Flavor Physics and CP Violation

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5 Mote pillo con carne & cafe $7.50
white hominy w/ grilled steak & coffee

6 Calentado con Huevos pericos $6.00
Mixed rice & beans w/ scramble eggs

7 Calentado con carne asada y huevos pericos $8.50
Mixed rice & beans w/ grilled steak & scramble eggs

8 Carne asada con arepa y queso $7.50
Grilled steak with corncake & cheese

9 Calentado con Chorizo O chicharron $5.75
Mixed rice and beans w/ sausage

10 Huevos pericos con arroz o arepa . $5.75
Scramble eggs with rice or corn cake

11 Humitas con cafe $4.00
Sweet corn patties with coffee

12 Bolon de verde mixto con Chicharron y queso / Mixed green plantain w/ pork & cheese $3.25

13 Pancakes con huevos revueltos, tocino & jugo de naranja $5.50
Pancakes w/ scramble eggs, bacon & Orange juice

Antojitos / Appetizers

14 Arepas Tostadas con mantequilla... $2.00
Toasted corncake w/ butter

15 Arepas Tostadas con queso................ $3.00
Toasted corncake w/ cheese

16 Arepa de choclo.................................. $2.50
Sweet corncake

17 Arepa de choclo con queso $3.00
Sweet corncake w/ cheese

18 Pan Ecuatoriano de todos los sabores, Pandebono, Bunuelos, empanadas de

22 Jamon & Queso $5.00
Ham & cheese

23 Tocino & Huevo/ Bacon & Egg $4.50

24 Pechuga de pollo/ Chicken cutlet $4.75

25 Pollo o carne a la milanesa.............. $5.00
Chicken or steak Milanese

Ensaladas / Salads

26 Ensalada con pollo Asado $8.50
Grilled chicken salad

27 Ensalada de Aguacate........................ $5.75
Avocado salad

28 Caesar salad / Caesar salad................. $5.00

29 Caesar salad con pollo $6.50
Caesar salad with grilled chicken

Por favor pregunte por su ensalada favorita!
Feel free to ask for your favorite salad

Especiales / Special dishes

(Sat & Sun) Sabados & Domingos

30 Encebollado de Pescado/ Fish stew... $9.00

31 Ceviche de camarones/Shrimp cocktail. $10.00

32 Arroz con guatita /Rice w/ tripe........... $8.00

33 Bandera / Typical Ecuadorian dish........ $11.00

34 Churrasco Ecuatoriano $11.00
Ecuadorian Steak w/ rice, f. fries, fries egg &avocado

35 Ceviche Mixto.. $13.00

36 Bistec encebollado $11.00
Stew steak w/ rice & beans, sweet plantains & salad

37 Pechuga la limon...................................... $11.00
Chicken breast in lemon sauce

38 Pechuga al Ajillo...................................... $11.00

Flavor Physics

What? Why? How?

- Flavor Physics? study the different types of quarks, a.k.a. “flavors,” their spectrum and transitions among them (interactions)
  - More generally: leptons too!
  - Transitions: strengths, symmetries (e.g., CP/P/T; continuous?)
- Why?
  - Richness (much to do & understand)
  - Stringent test of models/theory
  - Closely tied to all observed CP violation (CPV)
- Many/diverse methods involved. Main challenge: strong interactions (to uncover flavor)
  - EFT’s:
    - Electro-weak (Fermi)
    - Chiral-lagrangian
    - HQET
    - SCET
  - Symmetries
  - Non-perturbative (lattice)

Since the SM works well, we will adopt it as our standard (no pun) paradigm.
Review:

(three families of each of:)

$$q_L = \begin{pmatrix} \text{“}u_L\text{”} \\ \text{“}d_L\text{”} \end{pmatrix}, \begin{pmatrix} \text{“}c_L\text{”} \\ \text{“}s_L\text{”} \end{pmatrix}, \begin{pmatrix} \text{“}t_L\text{”} \\ \text{“}b_L\text{”} \end{pmatrix} \qquad u_R, \quad d_R, \quad \ell_L, \quad e_R;$$

$$(3,2)_{\frac{1}{6}} \qquad (3,1)_{\frac{2}{3}} \quad (3,1)_{-\frac{1}{3}} \quad (1,2)_{-\frac{1}{2}} \quad (1,1)_{-1}$$

$$\underset{(1,2)_{\frac{1}{2}}}{H,} \qquad \langle H\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \qquad D_\mu = \partial_\mu + i g_s A_\mu T^a + i g_2 W^a_\mu \frac{\sigma^a}{2} + i g_1 B_\mu Y$$

$$\mathcal{L} = \mathcal{L}_{\text{gauge}} + \sum_\psi \bar\psi i\gamma\cdot D\psi + |D_\mu H|^2 - \left[\lambda^U_{ij}\tilde H \bar u^i_R q^j_L + \lambda^D_{ij} H \bar d^i_R q^j_L + \lambda^E_{ij} H \bar e^i_R \ell^j_L + \text{h.c.}\right]$$

<u>Flavor “Symmetry:”</u> For $\lambda^{U,D,E} = 0$, $\mathcal{L}$ has $U(3)^5$ symmetry

(a U(1) is anomalous, but we will mostly be concerned with SU(3) factors)

$$q^i_L \to U_q{}^i{}_j q^j_L, \qquad u^i_R \to U_U{}^i{}_j u^j_R, \qquad d^i_R \to U_D{}^i{}_j d^j_R, \qquad \ell^i_L \to U_\ell{}^i{}_j \ell^j_L, \qquad e^i_R \to U_E{}^i{}_j e^j_R,$$

Tired of index gymnastics already? Use $q_L \to U_q q_L$ $\quad \tilde H \bar u_R \lambda^U q_L$ etc (with $q^i_L = \begin{pmatrix} u^i_L \\ d^i_L \end{pmatrix}$).

Flavor symmetry is broken explicitly by Yukawa interactions

Keep track of pattern of symmetry breaking : treat Yukawa couplings as "spurions"

(spurions: as if couplings were fields, but are constant in spacetime)

$$\tilde{H}\bar{u}_R\lambda_U q_L \to \tilde{H}\bar{u}_R U_U^\dagger \lambda'_U U_q q_L \quad \text{is invariant if} \quad \lambda_U \to \lambda'_U = U_U \lambda_U U_q^\dagger$$

check: $\bar{u}_R\lambda_U q_L \to \bar{u}_R\, U_U^\dagger U_U\, \lambda'_U\, U_q^\dagger U_q\, q_L = \bar{u}_R\lambda_U q_L$ and analogously for other couplings.

Summarize: under the flavor group $G_F = SU(3)_q \times SU(3)_U \times SU(3)_D$

the SM is invariant with the assignments

$$q_L : (3,1,1) \qquad \lambda_U : (\bar{3},3,1)$$
$$u_R : (1,3,1) \qquad \lambda_D : (\bar{3},1,3)$$
$$d_R : (1,1,3)$$

As we will see:

New interactions that break this "symmetry" tend to produce rates of flavor transformations that are inconsistent with experimental observation (absent tuning or large parametric suppression)

hence the usefulness of this symmetry

* This is a quark sector story. We will be mostly concerned with quarks.

KM-model of CPV and the CKM matrix (review)

Unitary gauge: $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$ gives fermion masses

$$-\mathcal{L}_m = \frac{v}{\sqrt{2}}\left[\bar{u}_R\lambda_U u_L + \bar{d}_R\lambda_D d_L + \bar{u}_L\lambda_U^\dagger u_R + \bar{d}_L\lambda_D^\dagger d_R\right]$$

Diagonalize mass matrices (for simpler computation and interpretation)
By Field redefinitions that:

- linear
- leave $\bar{\psi} i\gamma\cdot\partial\psi$ invariant (properly normalized kinetic terms)

Hence: linear-unitary transformations (not $G_F = SU(3)^3$, but larger !!)

$$u_L \to V_{u_L} u_L, \quad d_L \to V_{d_L} d_L, \quad u_R \to V_{u_R} u_R, \quad d_R \to V_{d_R} d_R,$$

chosen so that

$V_{u_R}^\dagger \lambda_U V_{u_L} = \lambda'_U =$ diagonal, real, positive and $V_{d_R}^\dagger \lambda_D V_{d_L} = \lambda'_D =$ diagonal, real, positive

Exercise: Show that this can always be done.

Then

$$-\mathcal{L}_m = \frac{v}{\sqrt{2}}\left[\bar{u}\lambda'_U u + \bar{d}\lambda'_D d\right] \quad \Rightarrow \quad m_U = \frac{1}{\sqrt{2}} v\lambda'_U, m_D = \frac{1}{\sqrt{2}} v\lambda'_D$$

It is <u>these</u> fields and associated particles that we identify with the "flavors:"

$$u, c, t, d, s, b$$

Note that under the field redefinition the kinetic term is unchanged, by design:

$$\bar{u}_L i\gamma^\mu \partial_\mu u_L \to \bar{u}_L V_{u_L}^\dagger i\gamma^\mu \partial_\mu V_{u_L} u_L = \bar{u}_L (V_{u_L}^\dagger V_{u_L}) i\gamma^\mu \partial_\mu u_L = \bar{u}_L i\gamma^\mu \partial_\mu u_L \qquad ☺ ☺ ☺$$

What about the gauge interactions?

$$-\bar{u}_R(g_3 \not{A}^a T^a + \tfrac{2}{3} g_1 \not{B}) u_R \to -\bar{u}_R V_{u_R}^\dagger (g_3 \not{A}^a T^a + \tfrac{2}{3} g_1 \not{B}) V_{u_R} u_R = -\bar{u}_R(g_3 \not{A}^a T^a + \tfrac{2}{3} g_1 \not{B}) u_R \qquad ☺$$

likewise for d_R, and also for

$$-\bar{q}_L(g_3 \not{A}^a T^a + \tfrac{1}{6} g_1 \not{B} + \tfrac{1}{2} g_2 \not{W}^3 \sigma^3) q_L = -\bar{u}_L(g_3 \not{A}^a T^a + \tfrac{1}{6} g_1 \not{B} + \tfrac{1}{2} g_2 \not{W}^3) u_L - \bar{d}_L(g_3 \not{A}^a T^a + \tfrac{1}{6} g_1 \not{B} - \tfrac{1}{2} g_2 \not{W}^3) d_L$$

But $W^\pm$ terms are off diagonal: $\sigma^\pm \equiv \dfrac{\sigma^1 \pm i\sigma^2}{\sqrt{2}}$ $\qquad W^\pm \equiv \dfrac{W^1 \mp iW^2}{\sqrt{2}}$ $\qquad \left(\sigma^+ = \sqrt{2}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\right)$

$$\bar{q}_L(\tfrac{1}{2} g_2 \sum_{a=1}^{2} \not{W}^a \sigma^a) q_L = \bar{u}_L \tfrac{1}{\sqrt{2}} g_2 \not{W}^+ d_L + \bar{d}_L \tfrac{1}{\sqrt{2}} g_2 \not{W}^- u_L \to \bar{u}_L V_{u_L}^\dagger V_{d_L} \tfrac{1}{\sqrt{2}} g_2 \not{W}^+ d_L + \text{h.c.}$$

