Searching for general relativistic signatures on large scales

Jinn-Ouk Gong

APCTP, Pohang 790-784, Korea

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CERN Theory Institute
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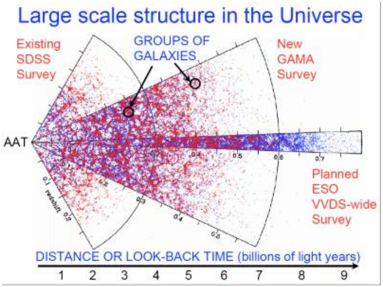
Based on collaborations with S. G. Biern, J.-c. Hwang, D. Jeong and H. Noh

Outline

Why GR in LSS?

Planned galaxy surveys: DESI, HETDEX, LSST, Euclid...



Larger and larger volumes, eventually accessing the scales comparable to the horizon: beyond Newtonian gravity, fully general relativistic approach (or any modification) is necessary

Why non-linearity and gauge in LSS?

- Non-linearity is prominent in large scale structure thus accurate modeling of of non-linearity is very important
- GR is a gauge theory, thus observational quantities only make sense after choosing the coordinate systems

On large scales where non-linearity can be probed by observations with improved accuracy, density contrast $\delta \equiv (\rho - \rho_0)/\rho_0$ deviates the Newtonian prediction

Q: how the deviations appear on large scales at non-linear level?

Setup and perturbation variables

We consider scalar metric pert in Einstein-de Sitter universe

$$g_{00} = -(1+2\alpha)dt^2\,, \quad g_{0i} = -a\beta_{,i}\,, \quad g_{ij} = a^2\left[(1+2\varphi)\delta_{ij} + \gamma_{,ij}\right]$$

The dynamical equations to be solved are:

$$\begin{aligned} \text{Energy conservation eq } &\rightarrow \text{Continuity eq} \\ \text{Trace of the Einstein eq } &\rightarrow \text{Euler eq} \end{aligned}$$

We identify the perturbation variables as

$$\delta \equiv \frac{\rho - \rho_0}{\rho_0} \quad \text{with} \quad \rho \equiv -T^0{}_0$$

$$\theta \equiv \frac{\nabla \cdot \boldsymbol{u}}{a} = 3H - K^i{}_i$$

Strategy for non-linear perturbations

With the linear solution the same as the standard one

$$\delta_1(\boldsymbol{k},t) = D(t)\delta_1(\boldsymbol{k},t_0)$$

we expand $\delta = \delta_1 + \delta_2 + \cdots$ using symmetric kernels

$$\delta(\boldsymbol{k},t) = \sum_{n=1}^{\infty} D^n(t) \int \frac{d^3 d_1 \cdots d^3 q_n}{(2\pi)^{3(n-1)}} \delta^{(3)}\left(\boldsymbol{k} - \boldsymbol{q}_1 - \cdots \boldsymbol{q}_n\right) \times F_n(\boldsymbol{q}_1, \cdots \boldsymbol{q}_n)\delta_1(\boldsymbol{q}_1)\cdots\delta_1(\boldsymbol{q}_n)$$

Then correlation functions are

$$\langle\delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_{12})P(k_1) \quad \text{with} \quad P = P_{11} + \underbrace{P_{22} + P_{13}}_{\text{1-loop}} + \cdots$$

$$\langle\delta(\boldsymbol{k}_1)\delta(\boldsymbol{k}_2)\delta(\boldsymbol{k}_3)\rangle = (2\pi)^3\delta^{(3)}(\boldsymbol{k}_{123})B(k_1,k_2,k_3)$$

$$\text{with} \quad B = B_{112} + \underbrace{B_{222} + B_{123} + B_{114}}_{\text{1-loop}} + \cdots$$

Comoving gauge

We set the gauge condition as

$$\gamma = 0 \quad \text{and} \quad T^0{}_i = 0$$

Kernels are found to be (Jeong, JG, Noh & Hwang 2011, Biern, JG & Jeong 2014)

