ÉCOLE POLYTECHNIQUE
FÉDÉRALE DE LAUSANNE

Living beyond the edge: Higgs inflation and vacuum metastability

Mikhail Shaposhnikov

based on :
Fedor Bezrukov, Javier Rubio, M.S., arXiv:1412.3811

Outline

- Motivation
- Renormalisation of non-renormalisable theories
- Low energy and high energy couplings in the SM
- Higgs inflation with metastable vacuum
- Conclusions

Motivation

Summary of the LHC findings

- The Standard Model in now complete: the last particle - Higgs boson, predicted by the SM, has been found

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  114 GeV $< m_H < 175$ GeV

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  114 GeV $< m_H <$ 175 GeV

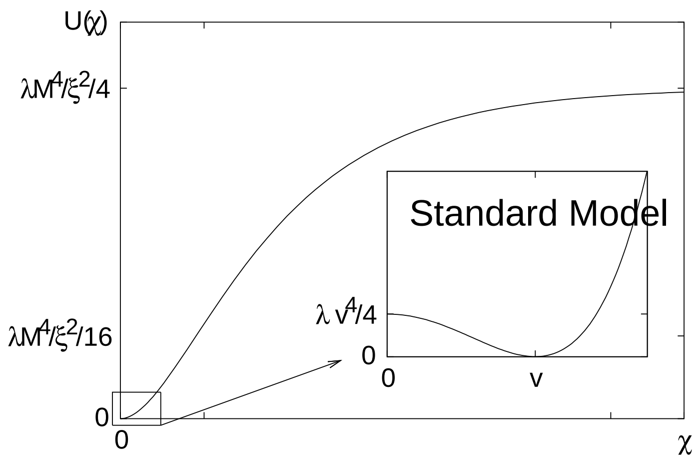
Therefore, we can try describe the evolution of the Universe from the very early stages till the present days within the SM!

Higgs boson as the inflaton

Higgs field in general must have non-minimal coupling to gravity:

$$S_G = \int d^4x \sqrt{-g} \left\{ -\frac{M_P^2}{2} R - \frac{\xi h^2}{2} R \right\}$$

Potential in Einstein frame: χ - canonically normalized scalar field in Einstein frame.



This form of the potential is universal for (Bezrukov, MS) $y_t(173.2) < y_t^{\rm crit}$:

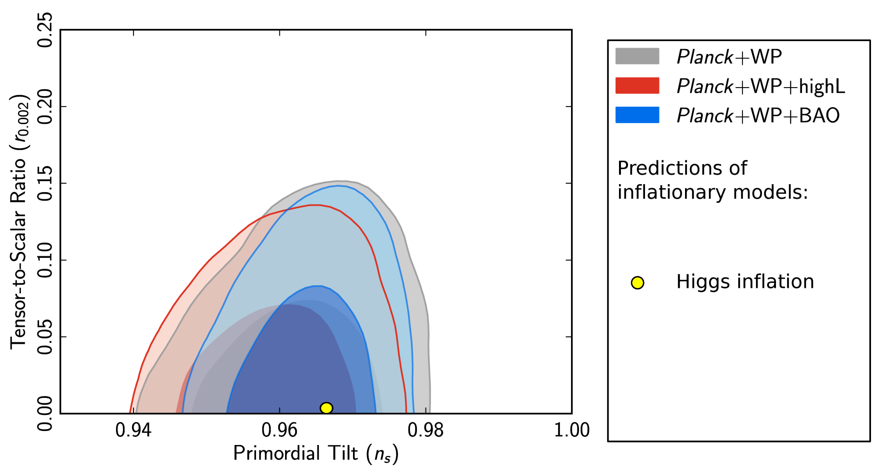
$$y_t^{\rm crit} = 0.9223 + 0.00118\left(\frac{\alpha_s - 0.1184}{0.0007}\right) + 0.00085\left(\frac{M_H - 125.03}{0.3}\right) + 0.0023\left(\frac{\log\xi}{6.9}\right)$$

$y_t(173.2)$ - top Yukawa coupling in $\overline{\rm MS}$- scheme at $\mu = 173.2$ GeV, $\alpha_s(M_Z)$ - strong coupling

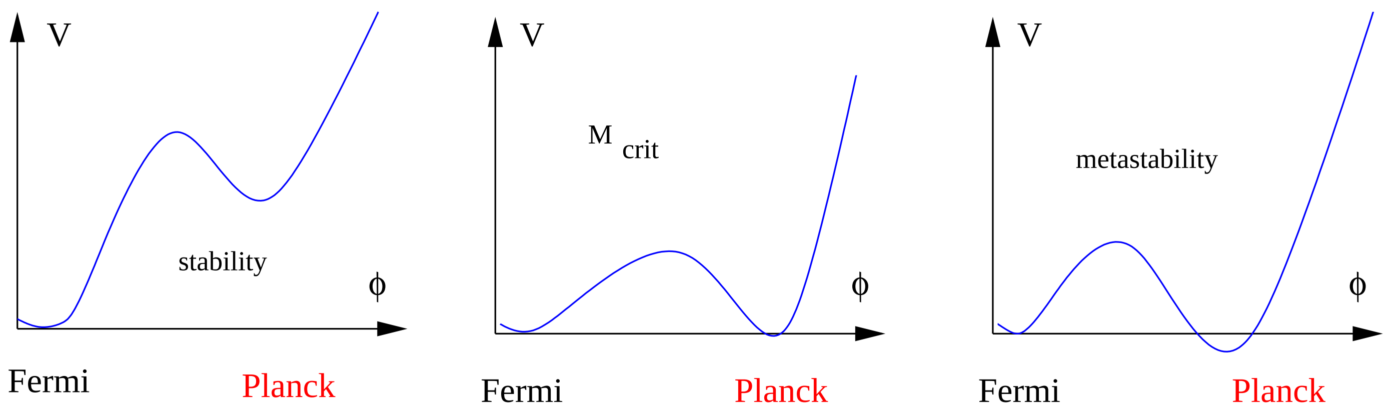
theoretical uncertainty: $\delta y_t / y_t \simeq 2 \times 10^{-4}$ equivalent to changing of M_H by ~ 70 MeV, or m_t by ~ 35 MeV Buttazzo et al