CKM matrix: $\quad V \equiv V_{u_L}^\dagger V_{d_L}$

Since this plays such a central role in flavor physics we will spend 6 slides on it! ☛

CKM matrix:

Unitary $V^\dagger V = V V^\dagger = 1$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

How many parameters?
Freedom (leaving rest of Lagrangian unchanged, including $m_{U,D}$ diagonal and positive)

$$u \to \mathrm{diag}(e^{i\alpha_1}, e^{i\alpha_2}, e^{i\alpha_3})u \qquad d \to \mathrm{diag}(e^{i\beta_1}, e^{i\beta_2}, e^{i\beta_3})\, d$$

Only phase differences $\alpha_i - \beta_j$ enter in off diagonal terms: 5 independent

Count parameters: 3x3 matrix, complex = 18; minus

Unitarity	9
Phases	5
Total	14

➠ 4 parameters: 3-ANGLES + 1-PHASE

Four Comments:

1. One irremovable phase $\Rightarrow$ CP is violated in $\quad \bar{u}_L V \not{W}^+ d_L + \bar{d}_L V^\dagger \not{W}^- u_L$

Under CP $\quad \bar{u}_L \gamma^\mu d_L \to \bar{d}_L \gamma_\mu u_L \quad$ and $\quad W^{+\mu} \to W^-_\mu$

$$\text{so CP} \Rightarrow V^\dagger = V$$

Exercise: If two entries in the (diagonal) matrix m_U (or in m_D) are equal, V can be brought into a real matrix (that is, in O(3), *the group of orthogonal matrices)*

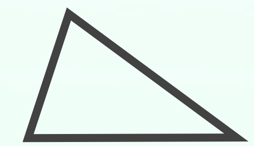
2. Precise knowledge of the elements of V is necessary to constrain new physics (or to test the validity of the SM/CKM theory)
Will describe later how well we know and how.
But for now, sketch the "texture:"

$$V \sim \begin{pmatrix} \epsilon^0 & \epsilon^1 & \epsilon^3 \\ \epsilon^1 & \epsilon^0 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon^0 \end{pmatrix} \quad \epsilon \sim 0.1$$

3. $V^\dagger V = V V^\dagger = 1$ ➠ rows (and columns) of V are ortho-normal vectors

$\sum_j V_{ij} V^*_{kj} = 0$ for $i \neq k$: the sum of 3 complex numbers vanish ⇨ triangle in z-plane

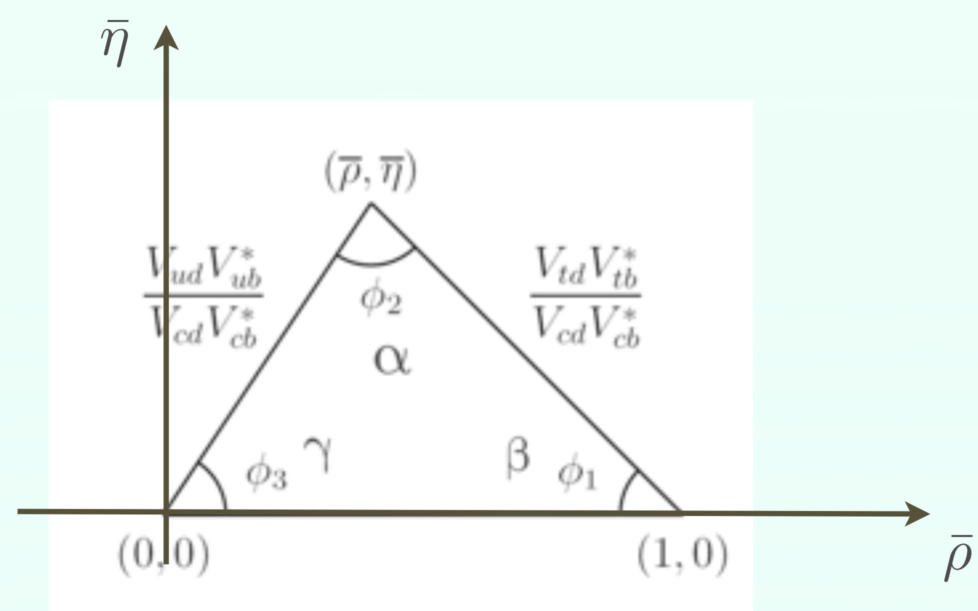
look more closely

ik	sum = 0	$\sim \epsilon^n$	shape (base normalized to 1)
12	$V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0$	$\epsilon + \epsilon + \epsilon^5 = 0$	 ϵ^4
23	$V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* = 0$	$\epsilon^4 + \epsilon^2 + \epsilon^2 = 0$	 ϵ^2
13	$V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0$	$\epsilon^3 + \epsilon^3 + \epsilon^3 = 0$	

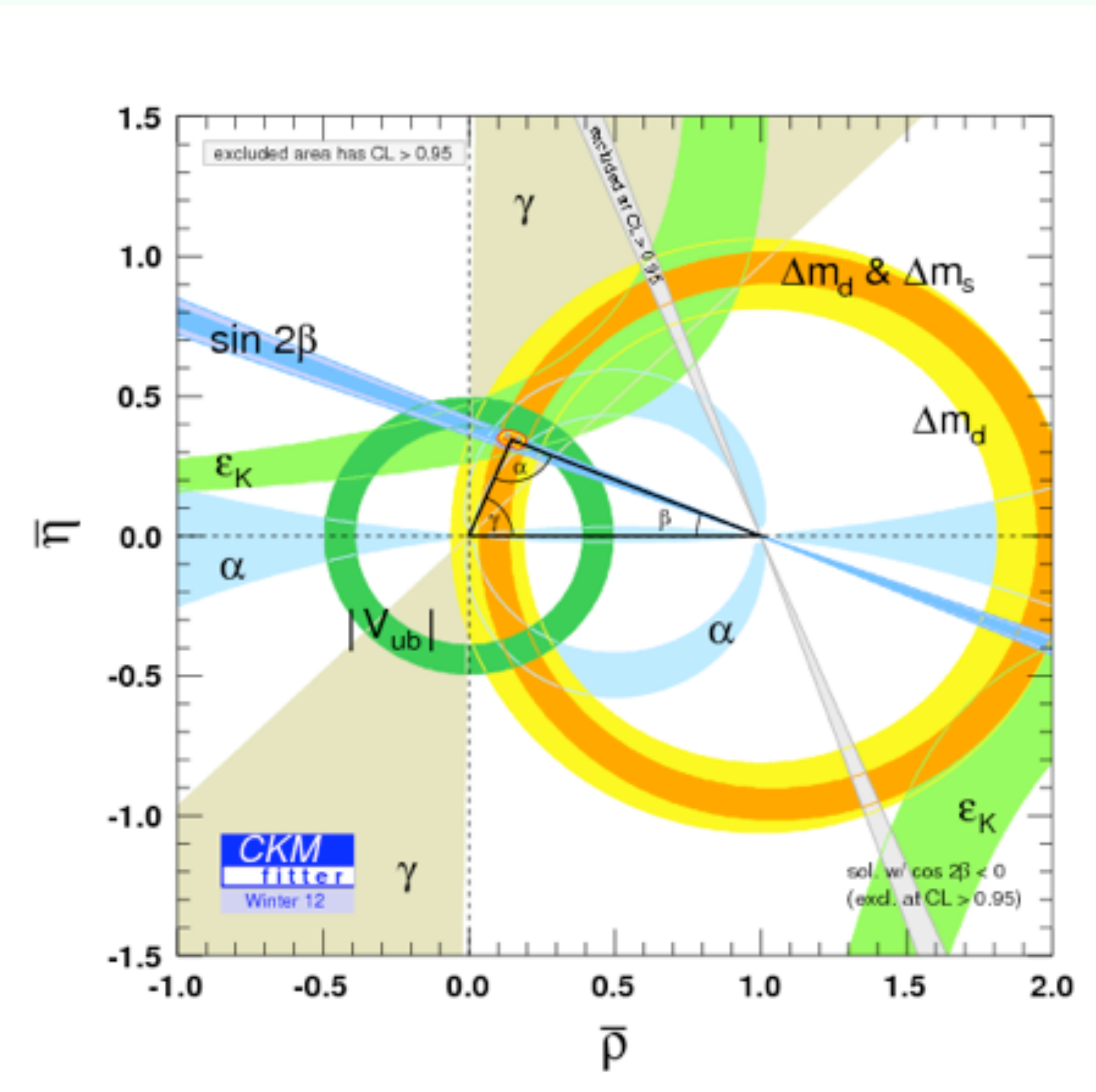
These are "Unitarity Triangles."
The most commonly discussed is the fat one in the 1-3 columns: $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$

Dividing my middle element in sum

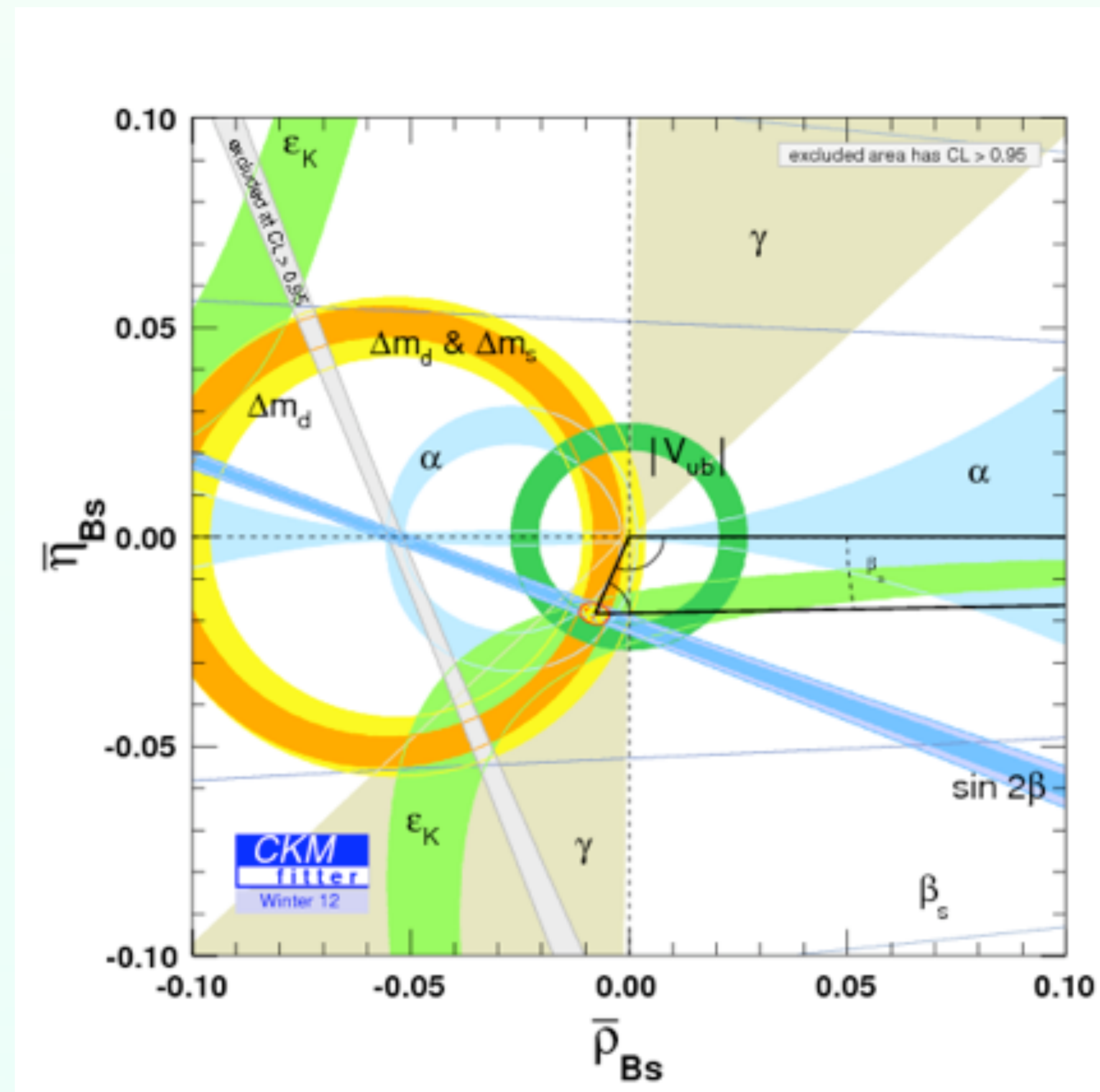
$$\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*} + 1 + \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*} = 0$$