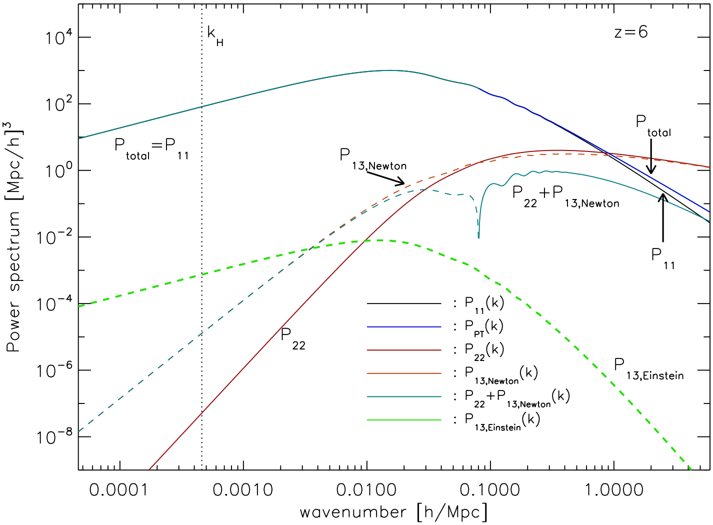
$$F_2 = \frac{5}{7} + \frac{\boldsymbol{q}_1 \cdot \boldsymbol{q}_2}{2q_1q_2}\left(\frac{q_1}{q_2} + \frac{q_2}{q_1}\right) + \frac{2}{7}\left(\frac{\boldsymbol{q}_1 \cdot \boldsymbol{q}_2}{q_1q_2}\right)^2$$

$$F_3 = F_{3\mathrm{N}} + F_{3\mathrm{GR}} \quad \text{where} \quad F_{3\mathrm{GR}} \propto k_H^2 \quad \text{with} \quad k_H \equiv aH$$

$$F_4 = F_{4\mathrm{N}} + (\cdots)k_H^2 + (\cdots)k_H^4$$

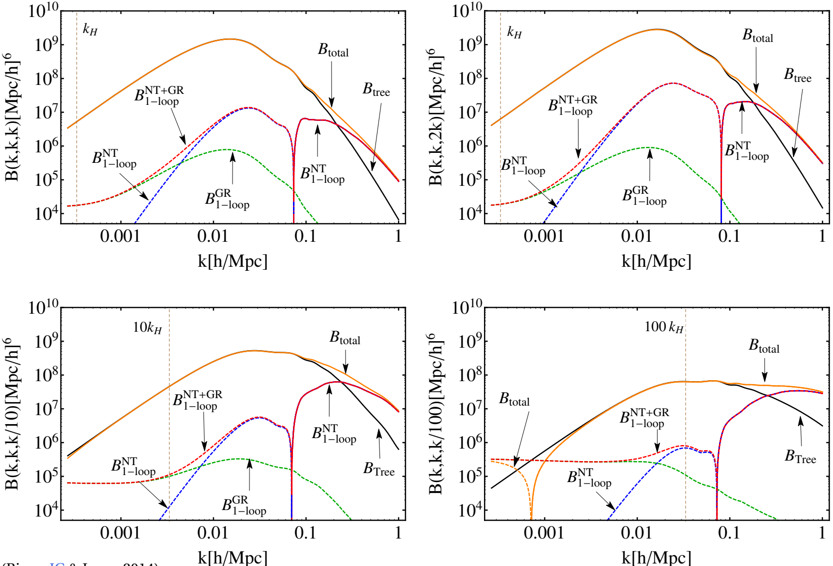
- Those w/o φ are identical to the Newtonian kernels
- Newtonian kernels are the same as those found in the standard perturbation theory based on the Newtonian gravity
- GR contributions appear from 3rd order, prop to $k_H \equiv aH$

Power spectrum with leading corrections in CG



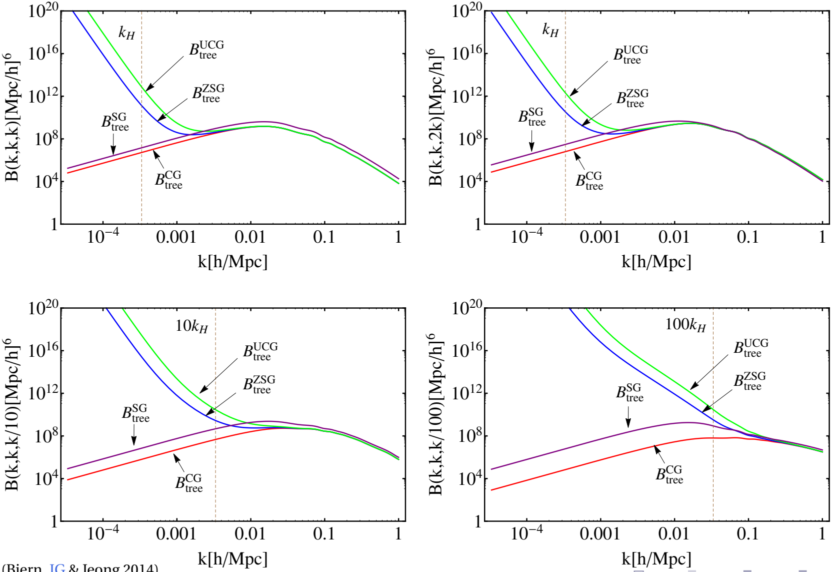
(Jeong, JG, Noh & Hwang 2011)

