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# Understanding the Lower Redshift Universe

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## Collaborators:

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- ❖ *Ziv Mikulizky (Student at Technion)*
- ❖ *Jim Peebles*
- ❖ *Steven Phelps*

“Happy families are all alike, every unhappy family is unhappy in its own way”

*–Tolstoy, Anna Karenina*

*- J. Diamond, The Anna Karenina Principle*

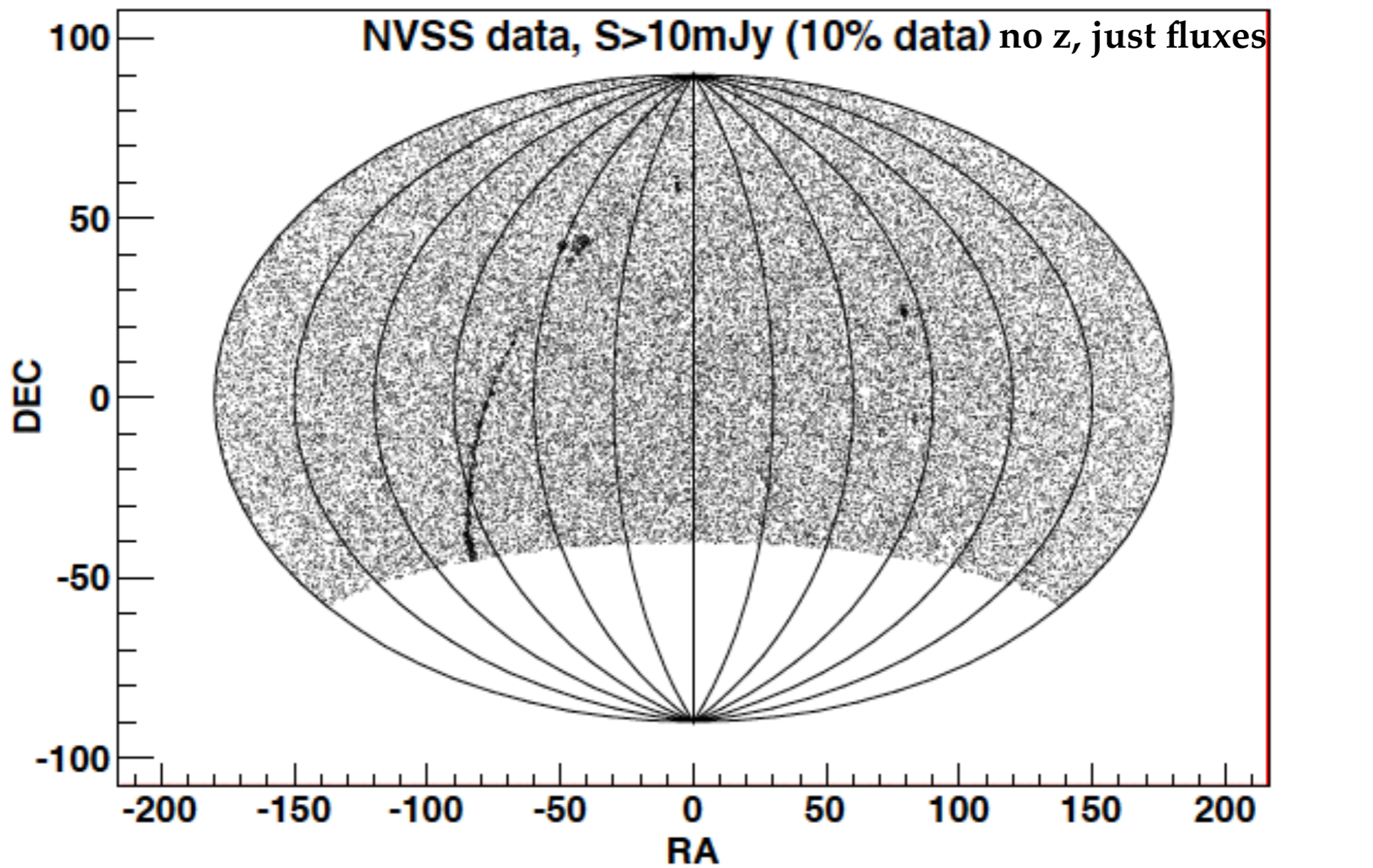
\* “Happy Community”:

- All reliable large scale data tell the same story.
- Very low level (but important) systematics.
- LSS from Local Group to ~150Mpc
  - traditional and New probes

\* The LCDM is a “Happy model” ... but a little “moody”

\* Therapy maybe required, perhaps by Dark sector physics





NRAO VLA Sky Survey,  $z \approx 1$ , 1.8 million sources in the entire survey



# The Cosmological Principle (Einstein 1917)

EINSTEIN: Zum kosmologischen Problem der allgemeinen Relativitätstheorie 235

## Zum kosmologischen Problem der allgemeinen Relativitätstheorie.

VON A. EINSTEIN.

Unter dem kosmologischen Problem wird die Frage über die Beschaffenheit des Raumes im großen und über die Art der Verteilung der Materie im großen verstanden, wobei die Materie der Sterne und Sternsysteme zur Erleichterung der Übersicht durch kontinuierliche Verteilung der Materie ersetzt gedacht wird. Seitdem ich kurz nach Aufstellung der allgemeinen Relativitätstheorie dieses Problem in Angriff nahm, sind nicht nur zahlreiche theoretische Arbeiten über diesen Gegenstand erschienen, sondern es sind durch HUBBELS Untersuchungen über den Dopplereffekt und die Verteilung der extra-galaktischen Nebel Tatsachen ans Licht getreten, welche der Theorie neue Wege weisen.

In meiner ursprünglichen Untersuchung ging ich von folgenden Annahmen aus:

1. Alle Stellen des Universums sind gleichwertig; im speziellen soll also auch die örtlich gemittelte Dichte der Sternmaterie überall gleich sein.
2. Räumliche Struktur und Dichte sollen zeitlich konstant sein.

Damals zeigte ich, daß man beiden Annahmen mit einer von Null verschiedenen mittleren Dichte  $\rho$  gerecht werden kann, wenn man das sogenannte kosmologische Glied an die Feldgleichungen der allgemeinen Relativitätstheorie einführt, so daß diese lauten:

$$(R_{ik} - \frac{1}{2} g_{ik} R) + \lambda g_{ik} = -k T_{ik} \dots \quad (1)$$

Diesen Gleichungen wird durch eine räumlich sphärische statische Welt vom Radius  $P = \sqrt{\frac{2}{\lambda \rho}}$  Genüge geleistet, wenn  $\rho$  die (druckfreie) mittlere Dichte der Materie bedeutet.

Nachdem nun aber durch HUBBELS Resultate klar geworden ist, daß die außer-galaktischen Nebel gleichmäßig über den Raum verteilt und in einer Dilatationsbewegung begriffen sind (wenigstens sofern man deren systematische Rotverschiebungen als Dopplereffekte zu deuten hat), hat die Annahme (2) von der statischen Natur des Raumes keine Berechtigung mehr, und es entsteht die Frage, ob die allgemeine Relativitätstheorie von diesen Befunden Rechenschaft zu geben vermag.

22\*

## Einstein 1931

*Allen Stellen des Universums sind gleichwertig; Im speziellen solo also such die örtlich gemittelte Dichte der Sternmaterie überall gleich seine*

## In English

*All places in the Universe are equivalent. In particular, the local, averaged density of stellar material ought to be the same everywhere.*

## Note

The second part of this statement is vague (not unusual for Einstein!) as the averaging process should refer to some physical scale. A more explicit Cosmological Principle is expressed by Milne (33,35).

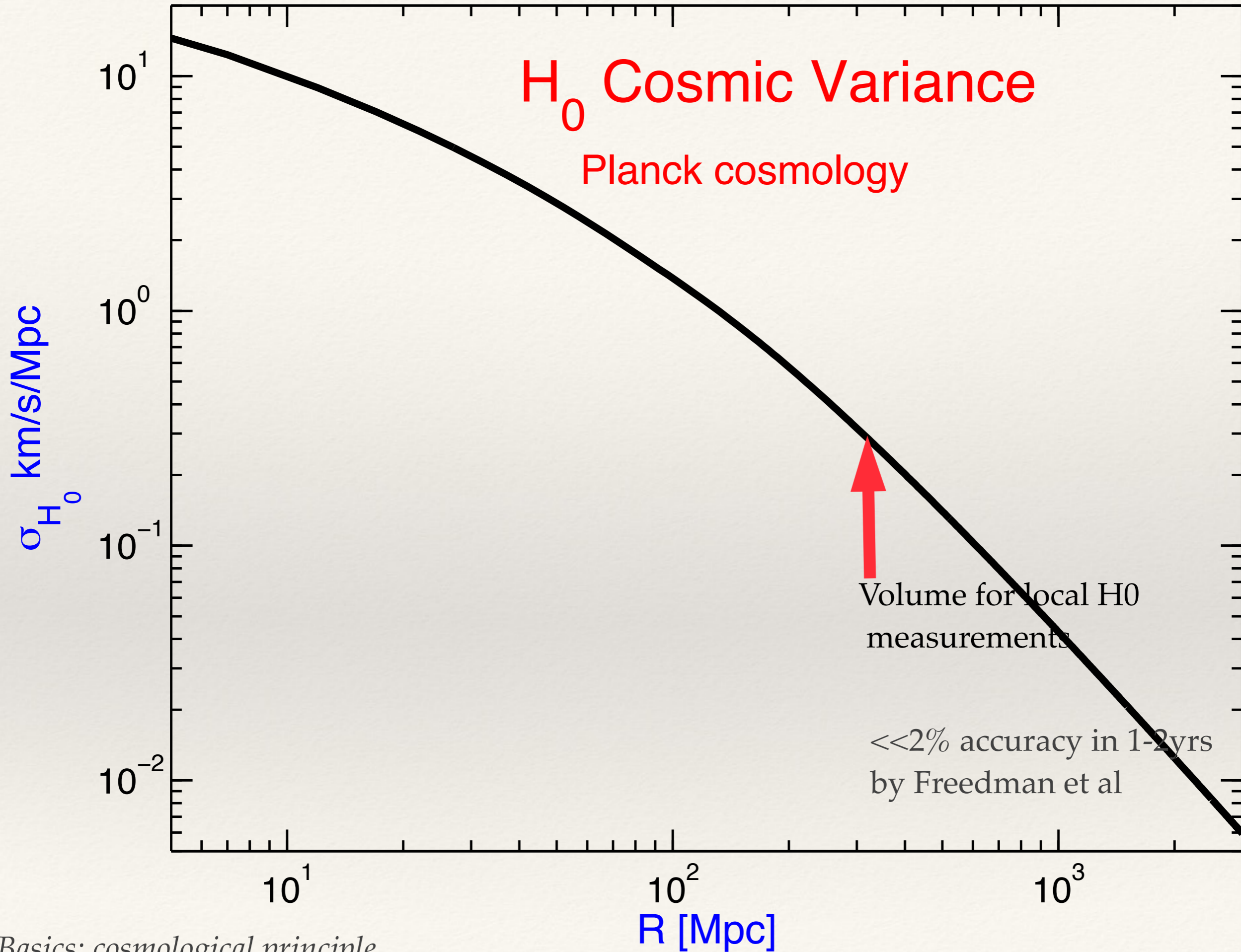
The name (Cosmological Principle) was given by Milne.

## In this talk, CP implies

- average  $\rho$  exists
- $\langle \delta \delta \rangle = \xi(r)$  approaches zero on the "largest scales".

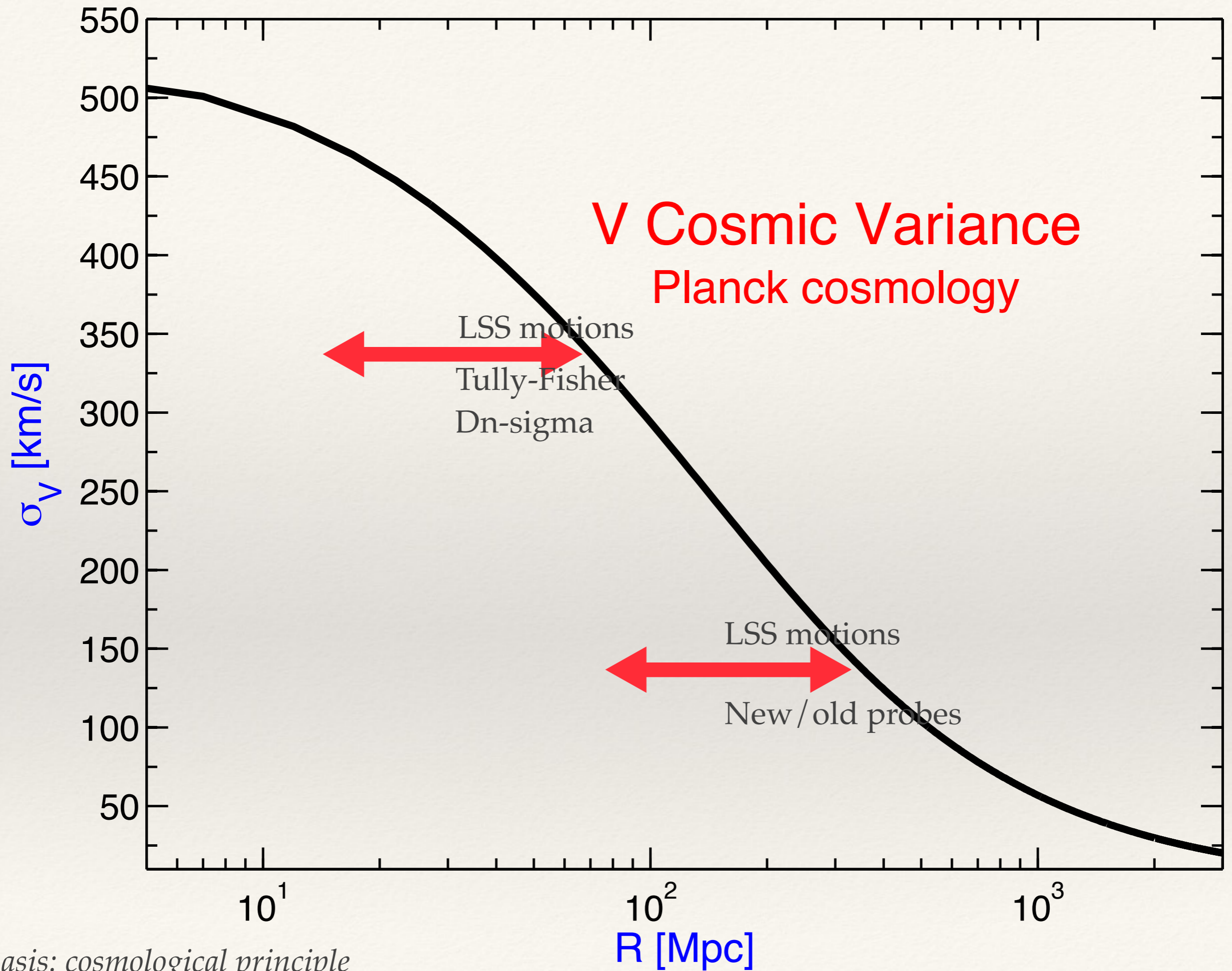


*Approaching homogeneity in LCDM*



Basics: cosmological principle



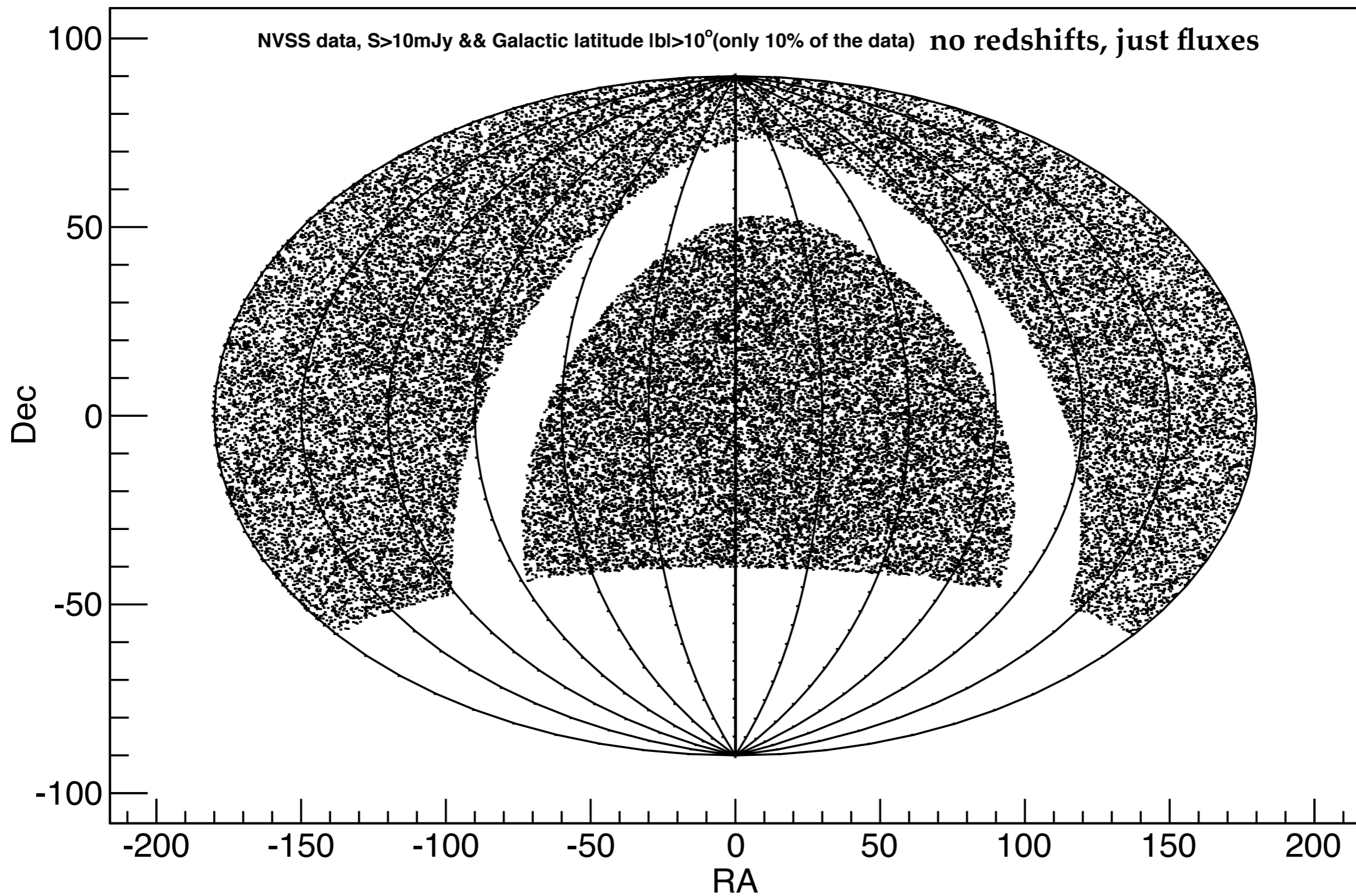


*Basis: cosmological principle*

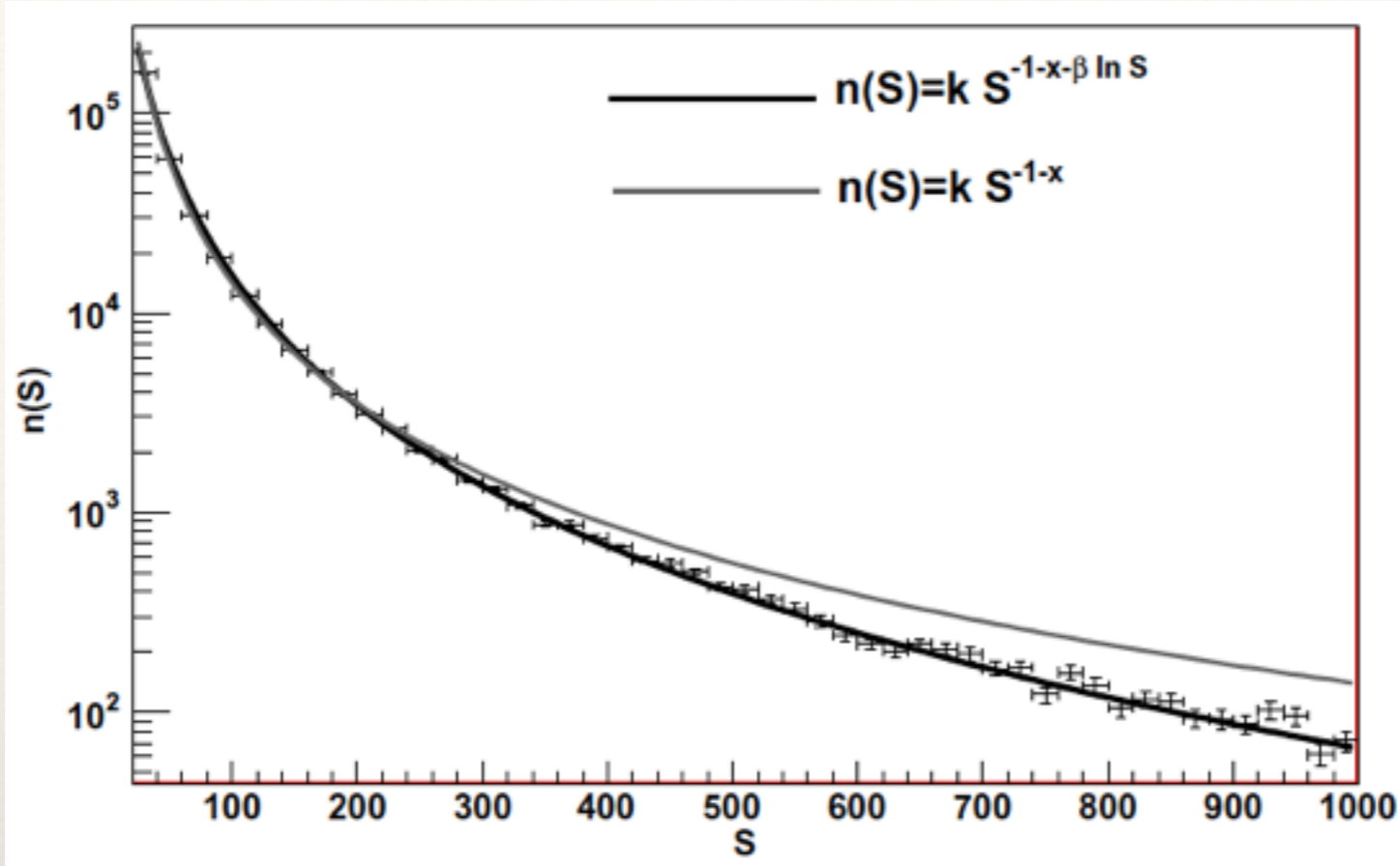
*Approaching homogeneity in observations*



