ Nc =3
- ☑ Quark Flavor thresholds

Compare TH to experimental data





R	2	10/3	11/3
n_F	3	4	5

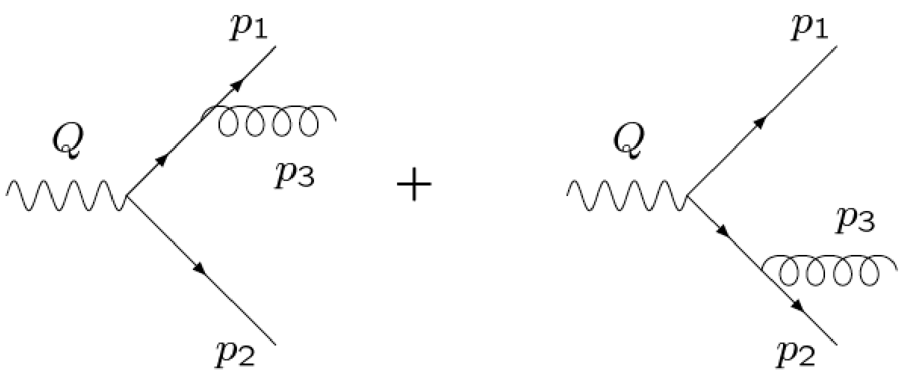
What about the next term in the expansion? $\mathcal{O}(\alpha_s)$

Coupling constant not so small : can lead to visible effect

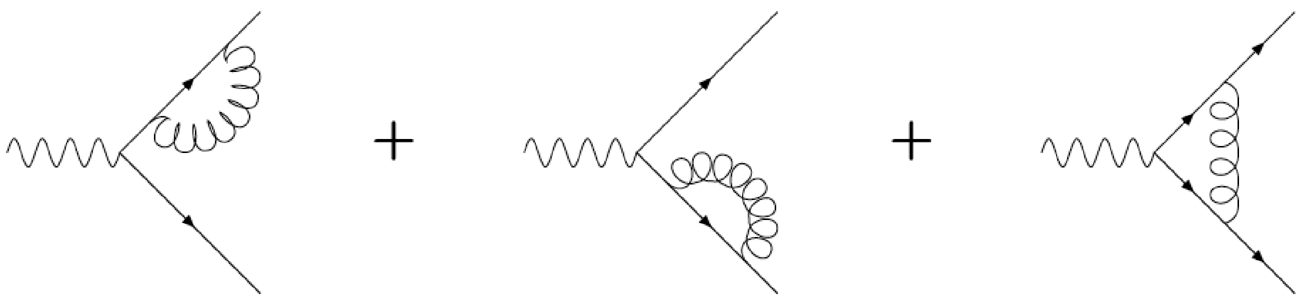
Two contributions: real and virtual gluon emission

Real included because we are interested in inclusive cross section, not in cross section with a fixed number of partons in final state (which by the way can not be computed...see later..)

Real



Virtual

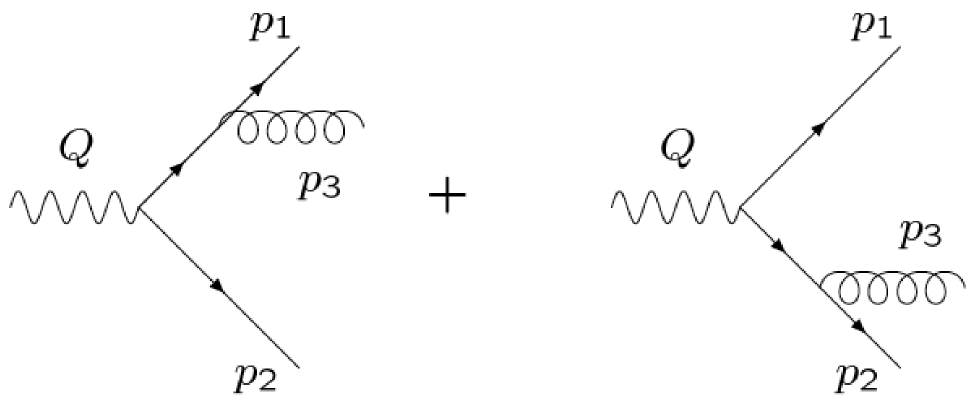


Real gluon emission (massless)

Best variables to describe the process

$$x_i = \frac{2p_i \cdot Q}{Q^2} \equiv \frac{2E_i}{Q}$$

$$0 \leq x_i \leq 1$$



Exercise: compute this!

$$|M_{\mathrm{real}}(x_{1,2}\,, x_3)|^2 = C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

Some more kinematics (angles between final state partons)

$$1 - x_1 = \frac{1}{2} x_2 x_3 (1 - \cos\theta_{qg})$$

$$1 - x_2 = \frac{1}{2} x_1 x_3 (1 - \cos\theta_{\bar{q}g}) \qquad x_1 + x_2 + x_3 = 2$$

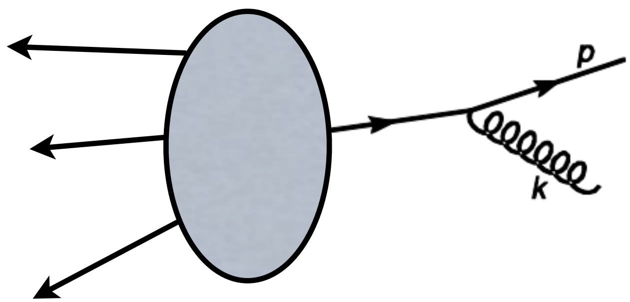
Integrate over phase space → real contribution to cross-section

Exercise: do this!

$$\sigma^R = \int_0^1 dx_1 dx_2 dx_3\, \delta(2 - x_1 - x_2 - x_3) |M_{\text{real}}(x_1, x_2, x_3)|^2$$ singular at $x_i = 1$

$$|M_{\text{real}}(x_{1,2}\,, x_3)|^2 = C_F \frac{\alpha_s}{2\pi} \frac{x_1^2 + x_2^2}{(1 - x_1)(1 - x_2)}$$

Origin of singular contributions: soft and collinear emission



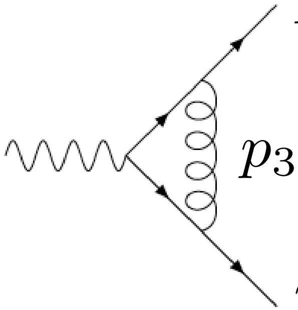
$$\frac{1}{(p+k)^2} = \frac{1}{2\,p \cdot k} = \frac{1}{2E_q E_g (1 - \cos\theta_{qg})}$$

soft (→ E_g) collinear (→ $1 - \cos\theta_{qg}$)

$$|M_{\text{real}}(x_{1,2}\,, x_3)|^2 \underset{x_1 \to 1}{\to} \frac{1}{(1 - x_1)} \frac{\alpha_s}{2\pi} C_F \frac{1 + x_2^2}{(1 - x_2)}$$

universal splitting kernel

$$q \longrightarrow qg$$



Virtual

$$\sigma^V = \int_0^1 dx_1 dx_2\, \delta(2 - x_1 - x_2) \int_0^\infty dx_3\, |M_{\rm virtual}(x_1, x_2, x_3)|^2$$