CMB parameters - spectrum and tensor modes, $\xi \gtrsim 1000$:

$$n_s = 0.97, \quad r = 0.003$$

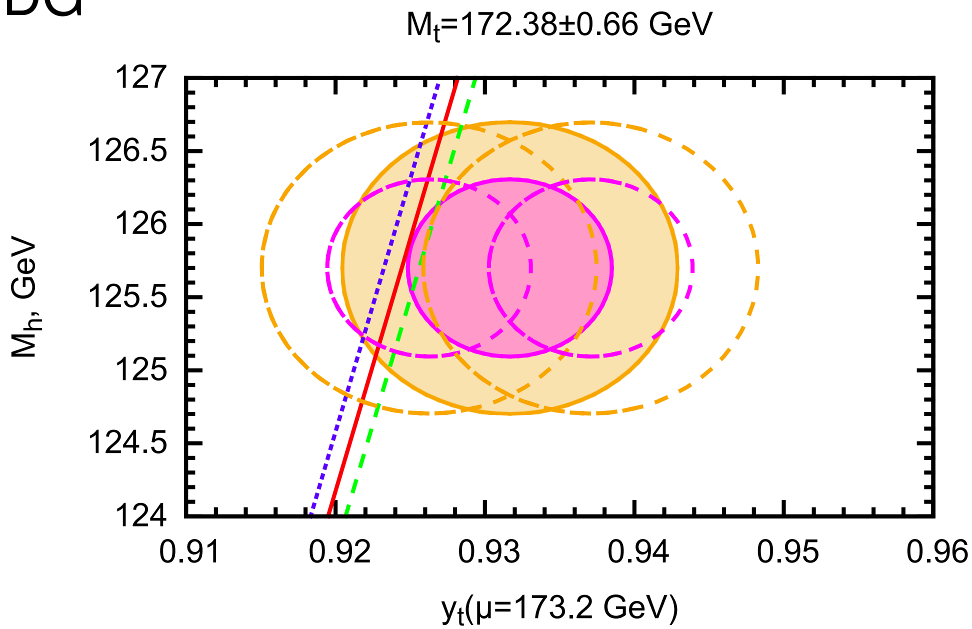


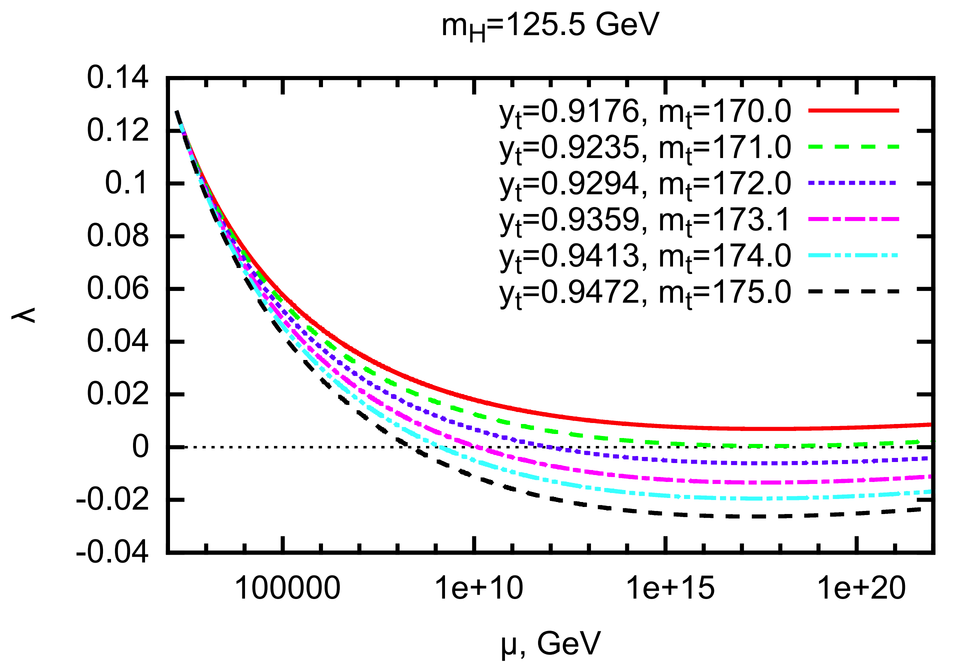
Important fact: Numerically for $\xi = 1, \quad y_t^{crit}$ coincides with the metastability bound on the top Yukawa coupling



MC top mass: CMS; 1 GeV uncertainty in MC-pole mass relation;

Higgs mass - PDG





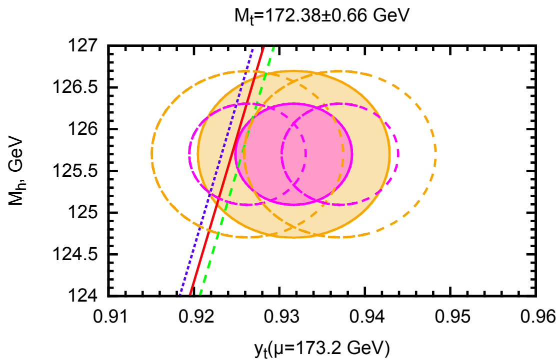
Determination of top quark mass

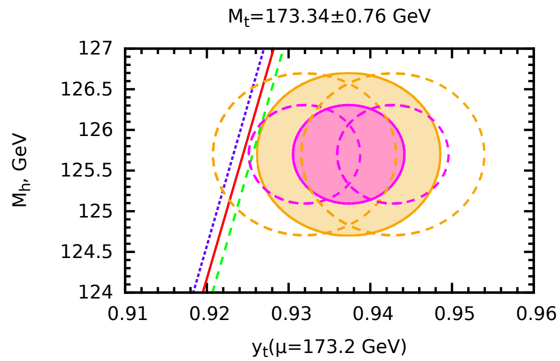
Monte Carlo mass:

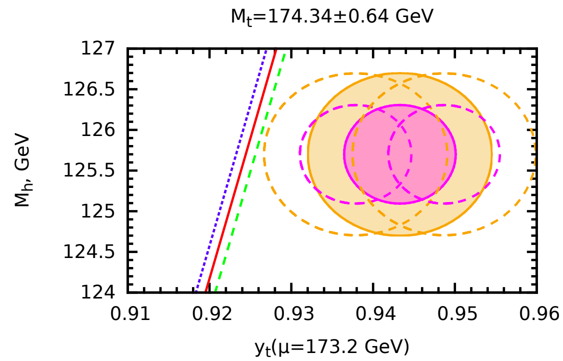
$$m_t = 172.38 \pm 0.10 \pm 0.65 \text{ GeV (CMS)}$$

$$m_t = 173.34 \pm 0.27 \pm 0.71 \text{ GeV (LHC Tevatron combined)}$$

$$m_t = 174.34 \pm 0.37 \pm 0.52 \text{ GeV (Tevatron)}$$







Suppose that future experiments will establish with certainty that $y_t > y_t^{\rm crit}$, meaning that our vacuum is metastable

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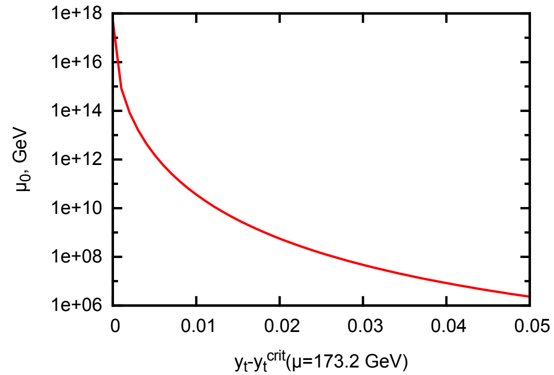
New physics is required at energies where λ crosses zero?