## Dipoles in the Sky



## Mean number as a function of flux at 1.4GHz



## Angular dependence:

$$\left(\frac{dN}{d\Omega}\right)_{obs} = k(S_{low})^{-x} \delta^{2+x(1+\alpha)} = \left(\frac{dN}{d\Omega}\right)_{rest} \delta^{2+x(1+\alpha)}$$

$$\delta \approx 1 + (v/c) \cos \theta$$

$$S \propto \nu^{-\alpha}$$



## The NVSS observed dipole:

$S_{\text{low}}$	Set	$ \vec{D}_N $	$\vec{v}$		
			$ \vec{v} $ (km/s)	RA (deg)	DEC (deg)
10	(a)	$0.0113 \pm 0.0018$	$1020 \pm 170$	$137 \pm 15$	$8 \pm 16$
	(b)	$0.0096 \pm 0.0026$	$810 \pm 220$	$145 \pm 20$	$20 \pm 17$
20	(a)	$0.0153 \pm 0.0032$	$1290 \pm 270$	$147 \pm 19$	$-22 \pm 19$
	(b)	$0.0125 \pm 0.0040$	$1000 \pm 320$	$159 \pm 27$	$-15 \pm 22$
30	(a)	$0.0163 \pm 0.0038$	$1320 \pm 310$	$149 \pm 22$	$-21 \pm 22$
	(b)	<u><math>0.0143 \pm 0.0048</math></u>	<u><math>1110 \pm 370</math></u>	<u><math>159 \pm 34</math></u>	<u><math>-14 \pm 25</math></u>
40	(a)	$0.0157 \pm 0.0040$	$1230 \pm 310$	$143 \pm 25$	$-38 \pm 24$
	(b)	$0.0136 \pm 0.0049$	$1030 \pm 370$	$156 \pm 38$	$-31 \pm 27$
50	(a)	$0.0172 \pm 0.0047$	$1320 \pm 360$	$156 \pm 28$	$-39 \pm 27$
	(b)	$0.0157 \pm 0.0059$	$1160 \pm 440$	$175 \pm 43$	$-33 \pm 28$

**Solar motion in CMB frame: 369km/s,**

**RA=168,**

**DEC=-7**

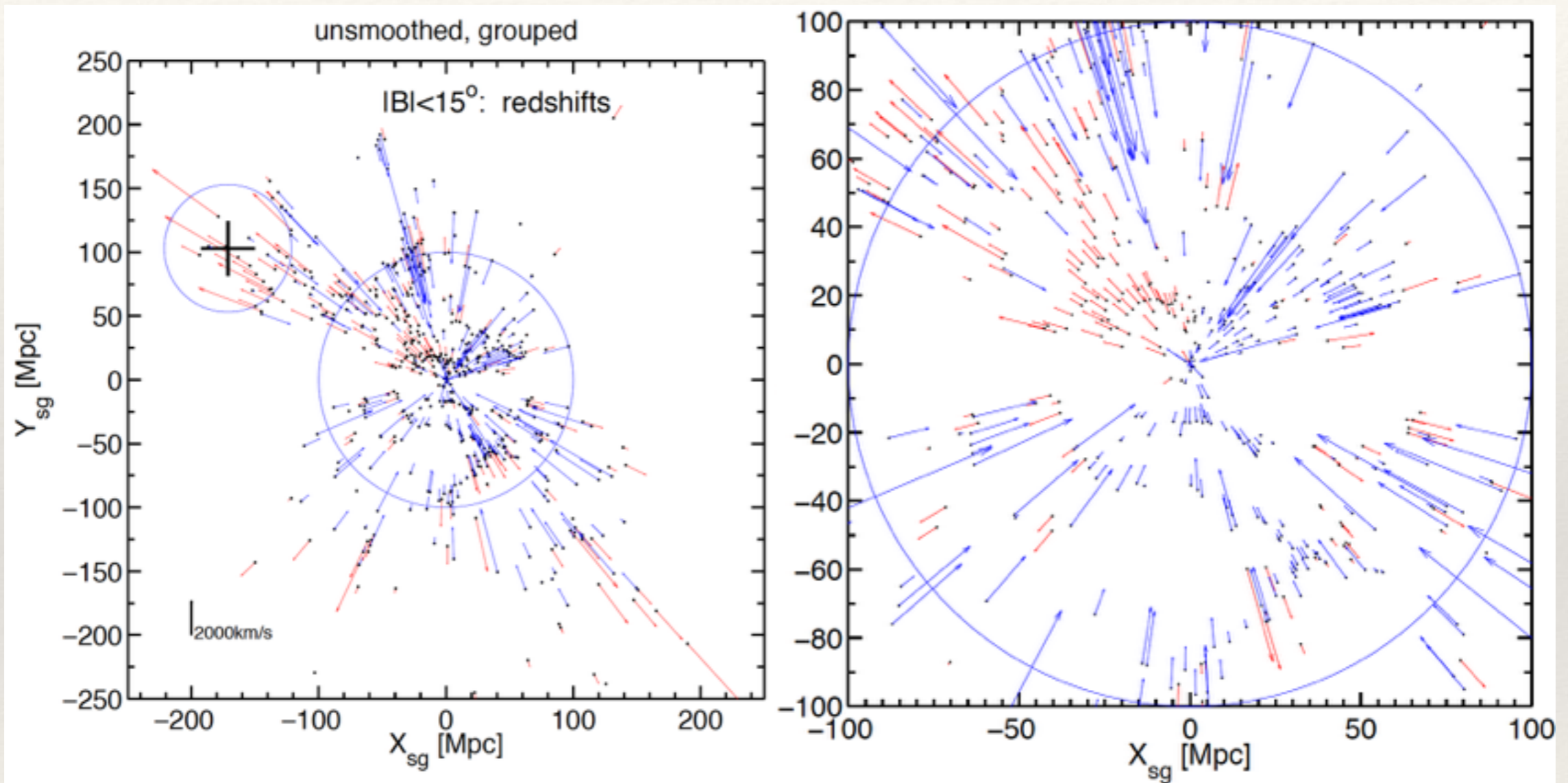
*How to Resolve this ``Discrepancy''?*



open big brackets



(radial) peculiar velocities of galaxies



SFI++, Cosmic Flows II  
Springob et al, Tully et al



The bulk flow on  $\lesssim 100 Mpc$

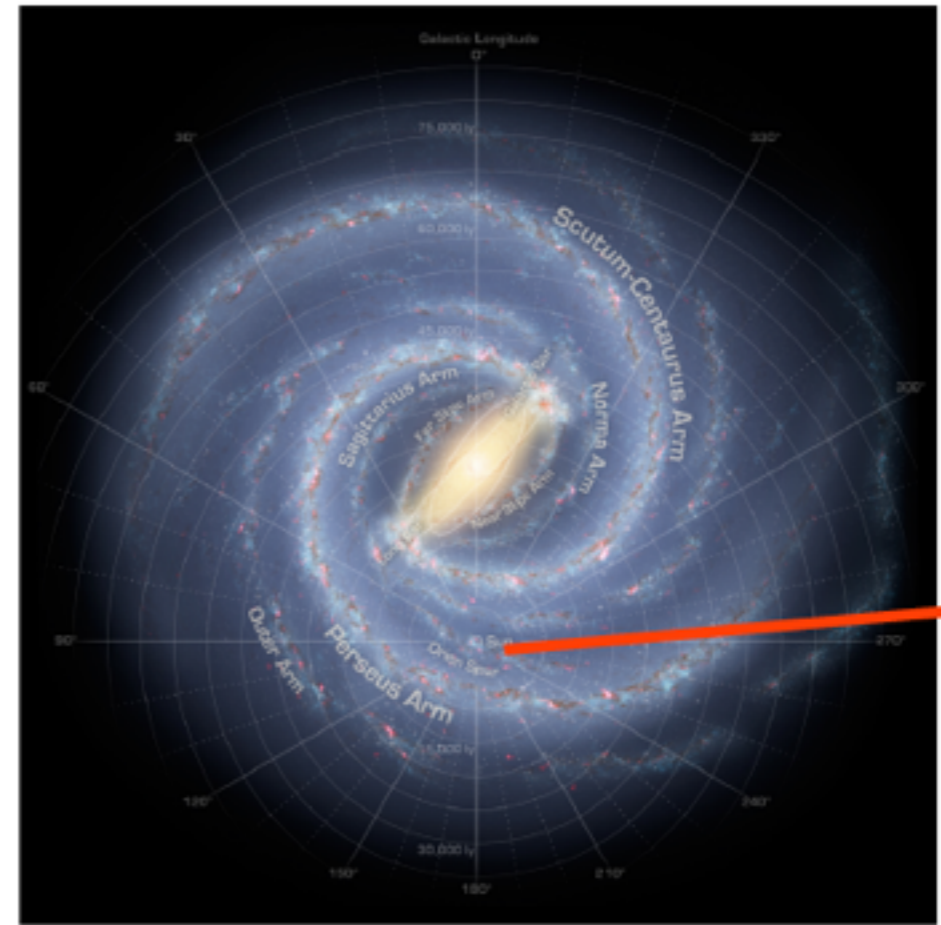
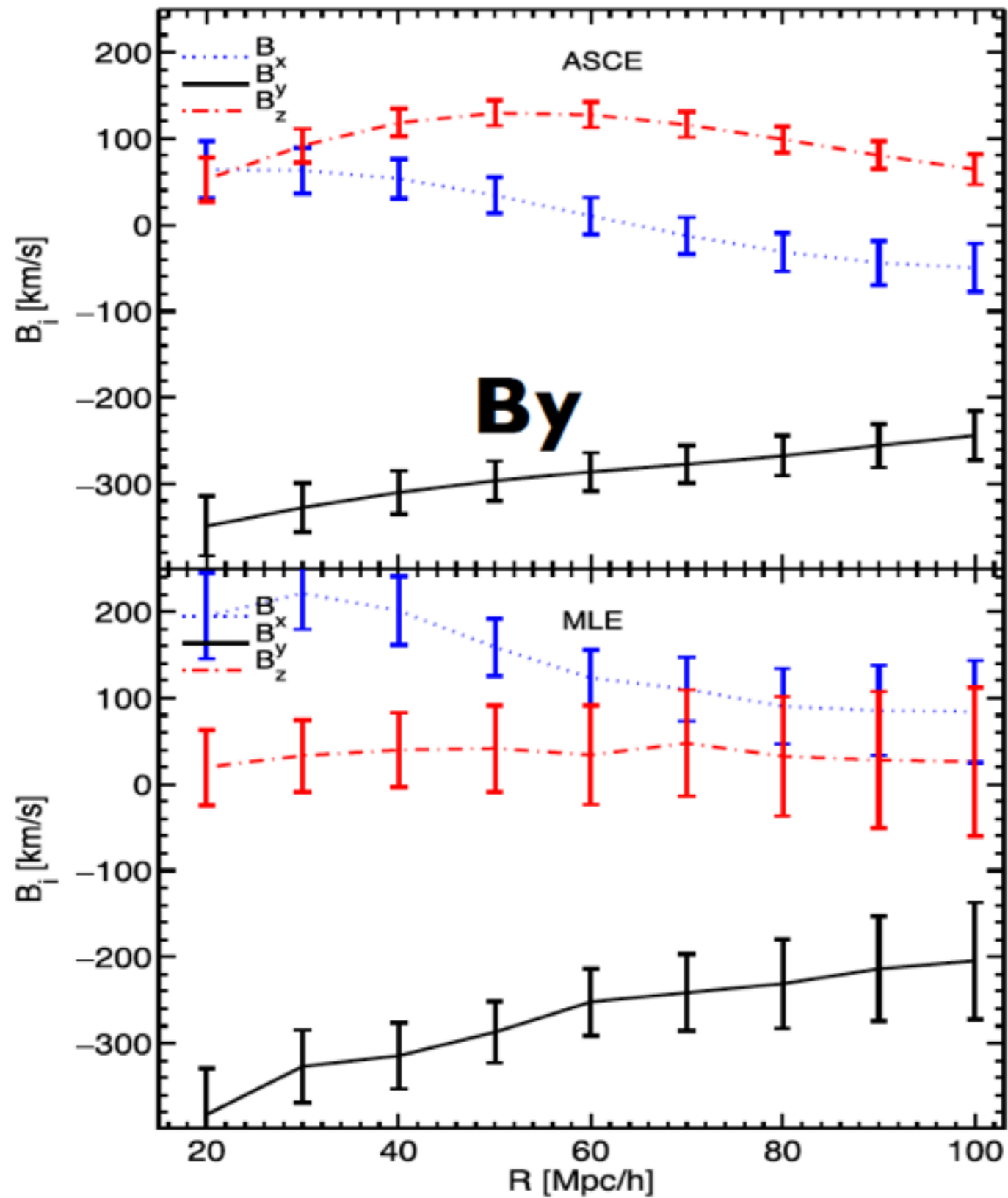
### Definition

$$\mathbf{B}(r) = \frac{3}{4\pi r^3} \int_{x < r} \mathbf{V}(\mathbf{x}) d^3x$$

$\mathbf{V}(\mathbf{x})$  is the 3D velocity field

**Simplest but most notorious moment of the observed velocities!**

# The 3 components (Galactic coordinates)



**B is at 40 deg to the SGP**  
**B is pretty featureless**



close big brackets



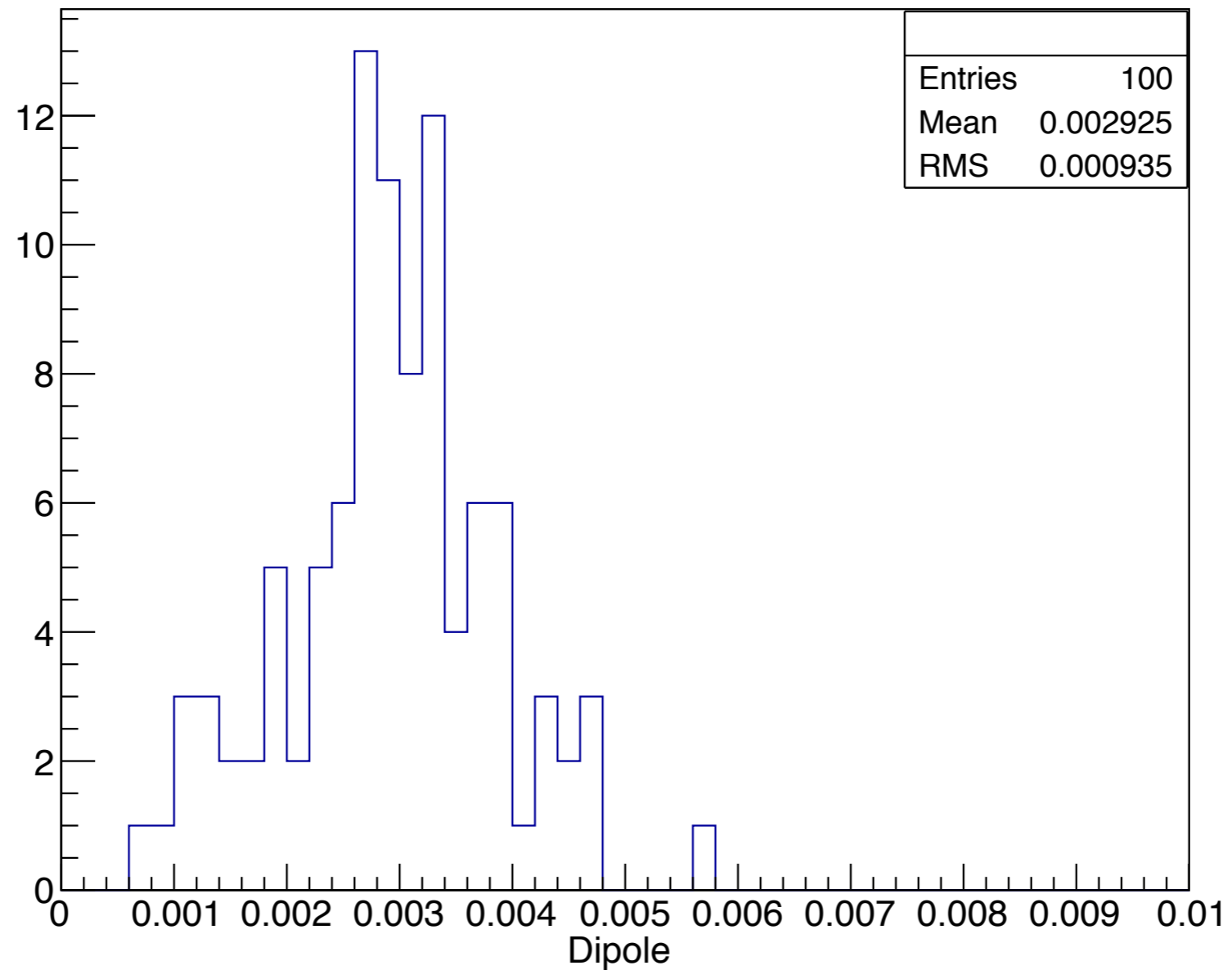
**B on 100Mpc/h is almost *parallel* to solar motion**

**There must be a dipole component of mass fluctuations at  $R > 100 \text{Mpc/h}$  to account for B.**

**This needs to be considered when we compare NVSS dipole component of  $n(S)$  to theoretical predictions, e.g. LCDM**



## “NVSS” Dipole in 100 mocks conditioned to yield proper B

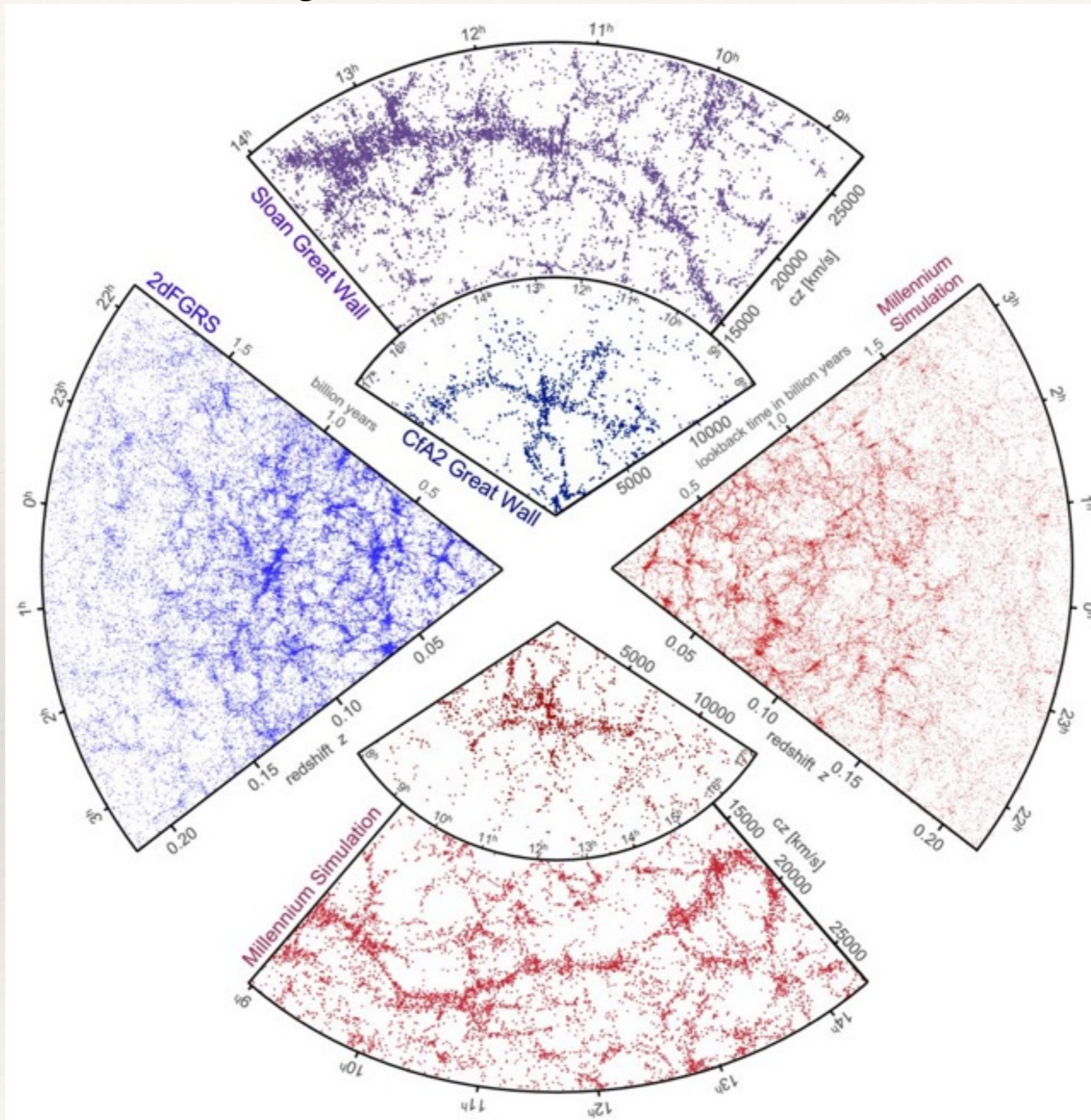


**This conditioning makes NVSS data consistent with LCDM at ~1sigma level**

# Dipoles in the Sky: *SDSS*



# Advantage of SDSS: measured redshifts!



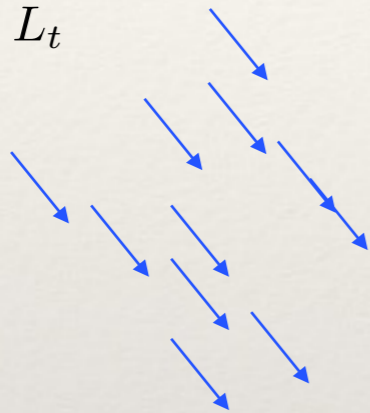


# A reincarnation of an old idea

Tammann, Yahil & Sandage 79 *and long before*

$$cz < Hr$$

$$L_0(cz) < L_t$$



$$cz > Hr$$

$$L_0(cz) > L_t$$

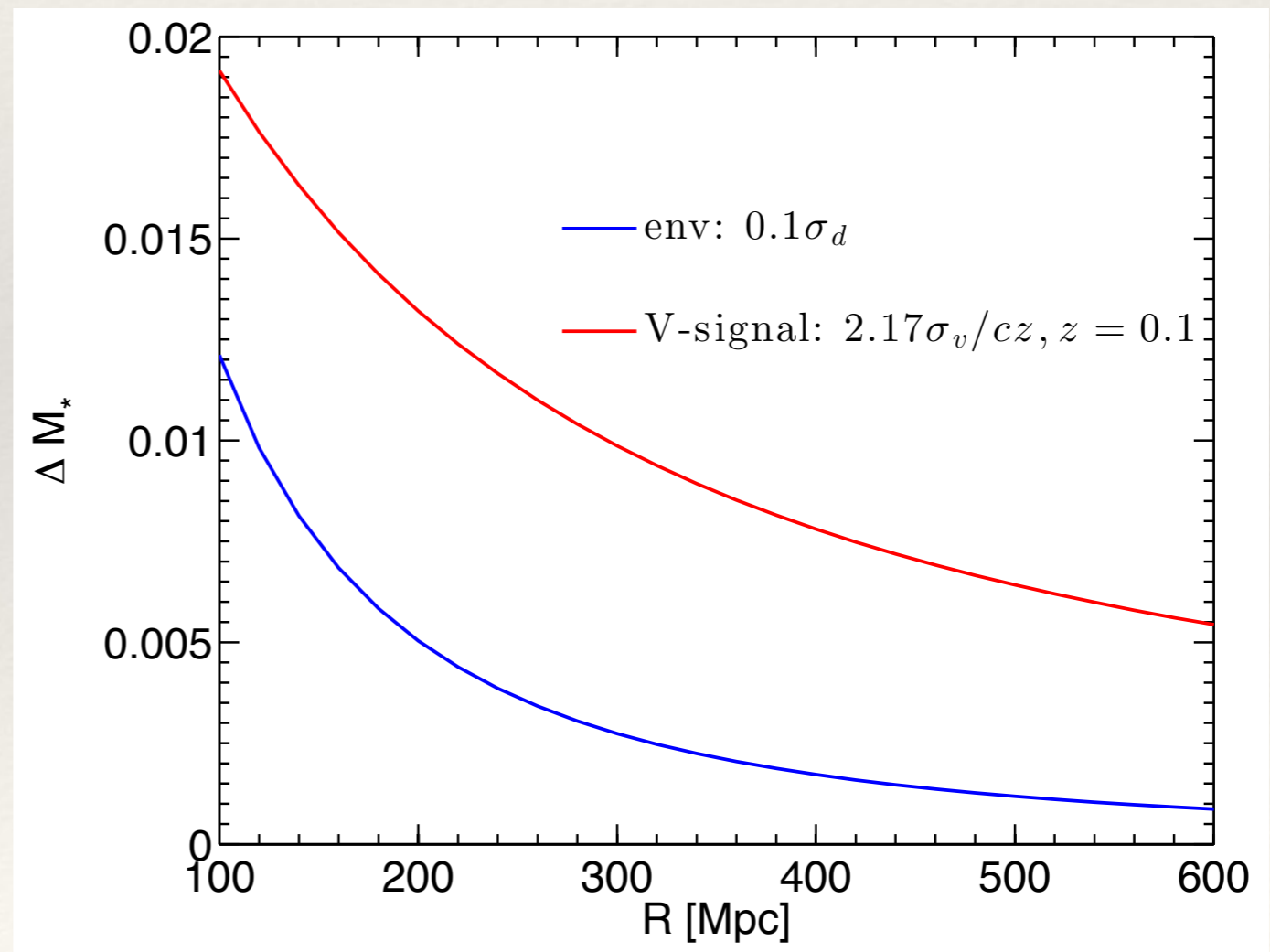


- take a very large redshift survey
- as estimate of magnitudes, compute
$$M_0 = m - 5\log(cz) = m - 5\log(Hr + V)$$
- true magnitudes are  $M_t = m - 5\log(Hr)$
- Simultaneously constrain a model for  $V$  and  $P(M_t)$  by maximizing  $P[M_0 = M_t - 2.17\log(1 - V/cz)]$ , assuming  $P(M_t)$  does not depend on velocity.