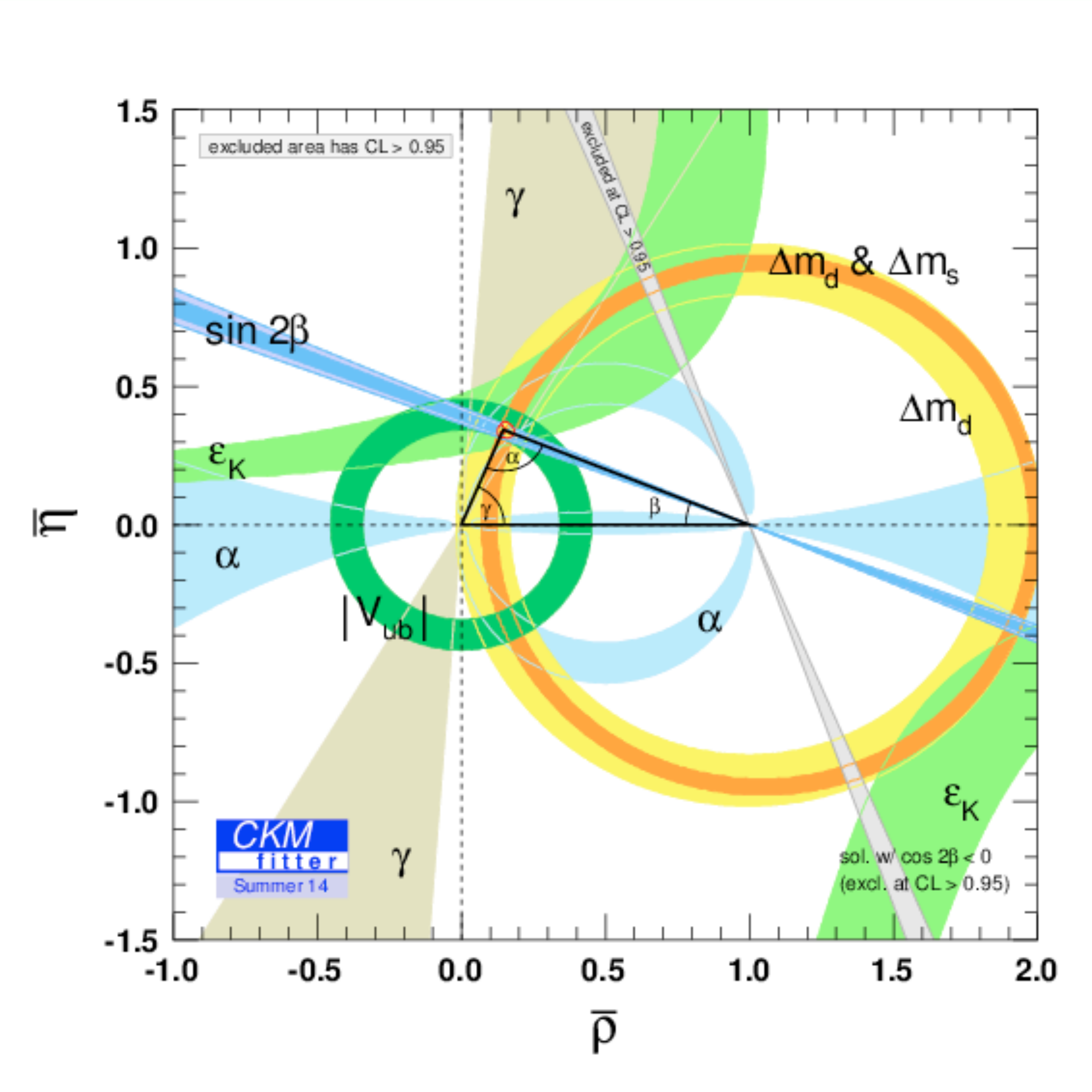
Fat


excluded area has CL > 0.95
excluded at CL > 0.95
γ
Δm_d & Δm_s
sin 2β
Δm_d
ε_K
α
β
|V_ub|
CKM fitter
Winter 12
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
1.5
1.0
0.5
0.0
-0.5
-1.0
-1.5
-1.0 -0.5 0.0 0.5 1.0 1.5 2.0
ρ̄
η̄

Skinny


excluded area has CL > 0.95
excluded at CL > 0.95
ε_K
γ
Δm_d & Δm_s
Δm_d
α
|V_ub|
sin 2β
β_s
CKM fitter
Winter 12
0.10
0.05
0.00
-0.05
-0.10
-0.10 -0.05 0.00 0.05 0.10
ρ̄_Bs
η̄_Bs

State of the art:



Exercise

i. Show that

$$\beta = -\arg\left(\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \qquad \alpha = -\arg\left(\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right) \qquad \gamma = -\arg\left(\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$

ii. Show that these are invariant under phase redefinitions of quark fields (e.g., under remaining arbitrariness). Hence they are physical.

iii. Define the Jarlskog invariant J through the last equality in this expression for the area of the unitarity triangle

$$\text{Area} = -\frac{1}{2}\text{Im}\left(\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) = -\frac{1}{2}\frac{1}{|V_{cd}V_{cb}^*|^2}\text{Im}\left(V_{ud}V_{ub}^*\,V_{cd}^*V_{cb}\right) \equiv -\frac{1}{2}\frac{1}{|V_{cd}V_{cb}^*|^2}J$$

Show that J is the common area of all the unitarity triangles (before we normalized the base to unity).

iv.The area of the normalized triangle is J divided by the square of the magnitude of the largest side

As we'll see, the area of the normalized triangle dictates the size of CP -asymmetries

4. Parametrization's of V

Standard:

$$V = ABC \qquad A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \quad B = \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \quad C = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

with $c_{ij} = \cos\theta_{ij}, s_{ij} = \sin\theta_{ij}$ and all angles in the first quadrant.

The texture of V then gives small angles.

Exercise: Get the order of $\varepsilon \sim 0.1$ of each of the angles θ_{ij}. Estimate δ.

Wolfenstein:

$$s_{12} = \lambda, \qquad s_{23} = A\lambda^2, \qquad s_{13}e^{i\delta} = A\lambda^3(\rho + i\eta) = \frac{a\lambda^3(\bar\rho + i\bar\eta)\sqrt{1 - A^2\lambda^4}}{\sqrt{1-\lambda^2}[1 - A^2\lambda^4(\bar\rho + i\bar\eta)]}$$

Exercise:

i. Show that $\bar\rho + i\bar\eta = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$, *hence it is field re-phasing invariant.*

ii. Expand in $\lambda \ll 1$ to show

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

Determination of CKM

Magnitudes

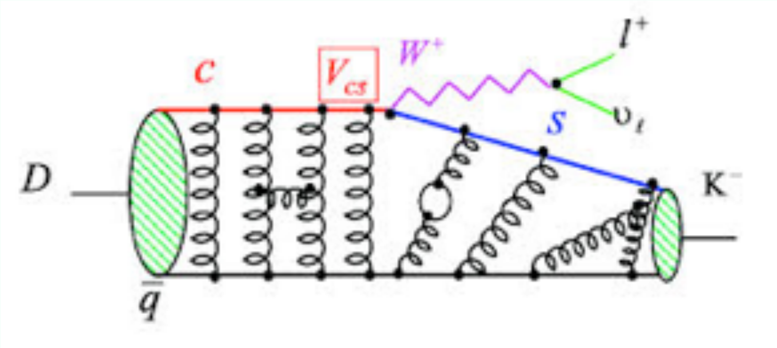
(i) $|V_{ud}|$ nuclear transitions,

(ii) $|V_{us}|, |V_{cd}|, |V_{cs}|, |V_{ub}|, |V_{cb}|$, semileptonic decays of mesons: $M \to M'\ell\nu$ *e.g.*, $K^+ \to \pi^0 e^+ \nu$

(iii) $|V_{tq}|$, $(q = d, s, b)$ through 1-loop, as above, with issues similar to meson decays;

or t decays, perturbative

Semileptonic decays: under much better theoretical control than purely hadronic decays.



1. Exclusive decays: $M \to M'\ell\nu$ (M, M' pseudo-scalar mesons)

$$\mathcal{A} = \langle M'(p')\ell\nu_L | \frac{g_2^2 V_{ij}}{M_W^2} \bar{u}_L^i \gamma^\mu d_L^j \bar{e}_L \gamma_\mu \nu | M(p) \rangle$$

$$V^\mu = \bar{u}\gamma^\mu d \qquad A^\mu = \bar{u}\gamma^\mu\gamma_5 d$$

Need: $\langle p' | V^\mu | p \rangle = f_+(q^2)(p+p')^\mu + f_-(q^2)(p-p')^\mu$ $\qquad q^\mu = (p-p')^\mu$

(No A^μ by P-sym of strong interactions)

Since $q^\mu(\bar{e}_L\gamma_\mu\nu_L) = 0$ for $m_e = 0$, no f_- in rate

Determination of CKM requires a priori knowledge of f_+

Symmetry plays a huge role

WARM-UP: EM form factor

Suppose V^μ is a conserved current $\quad \partial_\mu V^\mu = 0 \quad$ and take $M' = M$

e.g., $\quad \langle \pi(p') | J^\mu | \pi(p) \rangle = f_+(q^2)(p+p')^\mu + f_-(q^2)(p-p')^\mu$

Then $\quad \partial_\mu J^\mu = 0 \quad \Rightarrow \quad f_-(q^2) = 0 \quad$ and $\quad f_+(0) = Q$

Exercise: Prove these!

