



# Universality Classes and Inflationary Attractors

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Based on:

- "Universality classes of inflation", DR, 1309.1285 (JCAP)
- "A universal attractor for inflation at strong coupling", R. Kallosh, A. Linde, DR, 1310.395 (PRL)
- "The unity of cosmological attractors", M. Galante, R. Kallosh, A. Linde, DR (1412.3739)



CERN TH Institute  
*Understanding the Early Universe*  
Jan 7, 2015

# asses and tractors



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on at strong

OR, 1310.395 (PRL)

ctors", M. Galante, R.

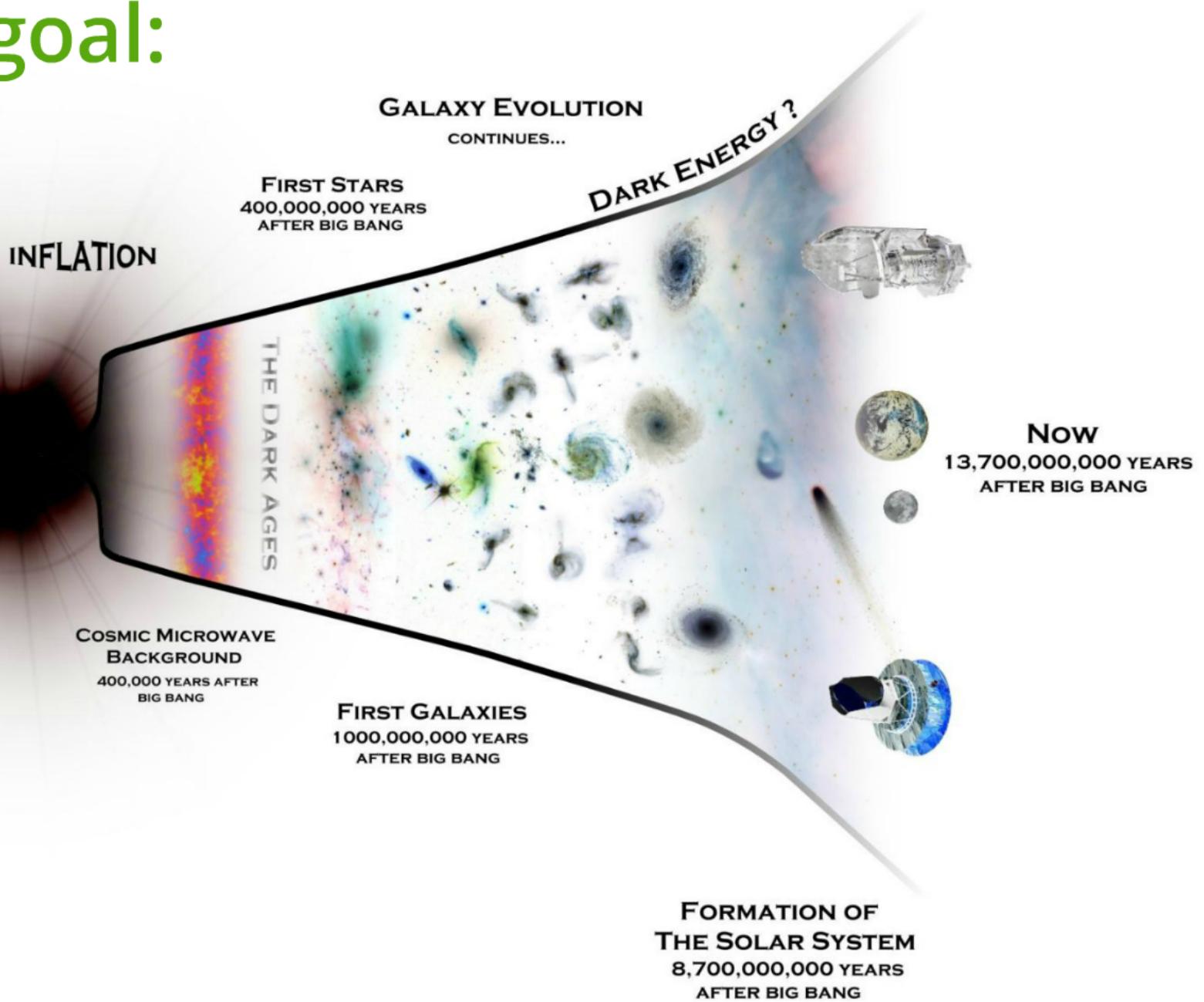
# Inflation in 2015



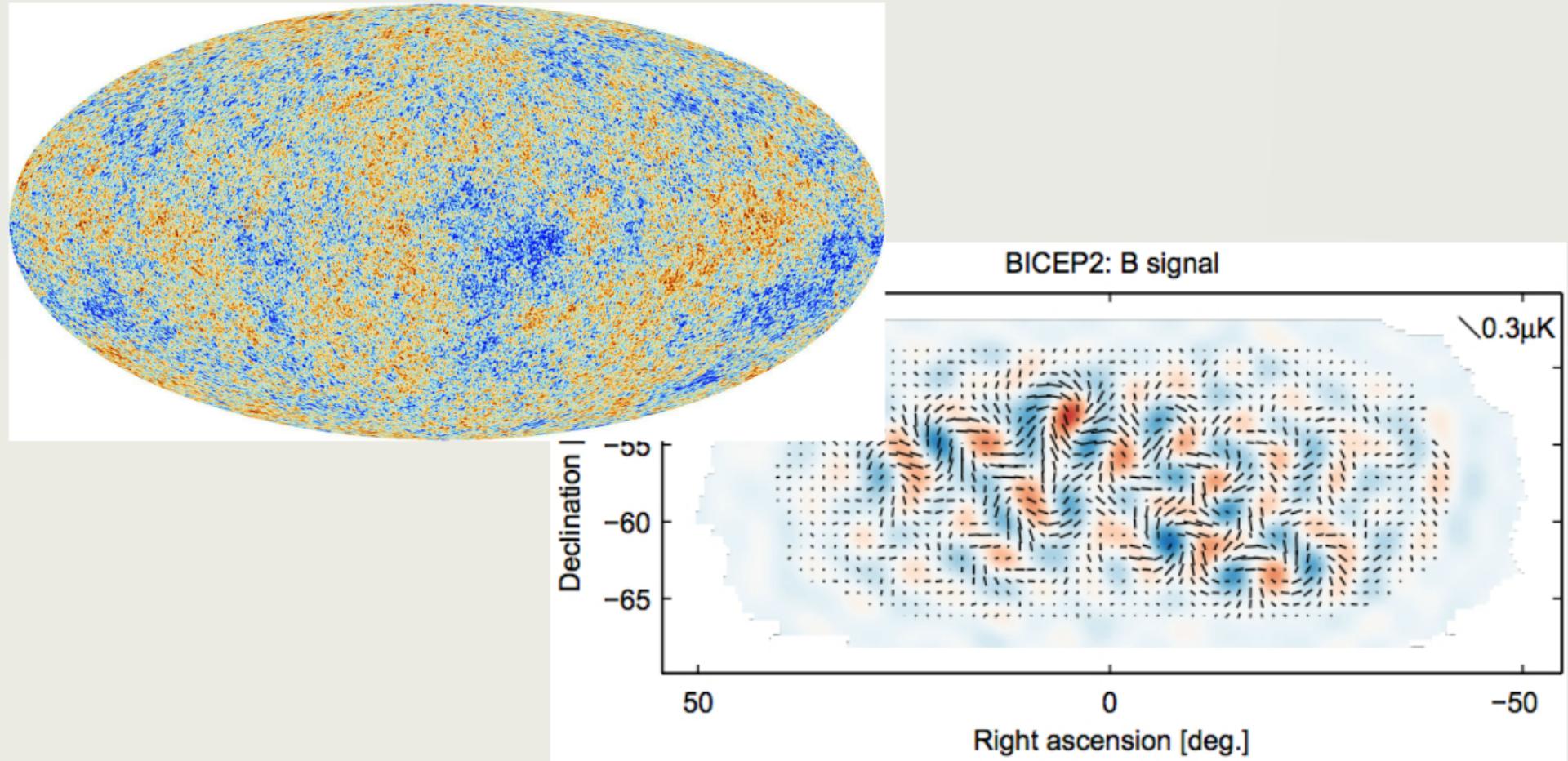
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# The goal:

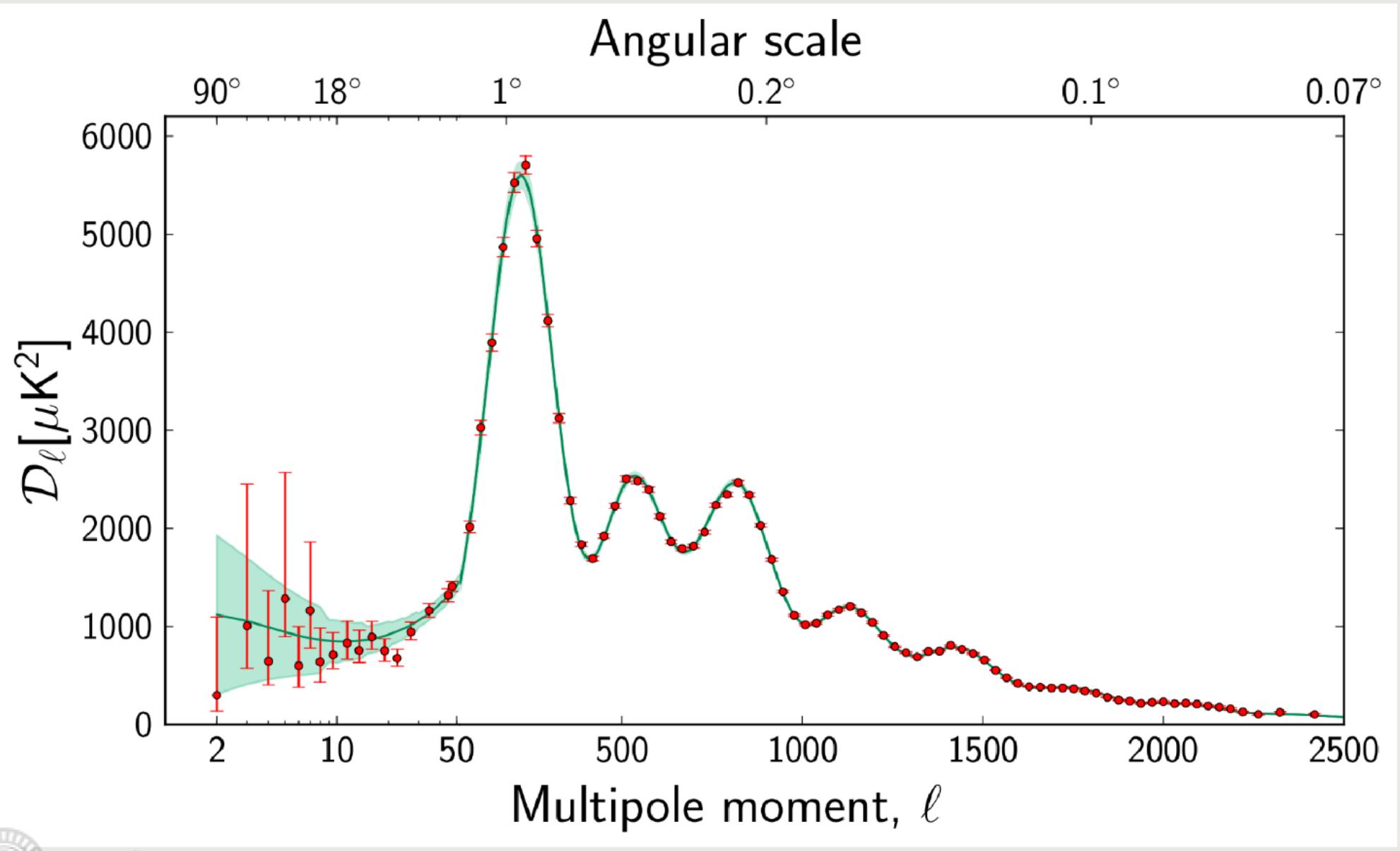
## THE BIG BANG

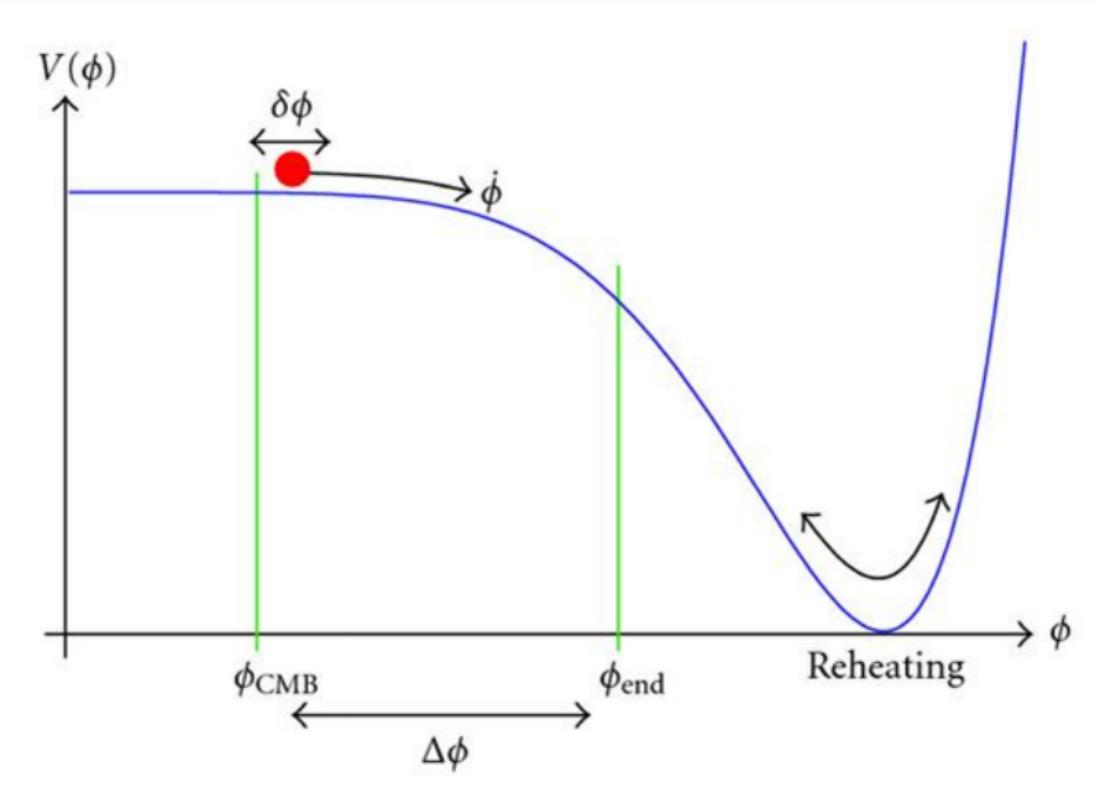


# The means:



## T-T power spectrum:





## Inflation

Modelled in terms of inflaton field with scalar potential.