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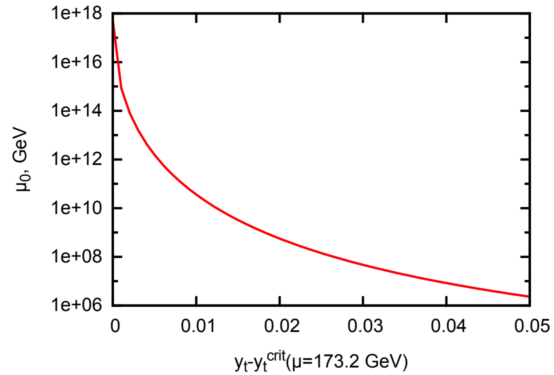

Should we abandon the Higgs inflation?

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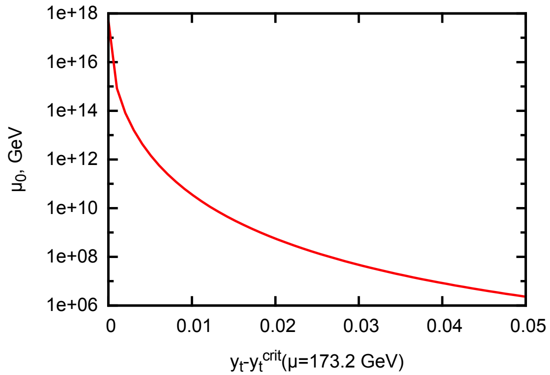

Should we abandon the Higgs inflation?

No!

Renormalisation of non-renormalisable theories

Any theory of inflation is non-renormalisable, as it includes gravity!

How to account for this fact in general, and for the Higgs inflation in particular ?

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(ii) Self-consistent approach to Higgs inflation: compute the onset of strong coupling Λ ("UV cutoff") by considering tree high energy scattering amplitudes Burgess, Lee, Trott ; Barbon and Espinosa in the Higgs-dependent background Bezrukov, Magnin, M.S., Sibiryakov; Ferrara, Kallosh, Linde, A. Marrani, Van Proeyen and add higher-dimensional operators suppressed by this cutoff.

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(iii) The most minimal setup: add to Lagrangian all counter-terms necessary to make the theory finite with all constant parts having the same structure as counter-terms. Bezrukov, Magnin, MS, Sibiryakov

Effective theory of Higgs inflation

The procedure must respect the classical symmetries of the theory (scale invariance in Jordan frame = shift symmetry in Einstein frame). Technically - use dimensional regularisation and $\overline{\text{MS}}$ subtraction procedure.

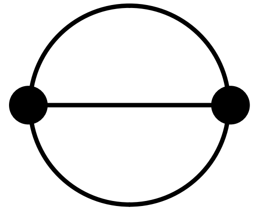
Starting Lagrangian:

$$L = \frac{(\partial_\mu \chi)^2}{2} - U(\chi)$$

where $U(\chi)$ has at large fields the generic form

$$U(\chi) = U_0 \left(1 + \sum_{n=1}^{\infty} u_n e^{-n\chi/M}\right)$$

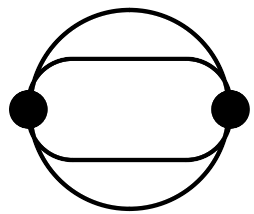
$$\frac{U_0 u_n}{M^3} e^{-n\bar{\chi}/M}$$



$$\frac{U_0 u_m}{M^3} e^{-m\bar{\chi}/M}$$

$$\propto \frac{1}{\epsilon} \cdot \frac{U_0^2}{M^8} u_n u_m (\partial_\mu \bar{\chi})^2 e^{-(n+m)\bar{\chi}/M} \quad ,$$

$$\frac{U_0 u_n}{M^4} e^{-n\bar{\chi}/M}$$



$$\frac{U_0 u_m}{M^4} e^{-m\bar{\chi}/M}$$

$$\propto \frac{1}{\epsilon} \cdot \frac{U_0^2}{M^8} u_n u_m \left(\frac{(\partial^2 \bar{\chi})^2}{M^2} + \frac{(\partial \bar{\chi})^4}{M^4} \right) e^{-(n+m)\bar{\chi}/M}$$

Effective action, incorporating radiative corrections:

$$L = f^{(1)}(\chi)\frac{(\partial_\mu\chi)^2}{2} - U(\chi) + f^{(2)}(\chi)\frac{(\partial^2\chi)^2}{M^2} + f^{(3)}(\chi)\frac{(\partial\chi)^4}{M^4} + \cdots$$

where dots stand for terms with more derivatives. The coefficient functions are (formal) series in the exponent,

$$f^{(i)}(\chi) = \sum_{n=0}^{\infty} f_n^{(i)} e^{-n\chi/M}$$

Important: asymptotic shift symmetry $\chi \to \chi + const$ in Einstein frame (or scale invariance in the Jordan frame).

Summary of assumptions

- We only add the higher dimensional operators that are generated via radiative corrections by the Lagrangian of the SM non-minimally coupled to gravity.
- The coefficients in front of these operators are assumed to be small and have the same hierarchy as the loop corrections producing them, i.e. the coefficients in front of the operators coming from two-loop diagrams are much smaller than those coming from one-loop diagrams, etc.
- The renormalisation scale is chosen as

$$\mu^2 \propto M_P^2/2 + \xi H^\dagger H \, .$$

This is equivalent to the requirement of scale invariance of the UV complete theory at large values of the Higgs field background, as in the classical theory

Low energy and high energy couplings in the SM

Bezrukov, Magnin, MS, Sibiryakov; related study: Burgess, Patil, Trott

Einstein frame Lagrangian

$$\frac{(\partial\chi)^2}{2} - \frac{\lambda}{4}F^4(\chi) + i\bar{\psi}_t \partial\!\!\!/ \psi_t + \frac{y_t}{\sqrt{2}} F(\chi)\bar{\psi}_t\psi_t \,.$$

The function $F(\chi) \equiv h(\chi)/\Omega(\chi)$ coincides with the Higgs field at low energies and encodes all the non-linearities associated to the non-minimal coupling to gravity in the large field regime

$$F(\chi) \approx \left\{ \begin{array}{ll} \chi & , \chi < \frac{M_P}{\xi} \\ \frac{M_P}{\sqrt{\xi}} \left(1 - e^{-\sqrt{2/3}\kappa\chi}\right)^{1/2} & , \chi > \frac{M_P}{\xi} \end{array} \right\} ,$$

with $\kappa = M_P^{-1}$.