Bispectrum with leading corrections in CG



(Biern, JG & Jeong 2014)

Leading bispectrum in various gauges



(Biern, JG & Jeong 2014)

Synchronous gauge

We set the gauge condition as

$$g_{00} = -1 \quad \text{and} \quad g_{0i} = 0$$

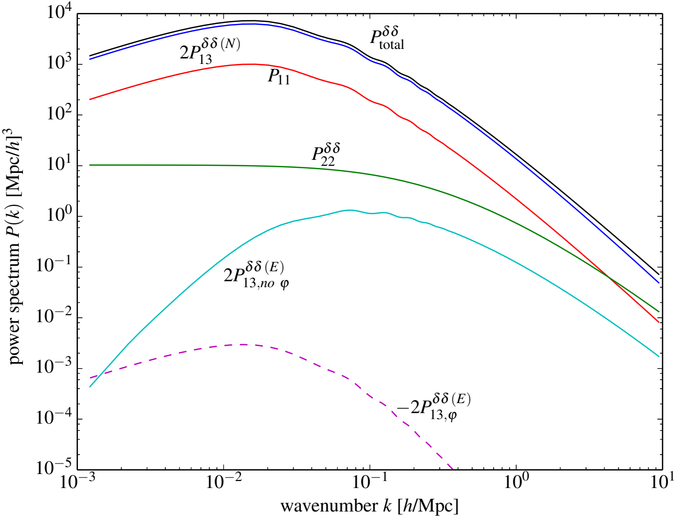
Kernels are found to be (Hwang, Noh, Jeong, JG & Biern 2014)

$$F_2 = \frac{5}{7} + \frac{2}{7}\frac{(\boldsymbol{q}_1 \cdot \boldsymbol{q}_2)^2}{q_1^2 q_2^2}$$

$$F_3 = F_{3\mathrm{N}} + F_{3\mathrm{GR},\varphi} + F_{3\mathrm{GR,no}\ \varphi}$$

- Newtonian kernels are *different* from standard ones
- Some GR contributions are not from φ but from non-linear coupling w/o k_H (thus time independent)

Power spectrum with leading corrections in SG

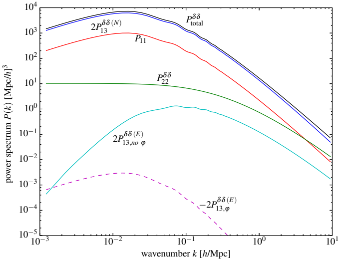


(Hwang, Noh, Jeong, JG & Biern 2014)

Newtonian interpretation of CG and SG

The problem lies in the Newtonian contributions

$$\dot{\delta} + \frac{1}{a}(1+\delta)\nabla\cdot\boldsymbol{u} = 0\,, \quad \dot{\theta} + 2H\theta + 4\pi G\rho\delta + \frac{1}{a^2}u^{i,j}u_{j,i} = \text{(NL terms)}$$



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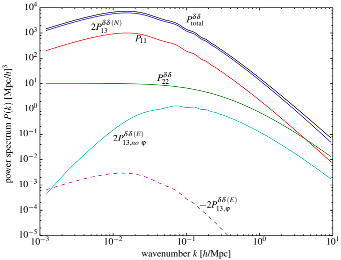
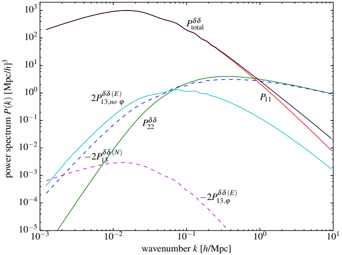
$$\Downarrow \quad \frac{d}{dt} \to \frac{d}{dt} + \frac{1}{a}\boldsymbol{u}\cdot\nabla \quad \text{transformation to convective derivative}$$

$$\dot{\delta} + \frac{1}{a}\nabla\cdot[(1+\delta)\boldsymbol{u}] = 0\,, \quad \dot{\theta} + 2H\theta + 4\pi g\rho\delta + \frac{1}{a^2}\nabla\cdot[(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}] = (\text{NL terms})$$





(Hwang, Noh, Jeong, JG & Biern 2014)