In slow-roll approximation:

Model:

$$V = V(\phi)$$

$$\epsilon = \frac{1}{2} \left( \frac{V'}{V} \right)^2$$

$$\eta = \frac{V''}{V}$$

Observables:

$$N = - \int_{\phi_{end}}^{\phi_{CMB}} \frac{V}{V'}$$

$$n_s = 1 + 2\eta - 6\epsilon$$

and amplitude!

$$r = 16\epsilon$$

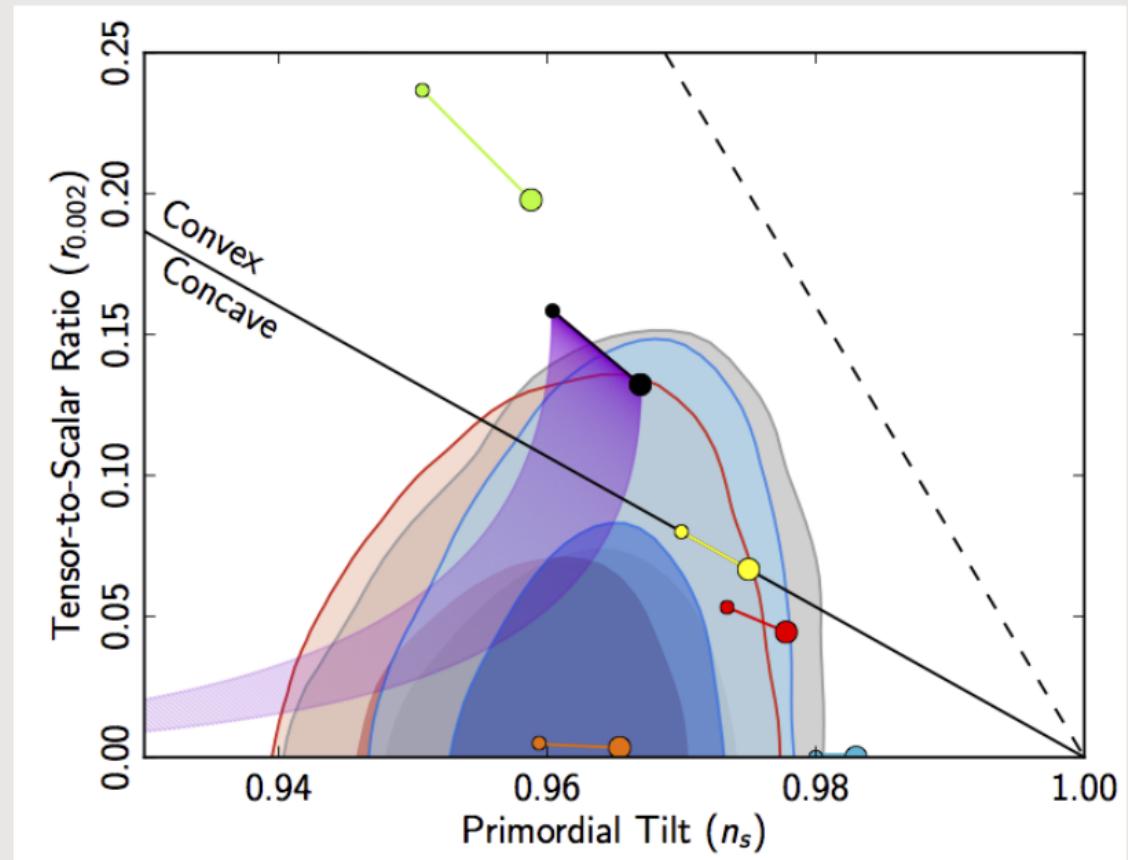
## Planck2013 results:

- measured spectral index different from one
- upper bound on tensor-to-scalar ratio
- constraints on other signatures, e.g.