Different phase space due to virtual gluon (instead of real)

$$\int_0^\infty dx_3 \ldots = \int_1^\infty dx_3 \ldots + \int_0^1 dx_3 \ldots$$

IR finite IR divergent

Looks similar to Real contribution (different kinematics)

$$\sigma^R = \int_0^1 dx_1 dx_2 dx_3\, \delta(2 - x_1 - x_2 - x_3)|M_{\rm real}(x_1, x_2, x_3)|^2$$

and also divergent...not UV, again due to soft and collinear emission

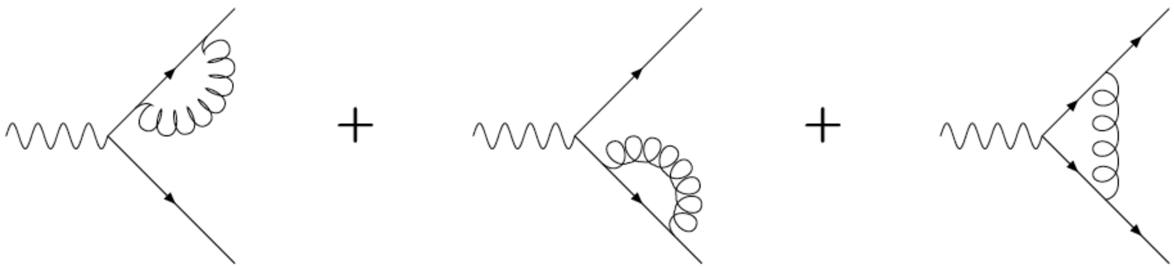
Looks bad: computing a physical quantity ... and diverges..

Lets regularize it by introducing a gluon mass m_g

$$\sigma^R = \sigma^{(0)} C_F \frac{\alpha_s}{2\pi} \left(\log^2 \frac{m_g^2}{Q^2} + 3 \log \frac{m_g^2}{Q^2} + 7 - \frac{\pi^2}{3} \right)$$

Double (log) singularities due to soft and collinear emission, one "log" per each

b) Add virtual contribution



$$\sigma^{(NLO)} = \sigma^{(0)} \left(1 + \frac{\alpha_s}{\pi} \right)$$

Same singularities but opposite sign!

$$\sigma^V = \sigma^{(0)} C_F \frac{\alpha_s}{2\pi} \left(-\log^2 \frac{m_g^2}{Q^2} - 3 \log \frac{m_g^2}{Q^2} - \frac{11}{2} + \frac{\pi^2}{3} \right)$$

Lets regularize by using dimensional regularization

Phase space and matrix elements computed in $d = 4 - 2\epsilon$

phase space

$$\int_0^1 \frac{1}{1-x}dx = \infty \quad \longrightarrow \quad \int_0^1 \frac{(1-x)^{-2\epsilon}}{1-x}dx = -\frac{1}{2\epsilon}$$

$$\sigma^R = \sigma^{(0)} C_F \frac{\alpha_S}{2\pi}\left(\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2\right)$$

finite real and virtual terms very different from previous slide (unphysical), but sum must be the same

$$\sigma^{(NLO)} = \sigma^{(0)}\left(1 + \frac{\alpha_s}{\pi}\right)$$

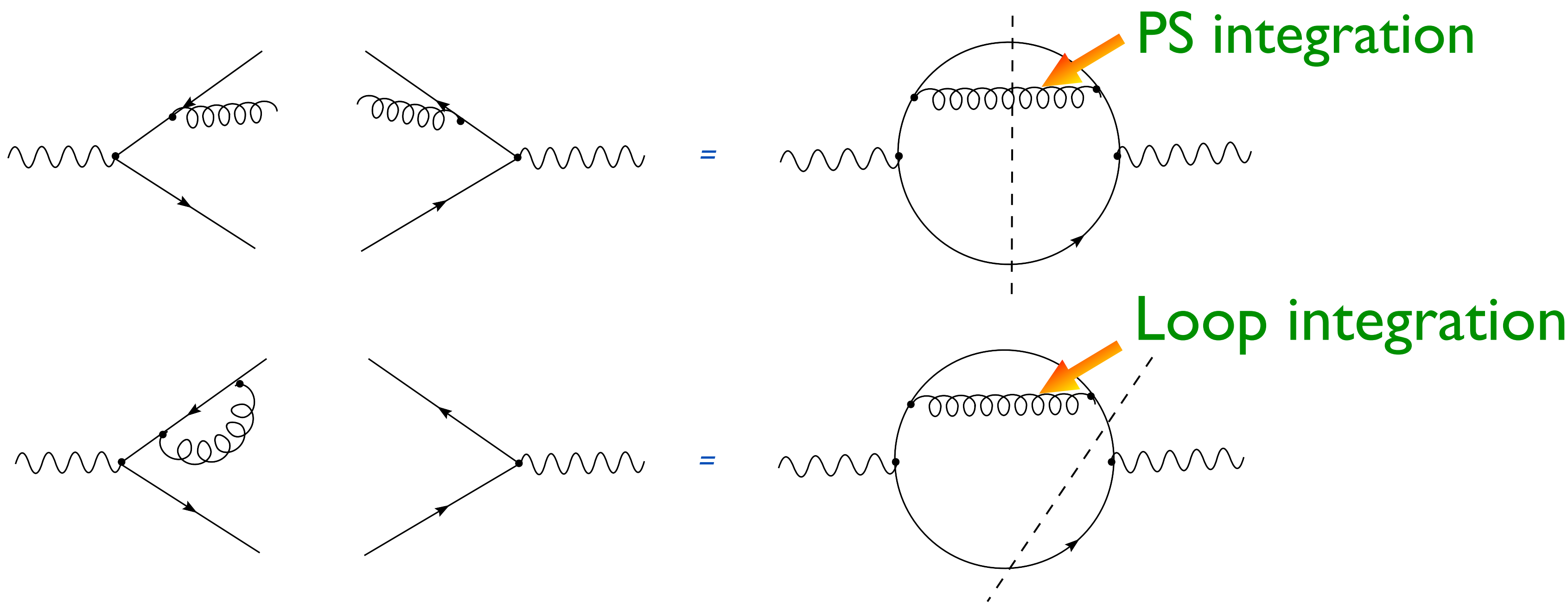
$$\sigma^V = \sigma^{(0)} C_F \frac{\alpha_S}{2\pi}\left(-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2\right)$$

Cancellation not by miracle

Since (Feynman, yes blame him!) we compute virtual and real separately: regularization needed until achieve cancellation

IR much worse than UV!