Putting dark energy on the table

Previous strategy is not complete

- ΛCDM power spectrum in EdS background
- Matter domination all the way

But we know the universe has been dominated by DE for a long time

$$\rho = \rho_m \longrightarrow \rho = \rho_m + \rho_{de}$$

For simplicity

1. No DE perturbation cf. Park, Hwang, Lee & Noh 2009
2. Comoving gauge

Effects of dark energy

Obviously BG changes

$$\mathcal{H}^2 = \frac{8\pi G}{3} a^2 \rho$$
$$\mathcal{H}' = -\frac{1}{2}\mathcal{H}^2 (1+3w) \quad \text{with} \quad w = [w_0 + w_a(1-a)]\,(1-\Omega_m)$$

DE also changes the energy constraint

$$-\Delta\varphi + \mathcal{H}\theta - \frac{3}{2}\mathcal{H}^2\delta = 0 \longrightarrow -\Delta\varphi + \mathcal{H}\theta - \frac{3}{2}\mathcal{H}^2 \Omega_m \delta = 0$$

Non-linear solutions also change!

$$\delta_2(\boldsymbol{k}, t) = D_{2a} \times (\text{from continuity}) + D_{2b} \times (\text{from Euler})$$

Only for EdS $D_1(t) = a(t)$ and $D_{2a} = \frac{3}{7} D_1^2$ $\left[\text{cf. } D_{2b} = \frac{1}{2}\left(D_1^2 - D_{2a}\right)\right]$

Relativistic kernels

Newtonian kernels are known Kamionkowski & Buchalter 1999 (2nd) and Takahashi 2008 (3rd) and pure GR terms consist of 7 contributions (Biern & JG)

$$\delta_{3\mathrm{GR}}(a;\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3)=\sum_{i=a}^{g}D_{3i}(a)\times(\text{momentum dependent parts})$$

Growth functions satisfy

$$\frac{d^2D}{da^2}+\frac{3}{2a}(1-w)\frac{dD}{da}-\frac{3}{2a^2}\Omega_m D=\text{couplings}$$

The coupling terms have H, dH/da and $d\Omega_m/da$ (Newtonian ones do not!) which are sensitive to BG, e.g.

$$\frac{d\Omega_m}{da}=\frac{3\left[w_0+w_a(1-a)\right](1-\Omega_{m0})\Omega_{m0}a^{-1+3w_0+3w_a}e^{3(1-a)w_a}}{\left\{1+\left[-1+a^{3(w_0+w_a)}e^{3(1-a)w_a}\right]\Omega_{m0}\right\}^2}$$

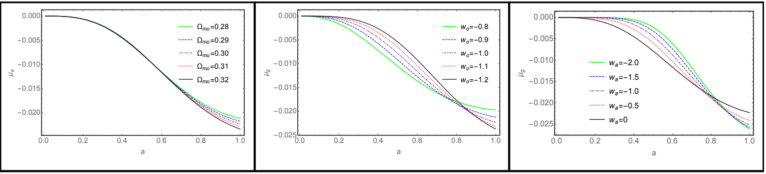
Kernel evolution

Kernel form of the solution is different from EdS case

$$\delta_{3\mathrm{GR}}(a,\boldsymbol{k}) = D_1^3(a)\int\frac{d^3q_1d^3q_2d^3q_3}{(2\pi)^{3\cdot 2}}\delta^{(3)}\left(\boldsymbol{k}-\boldsymbol{q}_{123}\right)\left[\sum_{i=a}^{g}\mu_i(a)F_{3i}(\boldsymbol{q}_1,\boldsymbol{q}_2,\boldsymbol{q}_3)\right]$$
$$\times\,\delta_1(\boldsymbol{q}_1)\delta_1(\boldsymbol{q}_2)\delta_1(\boldsymbol{q}_3)$$

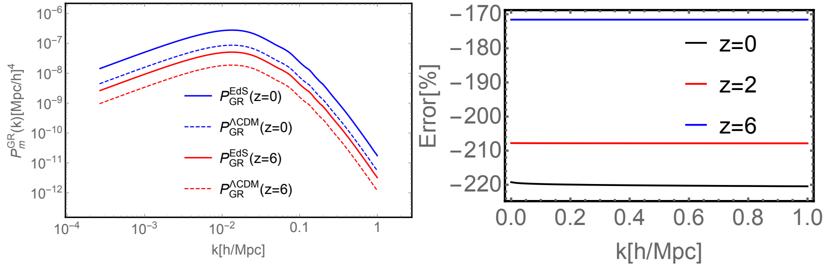
$$\mu_i(a)\equiv\frac{D_{3i}(a)}{D_1^3(a)}$$

