

*Phenomenology of a pseudoscalar glueball  
and charmed mesons  
in the extended Linear Sigma Model*

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# Introduction

- Quantum Chromodynamics: QCD

- Chiral Symmetry of the QCD Lagrangian:  
if all quark massless then we have chiral symmetry

$$U(N_f)_r \times U(N_f)_l = SU(N_f)_r \times SU(N_f)_l \times U(1)_V \times U(1)_A$$

- Spontaneous breaking of chiral symmetry by quark condensates
- Explicit breaking of global chiral symmetry by quark masses and chiral anomaly

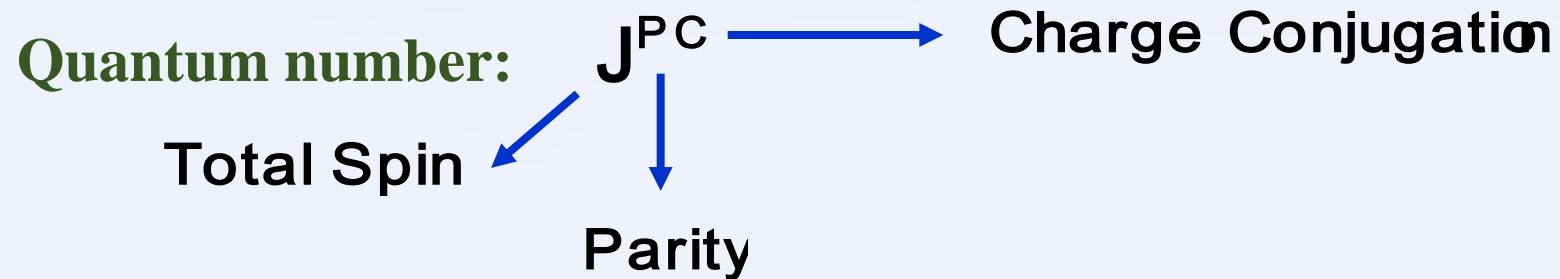
# Effective chiral model of (QCD)

- Development of a chirally symmetric model for mesons.
  - Nonlinear Sigma Model.
  - Linear Sigma Model extended  $\rightarrow$  extended Linear Sigma Model.
- Study of the model for  $T = \mu = 0$  (spectroscopy in vacuum).  
(Masses, decay, scattering length, ...)

# Fields of the model

- **Mesons: quark-antiquark states (  $q\bar{q}$  )**

(scalar, pseudoscalar, vector and axialvector quarkonia.)



- **Glueballs (additional mesons): The scalar and the pseudoscalar glueball**
- **Baryons: nucleon doublet and its partner**



# Scalar Glueball $G$

Mixing matrix of the three scalar fields ( $\sigma_N, \sigma_S, G$ )

$$\begin{pmatrix} f_0(1370) \\ f_0(1500) \\ f_0(1710) \end{pmatrix} = \begin{pmatrix} 0.94 & -0.17 & 0.29 \\ 0.21 & 0.97 & -0.12 \\ -0.26 & 0.18 & 0.95 \end{pmatrix} \begin{pmatrix} \sigma_N \equiv (\bar{u}u + \bar{d}d)/\sqrt{2} \\ \sigma_S \equiv \bar{s}s \\ G \equiv gg \end{pmatrix}$$

S. Janowski, F. Giacosa and D. H. Rischke, Phys.Rev. D90 (2014) 114005 [ arXiv:1408.4921 [hep-ph]].

## Pseudoscalar mixing angle

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos \theta_\eta & \sin \theta_\eta \\ -\sin \theta_\eta & \cos \theta_\eta \end{pmatrix} \begin{pmatrix} \eta_N \equiv \sqrt{1/2}(\bar{u}u + \bar{d}d) \\ \eta_S = \bar{s}s \end{pmatrix}$$

**Theory:**  $\theta_\eta = -44^\circ$

**Exp (KLOE):**  $\theta_\eta \cong -41^\circ$

# Construction of the eLSM

The construction of the so-called Extended Linear Sigma Model (eLSM) based on

- dilatation invariance

**Note that:** The breaking of the dilatation symmetry is only included in the “gluonic part” (scalar glueball and axial anomaly)

- chiral invariance

$$SU(N_f)_r \times SU(N_f)_l \times U(1)_V$$

Furthermore, the invariance under C and P is also taken into account.