Real and Virtual diagrams have very similar structure: cuts (dashed line)



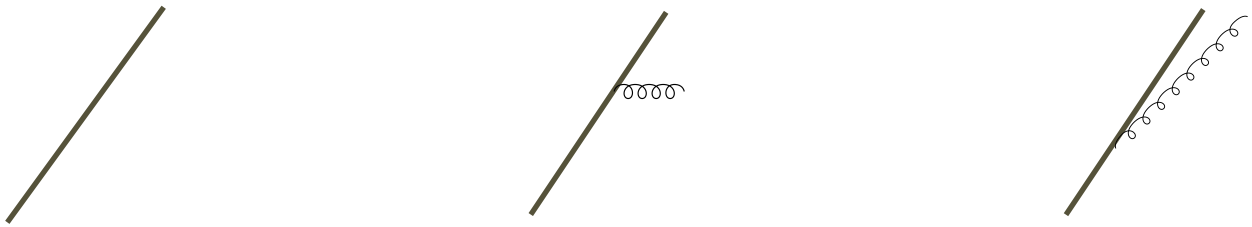
In the infrared region: virtual and real are kinematically equivalent

(-1) from Unitarity

KLN Theorem

Cancellation is a general feature: Kinoshita-Lee-Nauenberg theorem

Infrared singularities in massless theory cancel out after a sum over degenerate (initial and final) states

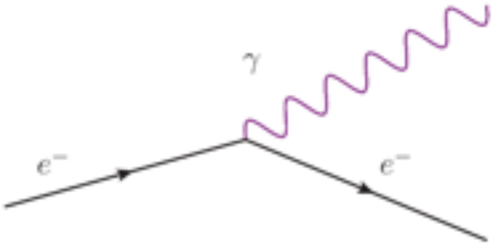


hard | hard + soft gluon | 2 collinear partons

Physically a hard parton can not be distinguish from a parton plus a soft gluon or two collinear partons : degenerate states. One should add over them (to some extent/resolution) to obtain a physically sound observable

KLN Theorem

In QED: Bloch-Nordsieck (only needs sum over final states), proved to all orders



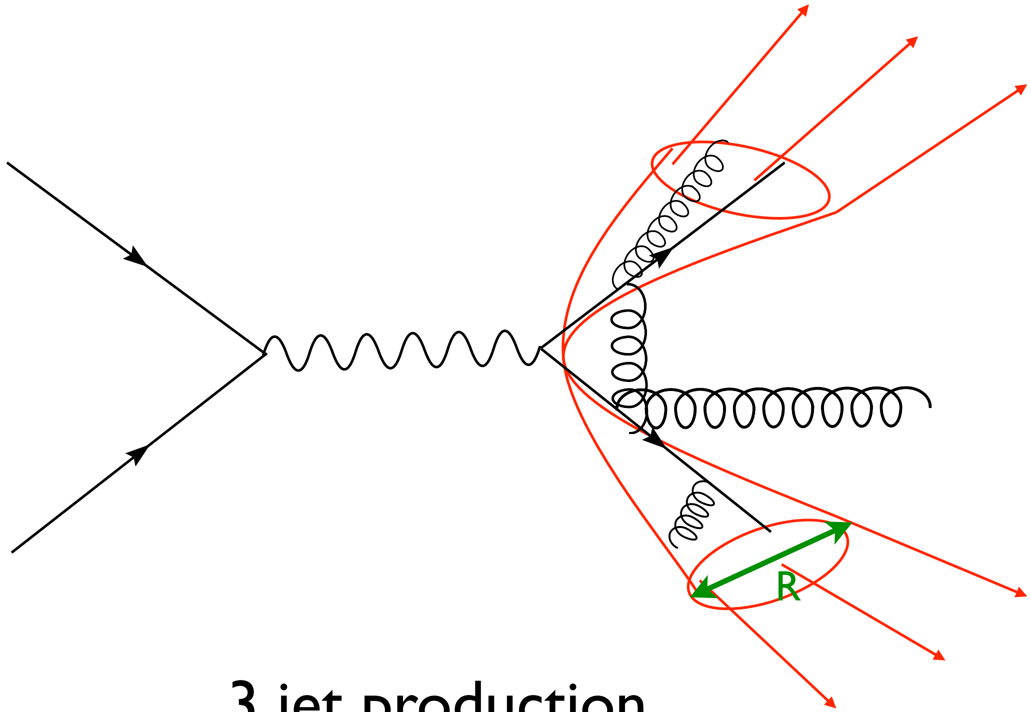
Solution of the well-known "infrared catastrophe" in QED (soft photon emission)

We can use QCD to compute observables corresponding "inclusive enough" processes

→ InfraRed safe (IRS)