$$f_{NL}^{local} = 2.7 \pm 5.8$$

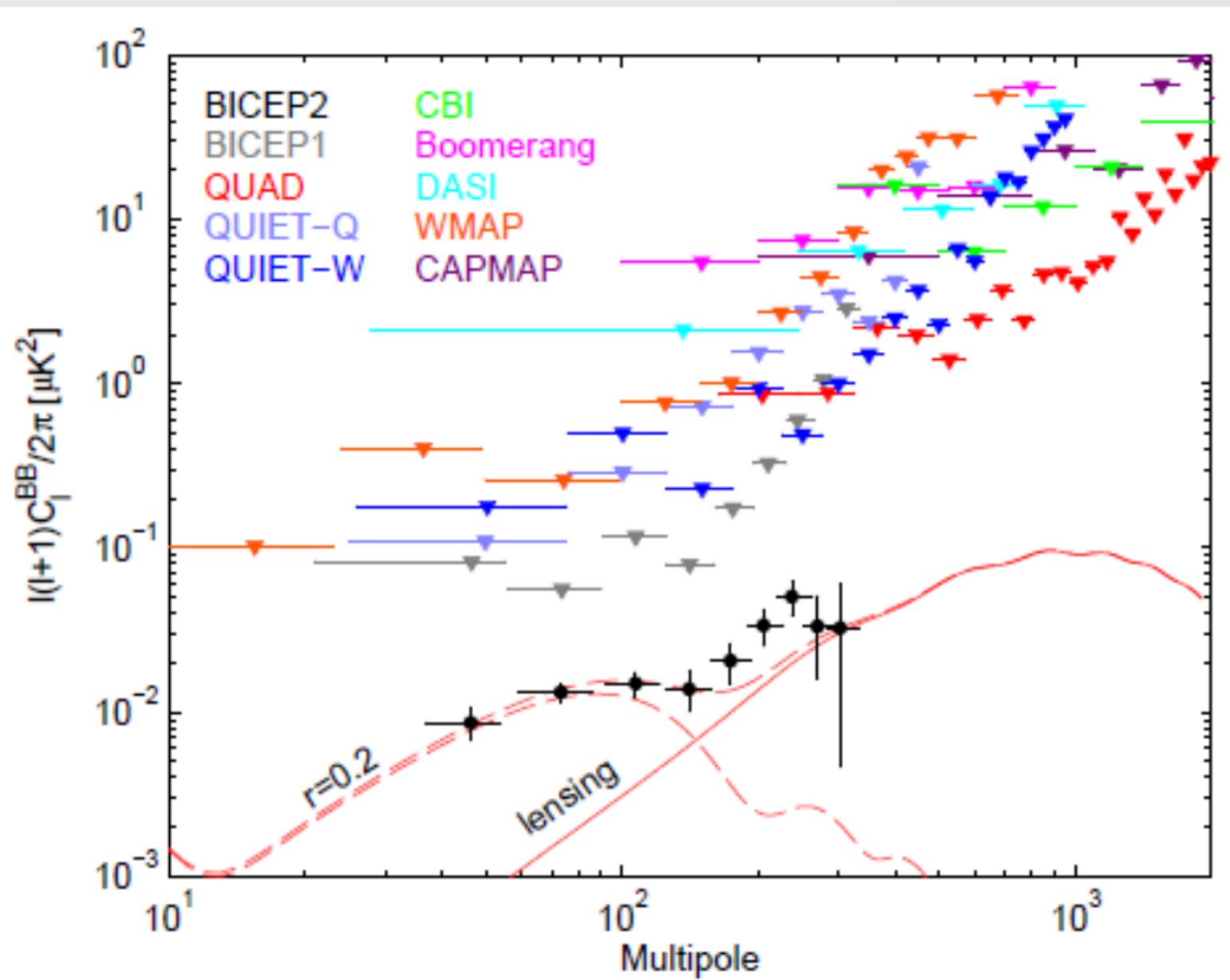
$$f_{NL}^{equil} = -42 \pm 75$$

$$f_{NL}^{ortho} = -25 \pm 39$$



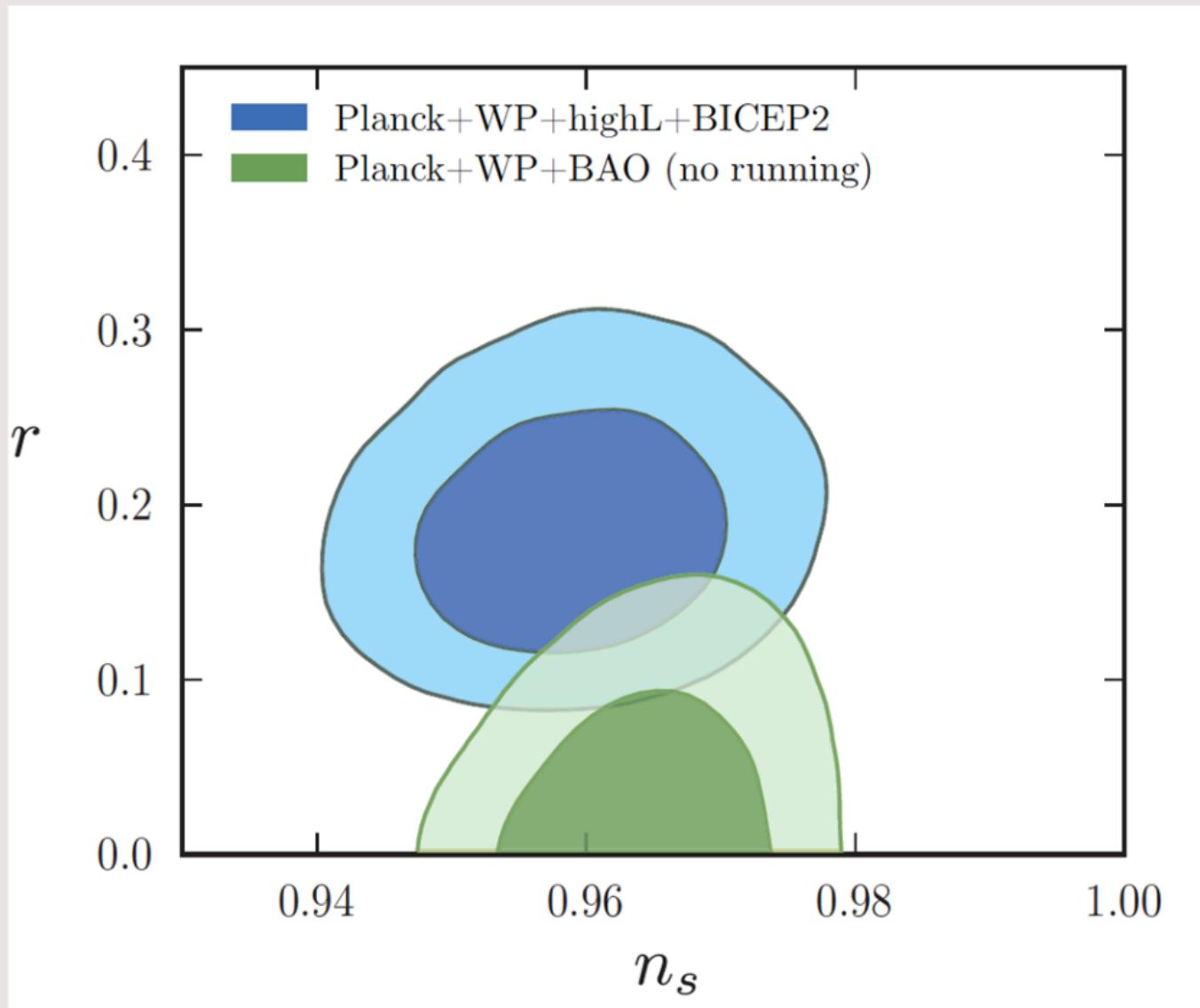
- similar prelim Planck2014/15 results
- fully consistent with single-field slow-roll

## B-B power spectrum...



## BICEP2 bomb shell:

- if primordial: tensor-to-scalar different from zero!

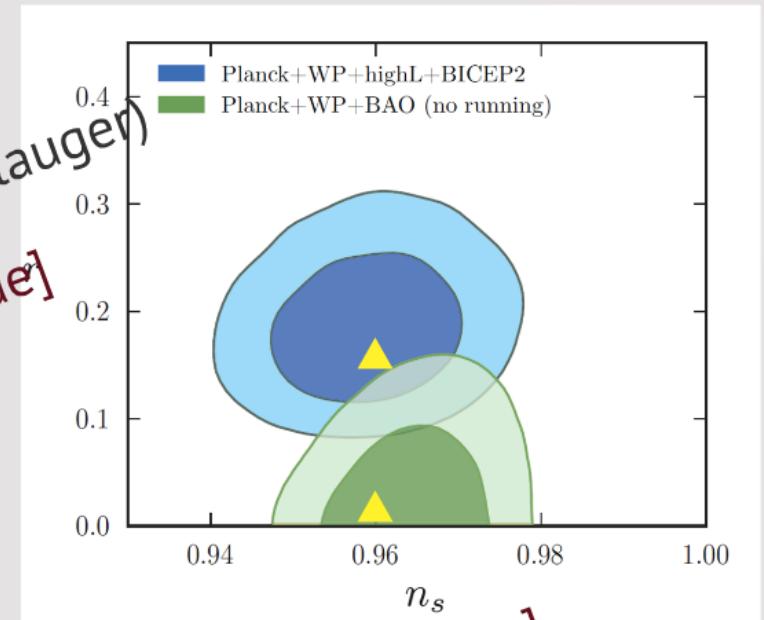


# Best fit?

(see talk by  
Raphael Flauger)

BICEP2: quadratic potential [Linde]

$$-\frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$$



Planck: quadratic potential in AdS box [Starobinsky]

$$-\frac{1}{2(\phi - 1)^2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2$$

Q: is there a deeper reason  
that such simple models arise?

# asses and tractors



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# Universality classes

[Universality classes of inflation", DR, 1309.1285, JCAP]



# Expansion in 1/N

Perturbation theory in number of e-foldings:

- only model-independent small number
- right order of magnitude for Planck
- subleading terms not unambiguously defined
- moreover, impossible to measure!

Expansion of slow-roll parameters:

$$n_s = 1 - \frac{p}{N} + \dots$$

Q: does this allow for arbitrary tensors?

A: very specific relations...

$$1 - n_s = \frac{r}{8} - \frac{r_{,N}}{r}$$

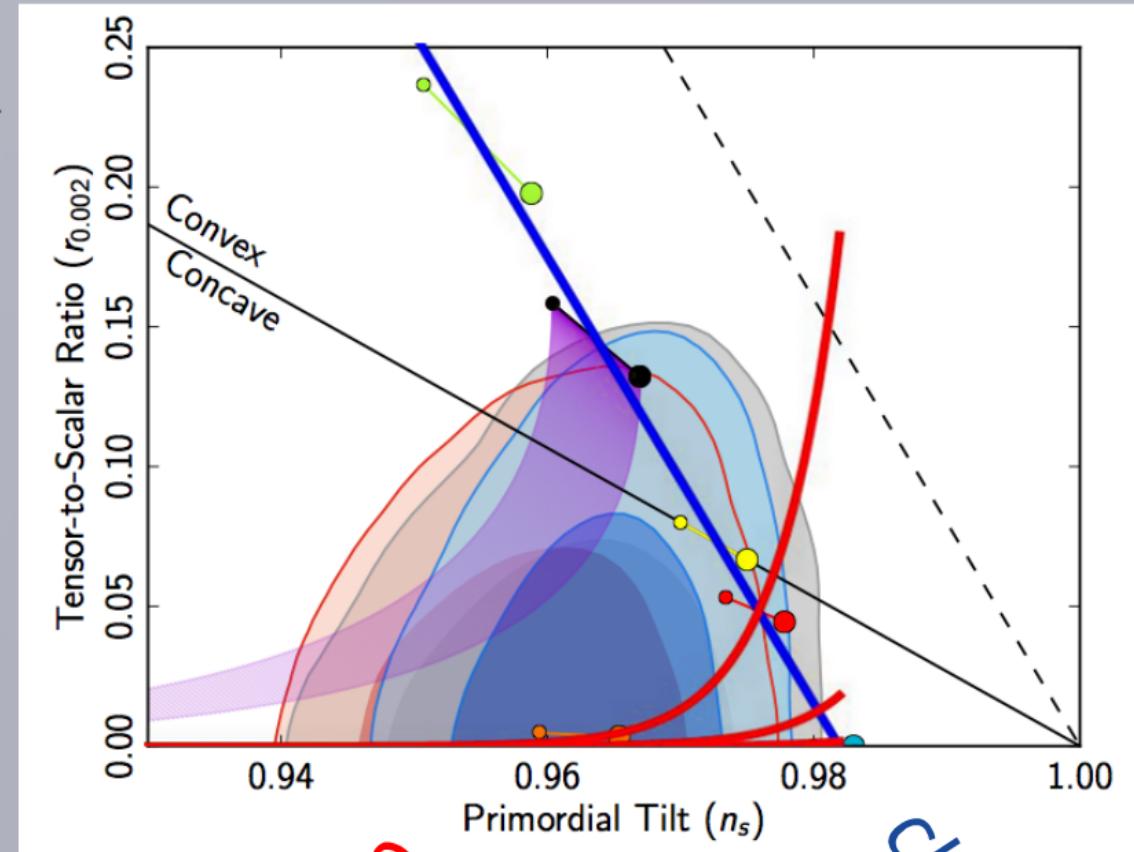
for speed of sound equal to unity.