<u>$K \to \pi$</u> Use Gell-Mann $SU(3)$ The triplet is: $\begin{pmatrix} u \\ d \\ s \end{pmatrix}$

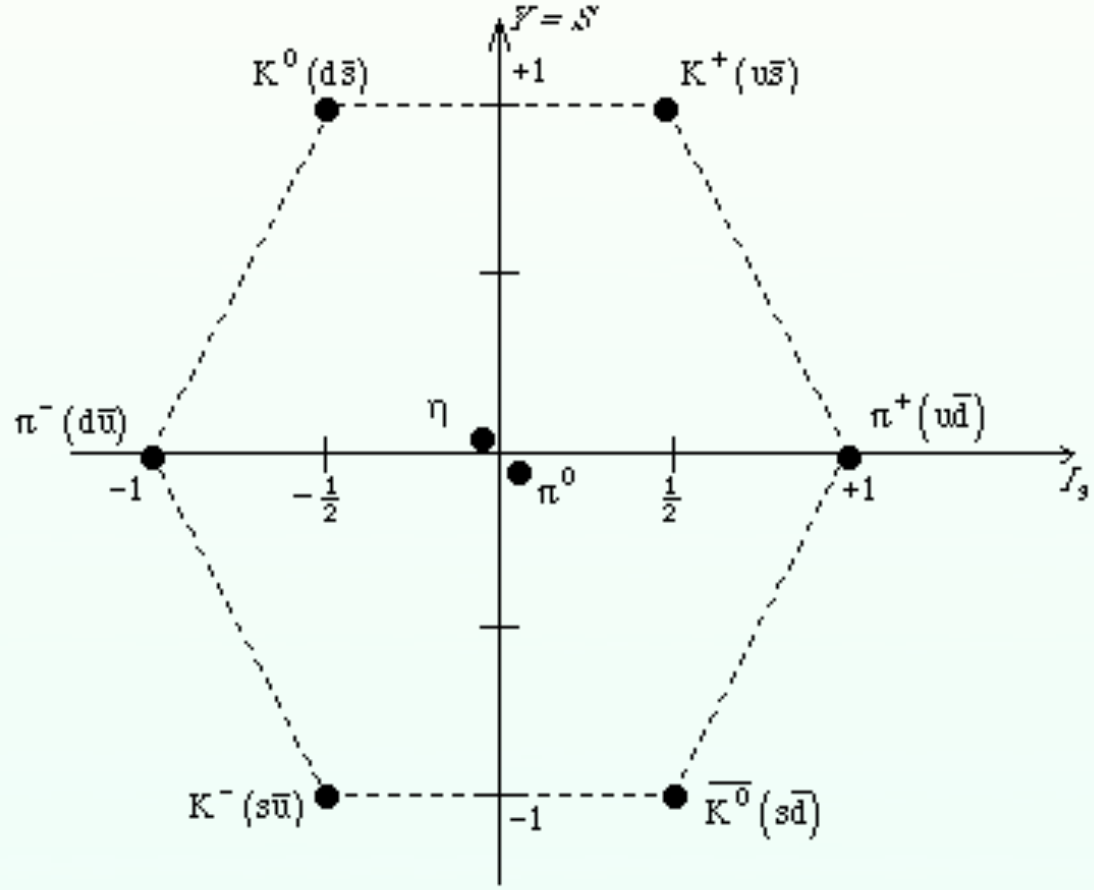
This is also called a *flavor* symmetry: don't get confused!

For processes with energies that cannot excite charm (or beauty or top or Ws....) and neglecting masses of *u,d,s*:

$$\mathcal{L} = \bar{u} i \not{D} u + \bar{d} i \not{D} d + \bar{s} i \not{D} s$$

where D is only QCD.

Masses and charges break the symmetry.



π, K, η Octet: degenerate in symmetry limit $f_-(q^2) = 0$ and $f_+(0) = 1$

Symmetry broken by quark masses $m_u \simeq m_d \neq m_s$ and by charge (smaller effect)
Away from symmetry limit, Ademolo-Gatto theorem

$$f_+(0) = 1 + \mathcal{O}(m_s) + \mathcal{O}(m_s^2) \qquad \frac{m_s^2}{\Lambda_\chi^2} \approx \left(\frac{0.1\text{GeV}}{1\ \text{GeV}}\right)^2 \sim 1\%$$

Combining data of neutral and charged semi-leptonic K decays: $|V_{us}| f_+(0) = 0.2163 \pm 0.0005$

Form factor from lattice QCD: $f_+(0) = 0.960 \pm 0.005$

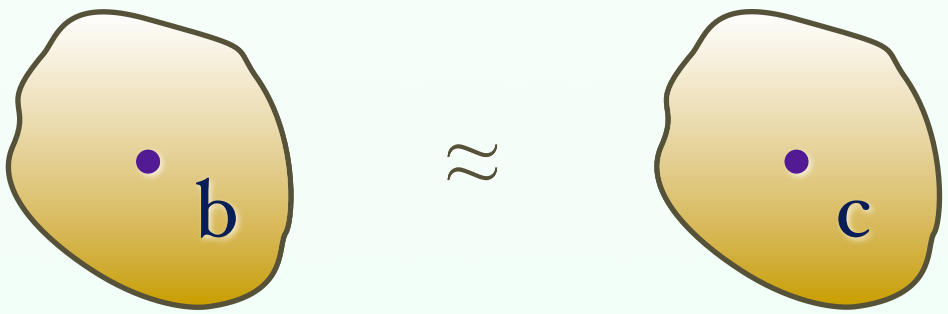
PDG: $|V_{us}| = 0.2253 \pm 0.0008$

$\bar{B} \to D$ $(b \to c\ell\nu)$

$\bar{B}^0(\bar{d}b), B^-(\bar{u}b), \bar{B}_s(\bar{s}b), D^0(\bar{u}c), D^+(\bar{d}c), D_s(\bar{s}c)$

Exclusive decays.

Heavy Quark (HQ) Symmetry (souped up with HQET)



The "brown muck" is in both cases bound by an infinitely heavy color triplet (static) source of color.

Heuristic: at zero recoil (max q^2 !!) state does not change, just like in $M' = M$ case

Exercise: Check that zero recoil is $q^2 = q^2_{max}$

For $m \to \infty$ use instead $v^\mu = \frac{p^\mu}{m}$ $|v\rangle = \frac{1}{\sqrt{m}}|p\rangle$

HQS: $\langle D(v')|V^\mu|B(v)\rangle = \xi(v \cdot v')(v + v')^\mu$ with $\xi(1) = 1$

Isgur-Wise function

($q^2 = q^2_{max}$ *is* $v'.v = 1$)

D is spin-0, *D** is spin 1, correspond to singlet and triplet, and are related by HQ-symmetry.

If this seemed to fast, it was. I left many details out.

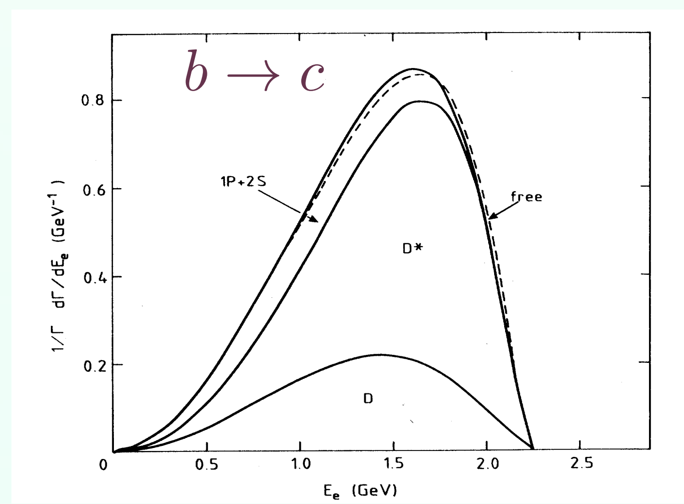
2. (semi-)Inclusive decays $\bar{B} \to X\ell\nu$ (as example, but also D decays)

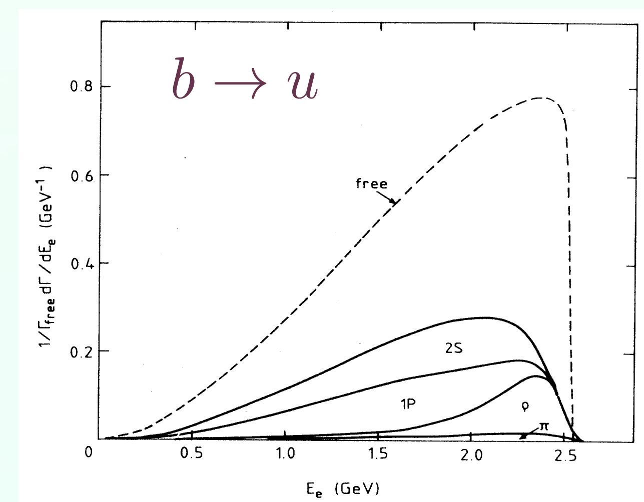
quark-hadron duality $\Gamma(\bar{B} \to X_c\ell\nu) = \Gamma(b \to c\ell\nu)$ ☞ 1% determination of $|V_{cb}|$

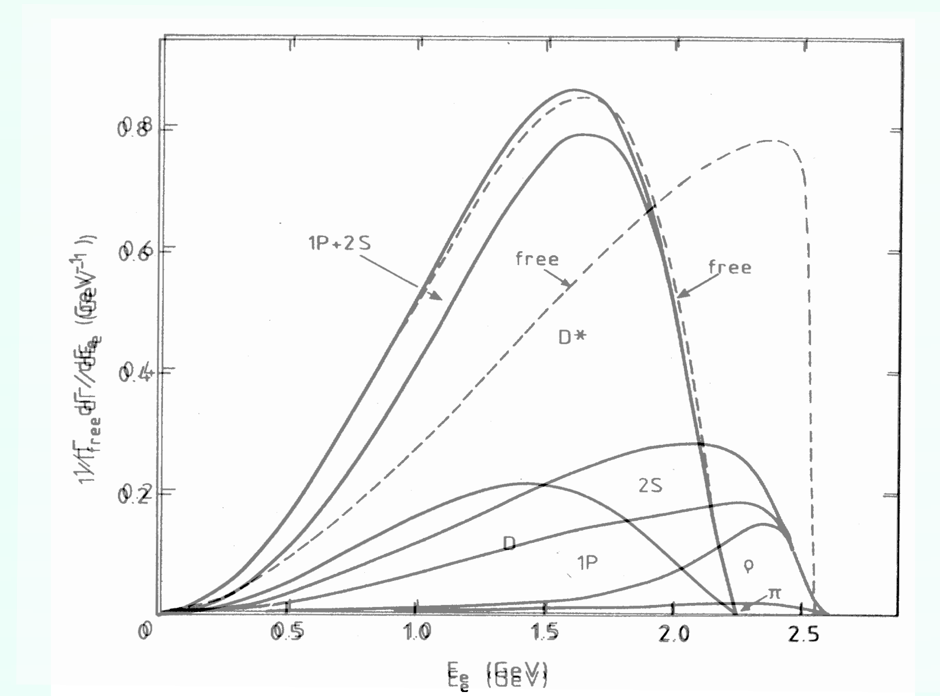
QCD+HQET+OPE provided sufficiently integrated $\Gamma(\bar{B} \to X_u\ell\nu) = \Gamma(b \to u\ell\nu)$