Observable "insensitive" to collinear and soft emission

$e^+e^- \to q\bar{q}$ is not IRS while $e^+e^- \to 2\,\text{jets}$ is

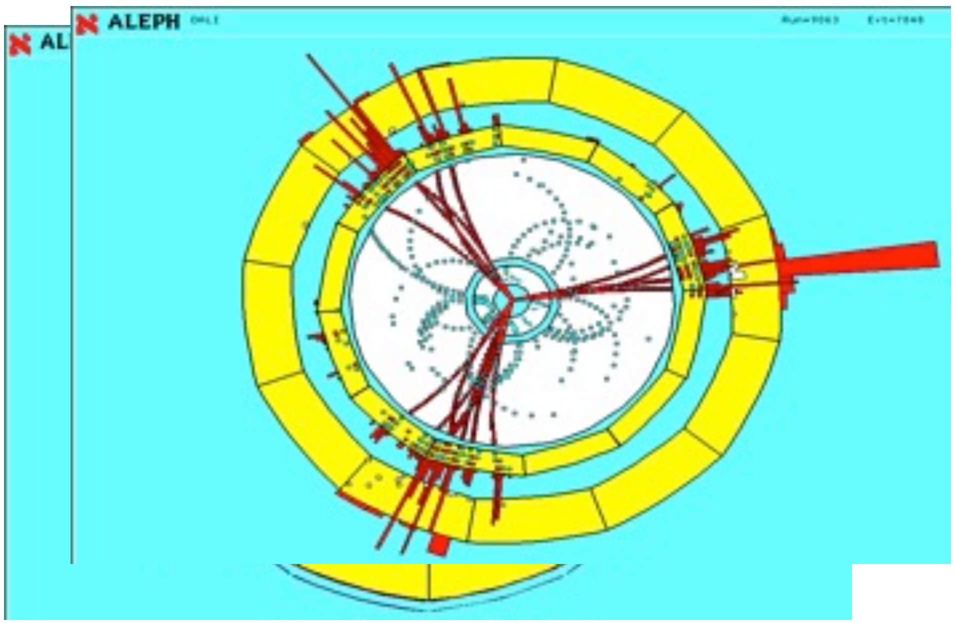


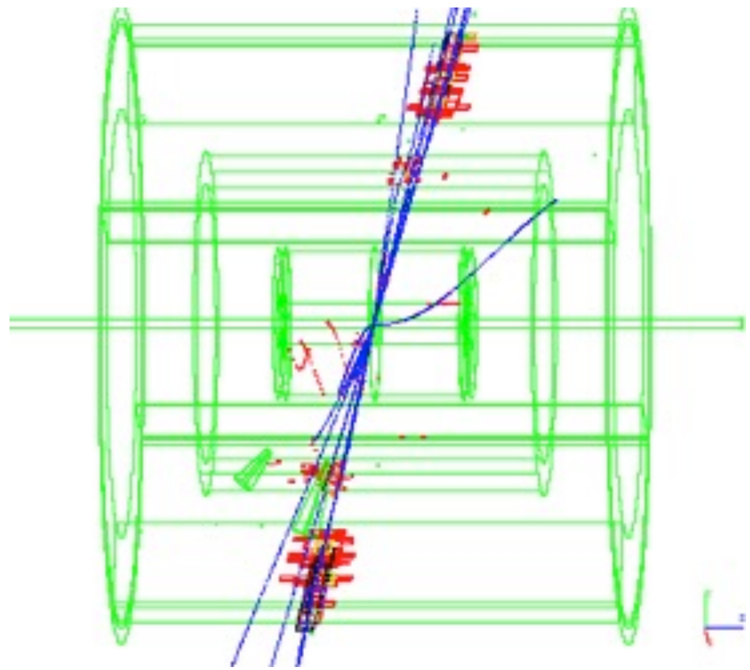
IR safe: KLN works

cancellation not as complete as for fully inclusive: some logs remain

$$\alpha_s \log R$$

3 jet production

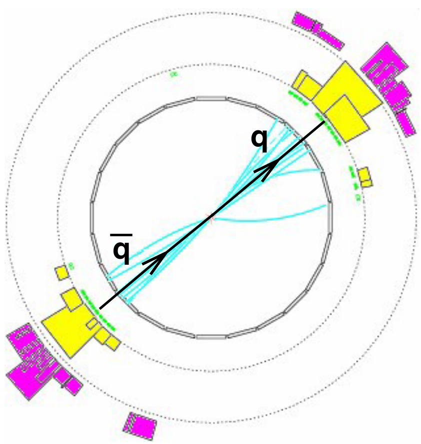


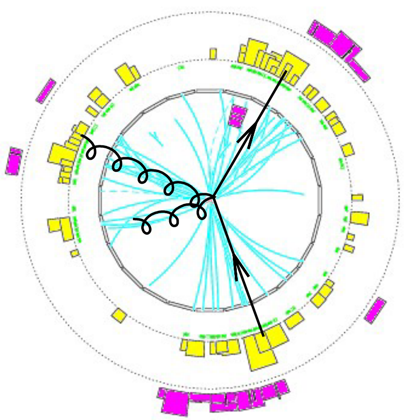


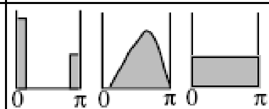
Infrared observables (beyond total cross sections)