# Inflation@large-N

$$n_s = 1 - \frac{p}{N} \quad p > 1$$

Class I:  $r \geq \frac{1}{Np}$

Class II:  $r = \frac{p-1}{N}$



hilltop  
Starobinsky  
inverse  
hilltop  
chaotic

# Predictions

Assuming 1/N spectral index: two universality classes, not all tensors possible.

[similar approaches: Mukhanov, 1303.3925,  
Garcia-Bellido, DR, 1402.2059  
Creminelli et al, 1412.0678]

Always minimum redshift:

$$n_s < 1 - \frac{1}{N} \approx 0.98$$

Fix spectral index and predict tensors:

$$n_s = 1 - \frac{2}{N} \quad \Rightarrow \quad r \approx 0.14 \quad / \quad r < 0.01$$

[Quadratic] [Starobinsky]

*"either unobservably small or close to the present observational limit"* (DR, 2013)

[Including a non-trivial speed of sound may change this (somewhat)  
- Zavala 1412.3732]

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# Inflationary attractors

[*"A universal attractor for inflation at strong coupling"*, R. Kallosh, A. Linde, DR, 1310.395, PRL]  
[*"The unity of cosmological attractors"*, M. Galante, R. Kallosh, A. Linde, DR, 1412.3739]





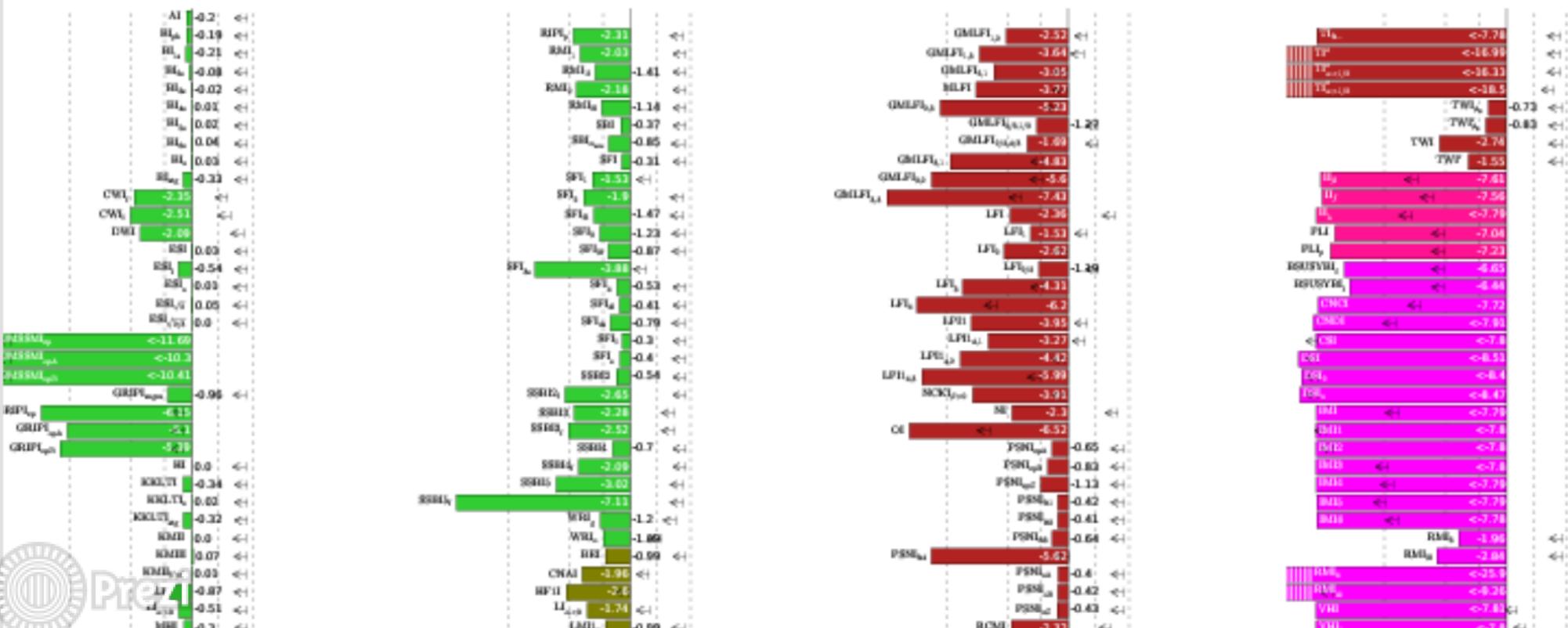
# The Best Inflationary Models After Planck

Jerome Martin, Christophe Ringeval, Roberto Trotta, Vincent Vennin

(Submitted on 12 Dec 2013 (v1), last revised 3 Jun 2014 (this version, v3))

We compute the Bayesian evidence and complexity of 193 slow-roll single-field models of inflation using the Planck 2013 Cosmic Microwave Background data, with the aim of establishing which models are favoured from a Bayesian perspective. Our calculations employ a new numerical pipeline interfacing an inflationary effective likelihood with the slow-roll library ASPIC and the nested sampling algorithm MULTINEST. The models considered represent a complete and systematic scan of the entire landscape of inflationary scenarios proposed so far. Our analysis singles out the most probable models (from an Occam's razor point of view) that are compatible with Planck data, while ruling out with very strong evidence 34% of the models considered. We identify 26% of the models that are favoured by the Bayesian evidence, corresponding to 15 different potential shapes. If the Bayesian complexity is included in the analysis, only 9% of the models are preferred, corresponding to only 9 different potential shapes. These shapes are all of the plateau type.