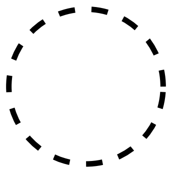
Exact h to χ relation:

$$\Omega^2 = 1 + \xi h^2/M_P^2$$

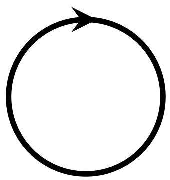
$$\frac{d\chi}{dh} = \sqrt{\frac{\Omega^2 + 6\xi^2 h^2/M_P^2}{\Omega^4}}$$

Higgs coupling

One loop effective potential : vacuum diagrams

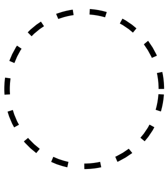


$$= \frac{1}{2}\mathrm{Tr}\ln\left[\Box - \left(\frac{\lambda}{4}(F^4)''\right)^2\right] ,$$

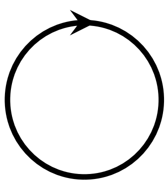


$$= -\mathrm{Tr}\ln\left[i\partial\!\!\!/ + y_t F\right] ,$$

whose evaluation, using the standard techniques, gives



$$= \frac{9\lambda^2}{64\pi^2}\left(\frac{2}{\bar{\epsilon}} - \ln\frac{\lambda(F^4)''}{4\mu^2} + \frac{3}{2}\right)\left(F'^2 + \frac{1}{3}F''F\right)^2 F^4,$$



$$= -\frac{y_t^4}{64\pi^2}\left(\frac{2}{\bar{\epsilon}} - \ln\frac{y_t^2 F^2}{2\mu^2} + \frac{3}{2}\right) F^4 .$$

Counter-terms

The divergencies in the loop diagrams are eliminated by adding counterterms with the definite coefficients in $1/\bar{\epsilon}$ and arbitrary finite parts $\delta\lambda_1$ and $\delta\lambda_2$

$$L_{\text{ct}} = \left(-\frac{2}{\bar{\epsilon}}\frac{9\lambda^2}{64\pi^2} + \delta\lambda_1\right)\left(F'^2 + \frac{1}{3}F''F^2\right)^2 F^4 + \left(\frac{2}{\bar{\epsilon}}\frac{y_t^4}{64\pi^2} - \delta\lambda_2\right) F^4. \qquad (1)$$

The structure of the counterterm involving $\delta\lambda_2$ coincides with that of the tree-level potential. This allows us to eliminate the constant $\delta\lambda_2$ by incorporating it into the definition of λ. The constant $\delta\lambda_1$, on the contrary, cannot be reabsorbed: new parameter and new term in the action!

Small versus large field values

- Small field values $F(\chi) \sim \chi \ll M_P/\xi$: the conformal factor $\Omega(\chi)$ equals to one and the theory becomes indistinguishable from the renormalizable SM. The new term turns into a simple power of the Higgs field, $\delta\lambda_1 \chi^4/4$, which allows to reabsorb the constant $\delta\lambda_1$ into the definition of λ.
- Large field values $F(\chi) \sim M_P/\sqrt{\xi}$, $\chi \gtrsim M_P$, the counter-term is exponentially suppressed $\sim \delta\lambda \frac{M_P^4}{\xi^4} e^{-4\chi/\sqrt{6}M_p}$ and the previously absorbed contribution to λ effectively disappears.

"Jump" of λ

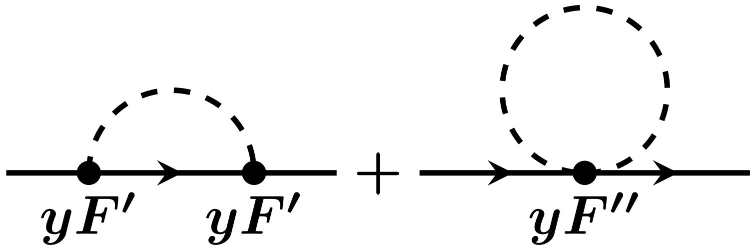
Neglecting the running of $\delta\lambda_1$ between the scales $\mu \sim M_P/\xi$ and $M_P/\sqrt{\xi}$, we can imitate this effect by a change

$$\lambda(\mu) \to \lambda(\mu) + \delta\lambda \left[\left(F'^2 + \frac{1}{3} F'' F \right)^2 - 1 \right],$$

which occurs at $\mu \simeq M_P/\xi$.

Top Yukawa coupling

Propagation of the top quark in the background χ



Cancelling divergencies in these diagrams requires the counter-terms of the form

$$
\begin{aligned}
L_{\text{ct}} \quad &\sim \quad \left(\# \frac{y_t^3}{\bar{\epsilon}} + \delta y_{t1}\right) F'^2 F \bar{\psi}\psi \\
&+ \quad \left(\# \frac{y_t \lambda}{\bar{\epsilon}} + \delta y_{t2}\right) F''(F^4)''\bar{\psi}\psi ,
\end{aligned}
$$