Definition insensitive to soft and collinear branching

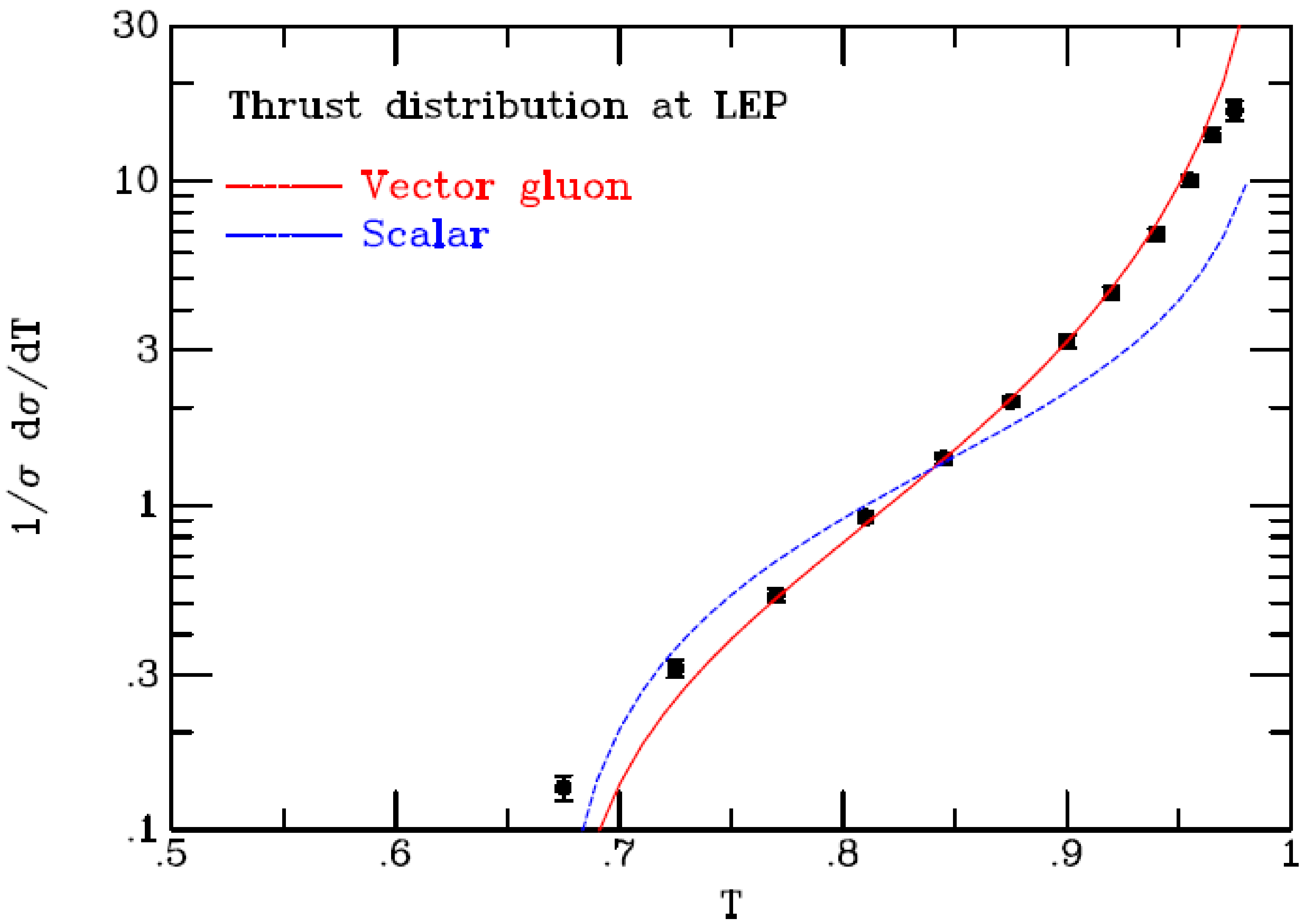
Event shape variables in e+e-



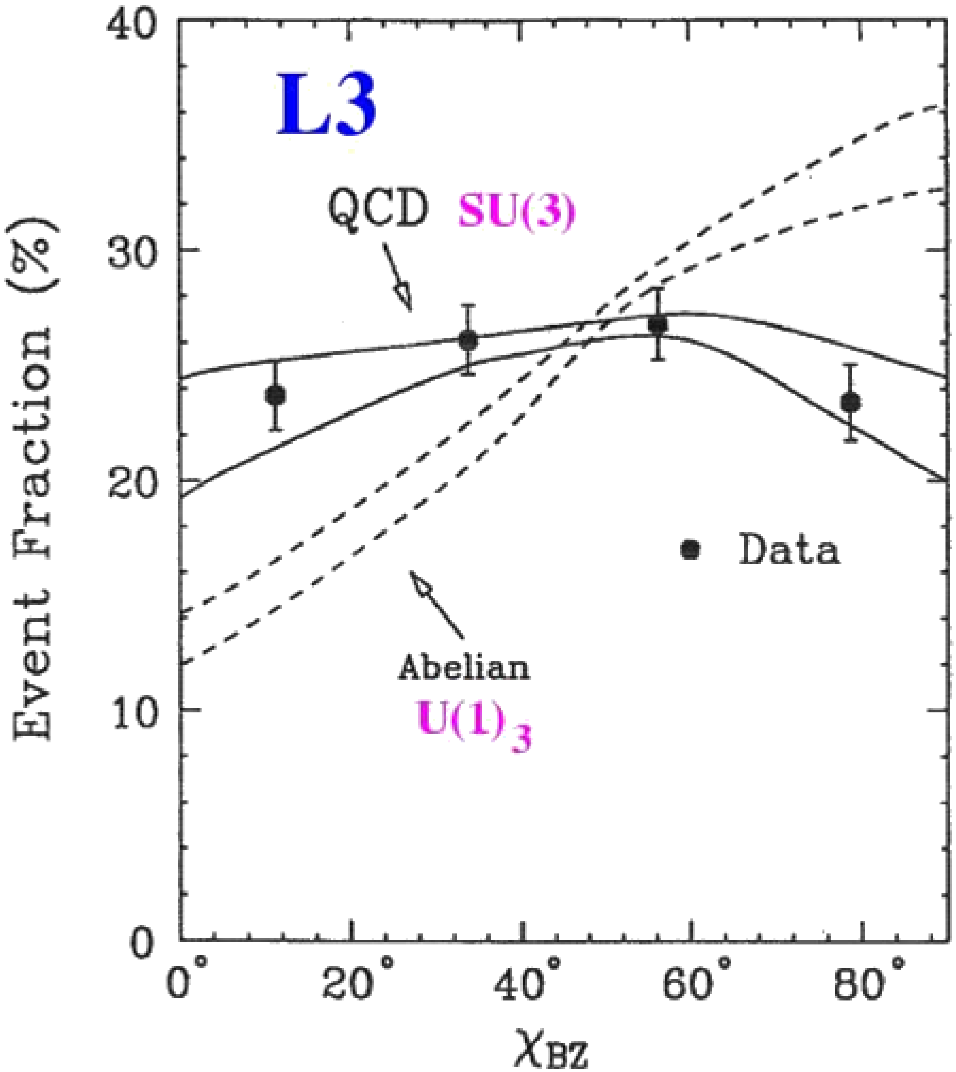


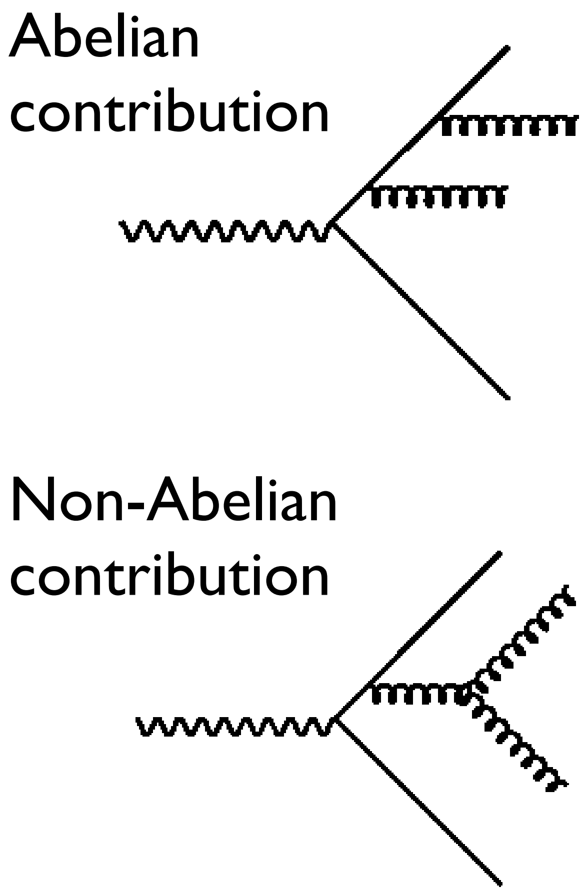
Name of Observable	Definition	Typical Value for:			QCD calculation
Thrust	$T = \max_{\vec{n}}\left(\frac{\sum_i \lvert\vec{p}_i\vec{n}\rvert}{\sum_i \lvert\vec{p}_i\rvert}\right)$	1	≥2/3	≥1/2	(resummed) $O(\alpha_s^2)$
Thrust major	Like T, however T_{maj} and $\vec{n}_{maj}$ in plane $\perp \vec{n}_T$	0	≤1/3	≤1/√2	$O(\alpha_s^2)$
Thrust minor	Like T, however T_{min} and $\vec{n}_{min}$ in direction $\perp$ to $\vec{n}_T$ and $\vec{n}_{maj}$	0	0	≤1/2	$O(\alpha_s^2)$
Oblateness	$O = T_{maj} - T_{min}$	0	≤1/3	0	$O(\alpha_s^2)$
Sphericity	$S = 1.5\,(Q_1 + Q_2)$; $Q_1 \le \ldots \le Q_3$ are Eigenvalues of $S^{\alpha\beta} = \frac{\sum_i p_i^\alpha p_i^\beta}{\sum_i p_i^2}$	0	≤3/4	≤1	none (not infrared safe)
Aplanarity	$A = 1.5\,Q_1$	0	0	≤1/2	none (not infrared safe)
Jet (Hemisphere) masses	$M_\pm^2 = \left(\sum_i E_i^2 - \sum_i \vec{p}_i^{\,2}\right)_{i \in S_\pm}$ ($S_\pm$: Hemispheres $\perp$ to $\vec{n}_T$) $M_H^2 = \max(M_+^2, M_-^2)$ $M_D^2 = \lvert M_+^2 - M_-^2\rvert$	0 0	≤1/3 ≤1/3	≤1/2 0	(resummed) $O(\alpha_s^2)$
Jet broadening	$B_\pm = \frac{\sum_{i \in S_\pm} \lvert\vec{p}_i \times \vec{n}_T\rvert}{2\sum_i \lvert\vec{p}_i\rvert}$; $B_T = B_+ + B_-$ $B_w = \max(B_+, B_-)$	0 0	≤1/(2√3) ≤1/(2√3)	≤1/(2√2) ≤1/(2√3)	(resummed) $O(\alpha_s^2)$
Energy-Energy Correlations	$EEC(\chi) = \sum_{events} \sum_{i,j} \frac{E_i E_j}{E_{vis}^2} \int_{\chi-\frac{\Delta\chi}{2}}^{\chi+\frac{\Delta\chi}{2}} \delta(\chi - \chi_{ij})$				(resummed) $O(\alpha_s^2)$
Asymmetry of EEC	$AEEC(\chi) = EEC(\pi - \chi) - EEC(\chi)$				$O(\alpha_s^2)$
Differential 2-jet rate	$D_2(y) = \frac{R_2(y - \Delta y) - R_2(y)}{\Delta y}$				(resummed) $O(\alpha_s^2)$