# Problems:

- coherent photometric mis-calibration: *affects all clustering studies on large scales*
- environmental dependences of the luminosity distribution: *affects certain velocity models*

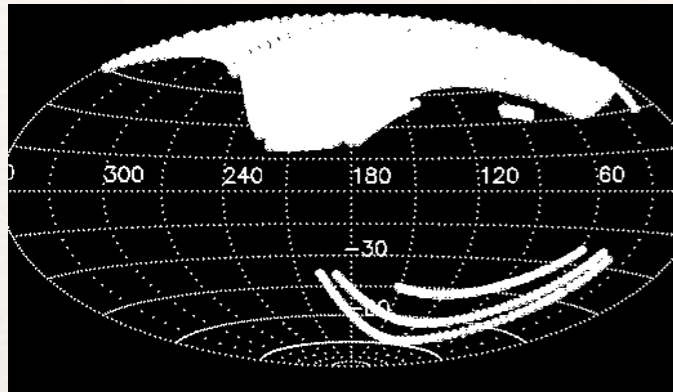
Observations give  $\Delta M_*/\Delta\delta \sim 0.1$





# Application to SDSS ( $z \sim 0.1$ )

*general velocity model*



Consider two bins:

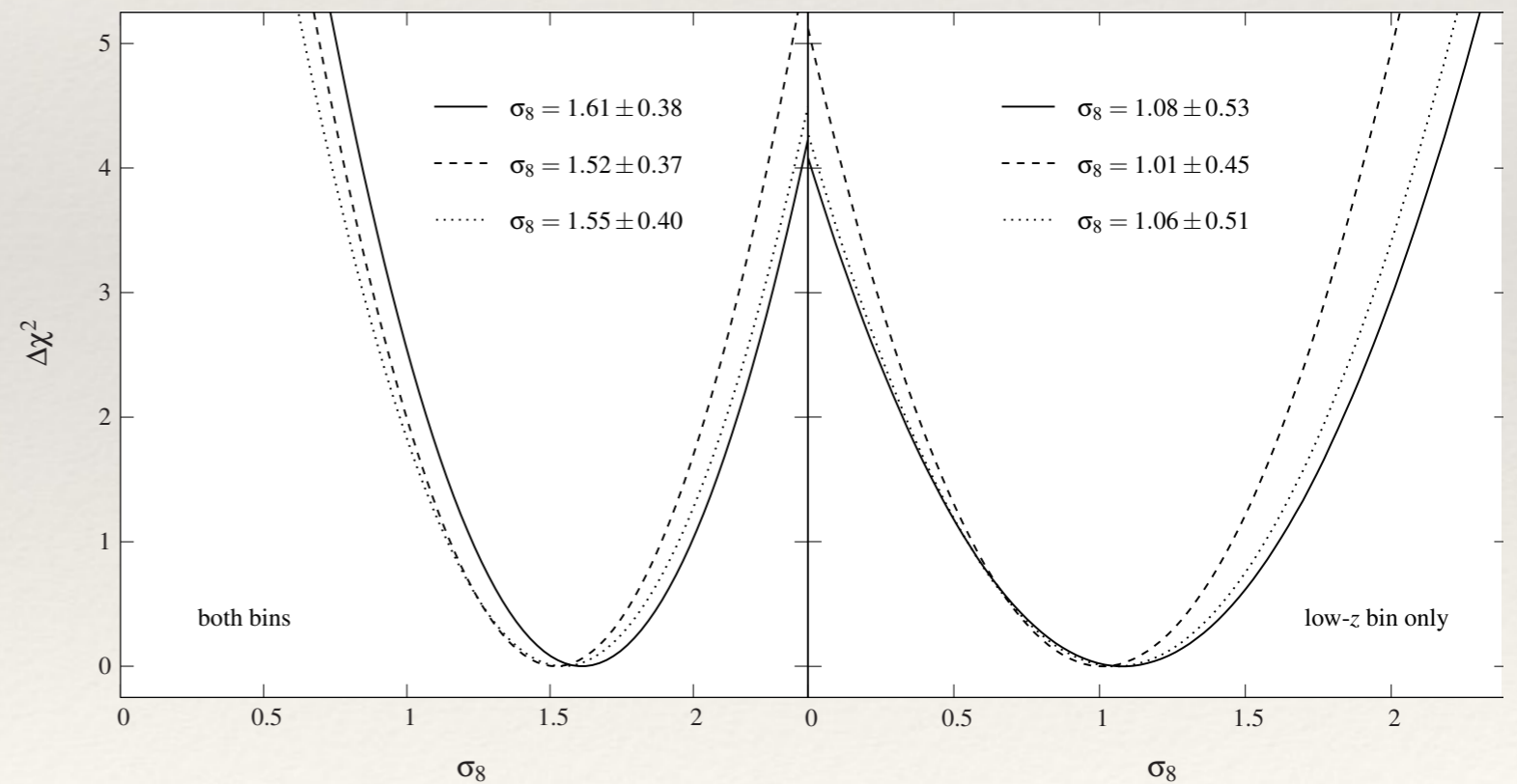
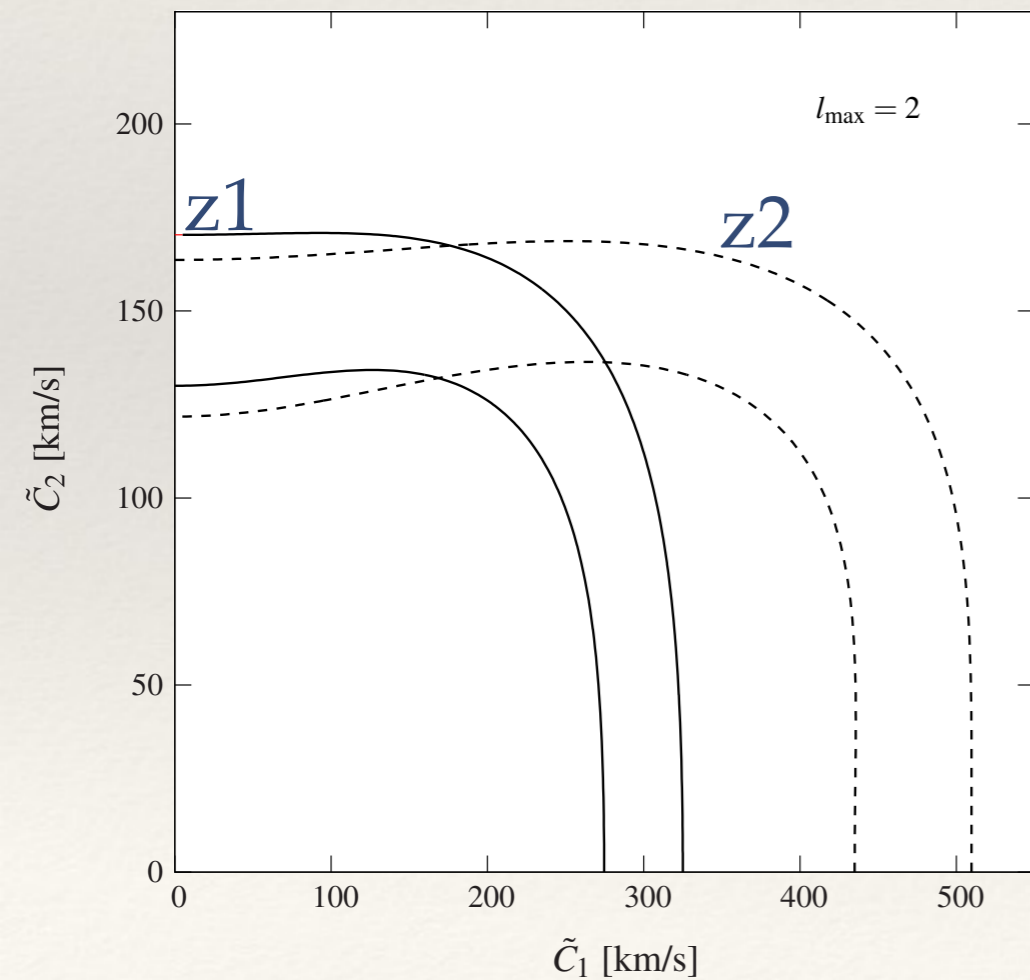
$$0.02 < z_1 < 0.07 < z_2 < 0.22$$

$$N_1 \sim 1.5 \times 10^5, N_2 \sim 3.5 \times 10^5$$

$$14.5 < m_r < 17.6$$

NYU Value-Added Catalog  
(Blanton et al 05)

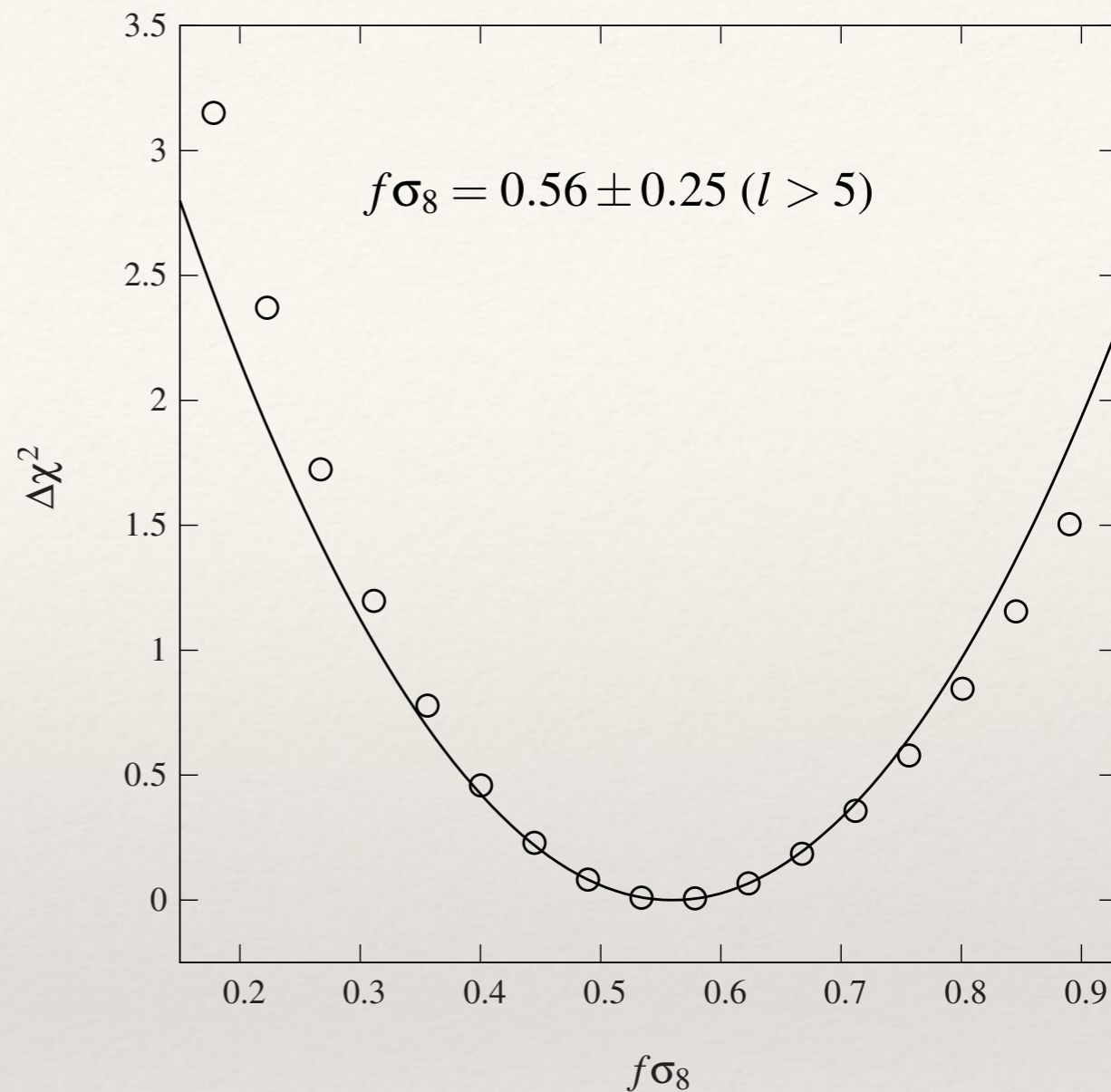
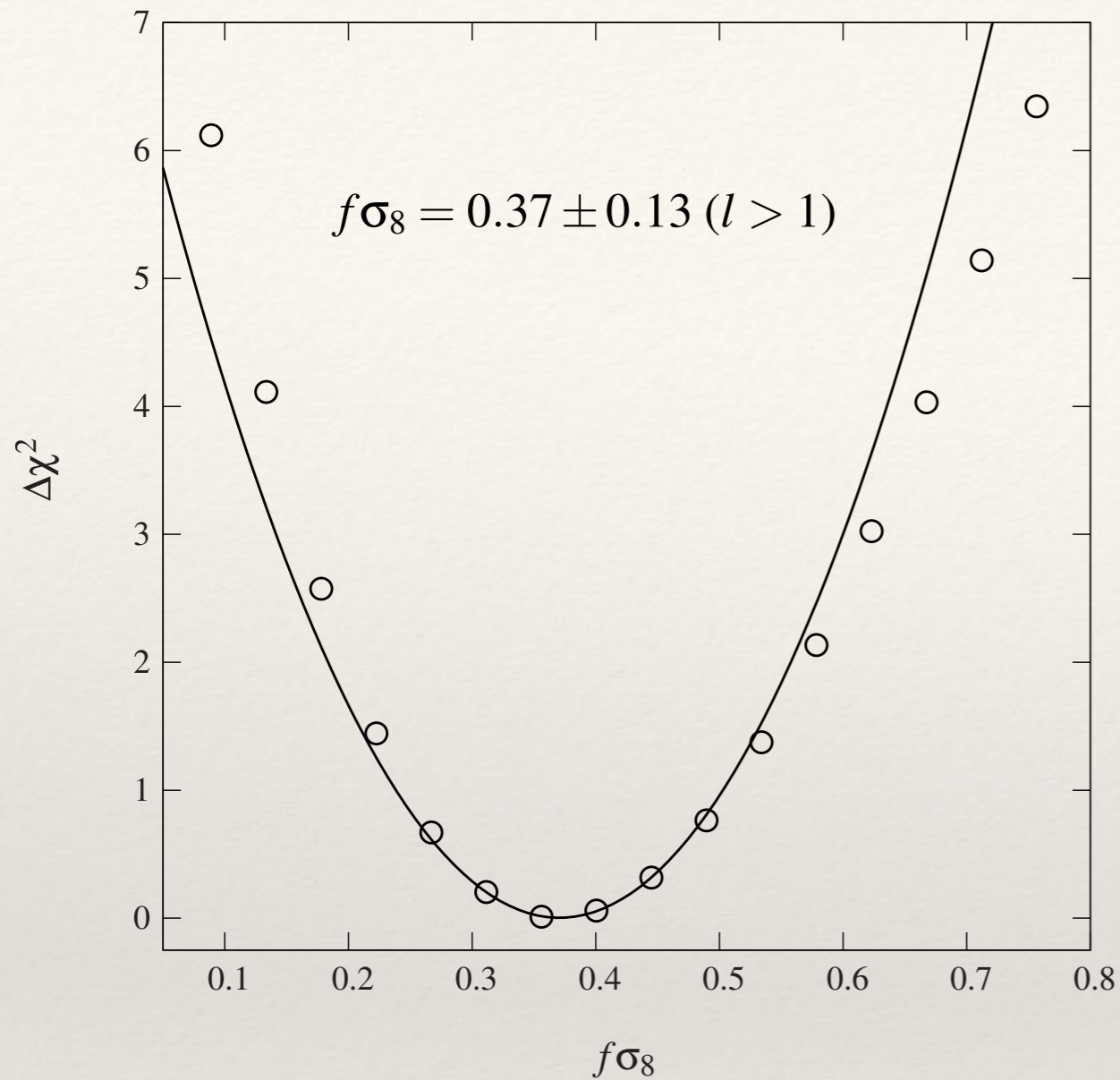
**1% tilt in photometry!!!**



## Growth Rate From SDSS

- As a velocity model, take  $V(\beta = f/b)$  from the SDSS galaxy distribution.
- Tune  $\beta$  such as  $P(M_0)$  is maximum.
  - This is basically an elaborate alternative for minimizing the variance in  $M_{est} = m - 5\log(cz - V(\beta))$  with respect to  $\beta$ .