“Jump” of y_t

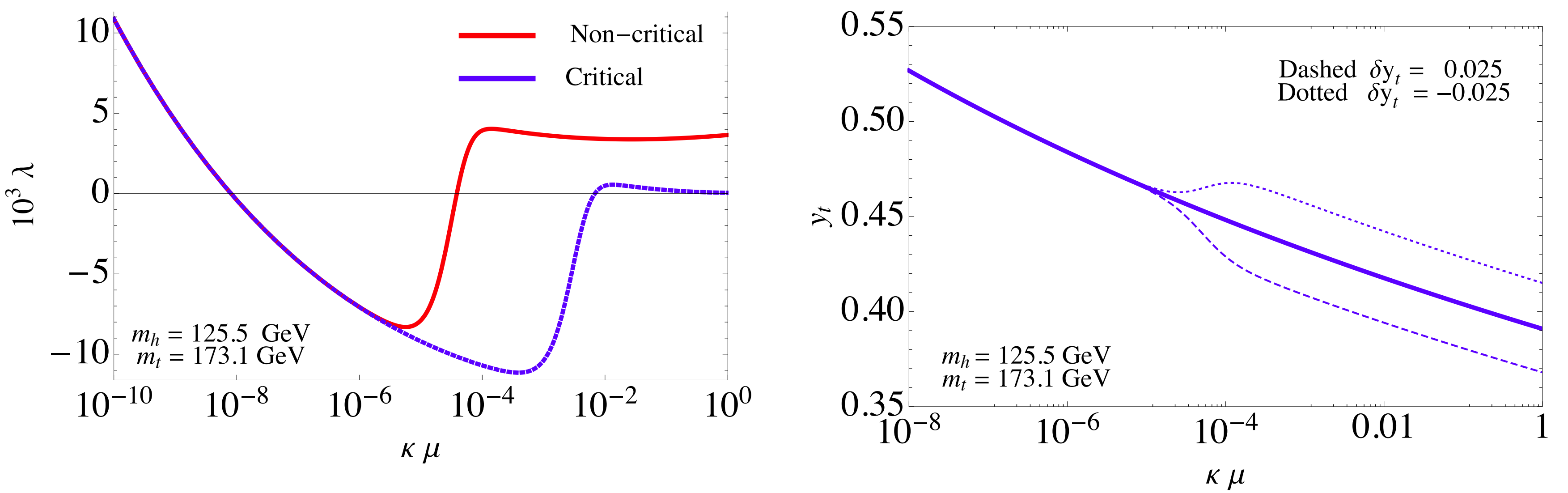
Effective change

$$y_t(\mu) \to y_t(\mu) + \delta y_t \left[F'^2 - 1 \right] \; + \; \text{higher orders},$$

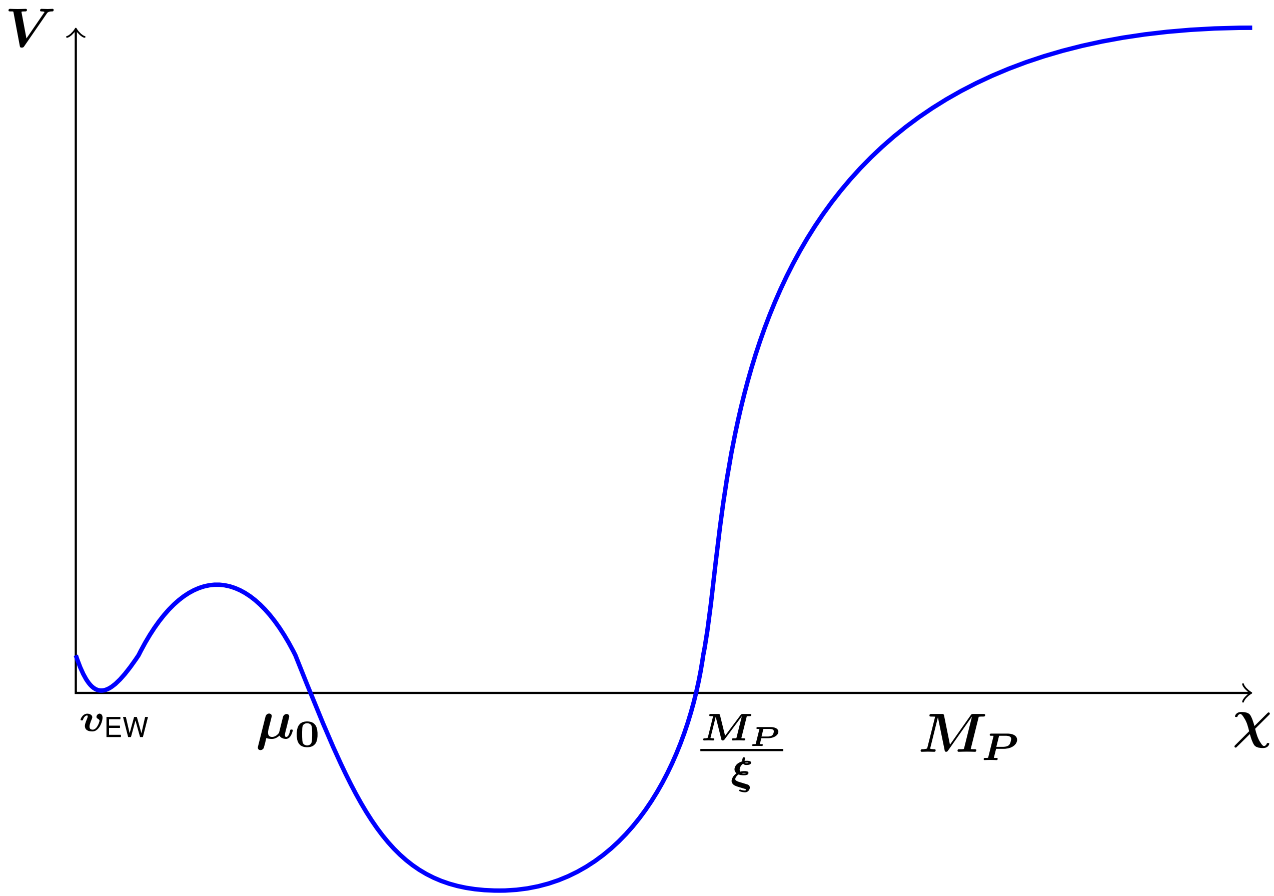
Higgs inflation with metastable vacuum

If $\delta\lambda \ll \lambda(M_P/\xi)$ and $\delta y_t \ll y_t(M_P/\xi)$ then the Higgs inflation can only take place if the SM vacuum is absolutely stable, i.e. only for $y_t < y_t^{\mathrm{crit}}$.