Thrust to determine spin of the gluon



Non-Abelian nature : 4 jets

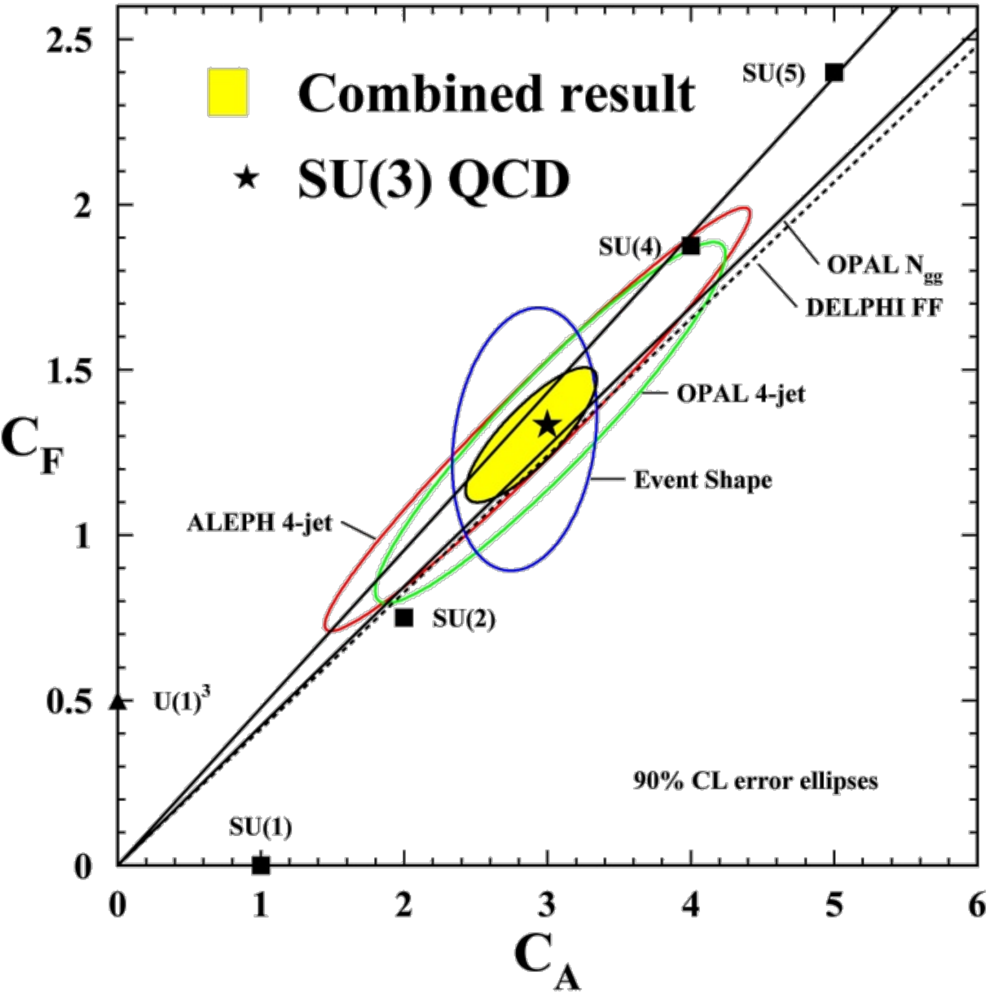




Bengtsson-Zerwas: angle between the planes containing the two highest and lowest energy jets