*Martin Feix, AN, Enzo Branchini 15*



$$f = \Omega^\gamma$$

$$\beta = f/b$$

$$\delta_{\text{galaxies}} \approx b\delta_{\text{mass}}$$



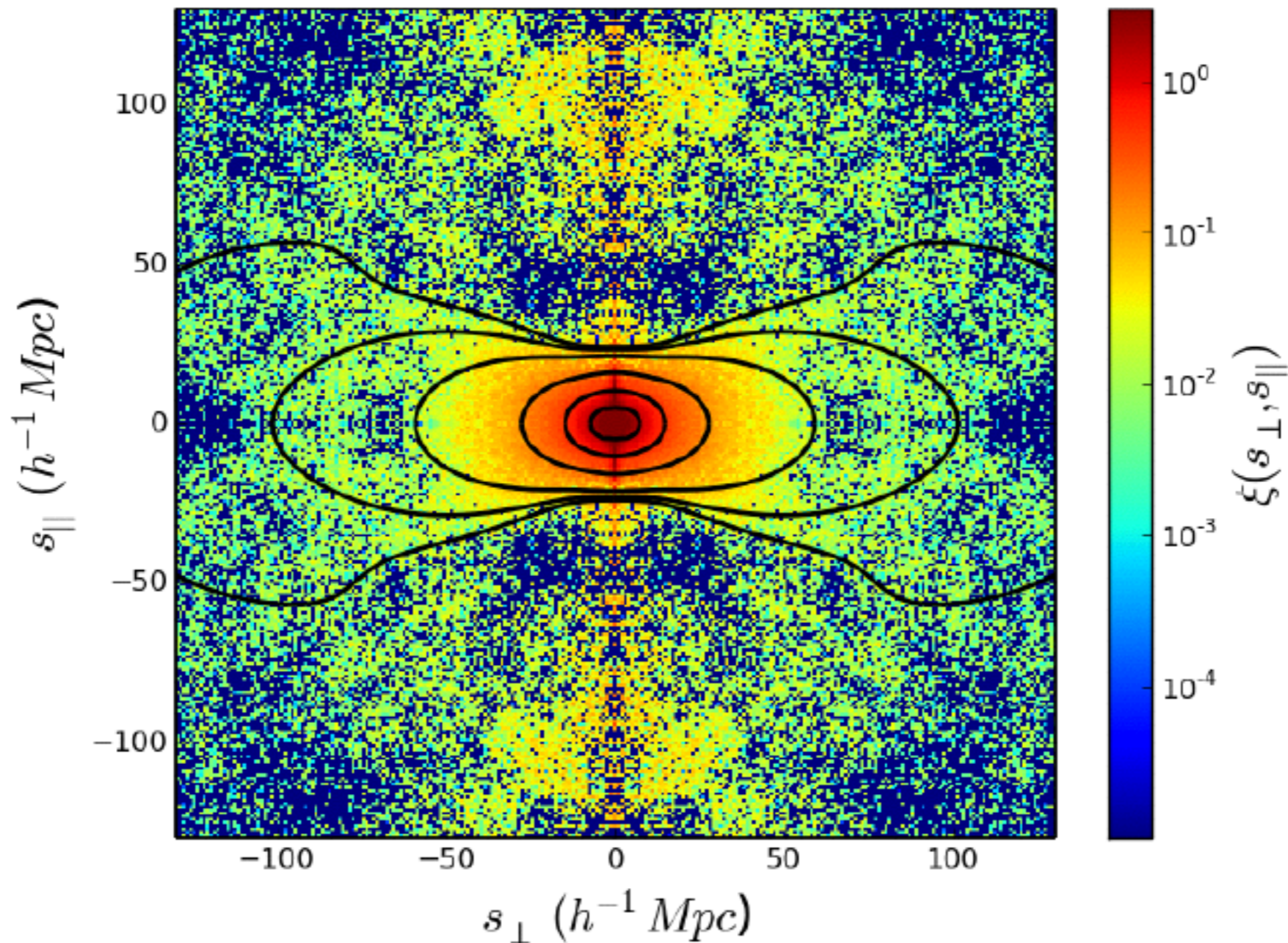
$$\beta\sigma_8^{\text{galaxies}} = f\sigma_8^{\text{mass}}$$



The result is competitive and completely independent of z-distortions

Cullan Howlett<sup>\*1</sup>, Ashley J. Ross<sup>1</sup>, Lado Samushia<sup>1,2,3</sup>, Will J. Percival<sup>1</sup>, Marc Manera<sup>4,1</sup>

the MGS correlation function, including both the RSD and the Alcock-Paczynski effect, the latter of which results in anisotropic distortions if the wrong distance-redshift relationship is used. This gives a measurement of  $f(z_{eff})\sigma_8(z_{eff}) = 0.53^{+0.19}_{-0.19}$ . If we assume a fixed fiducial cosmology for the distance-redshift relationship based on recent Planck results, our constraints on the growth rate tighten to  $f(z_{eff})\sigma_8(z_{eff}) = 0.44^{+0.16}_{-0.12}$ . Combining our mea-





# The value of $\Omega^\gamma/b$

$$f = \Omega^\gamma$$

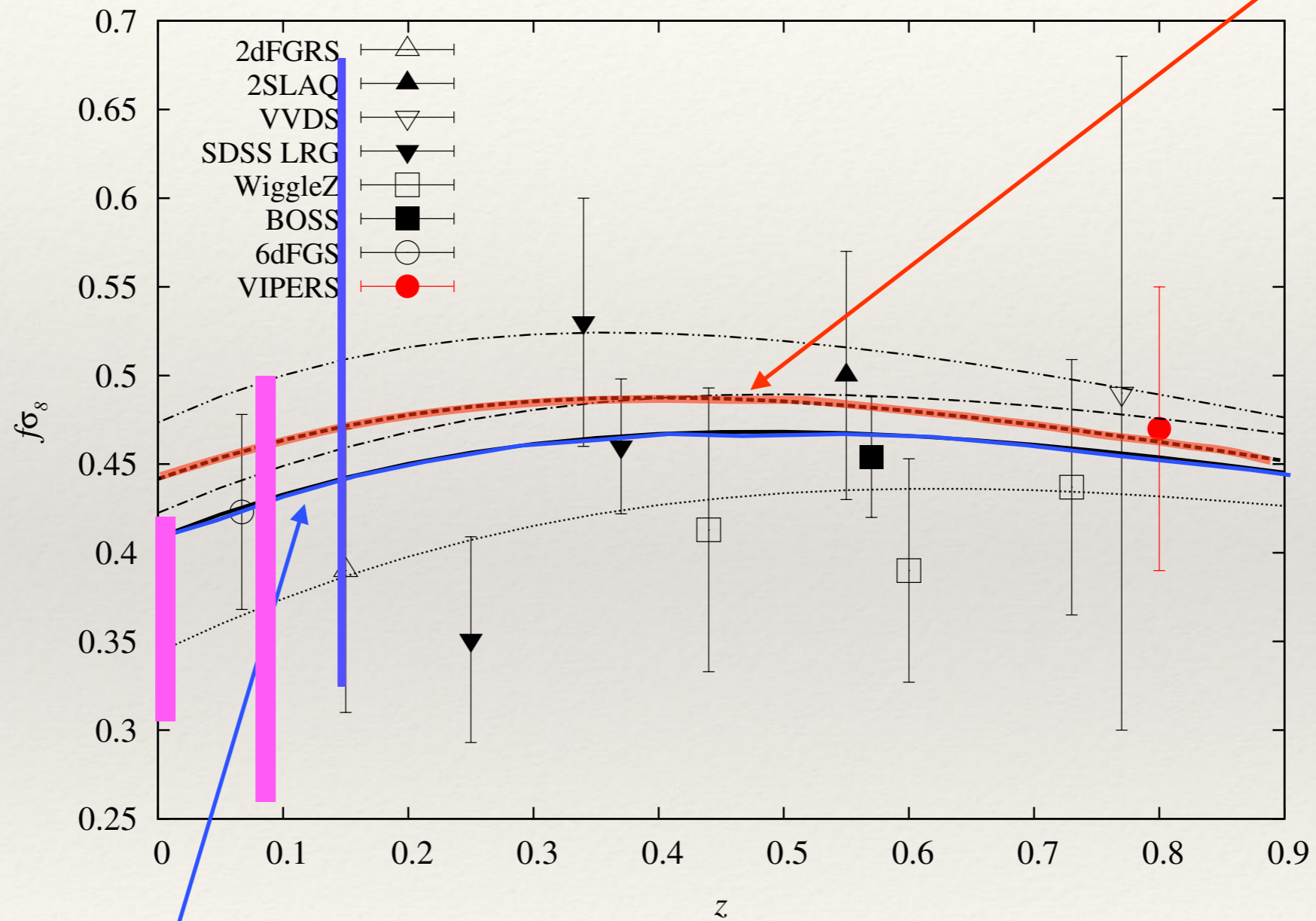
$$\beta = f/b$$

$$\delta_{\text{galaxies}} \approx b\delta_{\text{mass}}$$



$$\beta\sigma_8^{\text{galaxies}} = f\sigma_8^{\text{mass}}$$

Planck



de la Torre et al (VIPERS)

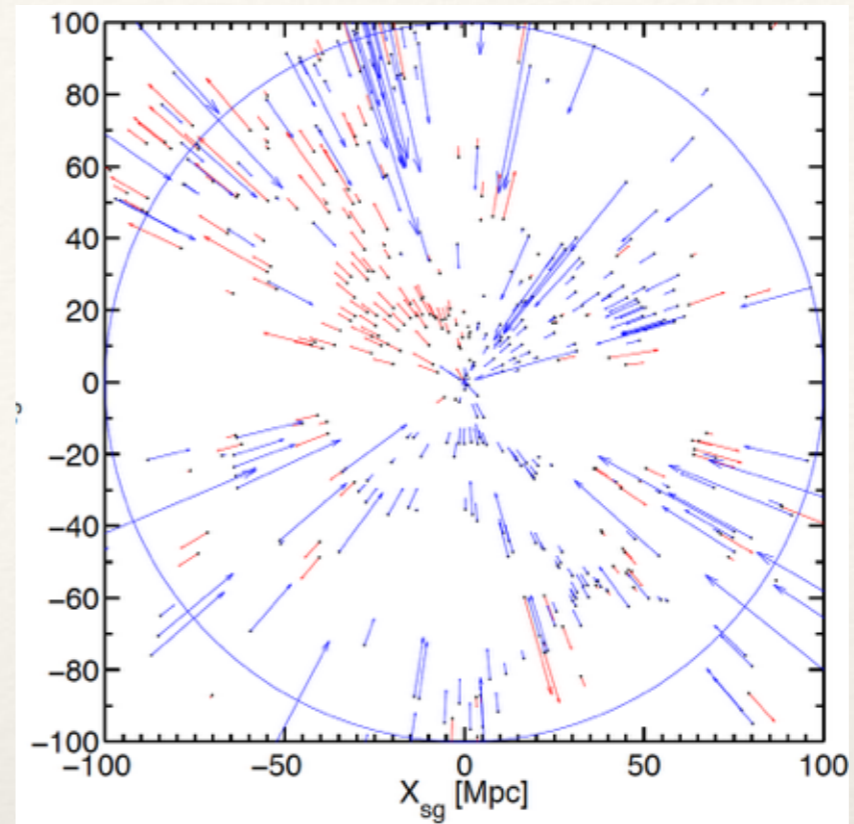
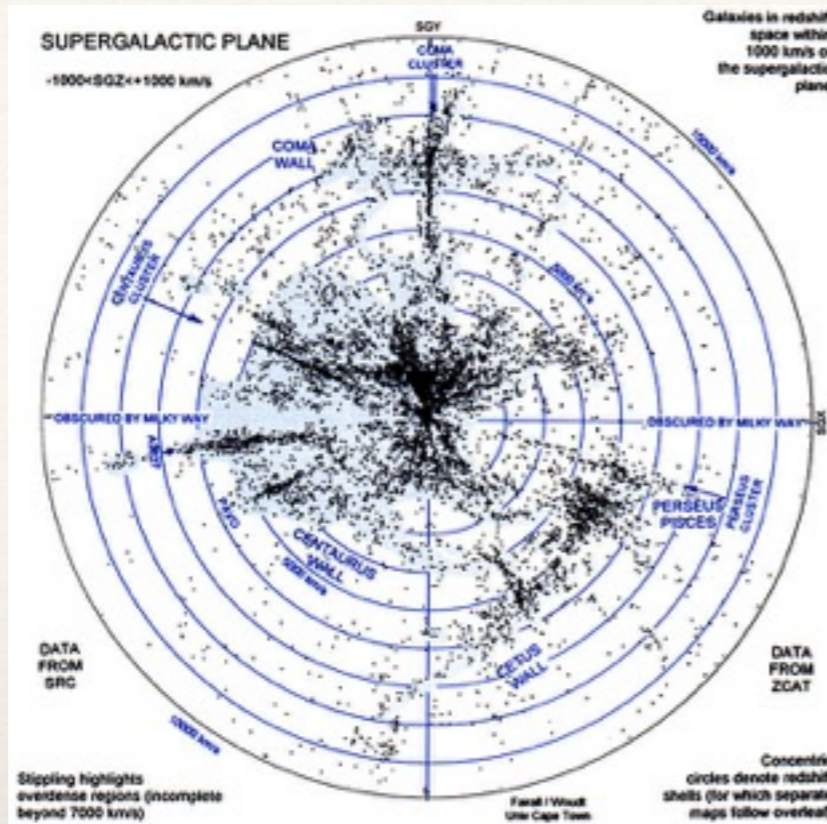
$V_{2mrs} \text{ vs } V_{TF}$

WMAP9

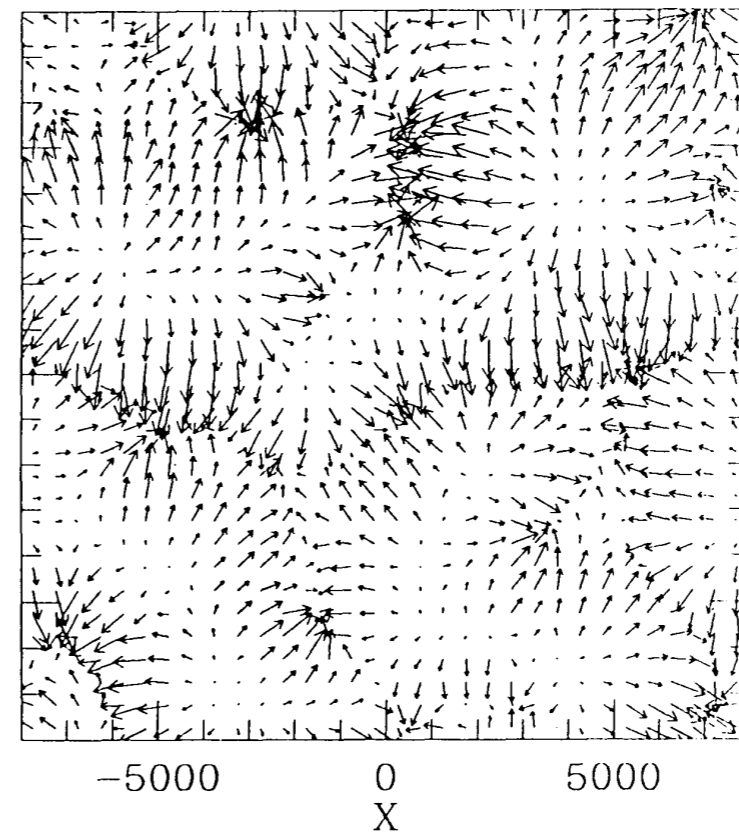
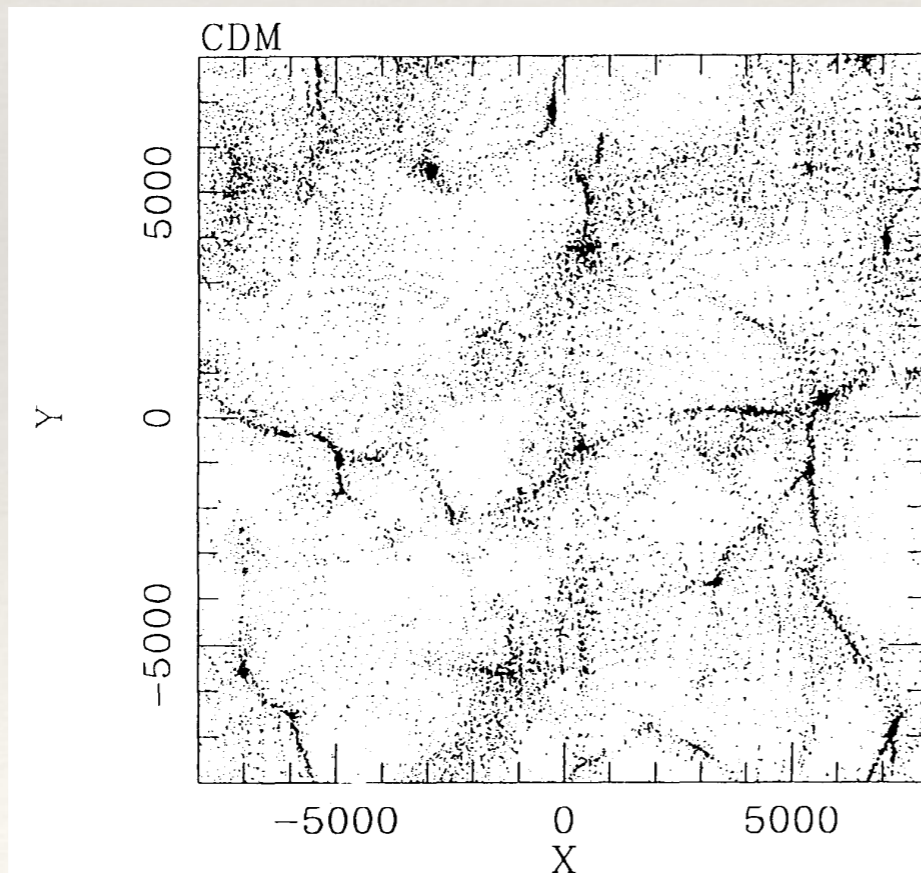
Do galaxy motion follow gravity?



Hence, it is good to do a combined analysis the two independent datasets:



observations



theory

matching z-surveys &  $V_{pec}$

## Linear theory

$$\delta = -\frac{1}{f(\Omega)} \nabla \cdot \mathbf{v}$$

$$f(\Omega) = \frac{d \ln D}{d \ln t} \approx \Omega^\gamma$$

## Solution

$$\begin{aligned} \mathbf{v} &= \frac{\Omega^\gamma}{4\pi} \int_{\text{all space}} d^3x' \delta(\mathbf{x}') \frac{\mathbf{x}' - \mathbf{x}}{|\mathbf{x}' - \mathbf{x}|^3} \\ &= \int_{\text{survey}} (\cdot) + \int_{\text{external}} (\cdot) \end{aligned}$$

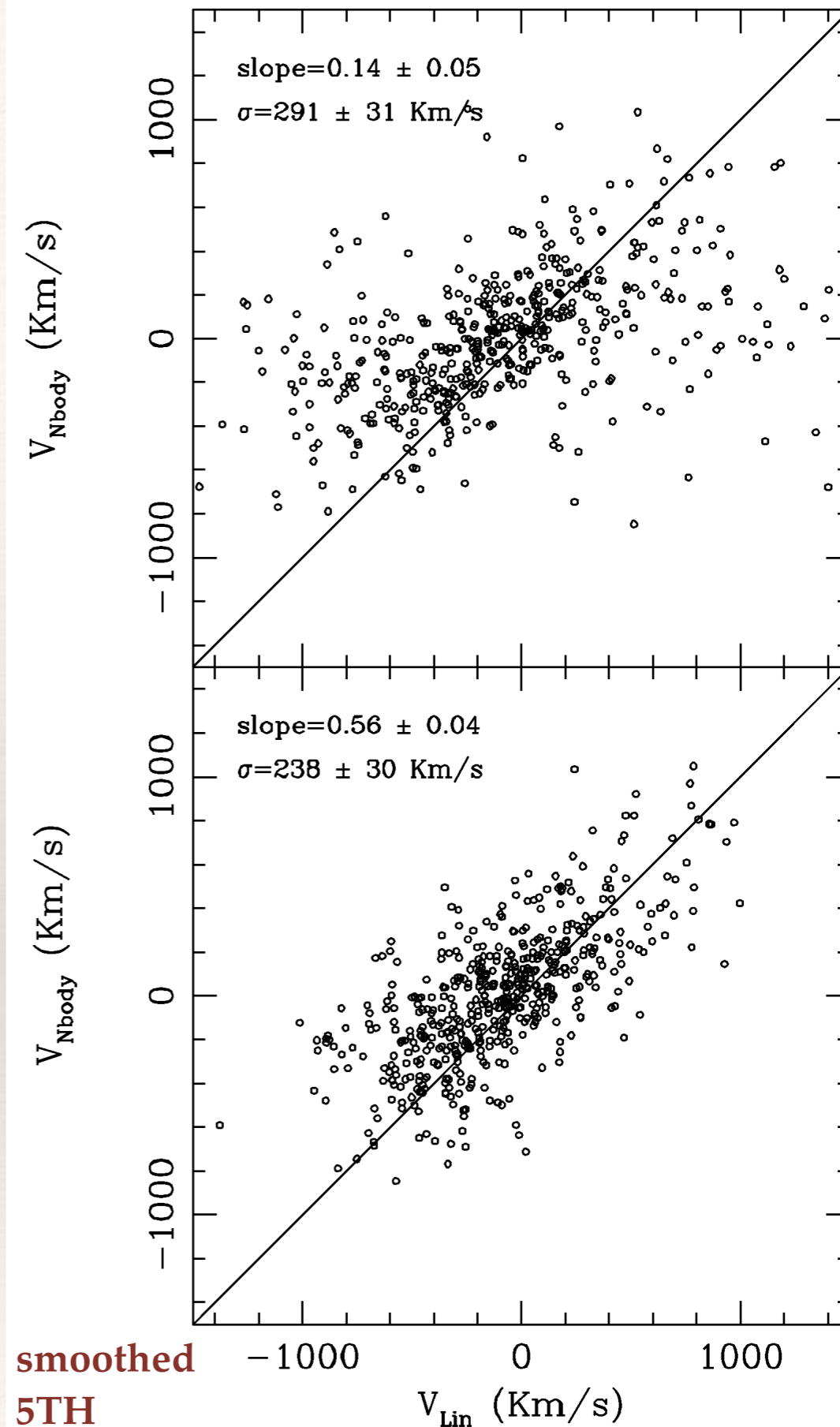
## In redshift space

$$s \equiv Hr + V^{\text{radial}}$$

$$\delta^S = -\frac{1}{f} \nabla \cdot \mathbf{v} - [\nabla \cdot \mathbf{v}]_{\text{radial}}$$

unsmoothed

Linear Theory

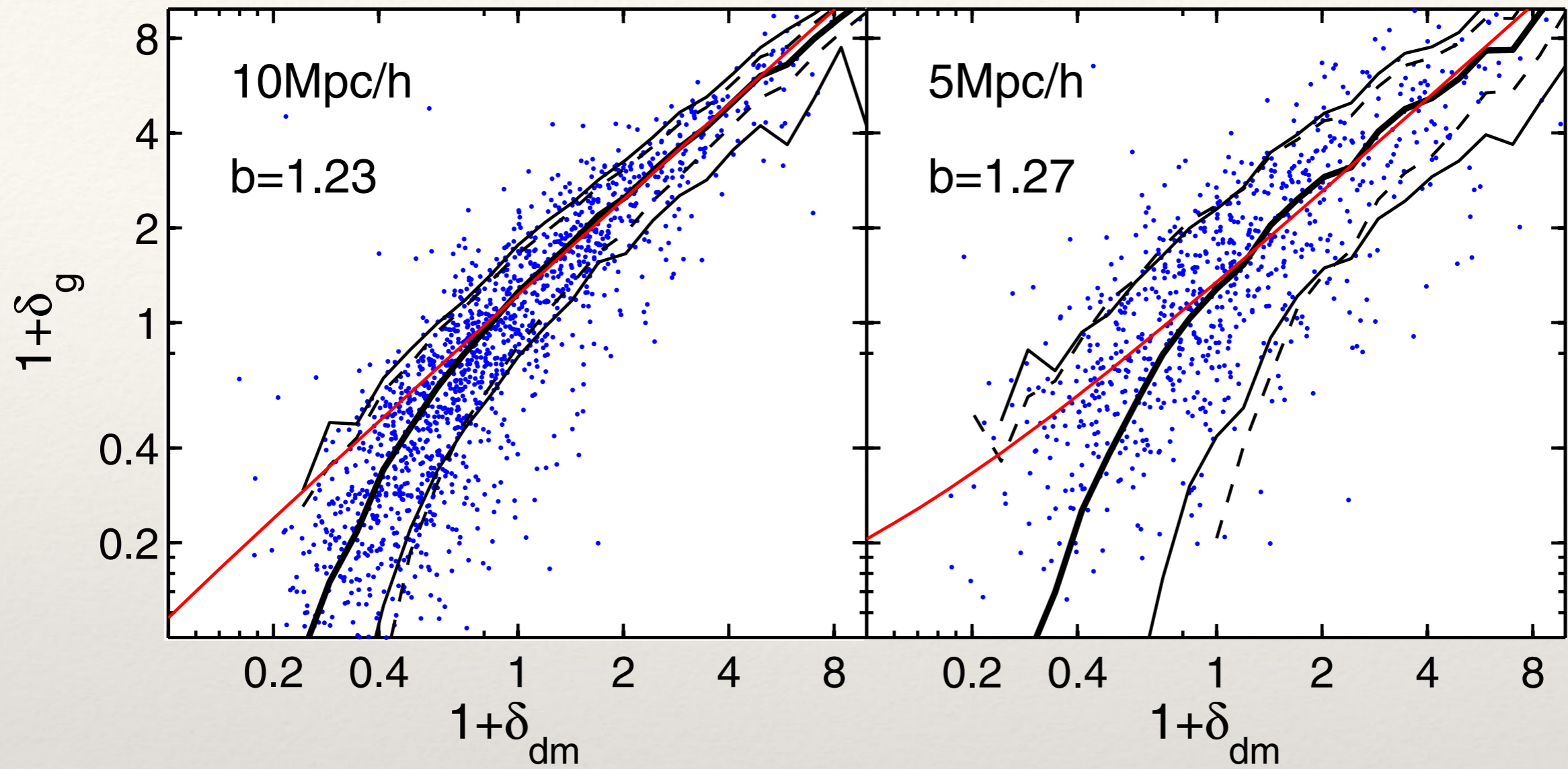


smoothed

5TH



*Based on Millennium 2MRS mocks (De Lucia & Blaizot)*



Scatter is mostly shot-noise

*AN, Davis & Branchini*



I will show next an excellent agreement between:

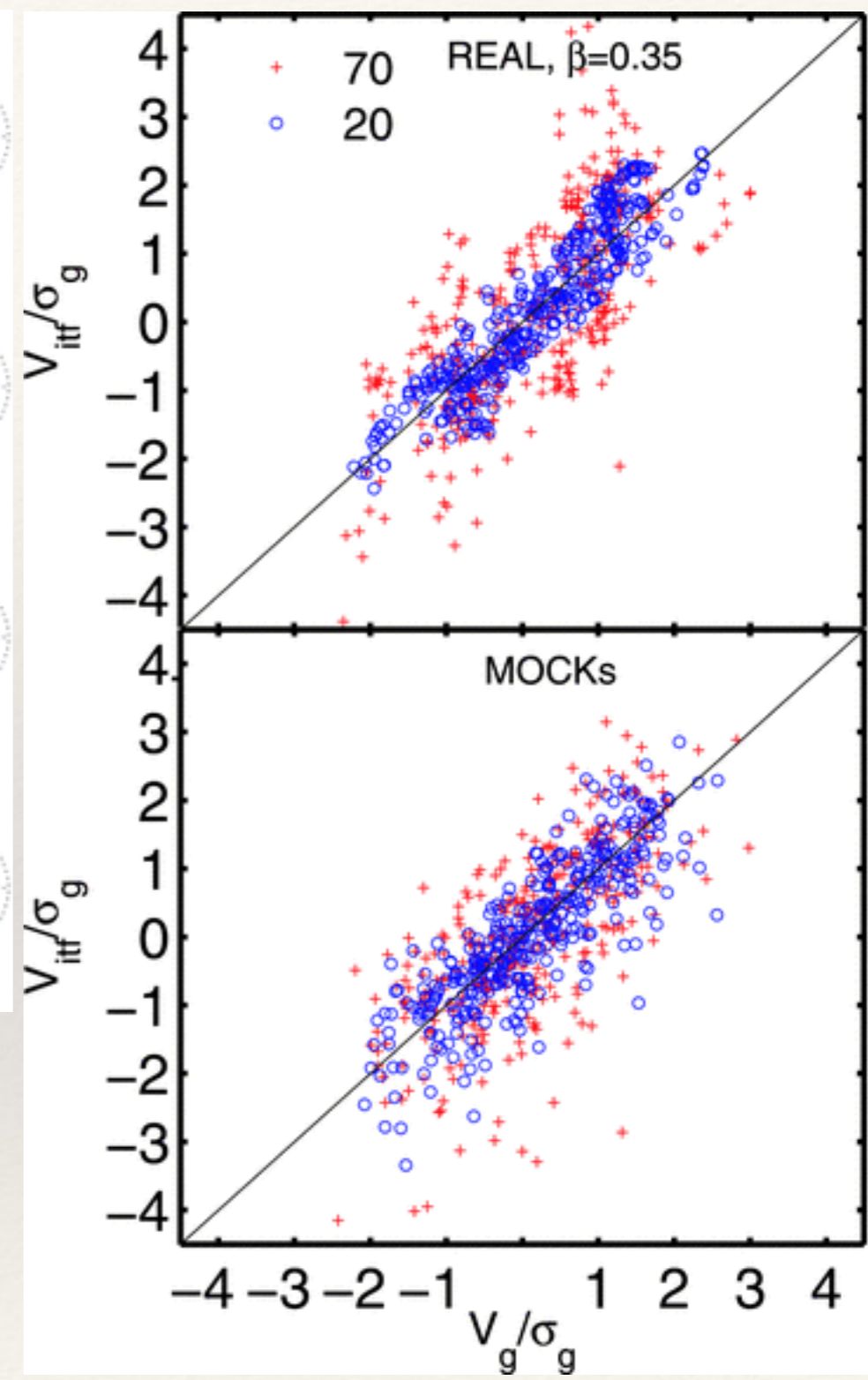
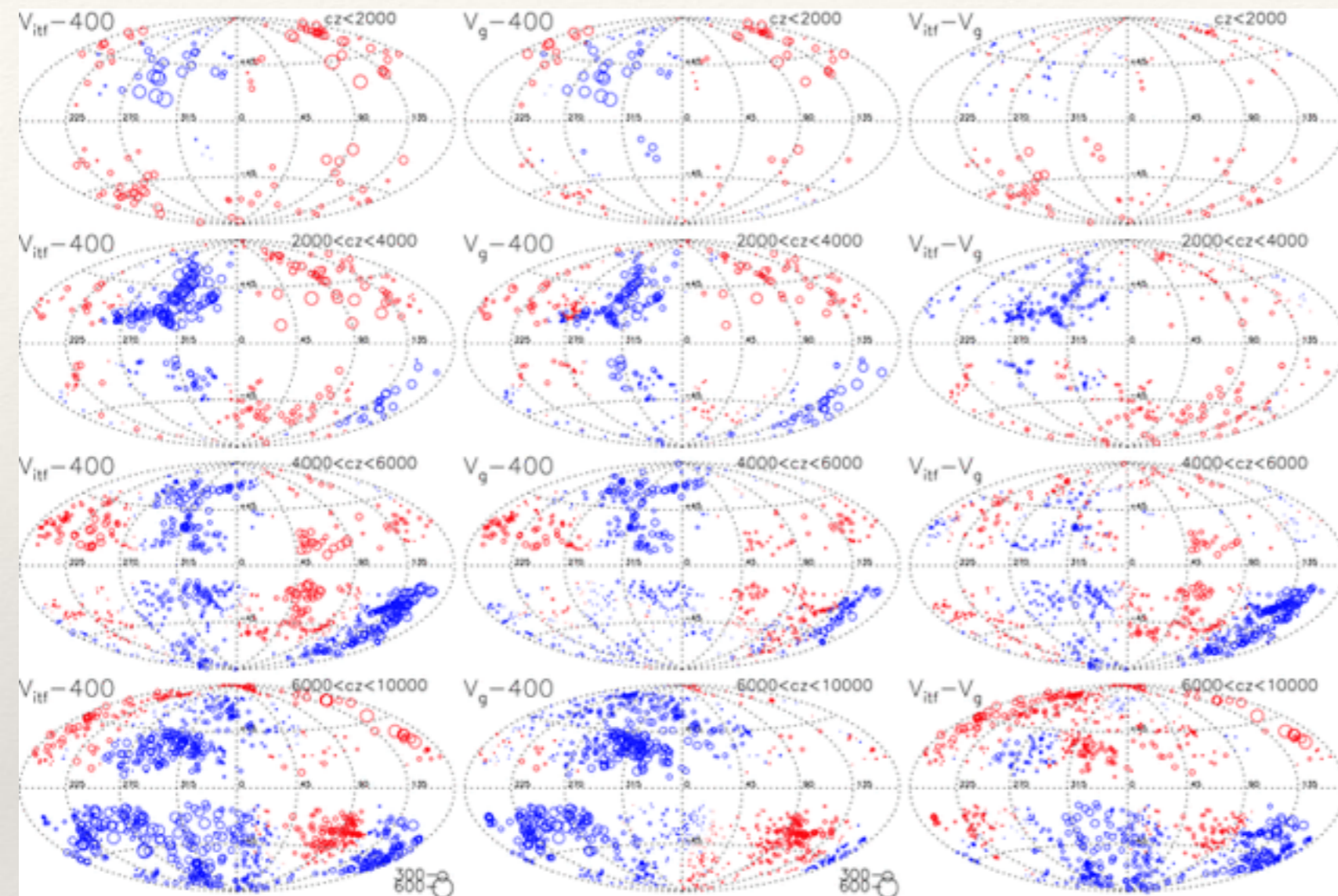
Peculiar motions derived (using linear theory) from the distribution of galaxies in the

*Two Mass Redshift Survey (2MRS)*

and

*The observed peculiar motions from the SFI++*

Marc Davis,<sup>1★</sup> Adi Nusser,<sup>2</sup> Karen L. Masters,<sup>3</sup> Christopher Springob,<sup>4</sup>  
 John P. Huchra<sup>5</sup> and Gerard Lemson<sup>6</sup>



$$\beta = \frac{\Omega^\gamma}{b}$$

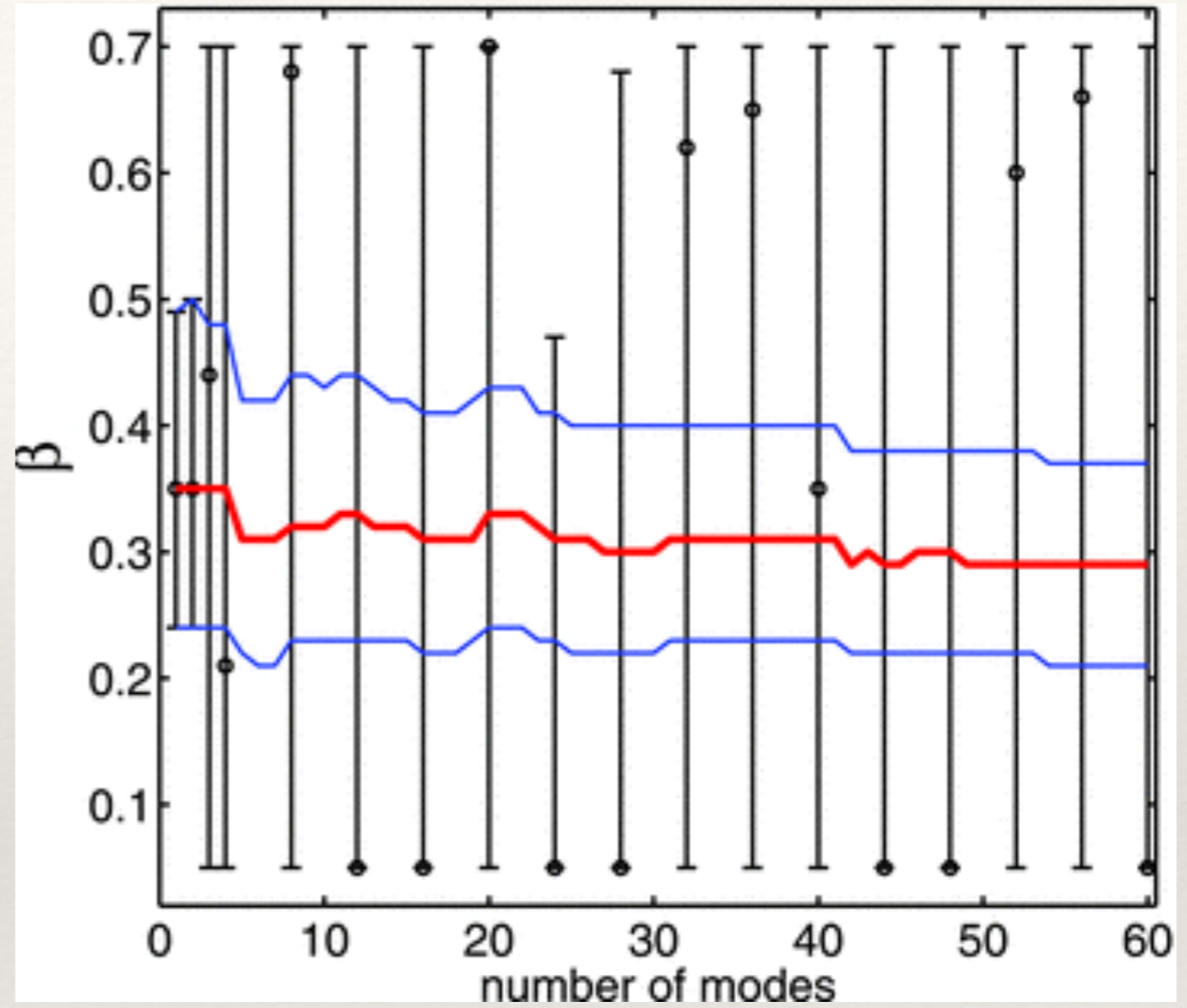
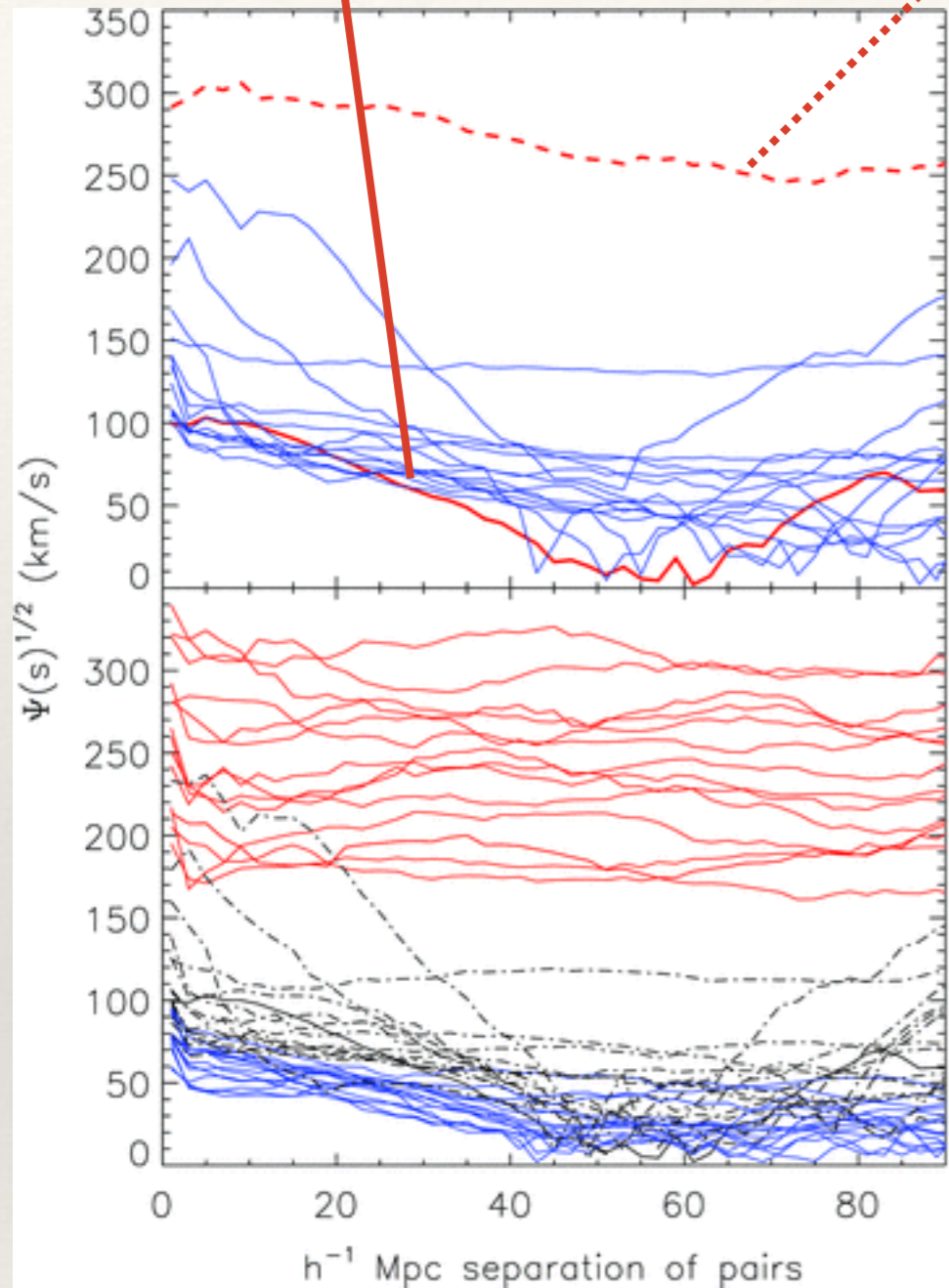
$$\delta_{\text{galaxies}} \approx b \delta_{\text{mass}}$$

*V<sub>2mrs</sub> vs V<sub>TF</sub>: visual*



correlation of SFI-2MRS

correlation of SFI (not to be compared with models)



$V_{2mrs}$  vs  $V_{TF}$ : quantitative



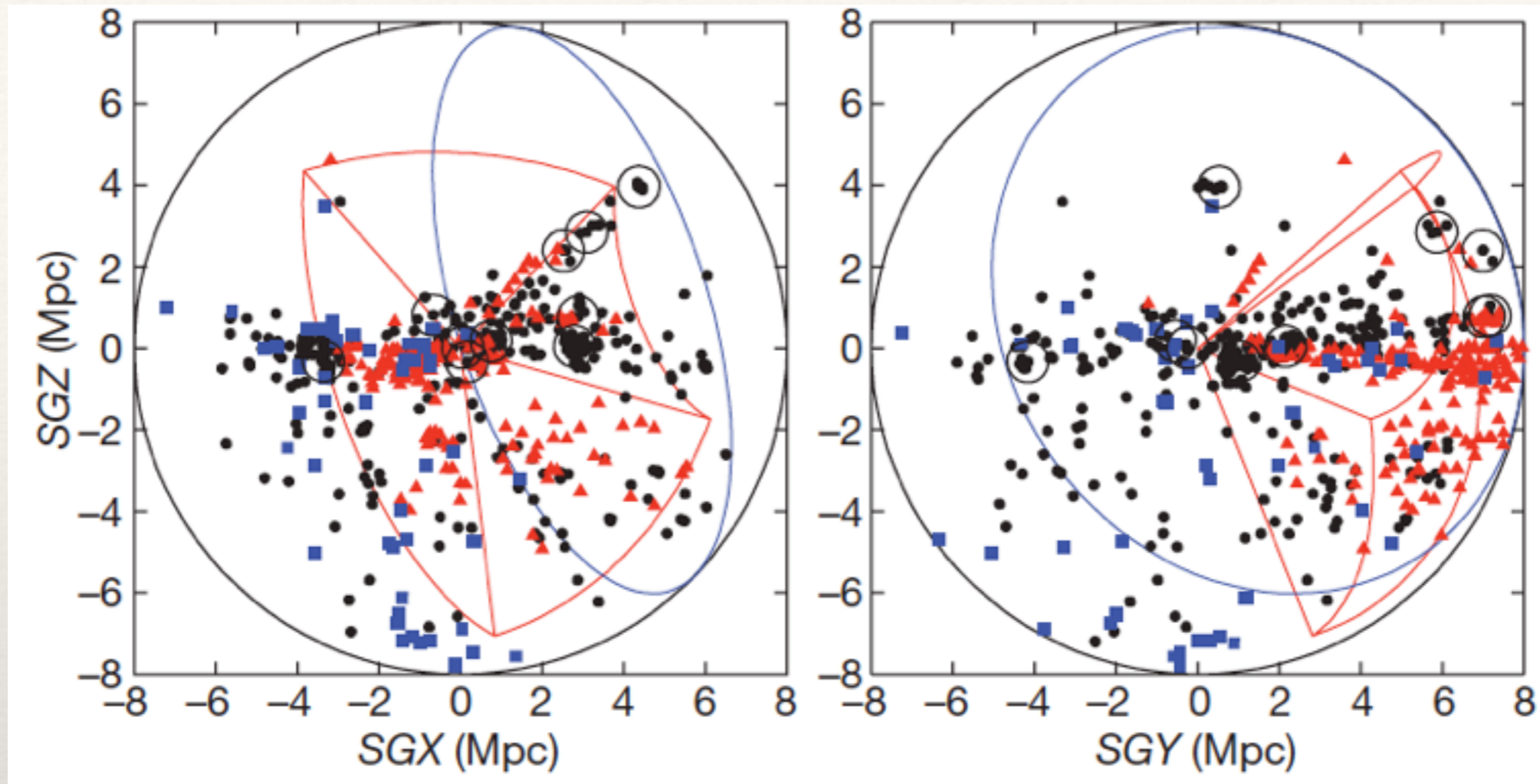
## Implications:

- finally, we have an excellent match.
  - *no cosmic variance uncertainty*
  - *Great job by the observers.*
- GI is confirmed with no indication for deviations on 30-70 Mpc scales.
- no scale dependence of  $\Omega^\gamma / b$ 
  - *likely to constraint alternative models*

Still, standard paradigm might have some problems!