However, $\lambda(M_P/\xi)$ is small due to cancellations between fermionic and bosonic loops: $\delta\lambda$ can be of the order of λ



Higgs potential

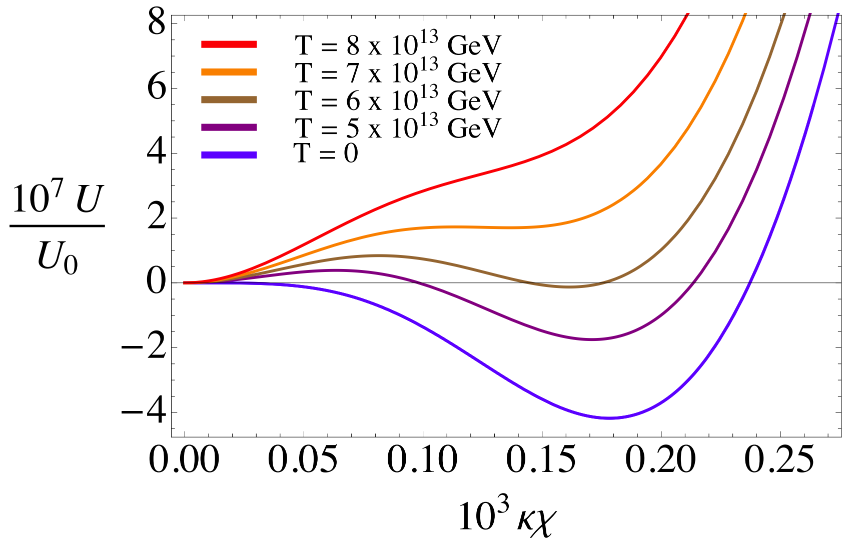


Time evolution

- Linde chaotic initial conditions: large Higgs field, slow-roll
- Oscillations of the Higgs field, particle creation, heating of the Universe
- Large Higgs value minimum of the effective potential disappears due to symmetry restoration
- The system is trapped in the SM vacuum and stays there till present time

Symmetry restoration

$$U_0 = (10^{-3} M_P)^4$$



Critical temperature for $\xi \sim 1000$, $T_c = 6 \times 10^{13}$ GeV

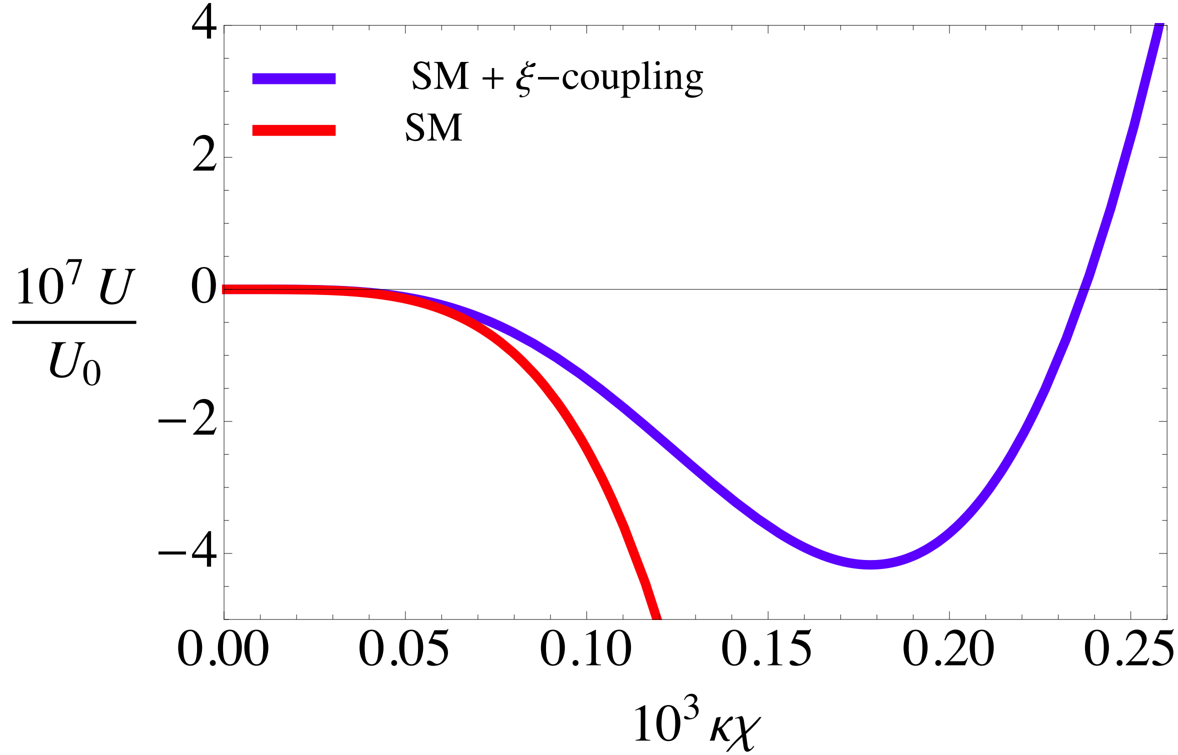
Temperature at which the vacuum with large vev disappears:

$T_+ \simeq 7 \times 10^{13}$ GeV

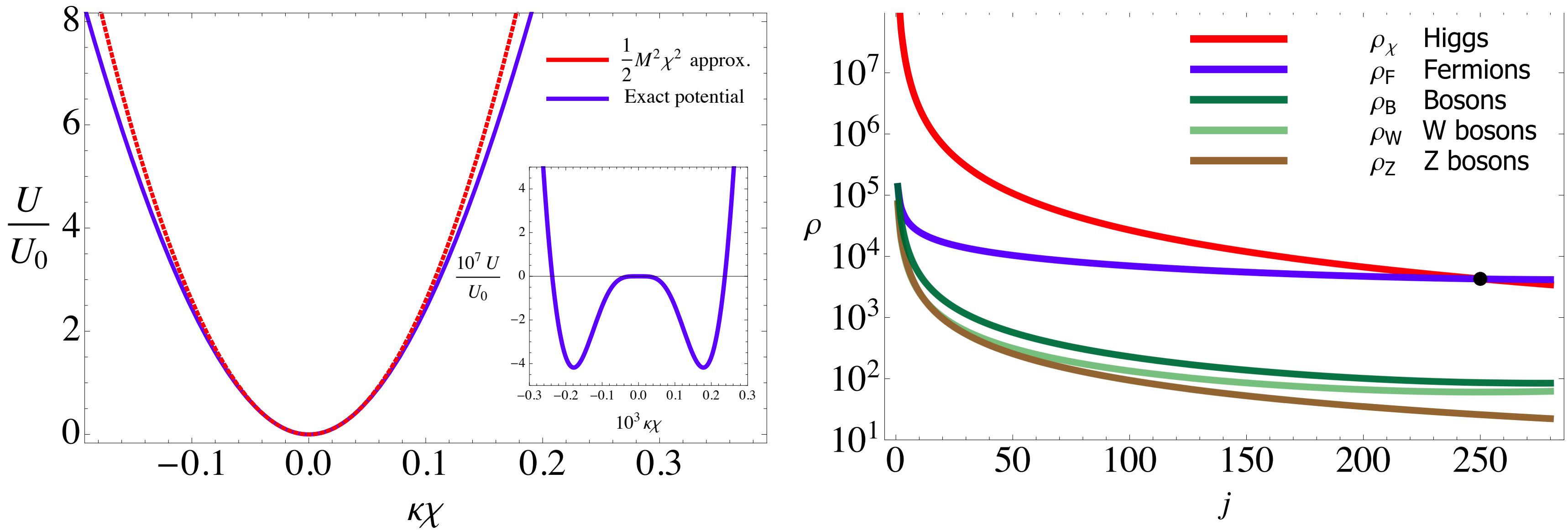
If the reheating temperature $T_R > T_+$ then the system relaxes in the minimum with $h = 0$