## Bayesian Evidences $\ln(\mathcal{E}/\mathcal{E}_{\text{HI}})$ and $\ln(\mathcal{L}_{\max}/\mathcal{E}_{\text{HI}})$

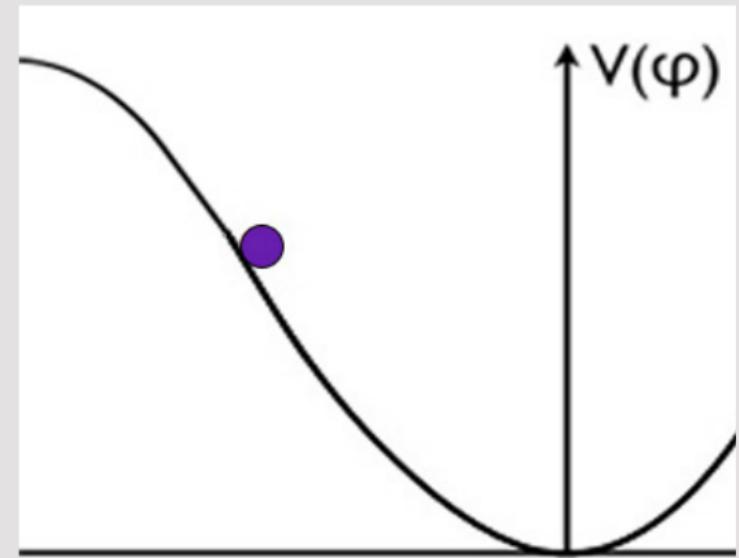


# Scalar potentials

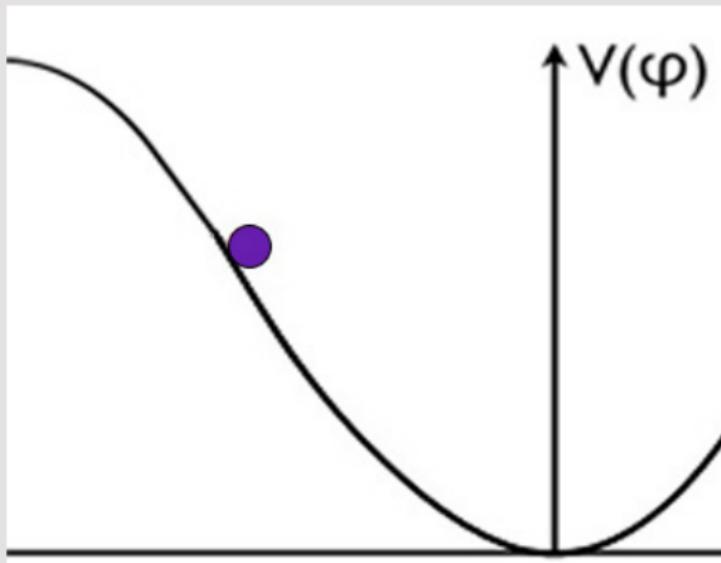
Planck is pointing towards plateau-like potentials:

$$\mathcal{L} = \sqrt{-g} \left[ \frac{1}{2} R - \frac{1}{2} (\partial\varphi)^2 - V(\varphi) \right]$$

Plateau at infinite / finite distance, with (inverse) polynomial / exp fall-off.



## Kinetic formulation



Redefinition to trivial potential:

$$\frac{1}{2}R - \frac{1}{2} \left( \frac{\partial \varphi}{\partial \rho} \right)^2 (\partial \rho)^2 - \frac{1}{2}m^2(\rho_0 - \rho)^2$$

Similar to  $V$  in  
terms of Hubble!

Plateau in potential implies a singularity in kinetic term!  
Behaviour close to singularity is crucial.

# Inflationary predictions

Behaviour at  $N=60$  determined by leading pole in Einstein-frame kinetic term:

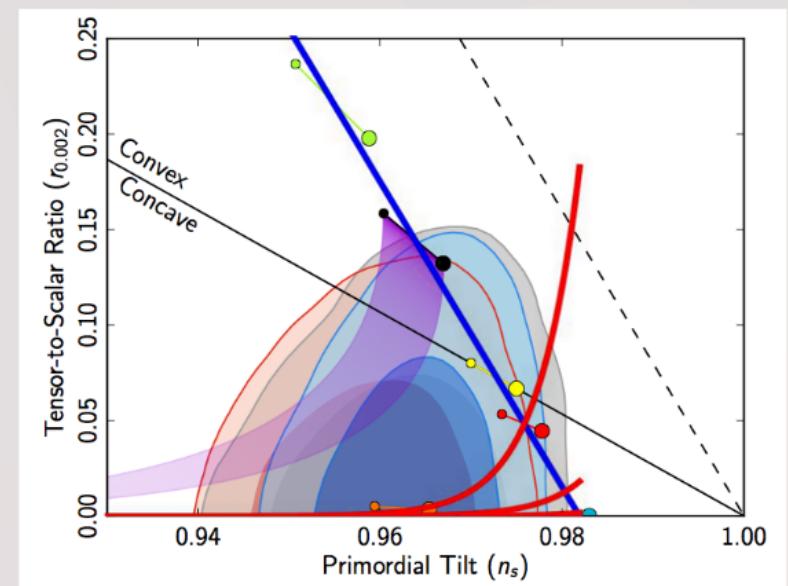
$$K_E = \frac{a_p}{\rho^p} + \dots$$

Independent of subleading terms in  $K$  and fully independent of  $V$ : robustness of universality class!

$$n_s = 1 - \frac{p}{p-1} \frac{1}{N}$$

$$r = \# \left( \frac{a_p}{N^p} \right)^{\frac{1}{p-1}}$$

Note: same coeff in  
 $n_s$  and power in  $r$



## Non-minimal coupling

$$\frac{1}{2}\Omega(\phi)R - \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{\xi^2}(\Omega - 1)^2$$

Jordan frame formulation:

Higgs inflation:

$$\Omega = 1 + \xi\phi^2$$

[Salopek, Bond, Bardeen  
'89, Bezrukov, Shaposhikov  
'07]

Universal attractor:

$$\Omega = 1 + \xi\phi^n$$

[Kallosh, Linde, DR '13]

Induced inflation:

$$\Omega = \xi\phi^2$$

[Giudice, Lee '14]

Einstein frame formulation, up to original kinetic term:

$$\frac{R}{2} - \frac{3}{4\rho^2}(\partial\rho)^2 - \frac{m^2}{2}(\rho - 1)^2$$

Always leads to quadratic potential in pole coordinate!

## Non-minimal coupling

$$\frac{1}{2}\Omega(\phi)R - \frac{1}{2}(\partial\phi)^2 - \frac{\lambda}{\xi^2}(\Omega - 1)^2$$

Jordan frame formulation:



Prezi

Higgs inflation:

$$\Omega = 1 + \xi\phi^2$$

Bond, Bardeen  
Chaposhikov

**Higgs inflation:**  $\Omega = 1 + \xi\phi^2$

Reformulation leads to quadratic potential plus

$$K_E = \frac{3}{2} \frac{1}{\rho^2} + \frac{1}{4\xi} \frac{1}{(\rho_0 - \rho)\rho^2} = \frac{3}{2} \left(1 + \frac{1}{6\xi}\right) \frac{1}{\rho^2} + \dots$$

Infinite coupling limit:

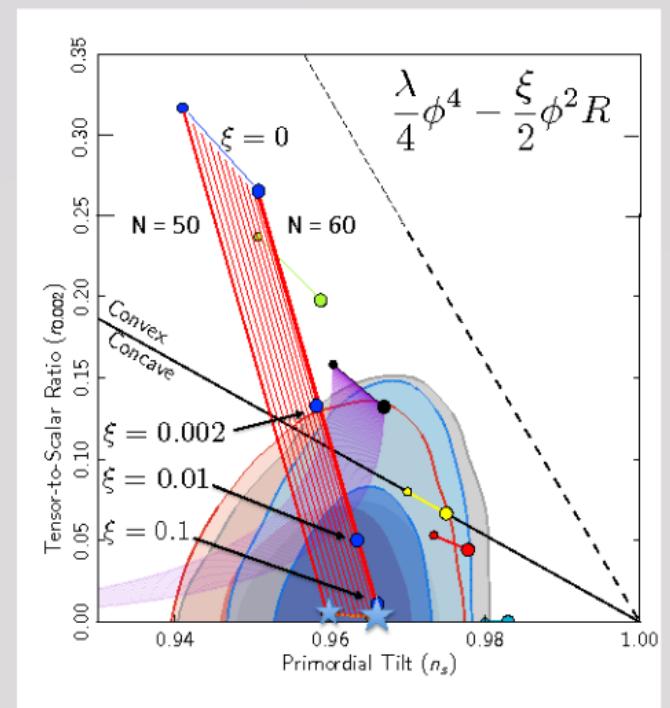
Starobinsky model = a pure pole

Large coupling:

increases residue and hence r

Small coupling:

subleading terms important



**Universal attractor:**  $\Omega = 1 + \xi\phi^n$

Reformulation leads to quadratic potential plus

$$K_E = \frac{3}{2\rho^2} + \frac{1}{n^2\xi^{2/n}\rho^{1+2/n}} +$$

Infinite coupling:

same pole

Large coupling:

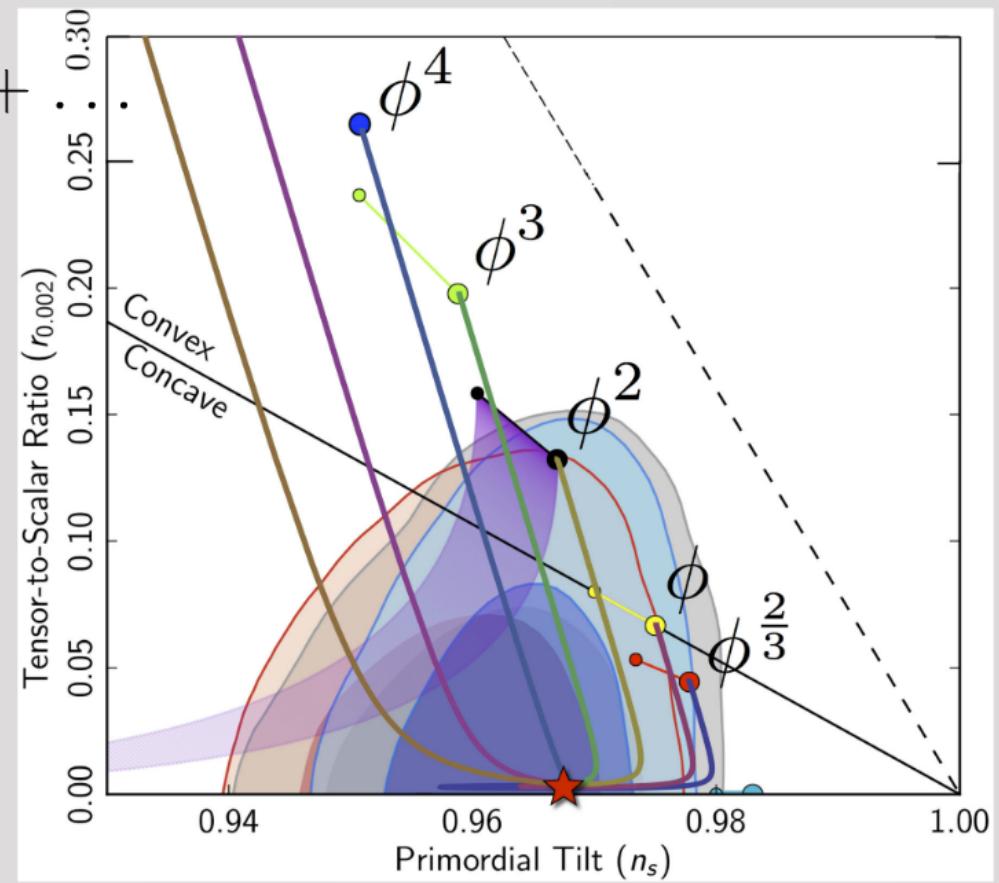
suppressed poles

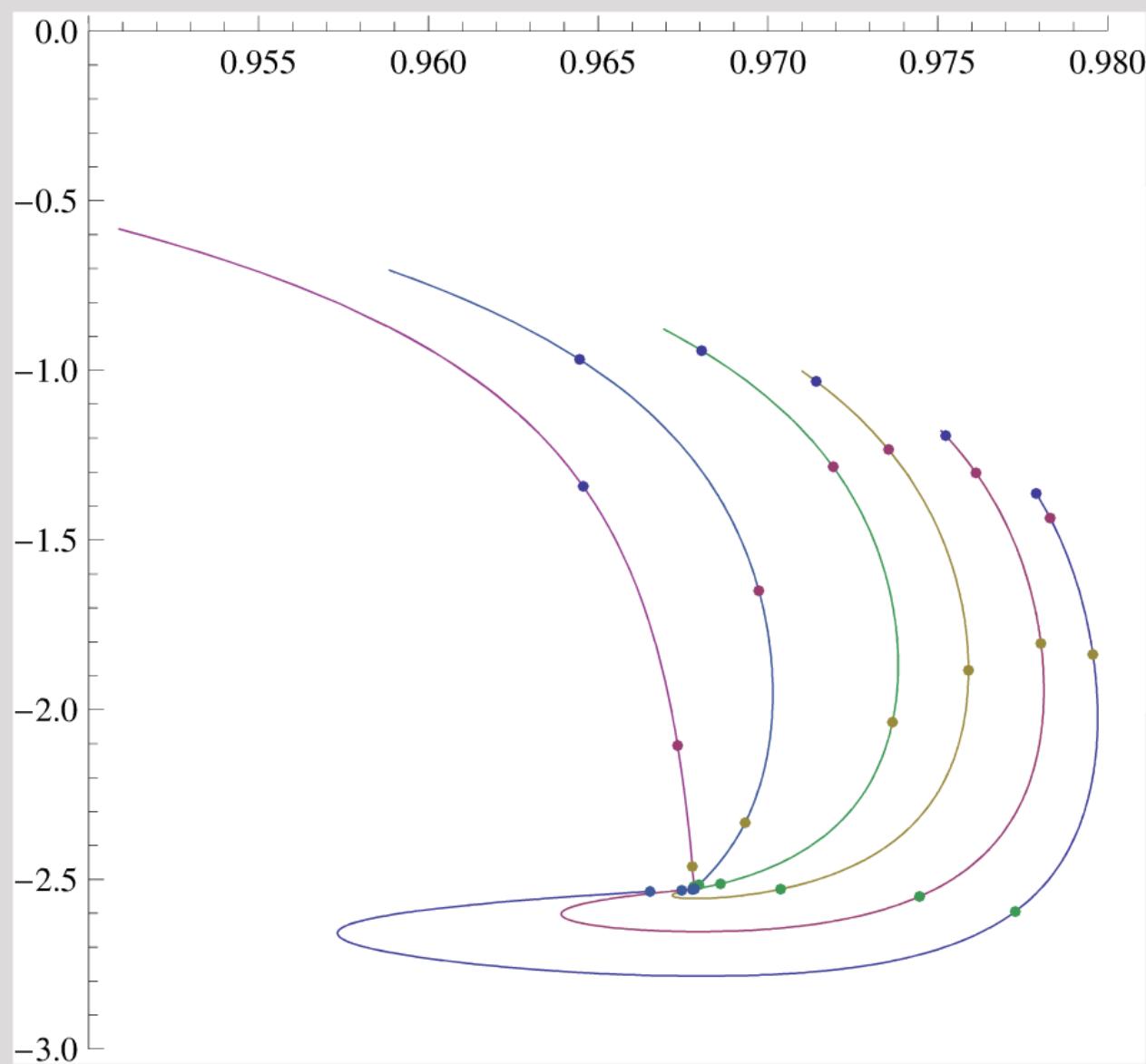
Order-one coupling:

other pole takes over

Small coupling:

subleading corrections





**Universal attractor:**  $\Omega = 1 + \xi\phi^n$

Reformulation leads to quadratic potential plus

$$K_E = \frac{3}{2\rho^2} + \frac{1}{n^2\xi^{2/n}\rho^{1+2/n}} +$$

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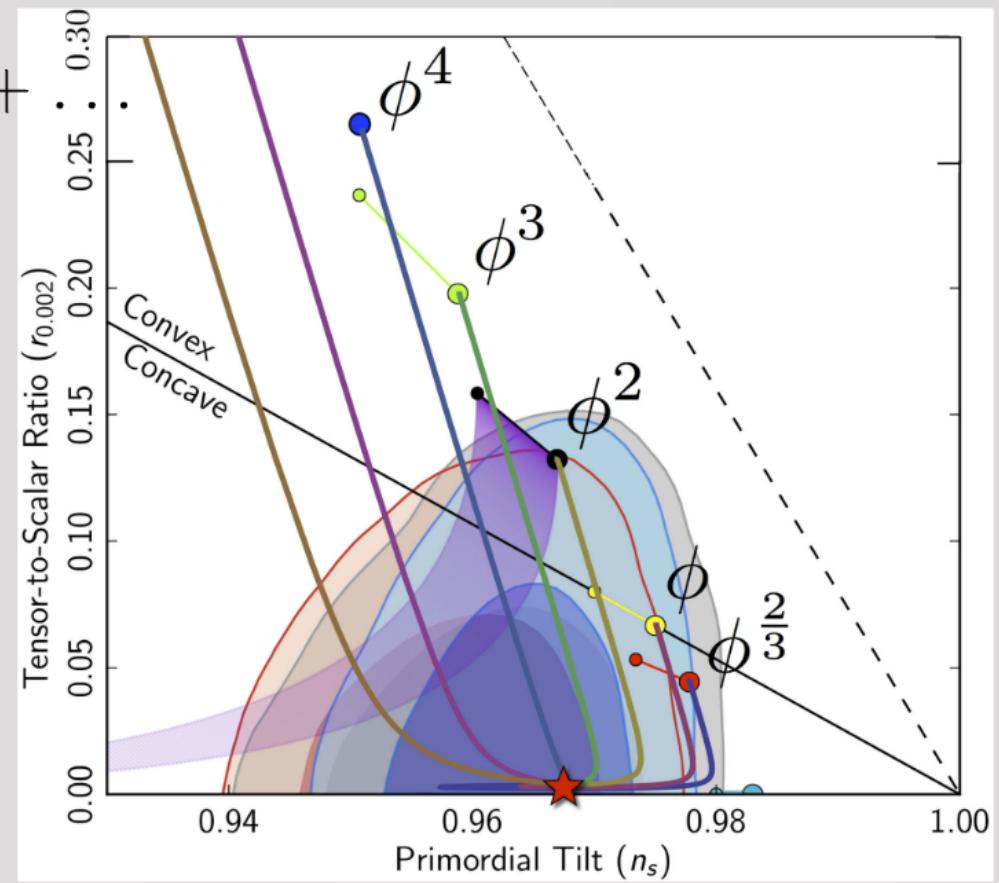
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Small coupling:

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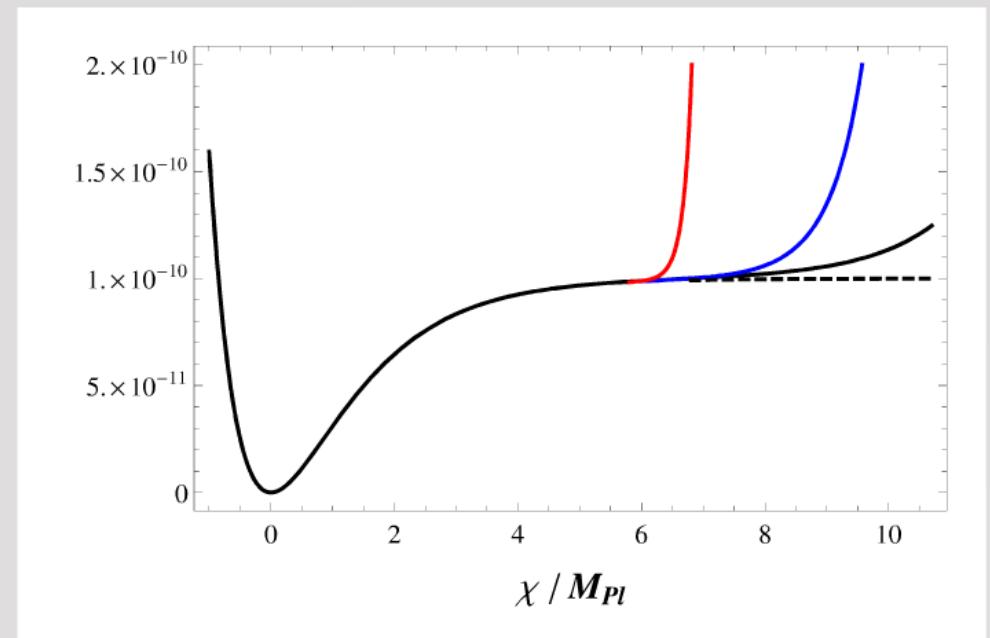


# Generic deformations

Setting  $\xi \sim 10^5$  for power spectrum amplitude yields at least 55 flat e-folds.

Percent-level power loss for larger N.

- Single parameter that sets
- spectral index
  - tensor-to-scalar ratio
  - power normalisation
  - number of flat e-folds



[Broy, DR, Westphal '14]

**Induced inflation:**  $\Omega = \xi\phi^2$

Reformulation leads to quadratic potential plus

$$K_E = \frac{3}{2}\left(1 + \frac{1}{6\xi}\right)\frac{1}{\rho^2}.$$

Two contributions feed into  $r$ :

- 1) positive offset from Jordan to Einstein transf.
- 2) second contribution due to Jordan kinetic term

- Coupling can be negative, but only in Einstein frame!
- Jordan frame imposes a lower bound  $r \sim 0.003$ .
- Conformal value predicts zero tensors.

# asses and tractors



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OR, 1310.395 (PRL)  
ctors", M. Galante, R.

# Conclusions

Model-independent 1/N expansion leads to simple expressions:

$$n_s = 1 - \frac{1}{(1+r)^2}$$

up to subleading corrections.

Convenient to model by pole structure in kinetic term (fully equivalent to previous refinement)

$$K_F = \frac{1}{r} + \dots$$

Higher-order corrections and potential energy only affect prefactors of subleading terms

Strong preference for case pm2 from theory & exp:  
non-minimal coupling leads to cosmological attractors with  
a pole at finite value r.

- different possible values of r
- different constraint values to coeff
- lower bound  $r > 0.003$  from Jordan frame
- relation between amplitude and # e-folds



Model-independent  $1/N$  expansion leads to simple expressions:

$$n_s = 1 - \frac{p}{p-1} \frac{1}{N}$$
$$r = \# \left( \frac{a_p}{N^p} \right)^{\frac{1}{p-1}}$$

up to subleading corrections.

Convenient to model by pole structure in kinetic term  
(fully equivalent to plateau inflation):

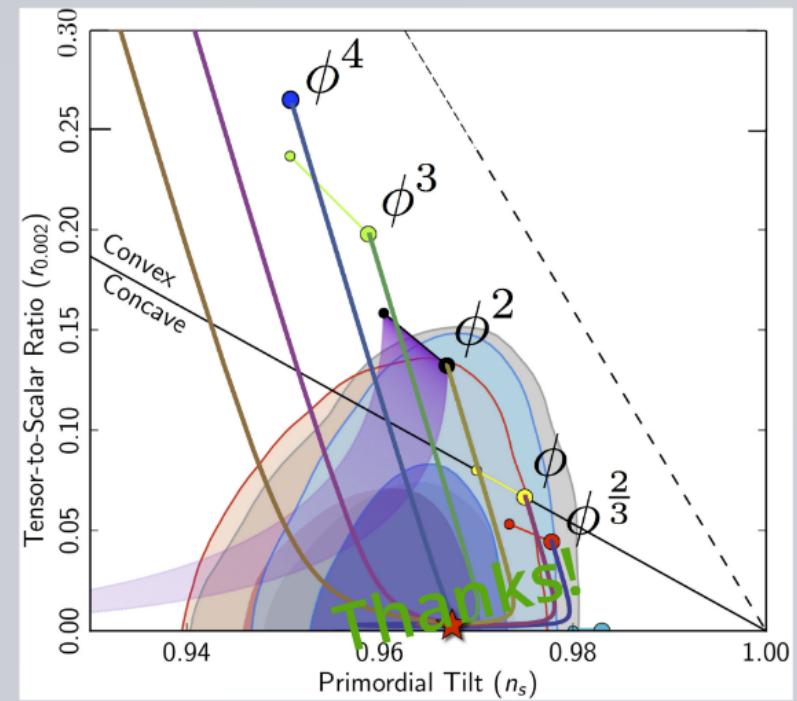
$$K_E = \frac{a_p}{\rho^p} + \dots$$

Higher-order corrections and potential energy only affect predictions at subleading terms.

Strong preference for case p=2 from theory & exp:

non-minimal coupling leads to cosmological attractors with a pole of order two:

- natural permille value of r
- different contributions to coeff
- lower bound  $r \sim 0.003$  from Jordan frame
- relation between amplitude and # e-folds



Thanks!

thanks!

0.96

0.