# The Curious Case of the Local Neighborhood

Peebles & AN

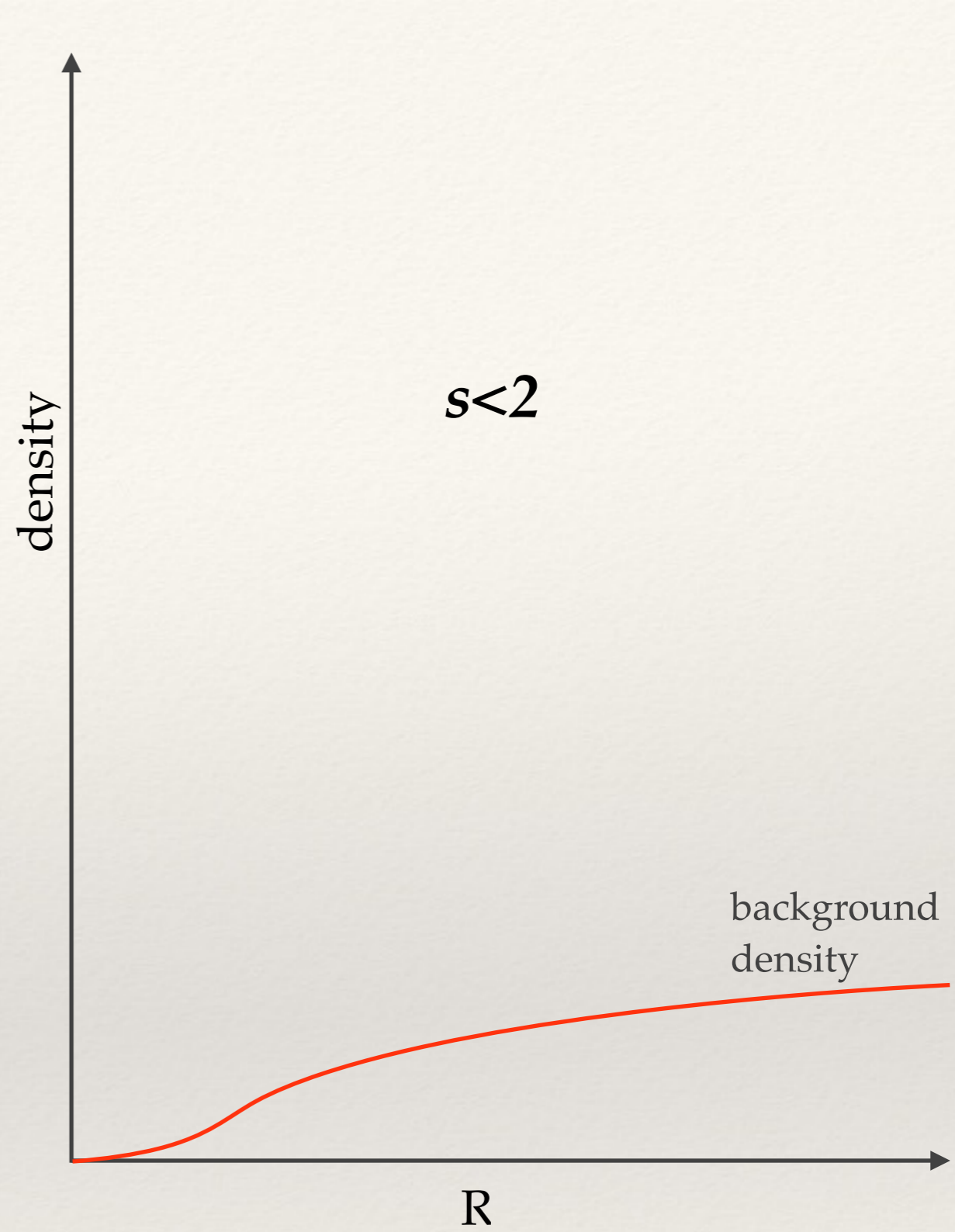
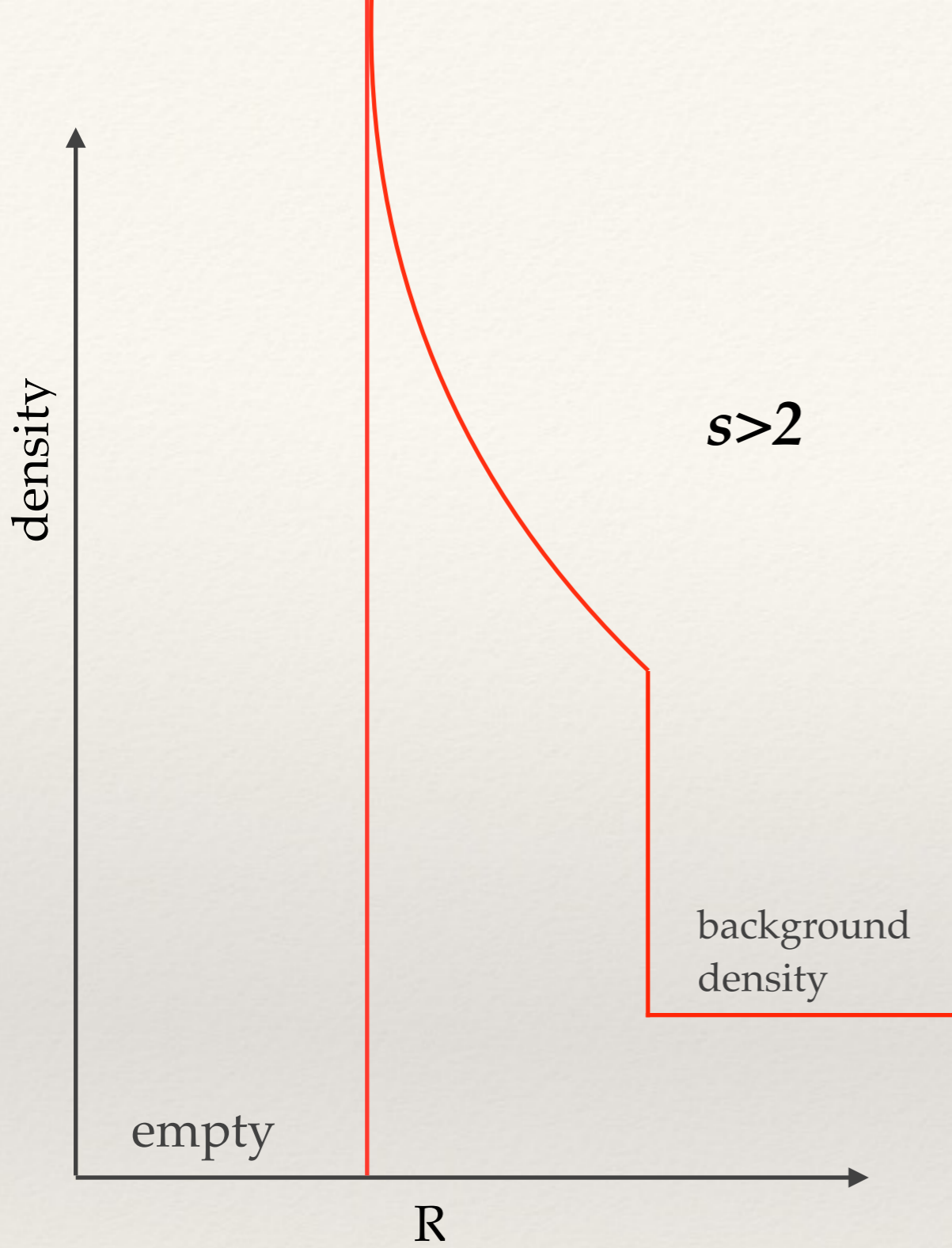


Note the impressive **Local Void** revealed by B. Tully and puzzled J. Peebles

- 337 galaxies with good distances
- ▲ 172 SDSS galaxies
- 53 HIPASS galaxies

*Nearby LSS*





initial profile:  $\frac{\delta M}{M} = -\left(\frac{r}{r_0}\right)^{-s}$

*Nearby: void*

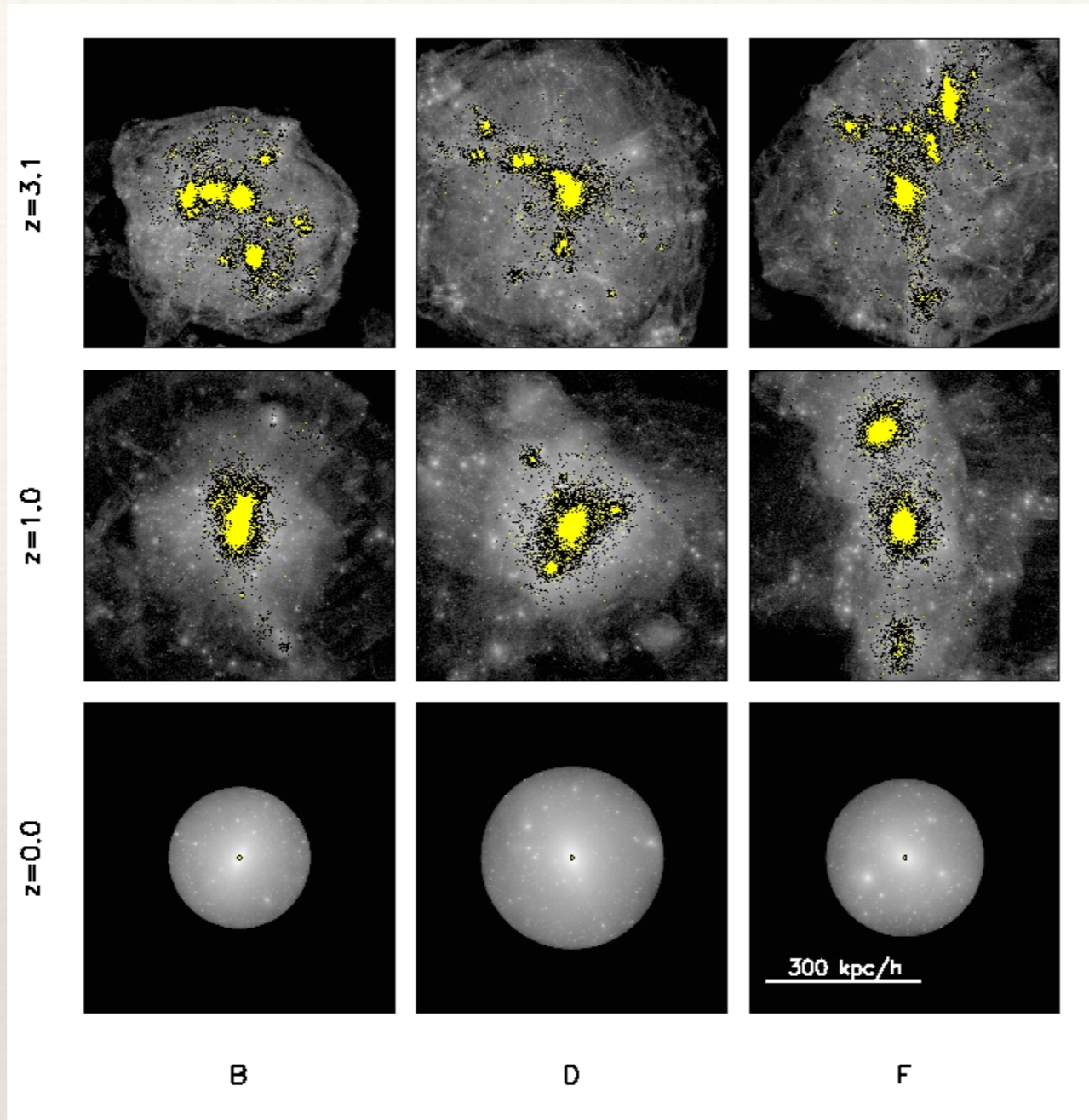
# About half of the brightest 20 nearby galaxies are pure disks just like the MW (Kormendy et al 10)

BULGE, PSEUDOBULGE, AND DISK INVENTORIES IN GIANT GALAXIES CLOSER THAN 8 MPC DISTANCE

Galaxy	Type	$D$ (Mpc)	S	$M_K$	$M_V$	$V_{\text{circ}}$ ( $\text{km s}^{-1}$ )	S	$B/T$	$PB/T$	S
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
NGC 6946	Scd	5.9	a,b	-23.61	-21.38	$210 \pm 10$	a,b	0	$0.024 \pm 0.003$	a
NGC 5457	Scd	7.0	c,d,e	-23.72	-21.60	$210 \pm 15$	c,d,e	0	$0.027 \pm 0.008$	a
IC 342	Scd	3.28	f,w	-23.23	-21.4 :	$192 \pm 3$	a,f	0	$0.030 \pm 0.001$	c,e,f
NGC 4945	SBcd	3.36	g	-23.21	-20.55	$174 \pm 10$	e	0	$0.036 \pm 0.009$	b
NGC 5236	SABc	4.54	d,i,j	-23.69	-21.0	$180 \pm 15$	e,i	0:	$0.074 \pm 0.016$	c,e
NGC 5194	Sbc	7.66	h	-23.94	-21.54	$240 \pm 20$	a,i,j	0:	$0.095 \pm 0.015$	d,e
NGC 253	SBc	3.62	g,k	-24.03	-20.78	$210 \pm 5$	a,f	0:	0.15	c
Maffei 2	SBbc	3.34	l	-23.0 :	-20.8 :	$168 \pm 20$	f	0:	$0.16 \pm 0.04$	b
Galaxy	SBbc	0.008	m,n,o	-23.7	-20.8 :	$220 \pm 20$	k,l	0:	$0.19 \pm 0.02$	g,h
Circinus	SABb:	2.8	a	-22.8	-19.8	$155 \pm 5$	o,p	0:	$0.30 \pm 0.03$	b,e
NGC 4736	Sab	4.93	h,p	-23.36	-20.66	$181 \pm 10$	e,q	0:	$0.36 \pm 0.01$	d,e
NGC 2683	SABb	7.73	h	-23.12	-19.80	$152 \pm 5$	g,h	$0.05 \pm 0.01$	0:	b
NGC 4826	Sab	6.38	h,u	-23.71	-20.72	$155 \pm 5$	m,n	0.10	0.10	d,e,f,i,j
NGC 2787	SB0/a	7.48	h	-22.16	-19.19	$220 \pm 10$	r,s,t	0.11	$0.28 \pm 0.02$	d,k
NGC 4258	SABbc	7.27	g,h,q	-23.85	-20.95	$208 \pm 6$	e,u	$0.12 \pm 0.02$	0:	b,d,e,l
M 31	Sb	0.77	c,h,r	-23.48	-21.20	$250 \pm 20$	e	$0.32 \pm 0.02$	0	b,m,n
M 81	Sab	3.63	d,r,s	-24.00	-21.13	$240 \pm 10$	e,v	$0.34 \pm 0.02$	0	d,e,f,i,o,p
Maffei 1	E	2.85	l	-23.1 :	-20.6 :	$(264 \pm 10)$	w	1	0	q
NGC 5128	E	3.62	e,h,t,v	-23.90	-21.34	$(192 \pm 2)$	x	1	0:	q



# Merging in Simulation



Wang, Peebles & AN



## **Andromeda has a huge bulge**





## pure disk galaxy



FIG. 14.— Color image of NGC 6503 taken with the *Hubble Space Telescope* Advanced Camera for Surveys. Colors are bland because the wavelength range available is small. Blue corresponds to the F650N filter ( $H\alpha$ ), red to F814W ( $I$  band) and green to their average. Brightness here is proportional to the square root of the brightness in the galaxy. North is up and east is at left. Like NGC 5457 and NGC 6946, this is a pure-disk galaxy. But NGC 6503 is smaller; it has a flat outer rotation curve with  $V_{\text{circ}} \simeq 115 \text{ km s}^{-1}$  compared with  $V_{\text{circ}} \simeq 210 \text{ km s}^{-1}$  for the previous galaxies. Like those galaxies, its Hubble type is Scd. And like them, a tiny, bright center visible in this image proves to be a pseudobulge that makes up 0.11 % of the  $I$ -band light of the galaxy (see text). The nucleus that we use to constrain  $M_{\bullet}$  makes up only 0.040 % of the  $I$ -band light of the galaxy. It is completely invisible here but is illustrated in Figure 15.



**Pure disk NGC4247**





## A pure disk with a pseudo-bulge

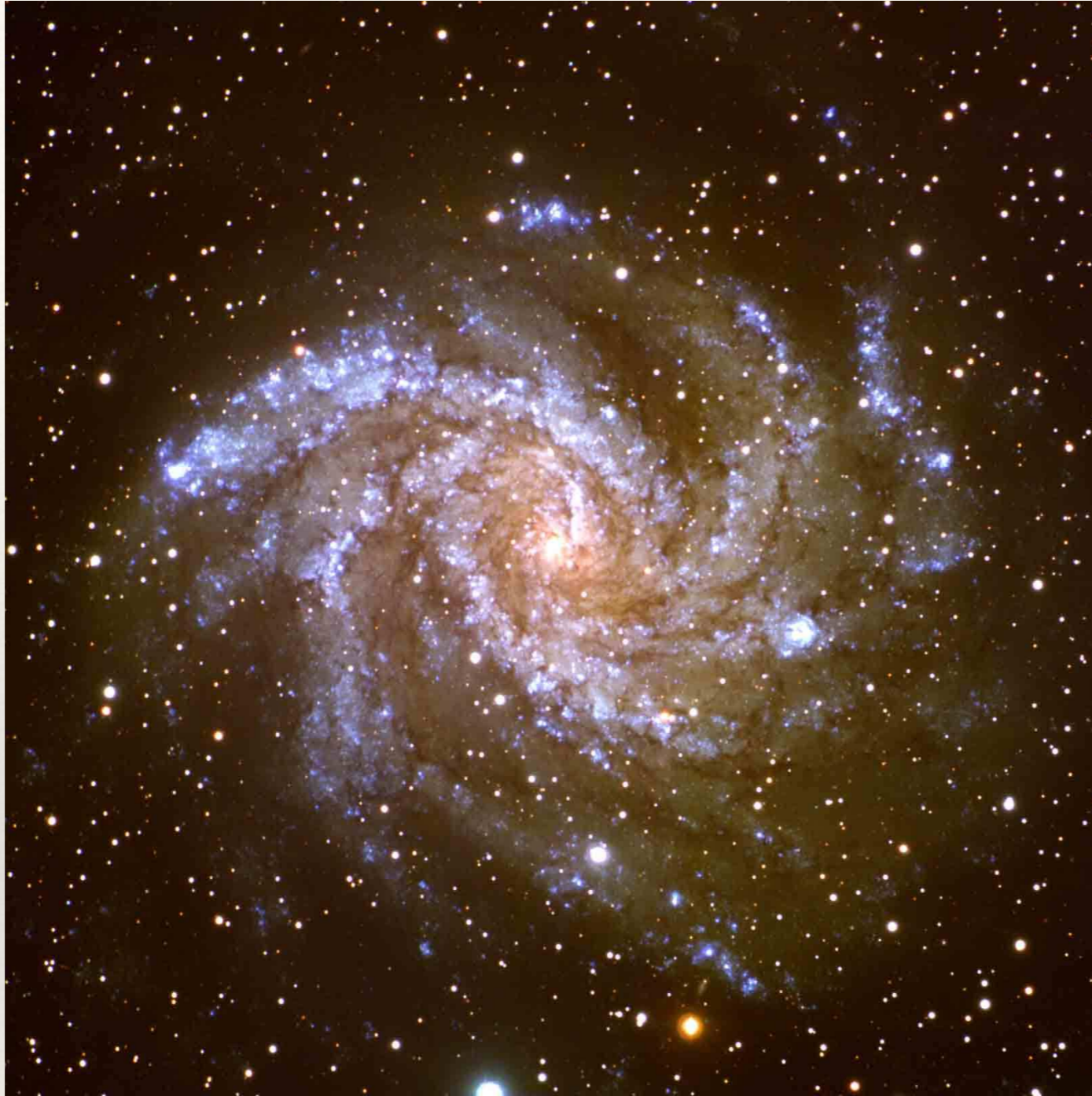


FIG. 7. — Color image of NGC 6946 taken with the Large Binocular Telescope (<http://medusa.as.arizona.edu/lbto/astromical.htm>). This galaxy is very similar to NGC 5457: it is a giant galaxy ( $V_{\text{circ}} \simeq 210 \pm 10 \text{ km s}^{-1}$ ; Table 2), but it is completely dominated by its disk (Hubble type Scd). As in NGC 5457, the tiny, bright center visible in this image proves to be a pseudobulge that makes up 2.4 % of the *I*-band light of the galaxy (see text). The nucleus whose dispersion we measure makes up only 0.12 % of the *I*-band light of the galaxy. It is completely invisible here but is illustrated in Figure 8.



# Tweaking

\ w Peebles, Gubser & Keselman

## Some motivations:

- Need for deeper voids:

*Local Void is too deep*

*ISW is too large in big voids (Granett et al)*

*Hotter voids in the  $\text{Ly}\alpha$  forest (Botlon et al)*

- Quieter, more isolated galaxies (*Kormendy et al*)
- fewer satellites
- many  $\Lambda$ CDM models include these modification

# Tweaking

- bellow is a simplified version which incorporates much of effects of more complicated models



# How do we do that?

$$V = -\frac{Gm^2}{r} \left( 1 + \beta e^{-r/r_s} \right)$$

**WARNING: ONLY BETWEEN DARK MATTER PARTICLES!!!!**

**The Baryons do not feel the extra term**

# Physical realization

Assume two species of dark matter particles of masses  $M_+(\Phi)$  and  $M_-(\Phi)$  that depend on a scalar field  $\Phi$ . Consider the action

$$S = \int \underbrace{d^3x dt \Phi_{,i} \Phi^{,i}}_{\text{kinetic term of the field}} - \sum_{\text{particles}} \int \left[ \underbrace{M_+(\Phi) dt \sqrt{1 - v_+^2}}_{\text{Lorentz factor}} + \underbrace{M_-(\Phi) dt \sqrt{1 - v_-^2}}_{\text{relativistic Lagrangian of a particle}} \right]$$

Ansatz:  $M_+ = M_{DM} - y\Phi$ ,  $M_- = M_s = y_s\Phi \approx 0$   
massive relativistic

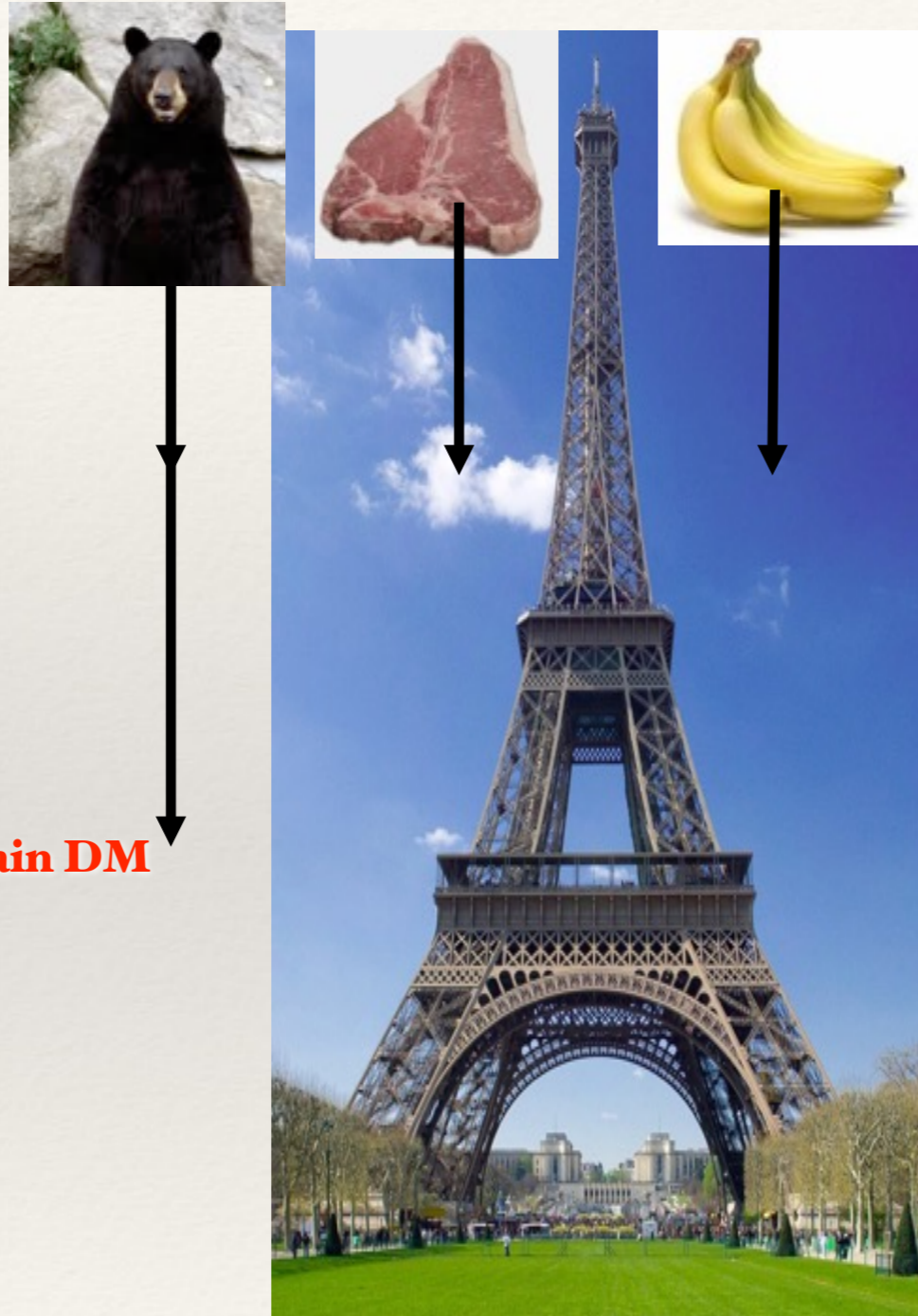


## Important:

- $r_s \sim$  a few Mpc **comoving**
- $\beta \sim 1$
- careful with element production