(Meta) stability of false vacuum

Computation for SM: Espinosa, Giudice, Riotto



Reheating



Reheating temperature $T_R \simeq 2 \times 10^{14}$ GeV $> T_+ \simeq 7 \times 10^{13}$ GeV
Main processes:

- W and Z creation in Higgs field oscillations, when the Higgs field crosses zero
- W and Z decays into fermions
- fermion scattering at later stages

Predictions for critical indexes n_s and r are the same as for non-crtitical Higgs inflation

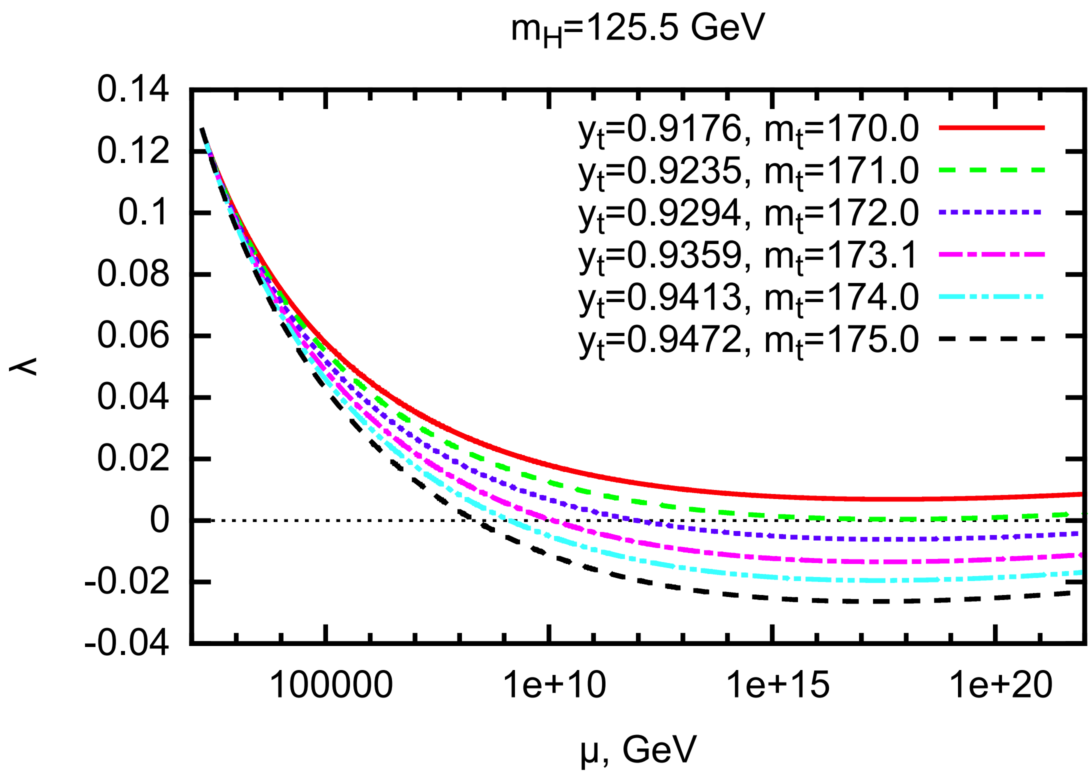
$$n_s = 0.97, \;\; r = 0.003$$

Critical Higgs inflation

Bezrukov, MS

For y_t very close to y_t^{crit} : critical Higgs inflation - tensor-to-scalar ratio can be large, $\xi \sim 10$

Behaviour of λ:



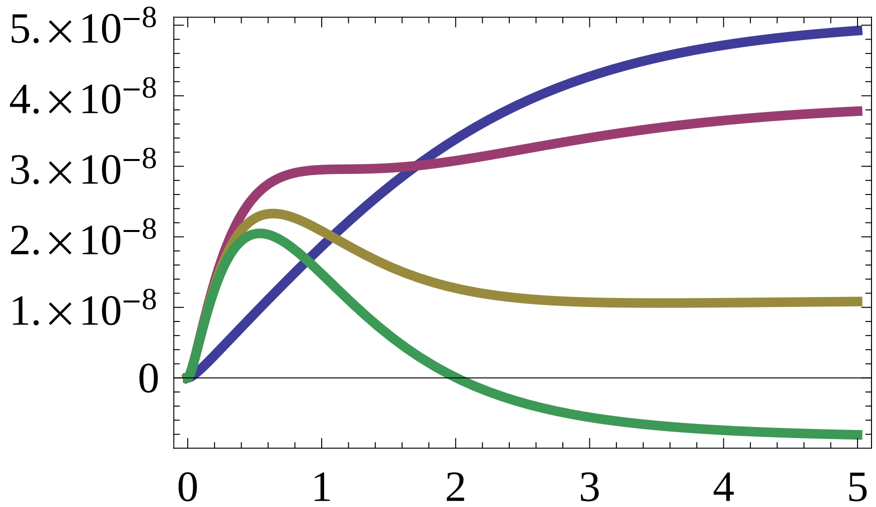
Effective potential

$$U(\chi) \simeq \frac{\lambda(z')}{4\xi^2}\bar{\mu}^4\,, \;\; z' = \frac{\bar{\mu}}{\kappa M_P},\;\; \bar{\mu}^2 = M_P^2\left(1 - e^{-\frac{2\chi}{\sqrt{6}M_P}}\right)$$