$b \to u\ell\nu$ headaches

- No HQS for
  $B \to \pi\ell\nu$
- Hides under charm for
  $b \to u\ell\nu$
  (except at endpoint)







PDG:

$$V_{\rm CKM} = \begin{pmatrix} 0.97427 \pm 0.00015 & 0.22534 \pm 0.00065 & 0.00351^{+0.00015}_{-0.00014} \\ 0.22520 \pm 0.00065 & 0.97344 \pm 0.00016 & 0.0412^{+0.0011}_{-0.0005} \\ 0.00867^{+0.00029}_{-0.00031} & 0.0404^{+0.0011}_{-0.0005} & 0.999146^{+0.000021}_{-0.000046} \end{pmatrix}$$

(magnitudes)

For phases we need more (soon to come). But here are the PDG results:

Jarlskog

$$J = (2.96^{+0.20}_{-0.16}) \times 10^{-5}$$

Wolfenstein

$$\lambda = 0.22535 \pm 0.00065\,, \qquad A = 0.811^{+0.022}_{-0.012}\,,$$

$$\bar\rho = 0.131^{+0.026}_{-0.013}\,, \qquad \bar\eta = 0.345^{+0.013}_{-0.014}\,.$$

FCNC

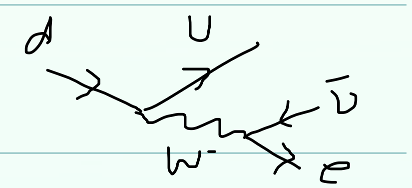
FCNC= Flavor Changing Neutral Currents

but is used more generally to mean FCN-transitions

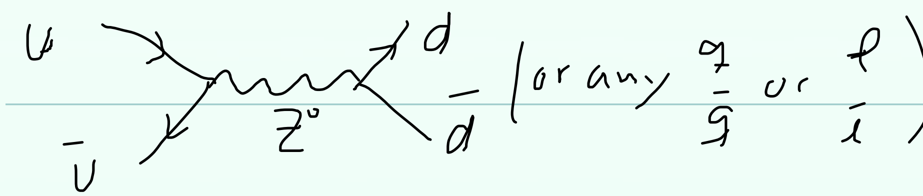
FC-transitions in the SM:

1. Tree level: Only $W^{\pm}$

e.g., $n \to pe\bar{\nu}$

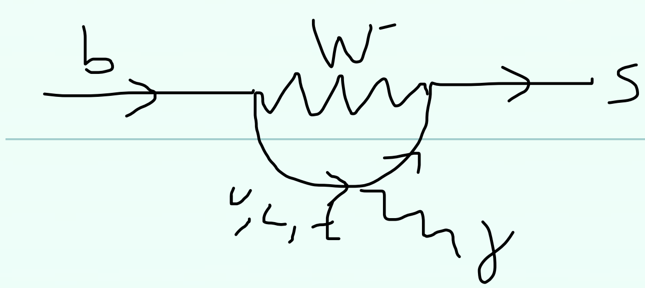


But Z^0 and h interactions are diagonal in flavor:



2. 1-loop: Can we have FCNC's? Say $b \to s\gamma$?

YES!!



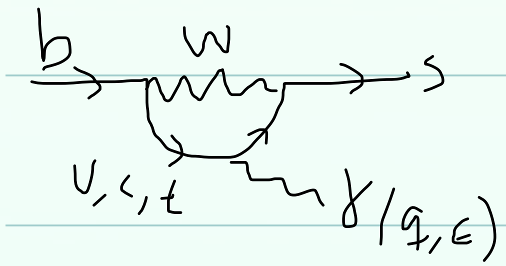
➠ FCNCs are suppressed in SM relative to tree level by $\sim \frac{g_2^2}{16\pi^2} = \frac{\alpha}{4\pi c_w^2}$

GIM

GIM-mechanism: more suppression of FCNC in SM!!
(GIM=Glashow-Iliopolus-Maiani)

(i) “Old:” Lets imagine a world with $m_u < m_c < m_t \ll M_W$

Without explicit computation of integrals we can see that



$$= e\, q_\mu\, \epsilon_\nu\, \bar{u}(p_s)\, \sigma^{\mu\nu} \left(\frac{1+\gamma_5}{2}\right) u(p_b)\, \frac{m_b}{M_W^2}\, \frac{g_2^2}{16\pi^2} \times I$$

(Dirac spinors)

where $I = \sum_{u,c,t} V_{ib} V_{is}^* F(\frac{m_i^2}{M_W^2})$

Expand in Taylor series $F(x) = F(0) + xF'(0) + \cdots$ and use $\sum_{u,c,t} V_{ib} V_{is}^* = 0$

$$I = \sum_{u,c,t} V_{ib} V_{is}^* F(0) + \sum_{u,c,t} V_{ib} V_{is}^* \frac{m_i^2}{M_W^2} F'(0) + \cdots$$

Moreover, since $\sum_{u,c} V_{ib} V_{is}^* = -V_{tb} V_{ts}^*$ then $I = F'(0) \sum_{u,c} V_{ib} V_{is}^* \frac{m_i^2 - m_t^2}{M_W^2} + \cdots$

➠ FCNCs suppressed, in addition to 1-loop, by $\sim V_{ub} V_{us}^* \frac{m_u^2 - m_t^2}{M_W^2} + V_{cb} V_{cs}^* \frac{m_c^2 - m_t^2}{M_W^2} \sim \epsilon^4 \frac{m_t^2}{M_W^2} + \epsilon^2 \frac{m_t^2}{M_W^2}$

That is, both by ϵ^2 and $\frac{m_t^2}{M_W^2}$

(ii) "Modern" GIM:

Of course $m_t \ll M_W$ is not a good approximation, but the suppression by ϵ^2 is still there

$$I = \sum_{u,c,t} V_{ib} V_{is}^* F(\frac{m_i^2}{M_W^2}) = -\sum_{u,c} V_{ib} V_{is}^* \left(F(\frac{m_t^2}{M_W^2}) - \frac{m_i^2}{M_W^2} \right) \sim \epsilon^2 \left(F(\frac{m_t^2}{M_W^2}) - \frac{m_c^2}{M_W^2} \right)$$

It turns out that *F(x)* is an increasing function, with *F(1) ~ O(1)*, so m_c^2/M_W^2 can be neglected

☛ virtual *t*-quark exchange dominates this amplitude.

Exercise: Show that for $s \to d\gamma$ *it is no longer true that t-quark exchange dominates the amplitude, in fact, that c and t quark exchange give numerically (roughly) the same amplitude.*

So why do we bother to explain "old" GIM? In models of new physics (NP) you will encounter examples of GIM-like cancellation.
A good tool in your toolbox!

Flavor Symmetry and New Physics. A First Look and Minimal Flavor Violation

Bounds on NP, by (rough) example

Extend SM by an interaction, *e.g.*,

$$\Delta\mathcal{L} = \frac{g}{\Lambda^2} B_{\mu\nu} H \bar{b}_R \sigma^{\mu\nu} \begin{pmatrix} c_L \\ s_L \end{pmatrix} \to \frac{ev}{\sqrt{2}\Lambda^2} F_{\mu\nu} \bar{b}_R \sigma^{\mu\nu} s_L$$

So, roughly, $$\frac{\mathcal{A}_{\rm NP}}{\mathcal{A}_{\rm SM}} \sim \frac{\frac{v}{\sqrt{2}\Lambda^2}}{V_{tb}V_{ts}^* \frac{\alpha}{4\pi s_w^2} \frac{m_b}{M_W^2}}$$

Since the SM prediction agrees well with experiment, requiring $\frac{\mathcal{A}_{\rm NP}}{\mathcal{A}_{\rm SM}} \lesssim 10\%$

$$\Lambda^2 \gtrsim \frac{vM_W^2}{\sqrt{2}m_b} \frac{s_w^2}{|V_{tb}V_{ts}^*| \frac{\alpha}{4\pi}} \frac{1}{0.1} \qquad \to \qquad \Lambda > 70 \text{ TeV}$$

Minimal Flavor Violation

- Standard Model Fields
- Extend by adding dim>4 (local, Poincare and gauge invariant, hermitian) operators

$$\Delta\mathcal{L} = \sum_i C_i O_i$$

- This is the Effective Field Theory (EFT) setup.
- Additionally: require invariance under $G_F = U(3)^3$ - including spurions λ_U and λ_D This is the "principle of **Minimal Flavor Violation** (MFV)."
- Take, for example:

$$O_1 = G^a_{\mu\nu}\tilde{H}\bar{u}_R T^a\sigma^{\mu\nu}\lambda_U q_L \qquad \text{and} \qquad O_2 = \bar{q}_L\gamma^\mu\lambda_U^\dagger\lambda_U q_L\,\bar{d}_R\gamma_\mu\lambda_D\lambda_D^\dagger d_R$$

Go to mass eigenstate basis, study flavor changing interactions:

$$O_1 \to G^a_{\mu\nu}\tilde{H}\bar{u}_R T^a\sigma^{\mu\nu}V^\dagger_{u_R}\lambda_U\begin{pmatrix}V_{u_L}u_L\\ V_{d_L}d_L\end{pmatrix}$$

$$= G^a_{\mu\nu}\tilde{H}\bar{u}_R T^a\sigma^{\mu\nu}V^\dagger_{u_R}\lambda_U V_{u_L}\begin{pmatrix}u_L\\ V^\dagger_{u_L}V_{d_L}d_L\end{pmatrix}$$

$$= G^a_{\mu\nu}\tilde{H}\bar{u}_R T^a\sigma^{\mu\nu}\lambda'_U\begin{pmatrix}u_L\\ V d_L\end{pmatrix}$$

The neutral interaction (*u* to *u*) does not change flavor ("flavor diagonal"); the charged interaction (*d* to *u*) changes flavor as determined by CKM matrix and mass matrix through $\lambda'_U V$

Similarly for $O_2 = \bar{q}_L \gamma^\mu \lambda_U^\dagger \lambda_U q_L \, \bar{d}_R \gamma_\mu \lambda_D \lambda_D^\dagger d_R$

$$O_2 \to \bar{q}'_L \gamma^\mu (\lambda'_U)^2 q'_L \, \bar{d}_R \gamma_\mu (\lambda'_D)^2 d_R \qquad \text{where} \qquad q'_L = \begin{pmatrix} u_L \\ V d_L \end{pmatrix}$$

(and we've used that the mass matrices are now real and diagonal)

Note there is now a flavor non-diagonal neutral transition, involving the current

$$\bar{d}_L \gamma^\mu [V^\dagger (\lambda'_D)^2 V] d_L$$

Exercise: Show that it is generally true that the CKM matrix determines the flavor changing interactions. More specifically, that flavor change is determined by

$$\lambda'_U V \text{ or } V \lambda'_D$$

Extensions of the SM in which the only breaking of $U(3)^3$ is by λ_U and λ_D automatically satisfy MFV.