μ_g



For EdS, $\mu_a=\cdots=\mu_d=\dfrac{2}{7}$, $\mu_e=\mu_f=-\dfrac{3}{7}$, $\mu_g=0$

Non-linear GR power spectrum



- EdS vs. ΛCDM: difference is $\mathcal{O}(100)\%$

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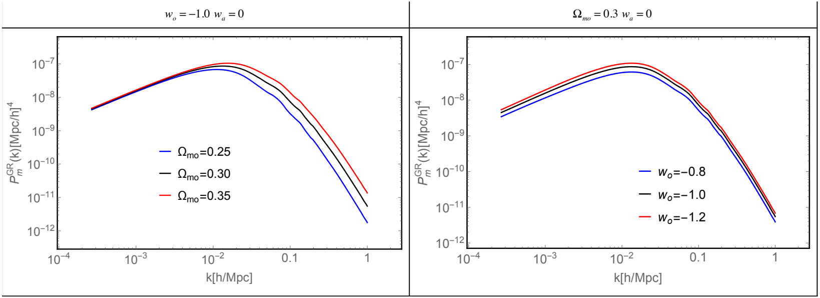
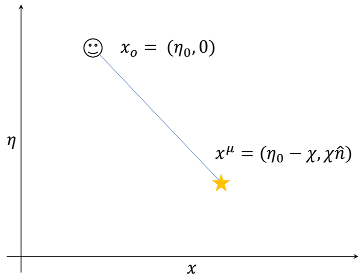


- EdS vs. ΛCDM: difference is $\mathcal{O}(100)\%$
- Parameter dependence: Ω_{m0} (change on large k) and w_0 (change on overall scales)

Observable galaxy number density

We observe as if photons come to us along a straight, unperturbed geodesic...



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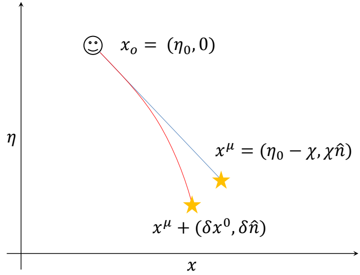
We observe as if photons come to us along a straight, unperturbed geodesic... but in fact the path is distorted due to perturbations at the locations of the observer and the source, and in between

(Yoo et al. 2009, Bonvin & Durrer 2011, Bertacca, Maartens & Clarkson 2014, Yoo & Zaldarriaga 2014...)



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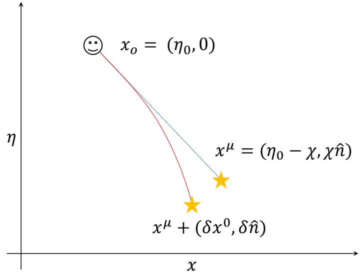
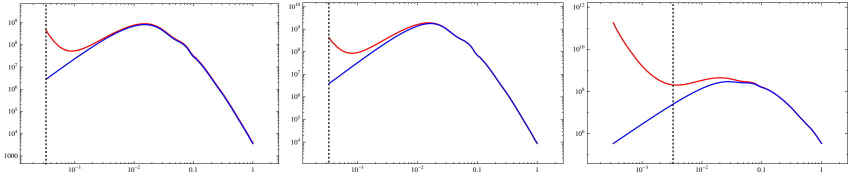


Observed number of galaxies N contained in vol $\widetilde{V}$

$$N = \int_{\widetilde{V}} \sqrt{-g} n_g \varepsilon_{\mu\nu\rho\sigma} u^\mu \frac{\partial x^\nu}{\partial \tilde{x}^1} \frac{\partial x^\rho}{\partial \tilde{x}^2} \frac{\partial x^\sigma}{\partial \tilde{x}^3} d^3\tilde{x} \quad \to \quad \text{Galaxy field } \delta_g = (\cdots)$$

Preliminary result

Galaxy bispectrum in different configurations (Biern, JG & Jeong; cf. Di Dio et al. 2014)



(Blue: Newtonian, red: Newtonian + GR contributions)

Conclusions

- As galaxy surveys are deeper and deeper, fully GR description is relevant
- Gauge dependence at non-linear order:
  - In CG the standard perturbation theory is reproduced
  - Pure GR corrections are heavily suppressed in almost all cases
  - Naively using SG leads to pathologies
  - Transformation by hands cures the problem
- Dark energy background greatly affects GR contributions
- Geodesic approach based on observable quantities should help