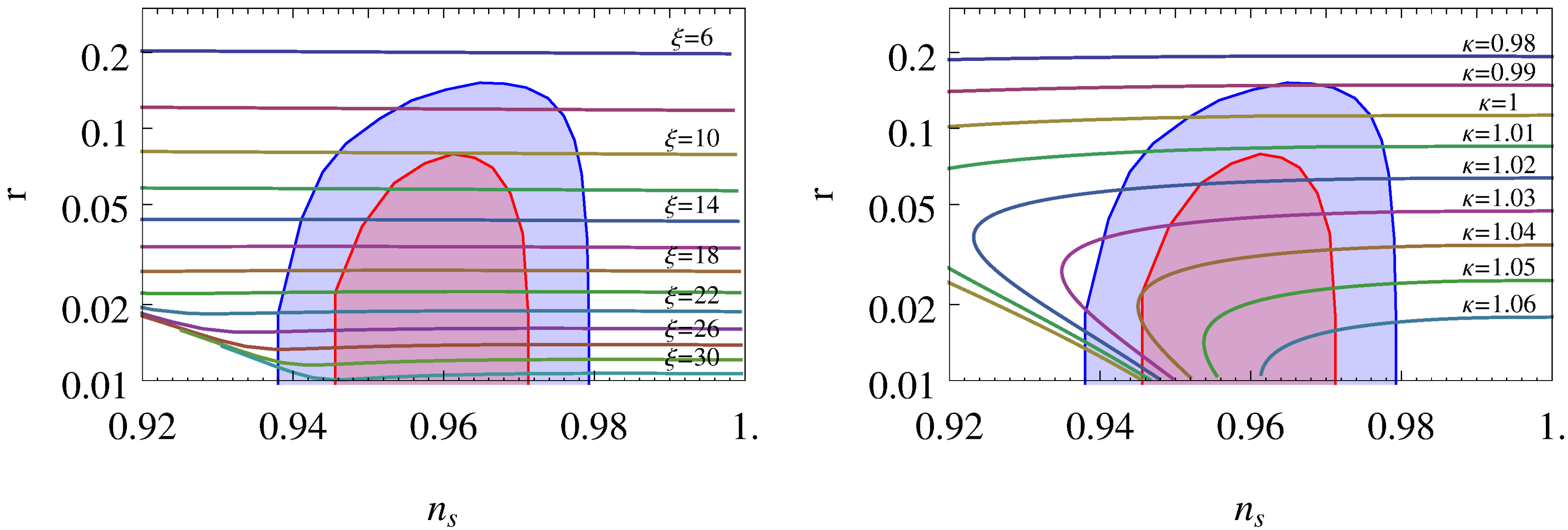
The parameter μ that optimises the convergence of the perturbation theory is related to $\bar{\mu}$ as

$$\mu^2 = \alpha^2\frac{y_t(\mu)^2}{2}\frac{\bar{\mu}^2}{\xi(\mu)}\,, \;\; \alpha \simeq 0.6$$

Behaviour of effective potential for $\lambda_0 \simeq b/16$:



The inflationary indexes



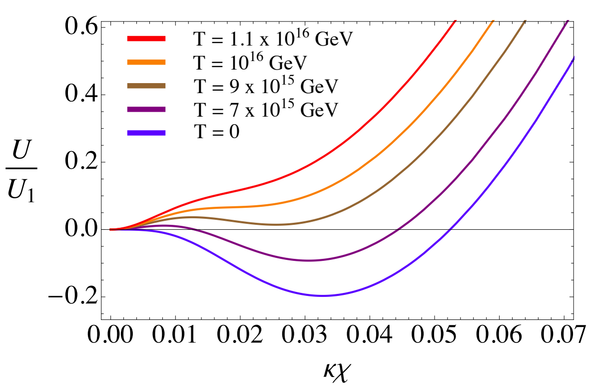
r can be large! BICEP 2?

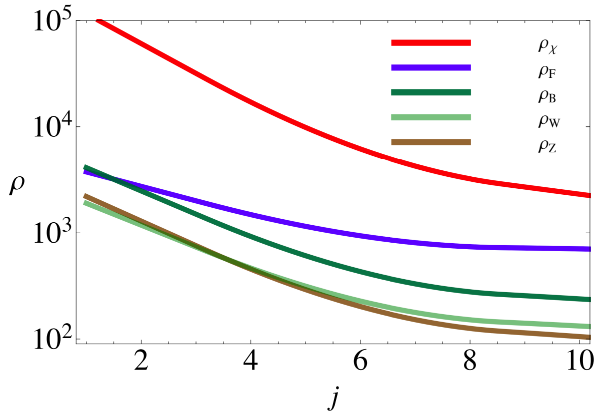
see also Hamada, Kawai, Oda and Park

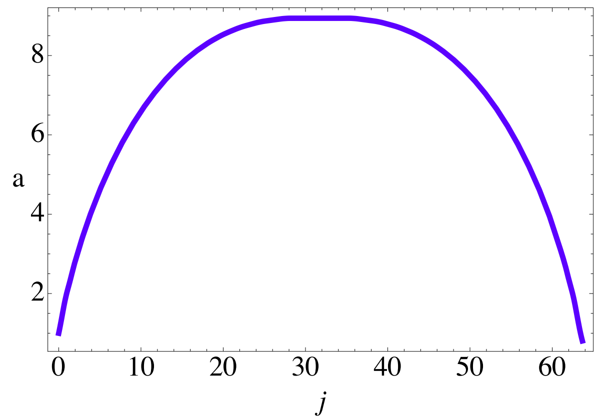
Critical Higgs inflation only works if both Higgs and top quark masses are close to their experimental values.

Critical Higgs inflation at $y_t > y_t^{\rm crit}$?

Critical Higgs inflation : small $\xi \sim 10$ - the depth of the large Higgs value vacuum is comparable with the energy stored in the Higgs after inflation: the required reheating temperature is too large, $T_+ \simeq 10^{16}$ GeV and cannot be achieved.







Conclusions

- Adding a minimal set of necessary counter-terms to the SM with non-minimal coupling to gravity does not spoil the flatness of the scalar potential for the large values of the Higgs field
- The relation between low energy parameters $(h < M_P/\xi)$ and high energy parameters $(h > M_P/\xi)$ is subject to uncertainties coming from addition of finite parts of counter-terms. This can be parametrised by "jumps" of coupling constants at $h \simeq M_P/\xi$: $\delta\lambda$, δy_t. The "jumps" cannot be found within the SM+gravity, and parametrise the ignorance of UV completion
- If these "jumps" are small in comparison with coupling constants taken at the scale M_P/ξ, the Higgs inflation is only possible for $y_t < y_t^{\mathrm{crit}}$, i.e. with the stable SM vacuum
- If $\delta\lambda \sim \lambda(M_P/\xi)$ the Higgs inflation can take place both for absolutely stable and metastable vacuum, with universal predictions $n_s = 0.97, \quad r = 0.003$ for a wide range of parameters
- For critical Higgs inflation corresponding to $y_t \approx y_t^{\mathrm{crit}}$ $\quad n_s$ and r can be substantially different from these values, but the stability of the SM vacuum is required.