They are least constrained by flavor changing and CPV observables.

Bounds on NP, by (rough) example (again)

(i) No MFV: extend SM by, eg, $\Delta\mathcal{L} = \frac{g}{\Lambda^2} B_{\mu\nu} H \bar{b}_R \sigma^{\mu\nu} \begin{pmatrix} c_L \\ s_L \end{pmatrix} \to \frac{ev}{\sqrt{2}\Lambda^2} F_{\mu\nu} \bar{b}_R \sigma^{\mu\nu} s_L$

So, roughly, $\frac{\mathcal{A}_{\rm NP}}{\mathcal{A}_{\rm SM}} \sim \frac{\frac{v}{\sqrt{2}\Lambda^2}}{V_{tb}V_{ts}^* \frac{\alpha}{4\pi s_w^2} \frac{m_b}{M_W^2}}$

and, since the SM prediction agrees well with experiment, requiring $\frac{\mathcal{A}_{\rm NP}}{\mathcal{A}_{\rm SM}} \lesssim 10\%$

$$\Lambda^2 \gtrsim \frac{v M_W^2}{\sqrt{2} m_b} \frac{s_w^2}{|V_{tb}V_{ts}^*| \frac{\alpha}{4\pi}} \frac{1}{0.1} \qquad \to \qquad \Lambda > 70 \text{ TeV}$$

(ii) With MFV: $\Delta\mathcal{L} = \frac{g}{\Lambda^2} B_{\mu\nu} H \bar{d}_R \lambda_D \sigma^{\mu\nu} q_L \to 0$ flavor diagonal!

But we can have $\Delta\mathcal{L} = \frac{g}{\Lambda^2} B_{\mu\nu} H \bar{d}_R \lambda_D (\lambda_U^\dagger \lambda_U) \sigma^{\mu\nu} q_L \to \frac{ev}{\sqrt{2}\Lambda^2} \lambda_b' (\lambda_t')^2 V_{tb} V_{ts}^* F_{\mu\nu} \bar{b}_R \sigma^{\mu\nu} s_L$

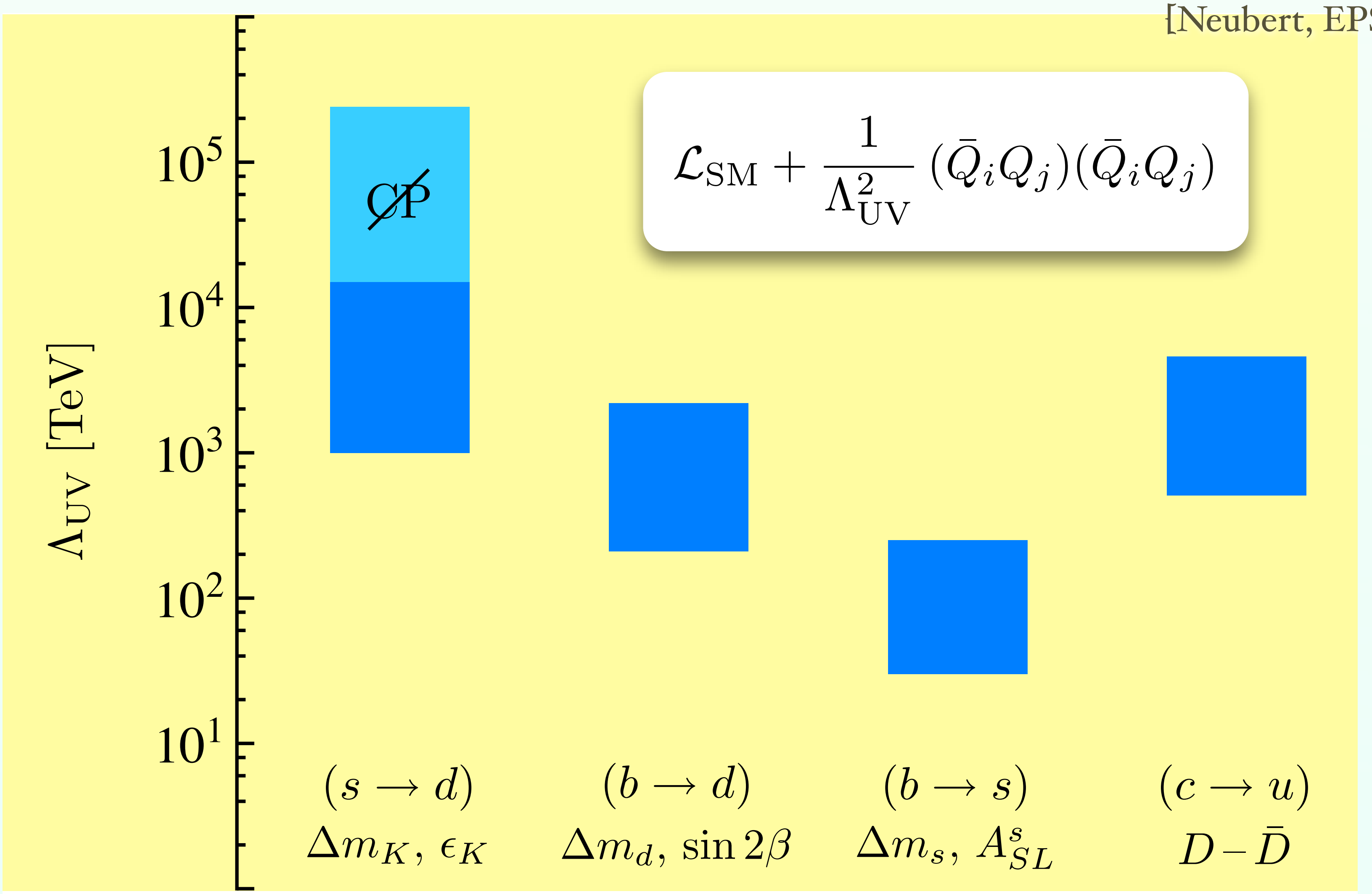
MFV is “protected” because it incorporates modern GIM.

Now $\frac{\mathcal{A}_{\rm NP}}{\mathcal{A}_{\rm SM}} \sim \frac{\frac{(\lambda_t')^2}{\sqrt{2}\Lambda^2}}{\frac{\alpha}{4\pi s_w^2} \frac{1}{M_W^2}} \to \Lambda^2 \gtrsim \frac{1}{\sqrt{2}} M_W^2 (\lambda_t')^2 s_w^2 \frac{4\pi}{\alpha} \frac{1}{0.1} \to \Lambda > 4 \text{ TeV}$

Flavor Physics: an important constraint on all new BSM models

Generic bounds without a flavor symmetry

[Neubert, EPS2011]



Exercise: from these determine bounds with MFV assumption

Note: CPV in K mixing gives strongest constraints: we should (will) spend time on it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THIS WASN T PREDICTED IN OUR MODEL — WHAT SHOULD WE DO?
DON T SAY ANYTHING. MAYBE NO ONE WILL NOTICE.
LARGE HADRON COLLIDER
CERN
CERN

Examples:

1. SUSY-SM

In the absence of SUSY-breaking this is MFV

$$\mathcal{L} = \int d^4\theta\, [\bar{Q} e^V Q + \bar{U} e^V U + \bar{D} e^V D] + \int d^2\theta W^\alpha W_\alpha + \text{other kin terms} - (\int d^2\theta W + \text{h.c.})$$

with superpotential:

$$W = H_1 U \lambda_U Q + H_2 D \lambda_D Q + \text{non-quark terms}$$

Here the chiral superfields are:

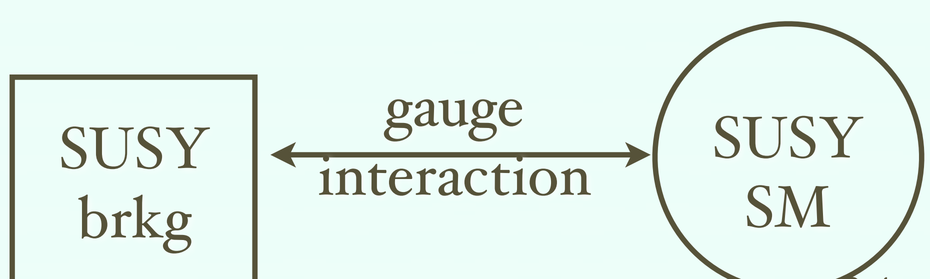
$$Q \sim (3,2)_{\frac{1}{6}} \qquad H_1 \sim (1,2)_{\frac{1}{2}}$$
$$U \sim (\bar{3},1)_{-\frac{2}{3}} \qquad H_2 \sim (1,2)_{-\frac{1}{2}}$$
$$D \sim (\bar{3},2)_{\frac{1}{3}}$$

Add soft SUSY-breaking: $\Delta\mathcal{L} = \phi_q^* \mathcal{M}_q^2 \phi_q + \phi_u^* \mathcal{M}_u^2 \phi_u + \phi_d^* \mathcal{M}_d^2 \phi_q + \phi_{h_1} \phi_u g_u \phi_q + \phi_{h_2} \phi_d g_d \phi_q + \text{h.c.}$

For generic $\mathcal{M}^2_{q,u,d}, g_{u,d}$ new flavor changing interactions are present and large (they can be suppressed by making the squarks heavy).

Not so if, e.g., $\mathcal{M}^2_{q,u,d} \propto \mathbf{1}, \quad g_{u,d} \propto \lambda_{u,d}$ (in accord with MFV)

This is the motivation for gauge mediated SUSY breaking



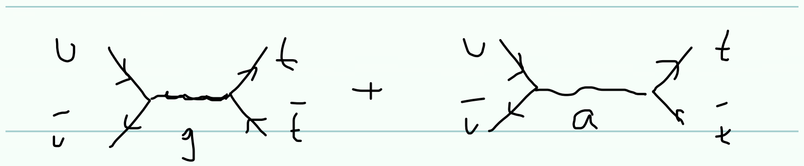
The gauge interactions are flavor blind

[Severe problem in gravity mediated SUSY-breaking]

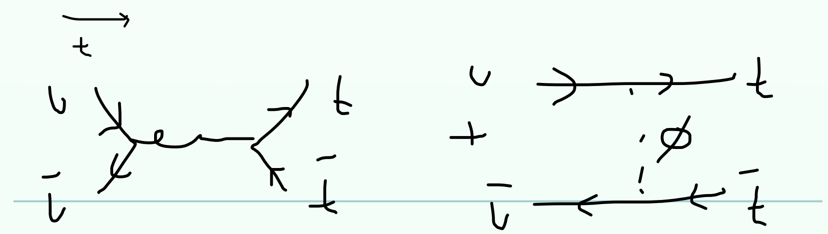
2. "MFV Fields"

Recently observed anomalous t-FB asymmetry. Possibly explained by

(i) *s*-channel, e.g., axigluon



(ii) *t*-channel, e.g., scalar



Won't explain why axigluon breaks U(3)^3 (roughly needs opposite sign couplings to *u* and *t*).
Concentrate on *t*-channel models: clearly $\phi t u$ coupling (flavor off-diagonal) breaks U(3)^3 .