Color Factors

From combinations of 4-jet events & event shapes



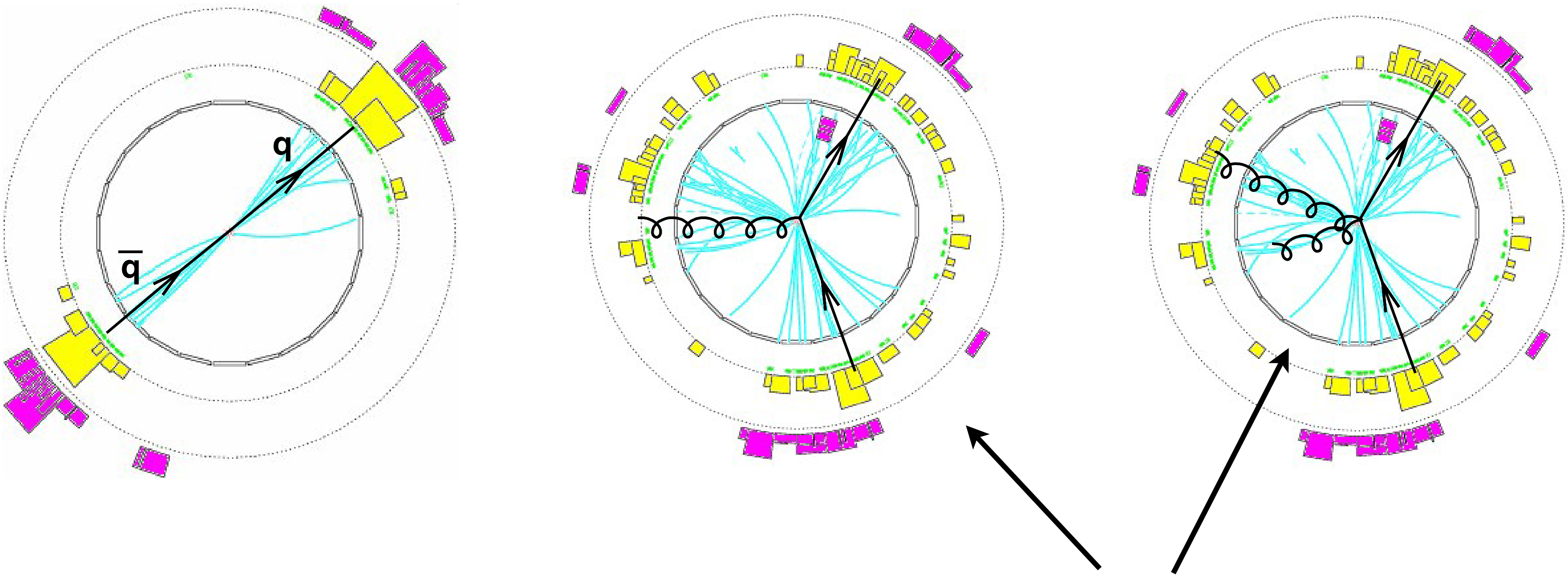
$C_A = 2.89 \pm 0.21$

$C_F = 1.30 \pm 0.09$

QCD
3
1.33

Jets : several definitions available

1. How do you group particles together in a common jet? : jet algorithm
2. How do you combine the momenta of particles inside the jet? : recombination scheme (E-scheme) add 4-vectors



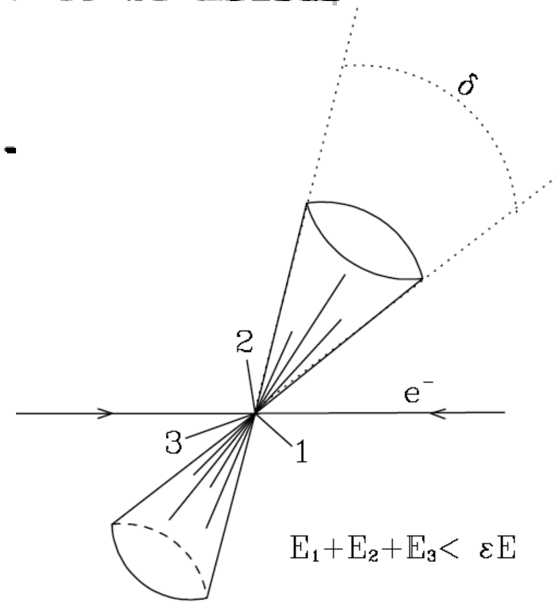
2-jets 3-jets same event!! 4-jets

number of jets depends on algorithm

First jet algorithm: Sterman-Weinberg (1977)

To study jets, we consider the partial cross section $\sigma(E,\theta,\Omega,\varepsilon,\delta)$ for e^+e^- hadron production events, in which all but a fraction $\varepsilon \ll 1$ of the total e^+e^- energy E is emitted within some pair of oppositely directed cones of half-angle $\delta \ll 1$, lying within two fixed cones of solid angle Ω (with $\pi\delta^2 \ll \Omega \ll 1$) at an angle θ to the e^+e^- beam line. We expect this to be measur-

$$\sigma(E,\theta,\Omega,\varepsilon,\delta) = (d\sigma/d\Omega)_0\,\Omega\left[1 - (g_E^2/3\pi^2)\left\{3\ln\delta + 4\ln\delta\,\ln 2\varepsilon + \frac{\pi^3}{3} - \right.\right.$$

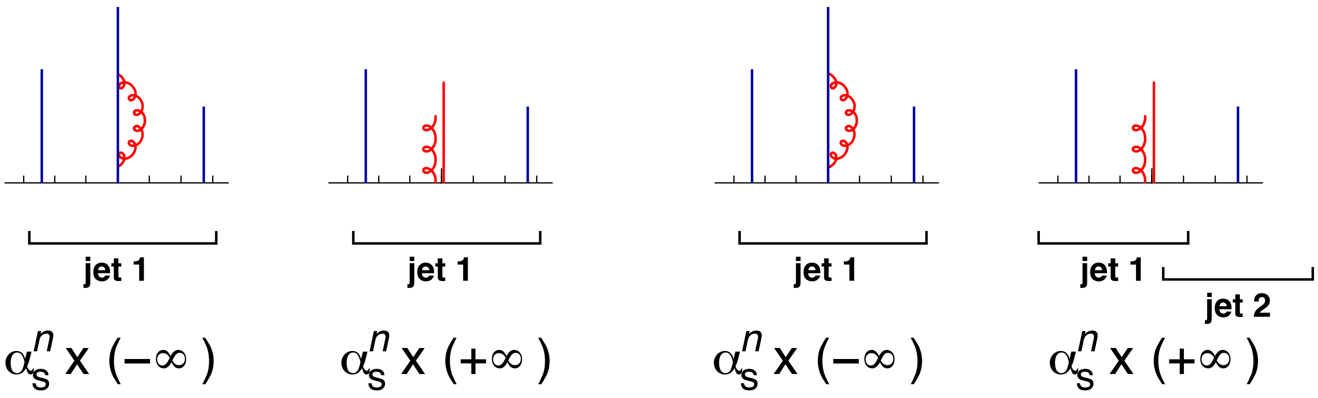


2-jets events if fraction $1-\epsilon$ of total energy contained in 2 cones of opening angle δ

Many since then, some with problems.... like infinities...