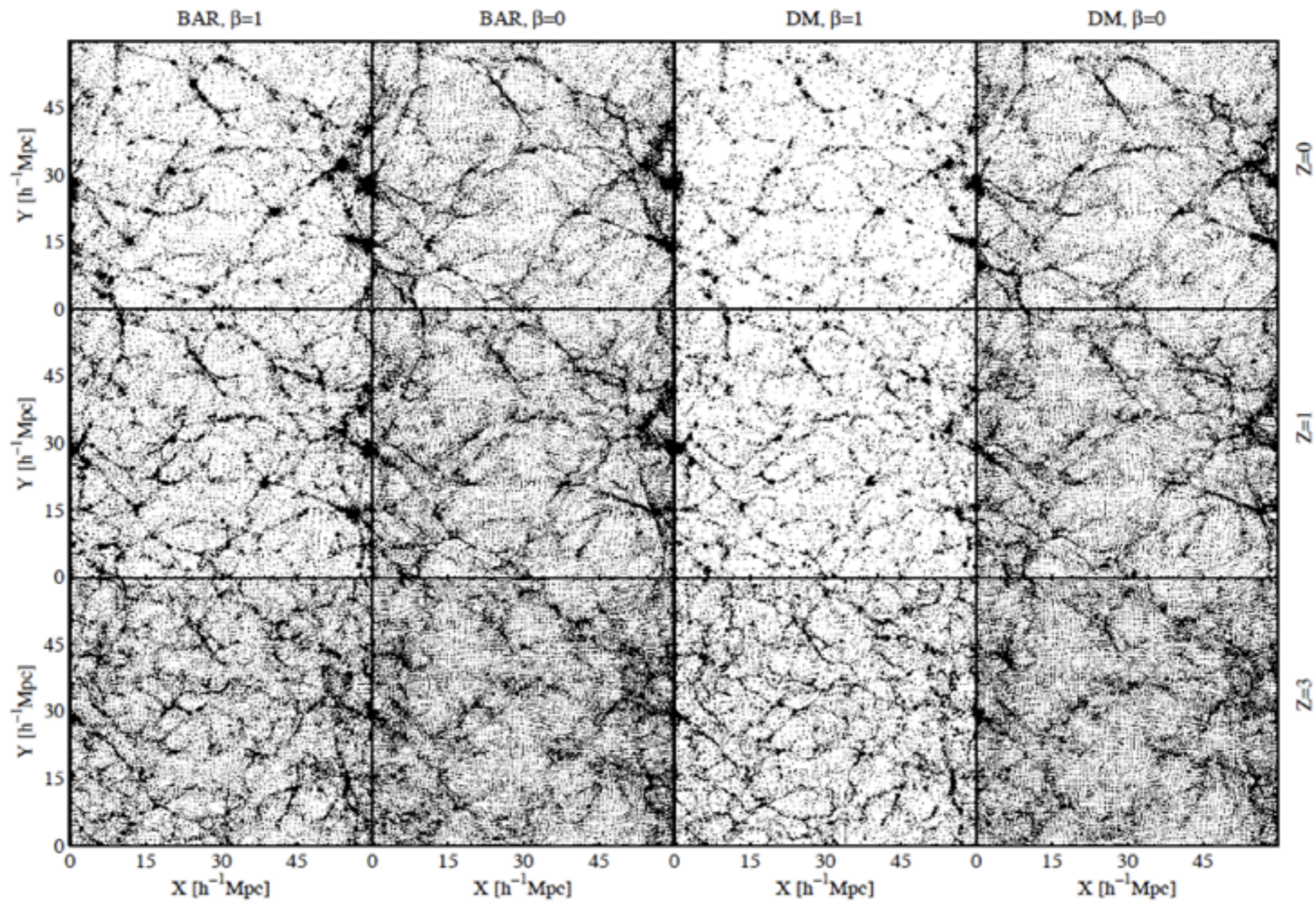
cf. Hamann, Hannestad, Raffelt & Wong 10  
Dvorkin, Wyman, Rudd, Douglas & Hu 14

# Violation of the Equivalence Principle



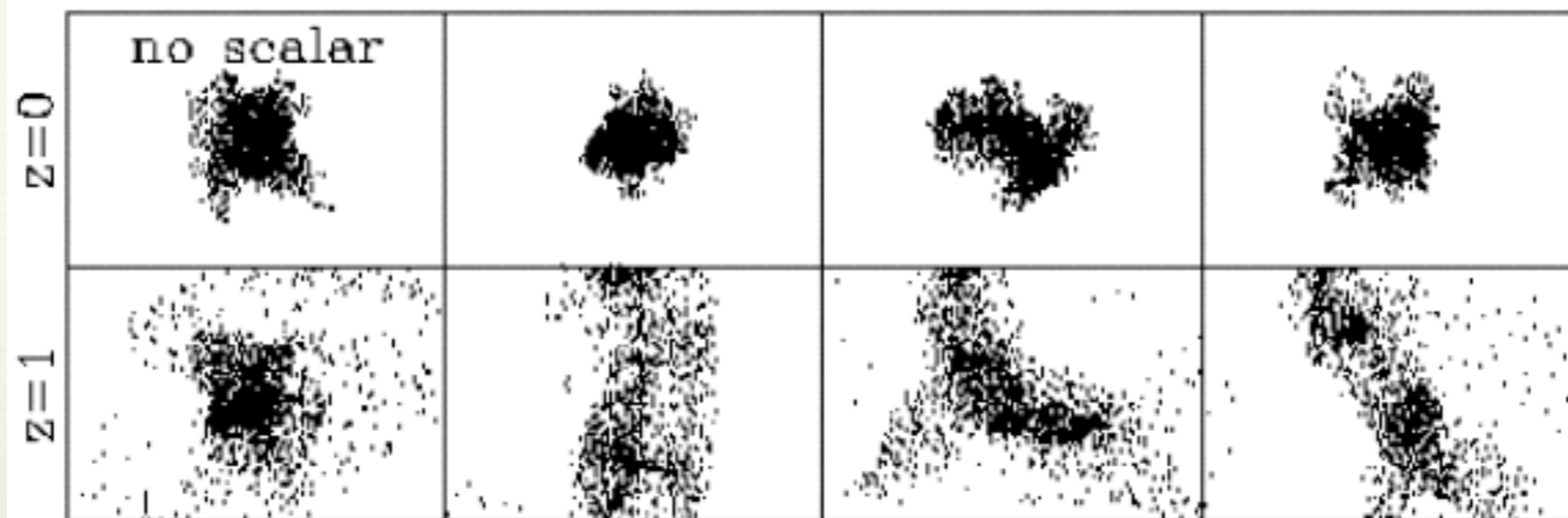
**If B and E contain DM**



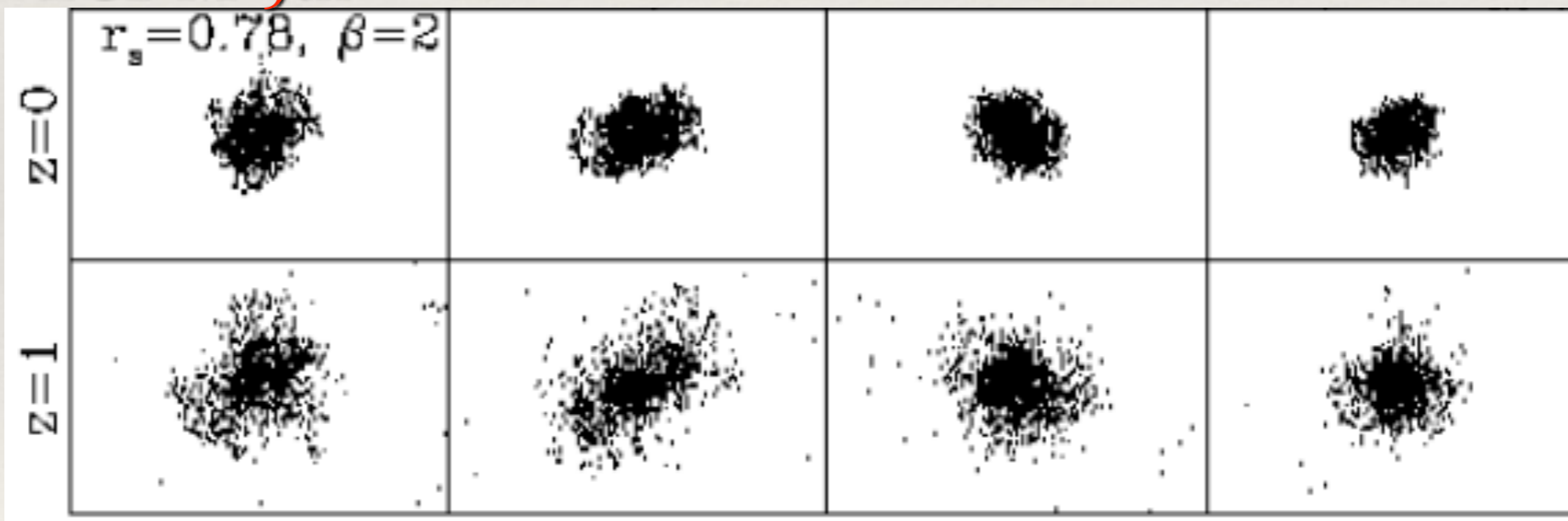




# LCDM



# LCDM+5th

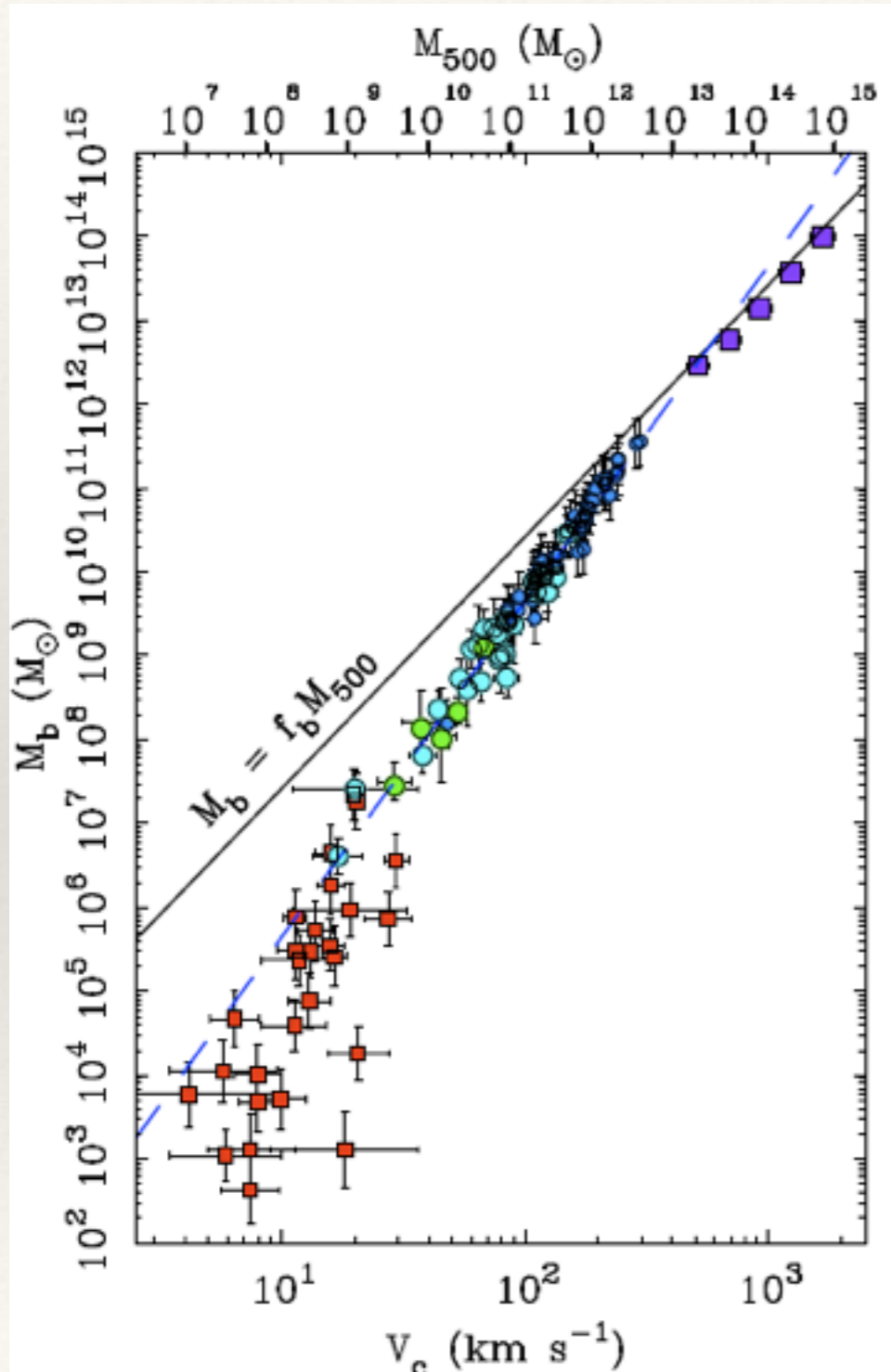




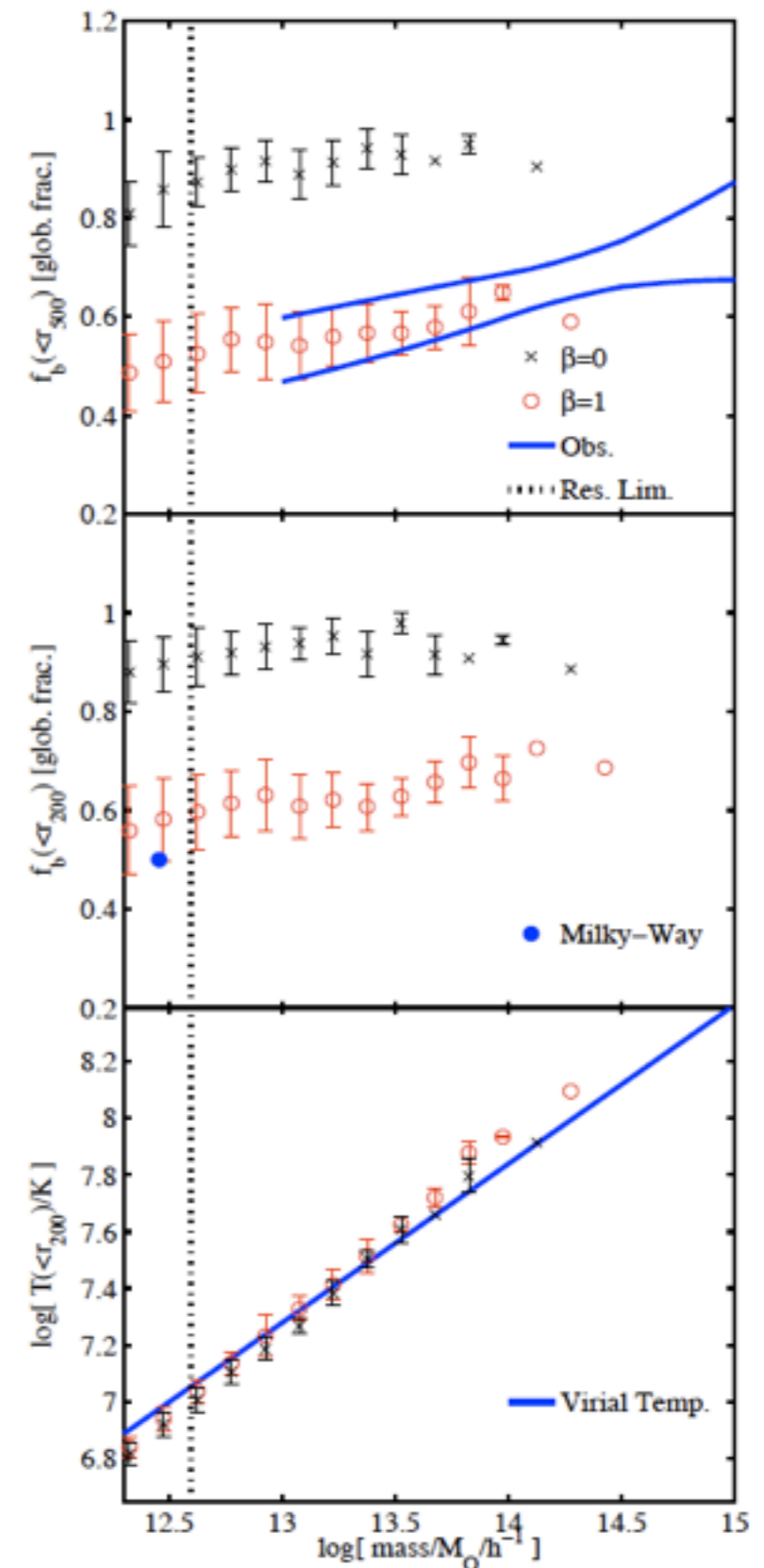
# Baryonic Fraction as a function of mass

## Observations

*McGaugh et. al. 09*



## Simulations with 5th

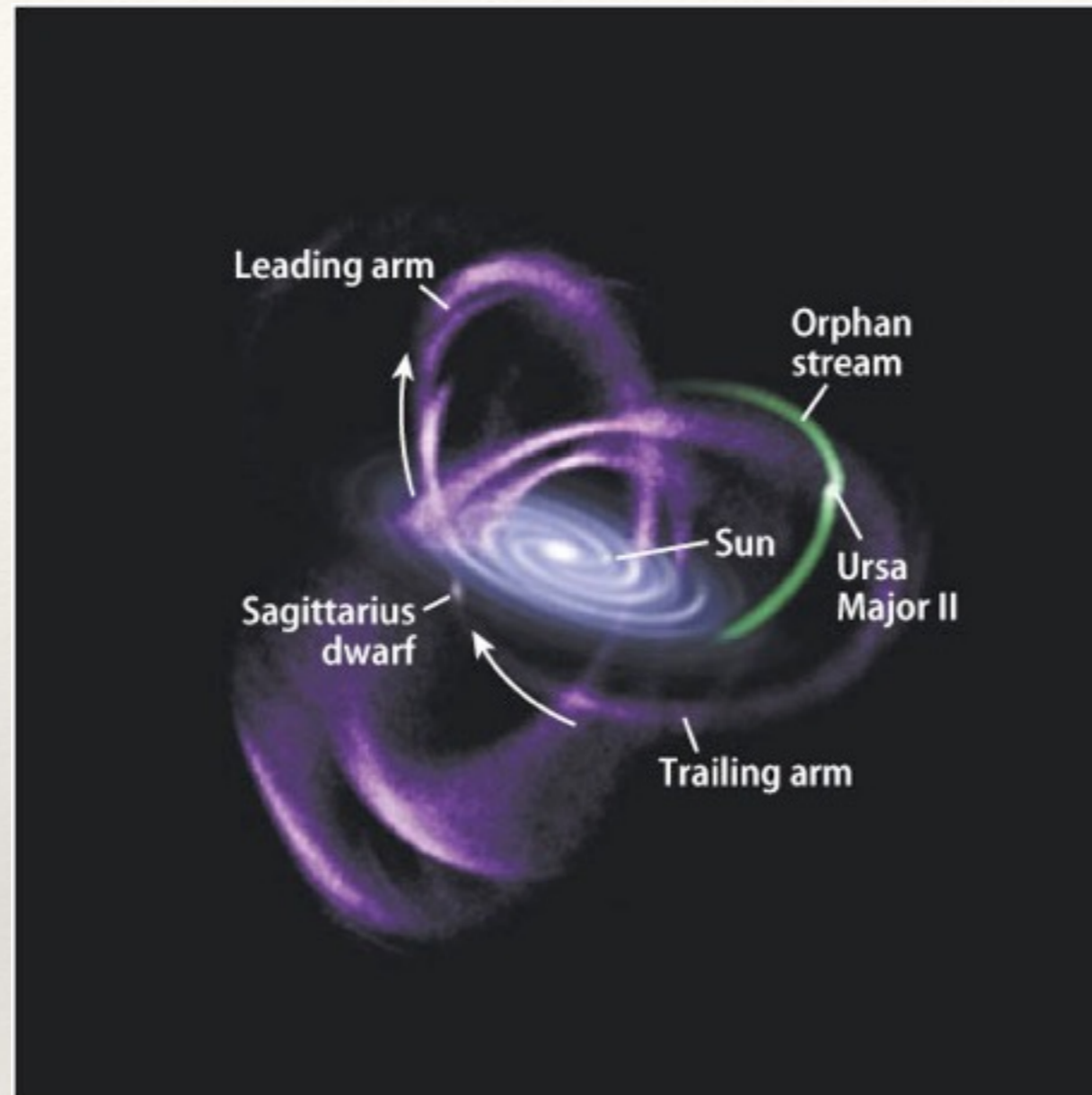


*Tweaking: implications*

# A test for ReBEL

Galactic satellites / streams

(Frieman & Gradwohl 93, Kesden & Kamionkowski 06)