Unless one fine-tunes, there are also large *uc* and *ct* couplings, and if coupling is to *L*-quarks also *ds*, *db* and *sb* couplings.

Solution: construct U(3)^3 symmetric model by introducing multiplet of scalars transforming under U(3)^3. For example, one can have

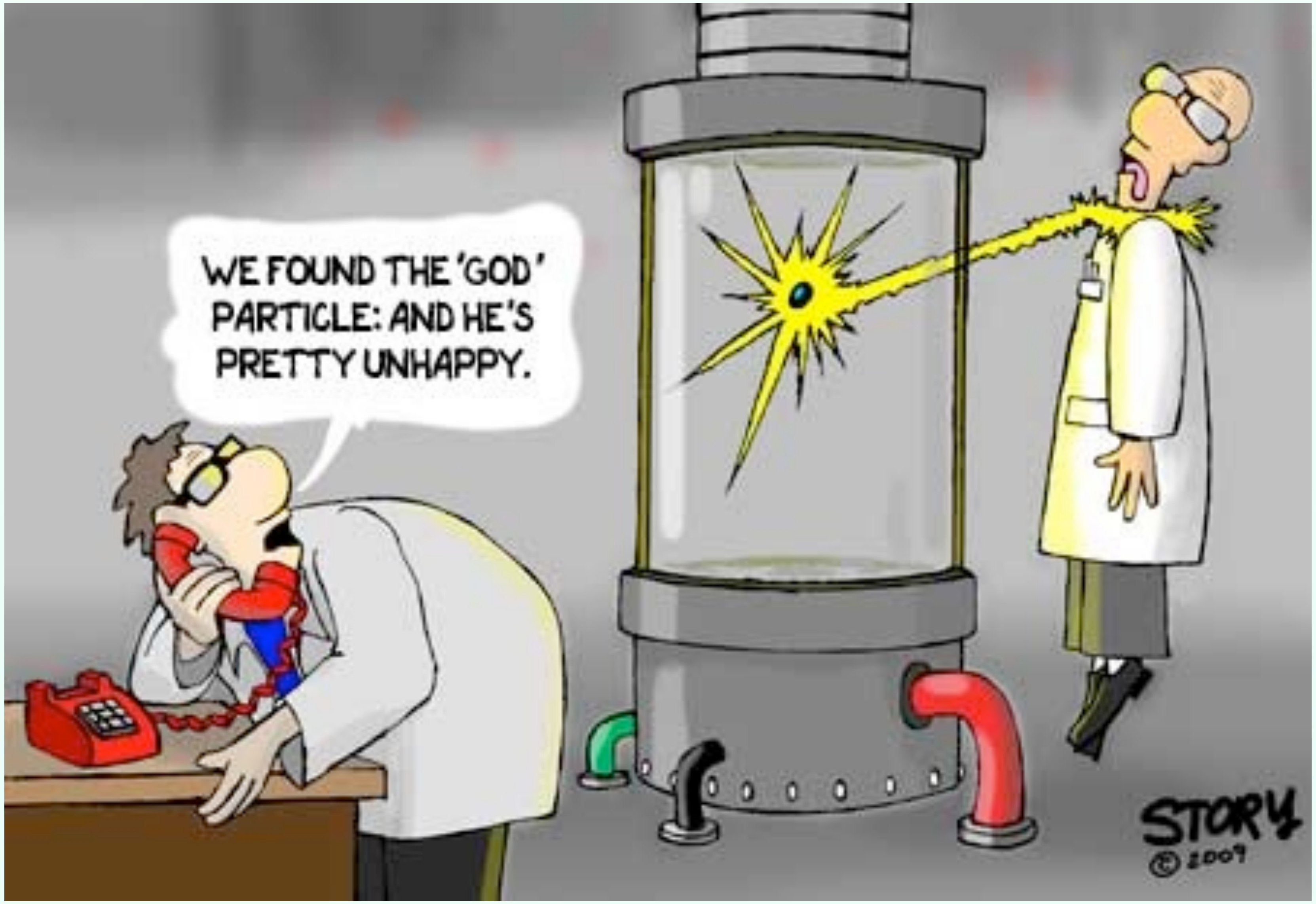
$$\bar{q}_L \phi u_R \quad \text{with} \quad \phi \to U_q \phi U_U^\dagger \qquad (\text{and a } 2_{-\frac{1}{2}} \text{ under } SU(2)_w \times U(1)_Y)$$

This actually works!

Exercise: classify all possible dim-4 interactions $\sim \phi\bar{\psi}\psi'$ *and corresponding transformation laws for the scalar field under* U(3)^3 *and the SM-gauge-group (i) to order* $(\lambda_{U,D})^0$ *and (ii) up to order* $(\lambda_{U,D})^1$

End Lecture 1

Flavor and Higgs: Standard and Nonstandard



Higgs Flavor couplings in SM

$$\mathcal{L}_{H\psi\psi} = \tilde{H}\,\bar{q}_L \lambda^U u_R + H\,\bar{q}_L \lambda^D d_R + H\,\bar{l}_L \lambda^E e_R \quad \left(+ H\,\bar{l}_L \lambda^N \nu_R + \bar{\nu}_L^c M_\nu \nu_R\right) + h.c.$$

Unitary gauge (Pich's lecture) $\quad H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}$

$$\mathcal{L} \to \frac{1}{\sqrt{2}}(v+h)\left[\bar{u}_L \lambda^U u_R + \bar{d}_L \lambda^D d_R + \bar{e}_L \lambda^E e_R \;\cdots\right]$$

$$= \left(1+\frac{h}{v}\right)\left[\bar{u}_L M^U u_R + \bar{d}_L M^D d_R \;\cdots\right]$$

As before $M^U \to V_{U_L}^\dagger M^U V_{U_R}$ = diagonal

$\Rightarrow$ In SM higgs couplings are flavor-diagonal

Suppose we add

Blankenburga, Ellis & Isidori, 1202. 5704
Harnik, Kopp & Zupan 1209.1397

$$\mathcal{L}_{\text{eff}} = \sum_{i,j=d,s,b\ (i\neq j)} c_{ij}\ \bar{d}^i_L d^j_R h + \sum_{i,j=u,c,t\ (i\neq j)} c_{ij}\ \bar{u}^i_L u^j_R h + \sum_{i,j=e,\mu,\tau\ (i\neq j)} c_{ij}\ \bar{\ell}^i_L \ell^j_R h + \text{H.c.}$$

Should we rush to look for, say, $h \to \bar{s} b$ or $t \to hc$?

Operator	Eff. couplings	95% C.L. Bound		Observables
		$\lvert c_{\text{eff}}\rvert$	$\lvert \text{Im}(c_{\text{eff}})\rvert$	
$(\bar{s}_R d_L)(\bar{s}_L d_R)$	$c_{sd}\ c^*_{ds}$	1.1×10^{-10}	4.1×10^{-13}	$\Delta m_K;\ \epsilon_K$
$(\bar{s}_R d_L)^2,\ (\bar{s}_L d_R)^2$	$c^2_{ds},\ \ c^2_{sd}$	2.2×10^{-10}	0.8×10^{-12}	
$(\bar{c}_R u_L)(\bar{c}_L u_R)$	$c_{cu}\ c^*_{uc}$	0.9×10^{-9}	1.7×10^{-10}	$\Delta m_D;\ \lvert q/p\rvert, \phi_D$
$(\bar{c}_R u_L)^2,\ (\bar{c}_L u_R)^2$	$c^2_{uc},\ \ c^2_{cu}$	1.4×10^{-9}	2.5×10^{-10}	
$(\bar{b}_R d_L)(\bar{b}_L d_R)$	$c_{bd}\ c^*_{db}$	0.9×10^{-8}	2.7×10^{-9}	$\Delta m_{B_d};\ S_{B_d\to\psi K}$
$(\bar{b}_R d_L)^2,\ (\bar{b}_L d_R)^2$	$c^2_{db},\ \ c^2_{bd}$	1.0×10^{-8}	3.0×10^{-9}	
$(\bar{b}_R s_L)(\bar{b}_L s_R)$	$c_{bs}\ c^*_{sb}$	2.0×10^{-7}	2.0×10^{-7}	Δm_{B_s}
$(\bar{b}_R s_L)^2,\ (\bar{b}_L s_R)^2$	$c^2_{sb},\ \ c^2_{bs}$	2.2×10^{-7}	2.2×10^{-7}	

Table 1: Bounds on combinations of the flavour-changing h couplings defined in (1) obtained from $\Delta F = 2$ processes [12], assuming that $m_h = 125$ GeV.

Here are some more:

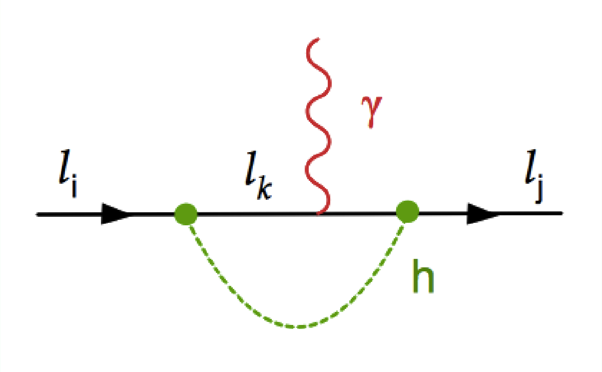
Eff. couplings	Bound	Constraint
$\|c_{sb}\|^2, \ \|c_{bs}\|^2$	2.9×10^{-5} [*]	$\mathcal{B}(B_s \to \mu^+\mu^-) < 1.4 \times 10^{-8}$
$\|c_{db}\|^2, \ \|c_{bd}\|^2$	1.3×10^{-5} [*]	$\mathcal{B}(B_d \to \mu^+\mu^-) < 3.2 \times 10^{-9}$

I want to spend some time in these lectures explaining the physics of these processes.

Your aim should be to be able to reproduce the entries in these tables, at least within an order of magnitude by an educated estimate.

My aim is to enable to do that.