G.Salam

Collinear Safe | **Collinear Unsafe**



Infinities cancel | **Infinities do not cancel**

Don't find infinities in experiment, but IR unsafety spoil calculations from certain orders

> introduces large sensitivity on non-perturbative physics

	Last meaningful order			
	JetClu, ATLAS cone [IC-SM]	MidPoint [IC_{mp}-SM]	CMS it. cone [IC-PR]	Known at
Inclusive jets	LO	NLO	NLO	NLO ($\rightarrow$ NNLO)
$W/Z + 1$ jet	LO	NLO	NLO	NLO
3 jets	**none**	LO	LO	NLO [nlojet++]
$W/Z + 2$ jets	**none**	LO	LO	NLO [MCFM]
m_{jet} in $2j + X$	**none**	**none**	**none**	LO

Popular algorithms for hadron colliders: k_T and anti-k_T

Sequential recombination (bottom-up approach)

k_T

$$d_{ij} = \min(p_{ti}^2, p_{tj}^2)\frac{\Delta R_{ij}^2}{R^2}$$ distance parameter for pairs

$$\Delta R_{ij}^2 = (\Delta\eta_{ij})^2 + (\Delta\phi_{ij})^2$$

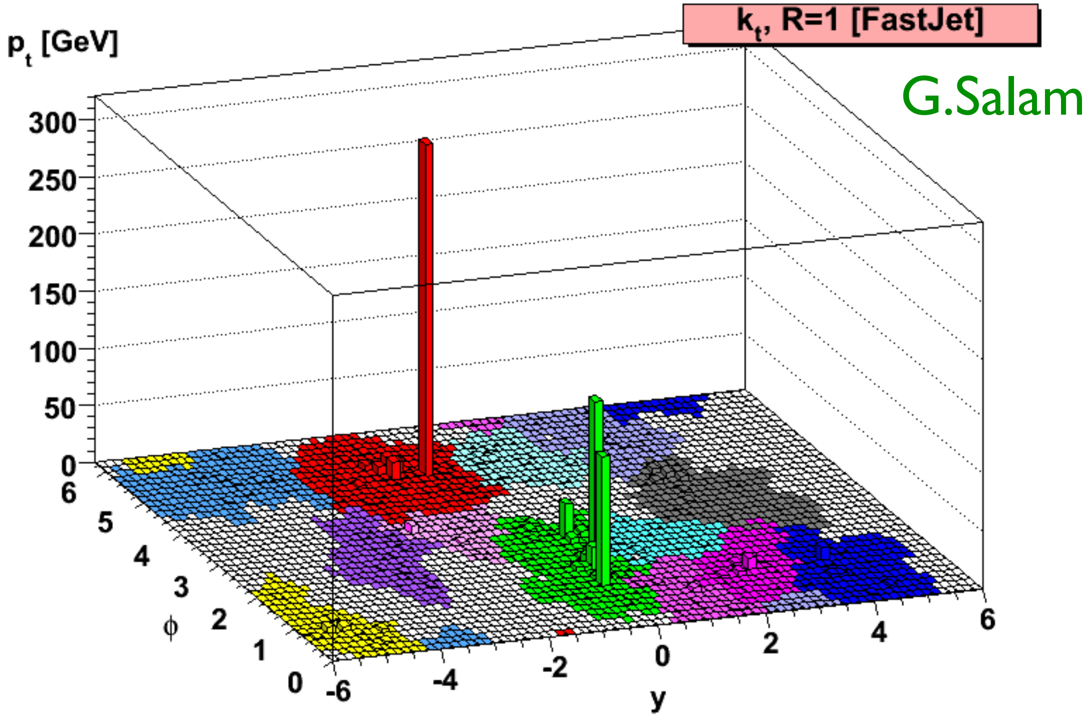
$$d_{iB} = p_{ti}^2$$ distance parameter to beam

Search for smallest distance among all possibilities

- if d_{iB} then particle i removed from list of particles and called a jet
- if d_{ij} then particles i and j are recombined in a single particle

Repeat until no particles remain

k_T



Jets irregular : soft particles recombine at the initial stages

- Acceptance corrections
- Underlying event corrections
- Energy calibration

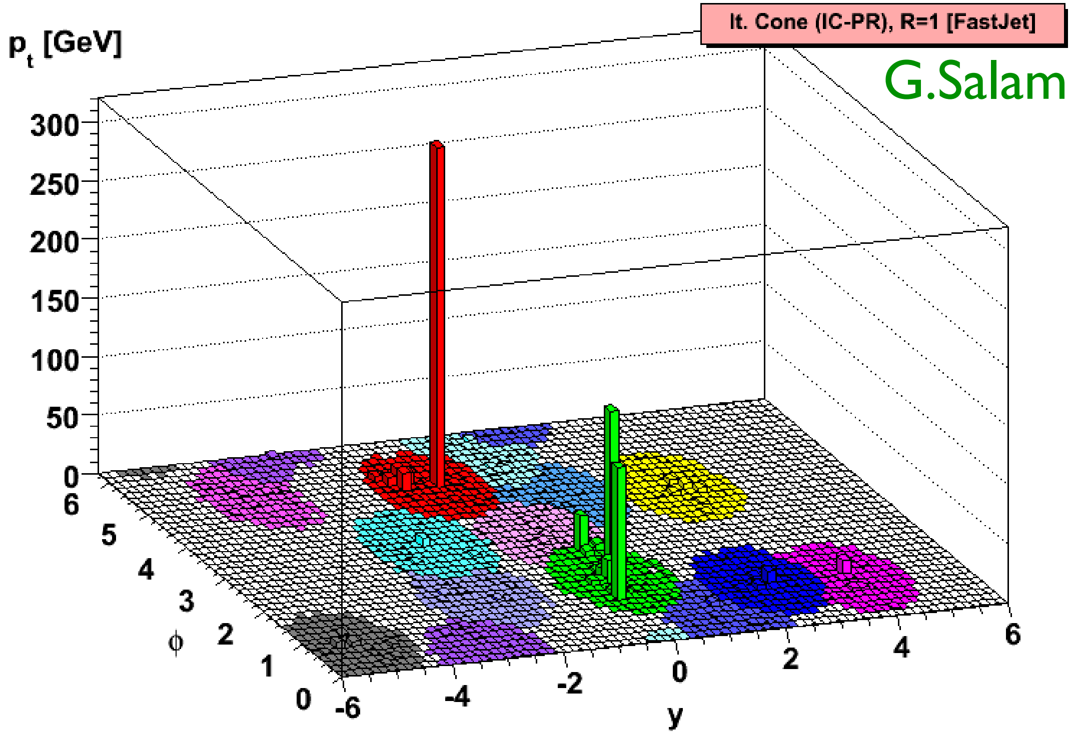
Can “undo” clustering sequence and look inside the jet

anti-k_T

"invert" distance measure

$$d_{ij} = \frac{1}{\max(p_{ti}^2, p_{tj}^2)} \frac{\Delta R_{ij}^2}{R^2}$$

$$d_{iB} = \frac{1}{p_{ti}^2}$$



Soft particles recombine early but preferably with hard particles : jets grow in concentric circles (like cone)

Can not look inside jet

Implemented in FastJet : default algorithm

Recap of first lecture

- Color "explains" hadron spectrum : charge of QCD

- QCD Lagrangian derived from gauge principle with non-abelian group SU(3) : Feynman rules for perturbative calculations

- There are UV divergences dealt by renormalization : as a result running coupling constant

- Two faces of QCD : asymptotically free and consistent with confinement

- There are also IR divergences that cancel when adding real and virtual contributions

- Jet algorithm is relevant to define IR safe observables

- QCD at work in e+e- : test the nature of SU(3) OK!