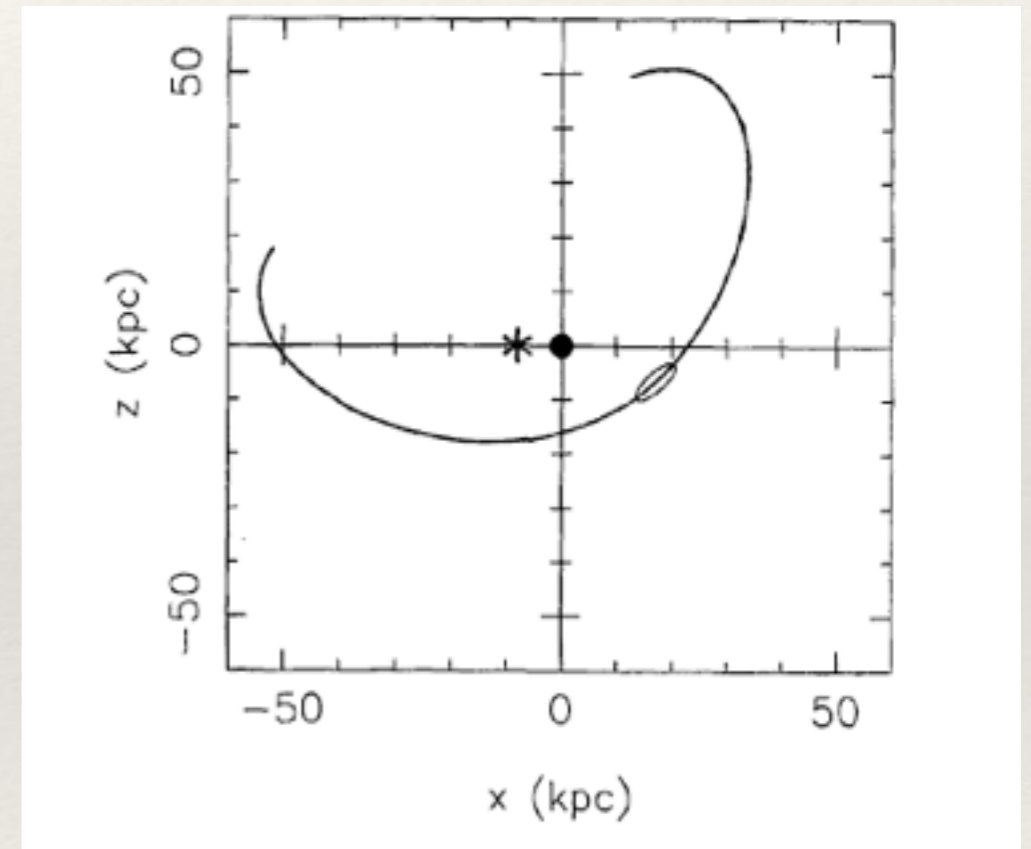
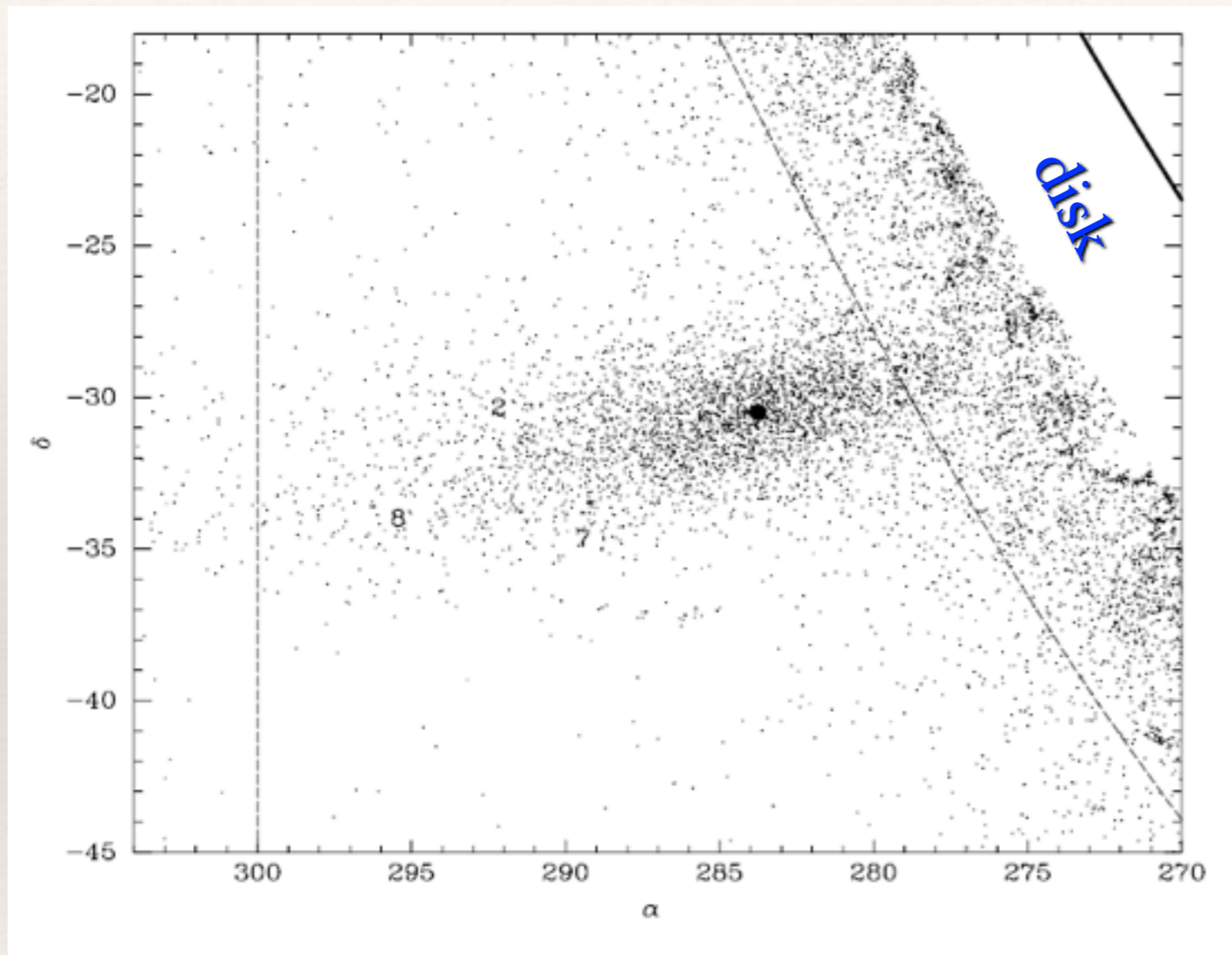
e.g. Sagittarius stream is ok

(Keselma, AN & Peebles in response to Kesden and Kamionkowski)

*Tweaking: falsify?*



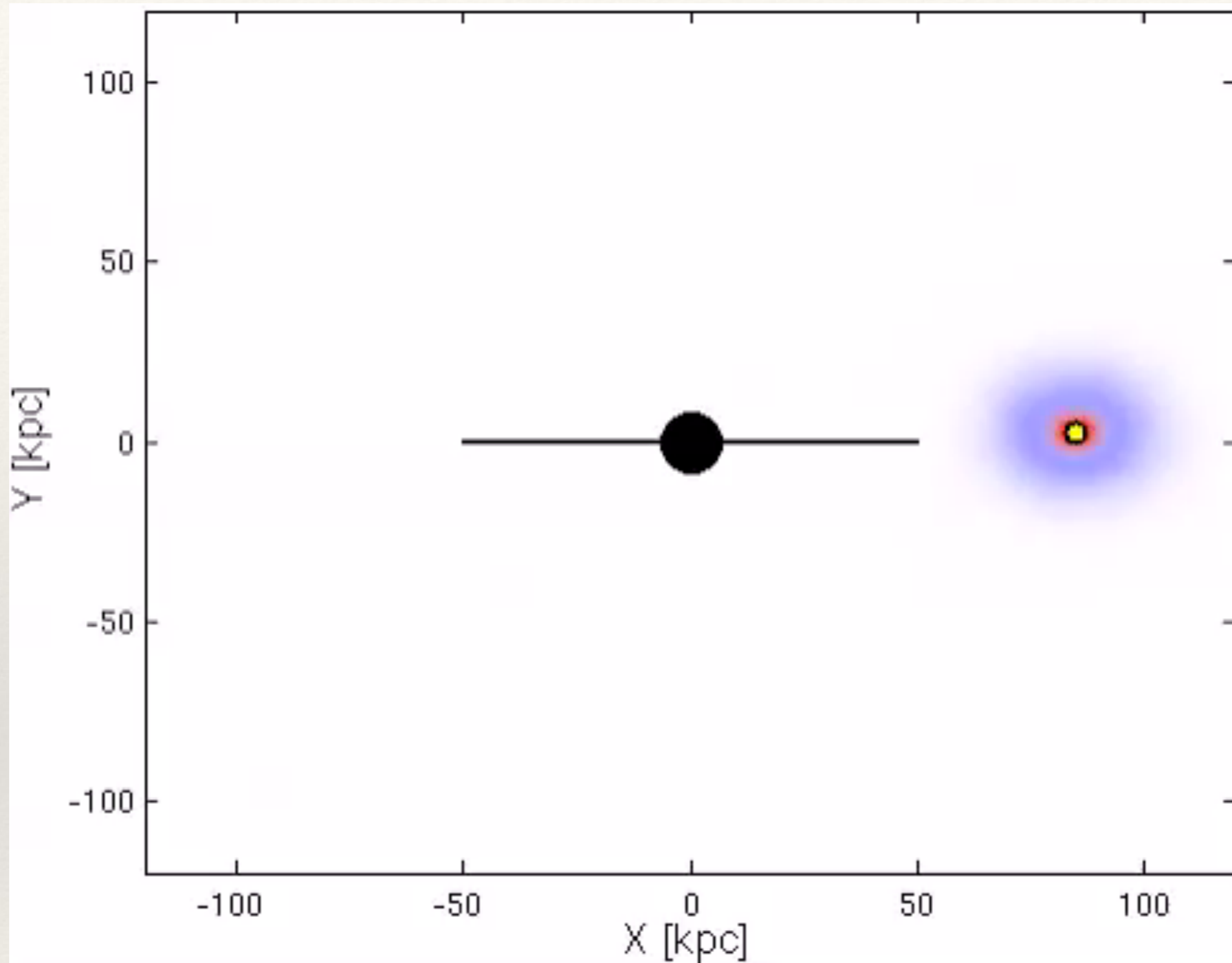
# The Sagittarius dwarf/stream



← 15kpc →

Ibata et al

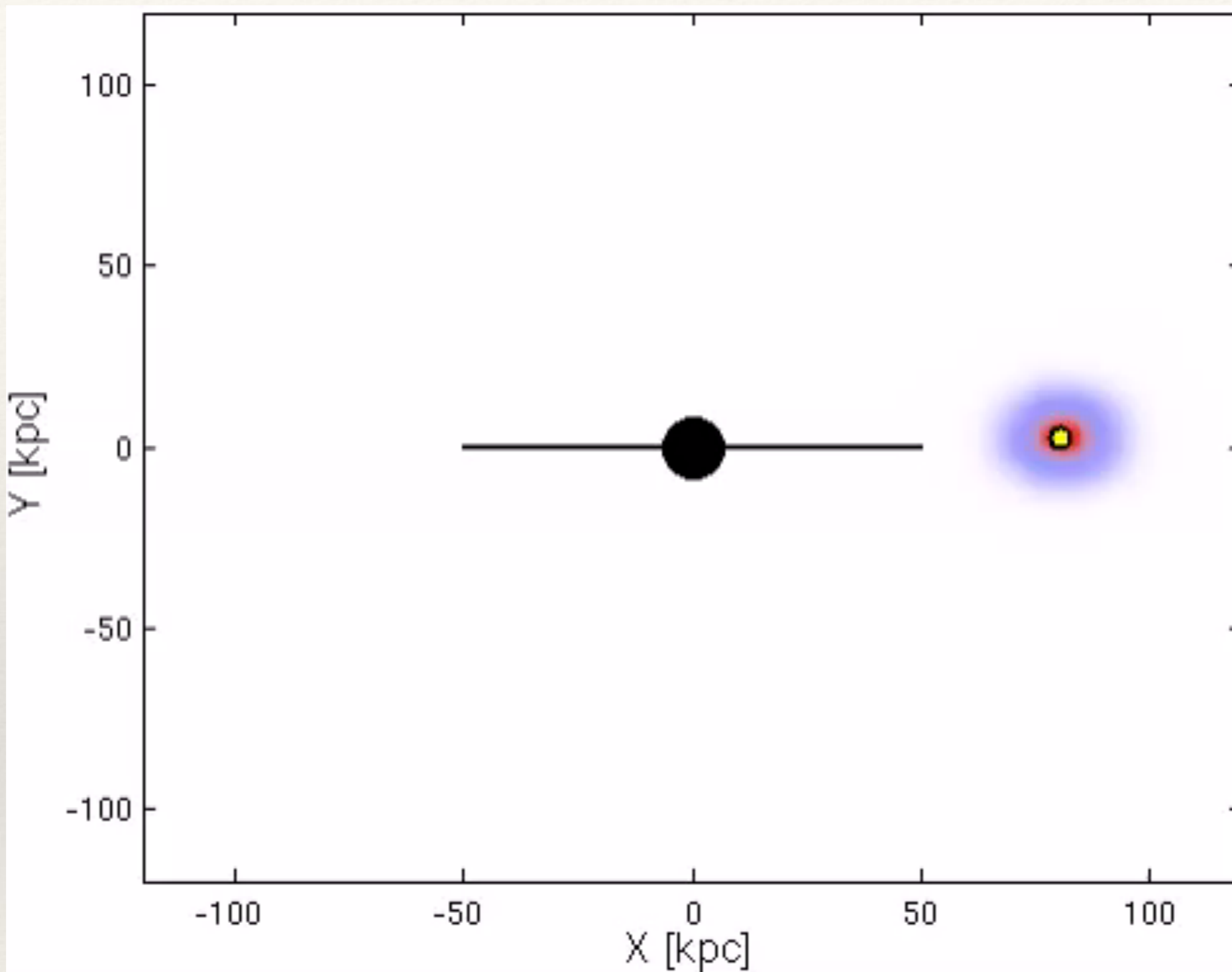
# Gravity alone



3 component gravitational potential: halo+disk+“bulge”

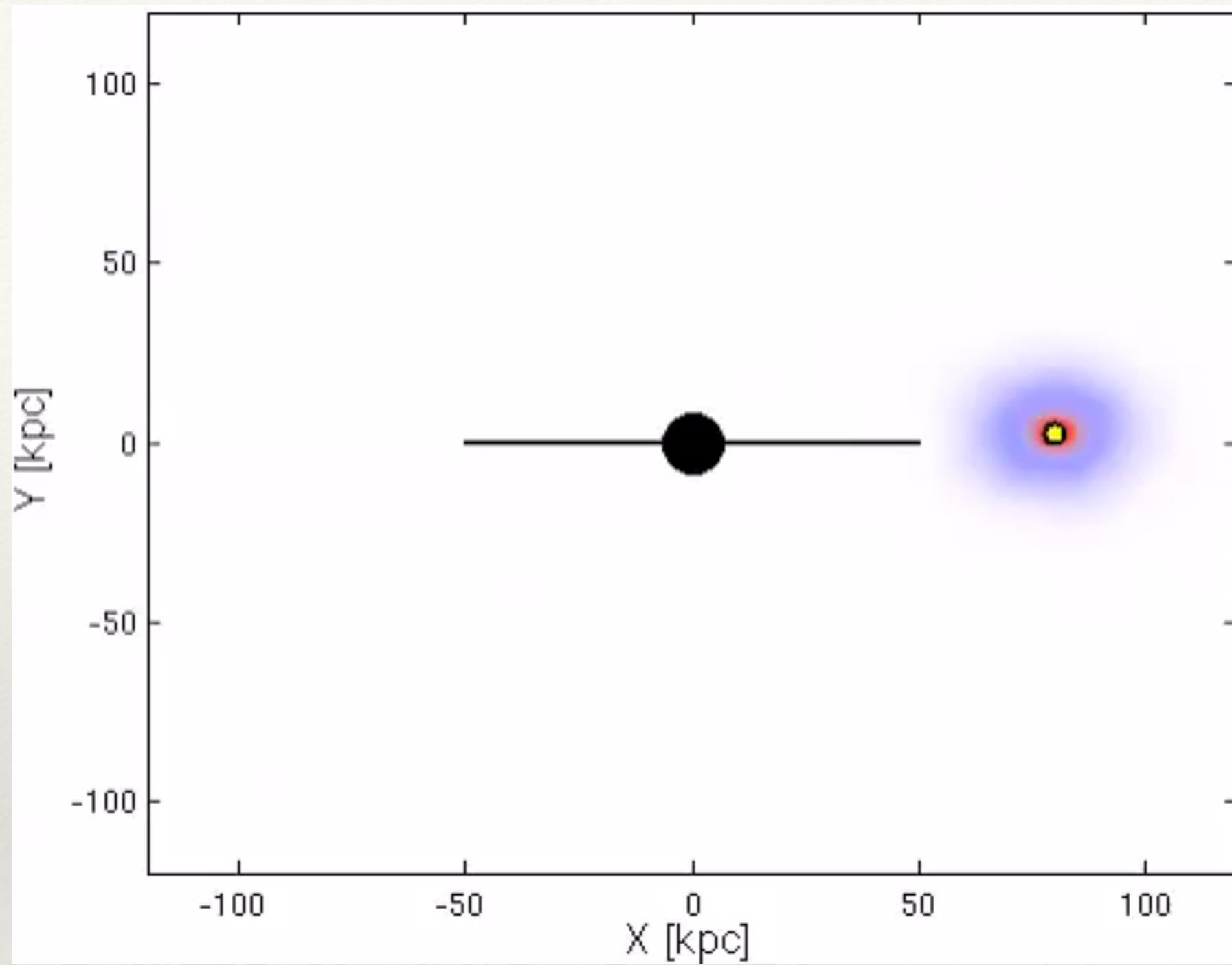


Gravity + weak 5th (beta=0.1)



A demonstration of the Frieman & Gradwohl 93 and Kesden & Kamionkowski 2006 test

# Gravity + strong ReBEL (beta=1)

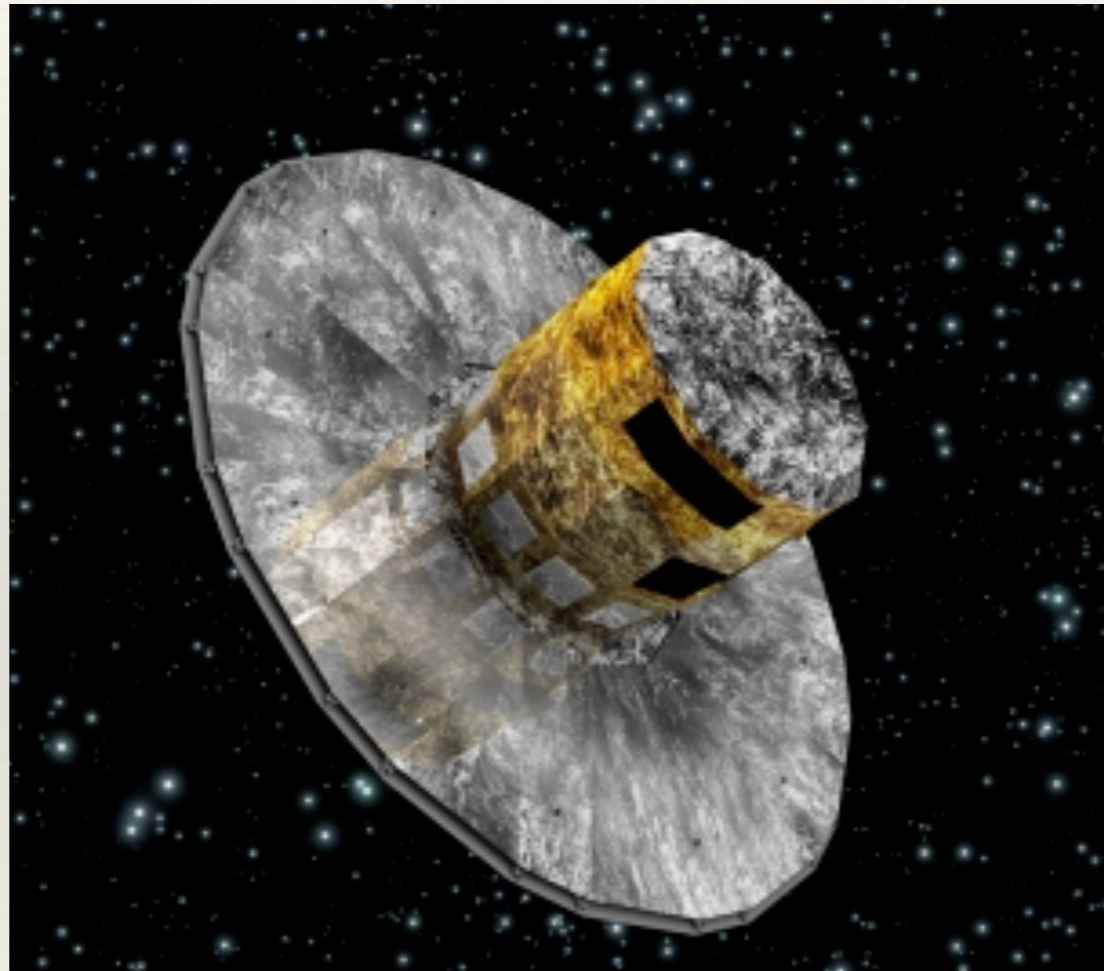




	Observations	Simulation (at 3 Gyrs)
Distance from center of galaxy	Close to pericenter, at 16 Kpc (Law et al., 2004)	Close to pericenter, at 18 Kpc
Luminosity, Assuming M/L=2.5 solar	2 to 5.8e7 (Ibata 97, Mateo 98)	8e7
Line of sight Velocity dispersion	9.6 Km/s, and constant (Bellazzini, 2008)	9.8 Km/s and constant

Observed core radius is about 1 Kpc (Majewski, 2003)  
 – matched well by the simulation.

# The Gaia prospect



Lunched Dec 2013 & doing well



# Problem I:

$$V_{\perp} = M d \rightarrow \text{Mpc}$$

$\swarrow$  km/s       $\downarrow$  as/yr

Error on  $d \sim d^2$  !!

→ disastrous for distant objects!

Solution: use  $d \approx cz/H_0$

$$\text{Error: } \Delta V_{\perp} = M \frac{V_{\text{rad}}}{H_0} \ll M d = V_{\perp}$$

## Problem II:

Gaia's onboard thresholding is optimized for point sources

But, a large number of galaxies have stellar light concentrated in compact regions, making them appear as point sources.

- For example, the nuclei of M87 and N5121 (both  $d=17.8\text{Mpc}$ ) should be detectable by Gaia with an end of mission accuracy of  $600\text{km/s}$  in  $V_{\perp}$ .
- Visual inspection of SB profiles of the Carnegie-Irvine Galaxy Survey (Ho et al 2011) shows that 70% of galaxies in this survey could be detected by Gaia. The majority of those nearby galaxies will be detected if placed at  $\gtrsim 500\text{Mpc}$  (early types) and  $\gtrsim 250\text{Mpc}$  (late type).



# End-of-mission (2018) expectations

$$v_{\parallel} = - \sum_{lm} \frac{d\Phi_{lm}}{ds} Y_{lm}$$

$$v_{\perp} = - \sum_{lm} \frac{\Phi_{lm}}{s} \Psi_{lm}$$

**Pros:** free of biases, allows tests of potential flow ansatz

*New probes: Gaia*