Although I will speak little about leptons, the story is analogous:



Eff. couplings	Bound	Constraint
$\|c_{e\tau}c_{\tau e}\| \quad (\|c_{e\mu}c_{\mu e}\|)$	$1.1\times10^{-2} \quad (1.8\times10^{-1})$	$\|\delta m_e\| < m_e$
$\|\mathrm{Re}(c_{e\tau}c_{\tau e})\| \quad (\|\mathrm{Re}(c_{e\mu}c_{\mu e})\|)$	$0.6\times10^{-3} \quad (0.6\times10^{-2})$	$\|\delta a_e\| < 6\times10^{-12}$
$\|\mathrm{Im}(c_{e\tau}c_{\tau e})\| \quad (\|\mathrm{Im}(c_{e\mu}c_{\mu e})\|)$	$0.8\times10^{-8} \quad (0.8\times10^{-7})$	$\|d_e\| < 1.6\times10^{-27}$ ecm
$\|c_{\mu\tau}c_{\tau\mu}\|$	2	$\|\delta m_\mu\| < m_\mu$
$\|\mathrm{Re}(c_{\mu\tau}c_{\tau\mu})\|$	2×10^{-3}	$\|\delta a_\mu\| < 4\times10^{-9}$
$\|\mathrm{Im}(c_{\mu\tau}c_{\tau\mu})\|$	0.6	$\|d_\mu\| < 1.2\times10^{-19}$ ecm
$\|c_{e\tau}c_{\tau\mu}\|,\ \|c_{\tau e}c_{\mu\tau}\|$	1.7×10^{-7}	$\mathcal{B}(\mu\to e\gamma) < 2.4\times10^{-12}$
$\|c_{\mu\tau}\|^2,\ \|c_{\tau\mu}\|^2$	0.9×10^{-2} [*]	$\mathcal{B}(\tau\to\mu\gamma) < 4.4\times10^{-8}$
$\|c_{e\tau}\|^2,\ \|c_{\tau e}\|^2$	0.6×10^{-2} [*]	$\mathcal{B}(\tau\to e\gamma) < 3.3\times10^{-8}$

Operator	Eff. couplings	Bound	Constraint
$(\bar\mu_R e_L)(\bar q_L q_R),\ (\bar\mu_L e_R)(\bar q_L q_R)$	$\|c_{\mu e}\|^2,\ \|c_{e\mu}\|^2$	3.0×10^{-8} [*]	$\mathcal{B}_{\mu\to e}(\mathrm{Ti}) < 4.3\times10^{-12}$
$(\bar\tau_R \mu_L)(\bar\mu_L\mu_R),\ (\bar\tau_L \mu_R)(\bar\mu_L\mu_R)$	$\|c_{\tau\mu}\|^2,\ \|c_{\mu\tau}\|^2$	2.0×10^{-1} [*]	$\Gamma(\tau\to\mu\bar\mu\mu) < 2.1\times10^{-8}$
$(\bar\tau_R e_L)(\bar\mu_L\mu_R),\ (\bar\tau_L e_R)(\bar\mu_L\mu_R)$	$\|c_{\tau e}\|^2,\ \|c_{e\tau}\|^2$	4.8×10^{-1} [*]	$\Gamma(\tau\to e\bar\mu\mu) < 2.7\times10^{-8}$
$(\bar\tau_R e_L)(\bar\mu_L e_R),\ (\bar\tau_L e_R)(\bar\mu_L e_R)$	$\|c_{\mu e}c^*_{e\tau}\|,\ \|c_{\mu e}c_{\tau e}\|$	0.9×10^{-4}	$\Gamma(\tau\to\bar\mu ee) < 1.5\times10^{-8}$
$(\bar\tau_R e_L)(\bar\mu_R e_L),\ (\bar\tau_L e_R)(\bar\mu_R e_L)$	$\|c^*_{e\mu}c^*_{e\tau}\|,\ \|c^*_{e\mu}c_{\tau e}\|$		
$(\bar\tau_R \mu_L)(\bar e_L\mu_R),\ (\bar\tau_L \mu_R)(\bar e_L\mu_R)$	$\|c_{e\mu}c^*_{\mu\tau}\|,\ \|c_{e\mu}c_{\tau\mu}\|$	1.0×10^{-4}	$\Gamma(\tau\to\bar e\mu\mu) < 1.7\times10^{-8}$
$(\bar\tau_R \mu_L)(\bar e_R\mu_L),\ (\bar\tau_L \mu_R)(\bar e_R\mu_L)$	$\|c^*_{\mu e}c^*_{\mu\tau}\|,\ \|c^*_{\mu e}c_{\tau\mu}\|$		

Digression 1: Higgs as Dilaton

SM LAGRANGIAN ONLY DIMENSIONAL SCALE : $-\mu^2$ (in $V = -\mu^2 H^\dagger H + \lambda (H^\dagger H)^2$)

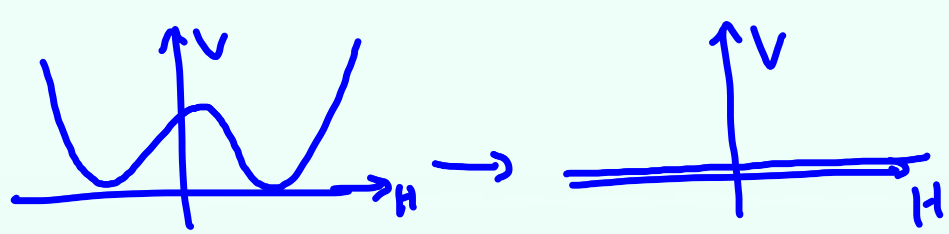
At classical level

SM is scale invariant as $\mu^2 \to 0$.

not interesting $\langle H \rangle = 0$

Better: $V = \lambda \left(H^\dagger H - \frac{1}{2} v^2\right)^2$, then classically

SM is scale invariant as $\lambda \to 0$



superb : $\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$

Scale invariance : $\phi(x) \to c^\Delta \phi(cx)$, eg, $\int d^dx\, \partial_\mu \phi' \partial^\mu \phi' = c^{2\Delta - (d-2)} \int d^dx\, \partial_\mu \phi\, \partial^\mu \phi$

$c \in \mathbb{R}^+$ → continuous symmetry ; $\langle H \rangle \neq 0$ → continuous symmetry spontaneously broken

→ Nambu Goldstone Boson ≡ "Dilaton"

of S.B. scale invariance

Where is it? (Who is it?).

For any SB symmetry, generator $Q = \int d^3x\, J^0$, $\partial_\mu J^\mu = 0 \Rightarrow$

(Goldstone's theorem) There is a (massless) spinless state $|\psi\rangle$ ∍ $\langle\psi| J^\mu |0\rangle \neq 0$

By L.I. $\langle\psi(\vec{p})| J^\mu(0) |0\rangle = f\, p_\mu$ $\quad f$ = constant called "decay constant" (historical, $\pi \to \mu\nu$).

For dilations S^μ is complicated, but $\partial_\mu S^\mu = T^\mu{}_\mu$ ($T^{\mu\nu}$ = improved stress-energy tensor)

Trick: introduce small breaking of symmetry, remove at end

$$T^{\mu\nu}(x) = \frac{2}{\sqrt{-g}} \frac{\delta S}{\delta g_{\mu\nu}(x)}, \quad S = \int d^4x \sqrt{-g}\left(\mathcal{L} + \tfrac{1}{2}\xi R \phi^2\right) \qquad \xi = \tfrac{1}{6} \text{ for improved } T^{\mu\nu}$$

Exercise: Determine $T^{\mu\nu}$ in SM. From it compute $T^\mu{}_\mu$. (Warning: simple answer, a lot of work).

Simpler example: single free scalar

Be a physicist: guess!

$T^{\mu\nu}$ should contain $\partial^\mu\phi\partial^\nu\phi$

but need $\partial_\mu T^{\mu\nu}=0$, but $\partial_\mu(\partial^\mu\phi\partial^\nu\phi) = (\partial^2\phi)\partial^\nu\phi + \partial^\mu\phi\,\partial_\mu\partial^\nu\phi$ ($\partial^2\phi \to 0$ (E.O.M.))

$$\Rightarrow \quad T^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - \tfrac{1}{2}\eta^{\mu\nu}\partial^\lambda\phi\partial_\lambda\phi$$

Not good enough, $T^\mu_\mu = -\partial^\lambda\phi\partial_\lambda\phi \neq 0$

Improvement

$$T^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - \tfrac{1}{2}\eta^{\mu\nu}\partial^\lambda\phi\partial_\lambda\phi + \boxed{\xi(\partial^\mu\partial^\nu - \eta^{\mu\nu}\partial^2)\phi^2}$$

automatically conserved

total derivative (does not change "charge" $\int d^3x\, T^{0\mu}$).

Find ξ: $T^\mu_\mu = 0 = -\partial^\lambda\phi\partial_\lambda\phi - 3\xi\partial^2\phi^2 = -(1+6\xi)\partial^\lambda\phi\partial_\lambda\phi$

$$T^{\mu\nu} = \partial^\mu\phi\partial^\nu\phi - \tfrac{1}{2}\eta^{\mu\nu}\partial^\lambda\phi\partial_\lambda\phi + \tfrac{1}{6}(\partial^2\eta^{\mu\nu} - \partial^\mu\partial^\nu)\phi^2$$

Finally, replace $\phi \to \frac{1}{\sqrt{2}}(v+h)$

$$T^{\mu\nu} = \frac{v}{6}(\partial^2\eta^{\mu\nu} - \partial^\mu\partial^\nu)h + \text{quadratic}$$

ONLY term in $T^{\mu\nu}$ LINEAR in SM fields

SM-DILATON = higgs

Digression 2: Dilaton as Higgs.

- Higgsless models of EW symmetry breaking
- Generically no "higgs"
- Suppose model is approximately scale invariant (Pica's lectures)

⇒ scale invariance + EWSB ⇒ dilaton

inexact scale invariance ⇒ mass for dilaton (pseudo GB).

AWESOME.

BUT

COUPLINGS ?

(Find this in Coleman's Aspects of Symmetry)

- Couplings of NGBs fixed by symmetry
- Easiest: in effective lagrangian
- Recipe: (2 cups of faith, 1 tsp of good luck):

If $\mathcal{L} = \sum_n c_n \mathcal{O}_n$ and $\dim \mathcal{O}_n = d_n$

(c_n ← coefficient; $\mathcal{O}_n$ ← operator (fields))

THEN $$\mathcal{L} \to \mathcal{L}_h = \sum_n c_n \left(e^{h/v}\right)^{4-d_n} \mathcal{O}_n$$

For example

$$\mathcal{L} = \bar{u}_L m^u u_R \to e^{h/v}\, \bar{u}_L m^u u_R = \left(1 + \frac{h}{v} + \cdots\right) \bar{u}_L m^u u_R$$

Corollary: Still diagonal in flavor In fact SAME AS FOR HIGGS in SM

Exercise: Compute $T^{\mu\nu}$ for single scalar with $V = \lambda(\phi^2 - v^2)$. If $\lambda \ll 1$ find pseudo-NGB and compute $\langle NGB | \partial_\mu S^\mu | 0 \rangle$. Compare with Pica's lectures.

END DIGRESSIONS