# The eLSM Lagrangian with (axial-)vector mesons

$$\mathcal{L} = \mathcal{L}_{\text{meson}} + \mathcal{L}_{\text{baryon}} + \mathcal{L}_{\text{dilaton}}$$

$$\begin{aligned} \mathcal{L}_{\text{meson}} = & \text{Tr}[(D_\mu \Phi)^\dagger (D^\mu \Phi)] - m_0^2 \left(\frac{G}{G_0}\right)^2 \text{Tr}(\Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 \\ & - \frac{1}{4} \text{Tr}[(L^{\mu\nu})^2 + (R^{\mu\nu})^2] + \text{Tr} \left[ \left( \left(\frac{G}{G_0}\right)^2 \frac{m_1^2}{2} + \Delta \right) (L_\mu^2 + R_\mu^2) \right] + \text{Tr}[H(\Phi + \Phi^\dagger)] \\ & + c_1 (\det \Phi - \det \Phi^\dagger)^2 + \frac{g_2}{2} \{ \text{Tr}(L_{\mu\nu} [L^\mu, L^\nu]) + \text{Tr}(R_{\mu\nu} [R^\mu, R^\nu]) \} \\ & + \frac{h_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr} (L_\mu^2 + R_\mu^2) + h_2 \text{Tr}[|L_\mu \Phi|^2 + |\Phi R_\mu|^2] \\ & + 2h_3 \text{Tr}(L_\mu \Phi R^\mu \Phi^\dagger) + \text{chirally invariant vector and axialvector four-point interaction vertices} \end{aligned}$$

$$\begin{aligned} \mathcal{L}_{\text{baryon}} = & \bar{\Psi}_{1L} i \gamma_\mu D_{1L}^\mu \Psi_{1L} + \bar{\Psi}_{1R} i \gamma_\mu D_{1R}^\mu \Psi_{1R} + \bar{\Psi}_{2L} i \gamma_\mu D_{2R}^\mu \Psi_{2L} + \bar{\Psi}_{2R} i \gamma_\mu D_{2L}^\mu \Psi_{2R} \\ & - \hat{g}_1 (\bar{\Psi}_{1L} \Phi \Psi_{1R} + \bar{\Psi}_{1R} \Phi^\dagger \Psi_{1L}) - \hat{g}_2 (\bar{\Psi}_{2L} \Phi^\dagger \Psi_{2R} + \bar{\Psi}_{2R} \Phi \Psi_{2L}) \\ & - (a\chi + bG) (\bar{\Psi}_{1L} \Psi_{2R} - \bar{\Psi}_{1R} \Psi_{2L} - \bar{\Psi}_{2L} \Psi_{1R} + \bar{\Psi}_{2R} \Psi_{1L}), \end{aligned}$$

$$\mathcal{L}_{\text{dilaton}} = \frac{1}{2} (\partial^\mu G)^2 - \frac{1}{4} \frac{m_G}{\Lambda^2} \left( G^4 \ln \left| \frac{G}{\Lambda} \right| - \frac{G^4}{4} \right)$$

D.Parganlija, P.Kovacs, G.Wolf, F.Giacosa and D.H. Rischke, Phys. Rev. D 87 (2013) 014011 [arXiv:1208.0585 [hep-ph]];

S.Gallas, F.Giacosa and D.H.Rischke, Phys. Rev. D82 (2010) 014004 [arXiv: 0907.5084 [hep-ph]].

# *Decays of the pseudoscalar glueball*

# A globally chirally invariant Lagrangian for three flavours

Interaction Lagrangian for the pseudoscalar glueball with scalar and pseudoscalar mesons

$$\mathcal{L}_{\tilde{G}}^{int} = ic_{\tilde{G}\Phi} \tilde{G} (\det\Phi - \det\Phi^\dagger)$$

where  $c_{\tilde{G}\Phi}$  is a dimensionless coupling constant and  $\Phi$  reads for three flavours,  $N_f = 3$  :

$$\Phi = (S^a + iP^a) t^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{(\sigma_N + a_0^0) + i(\eta_N + \pi^0)}{\sqrt{2}} & a_0^+ + i\pi^+ & K_S^+ + iK^+ \\ a_0^- + i\pi^- & \frac{(\sigma_N - a_0^0) + i(\eta_N - \pi^0)}{\sqrt{2}} & K_S^0 + iK^0 \\ K_S^- + iK^- & \bar{K}_S^0 + i\bar{K}^0 & \sigma_S + i\eta_S \end{pmatrix}$$

# Predictions for a pseudoscalar glueball

- Predict branching ratios for decays into three pseudoscalar mesons  $\tilde{G} \rightarrow PPP$

Quantity	Case (i): $M_{\tilde{G}} = 2.6$ GeV	Case (ii): $M_{\tilde{G}} = 2.37$ GeV
$\Gamma_{\tilde{G} \rightarrow KK\eta} / \Gamma_{\tilde{G}}^{tot}$	0.049	0.043
$\Gamma_{\tilde{G} \rightarrow KK\eta'} / \Gamma_{\tilde{G}}^{tot}$	0.019	0.011
$\Gamma_{\tilde{G} \rightarrow \eta\eta\eta} / \Gamma_{\tilde{G}}^{tot}$	0.016	0.013
$\Gamma_{\tilde{G} \rightarrow \eta\eta\eta'} / \Gamma_{\tilde{G}}^{tot}$	0.0017	0.00082
$\Gamma_{\tilde{G} \rightarrow \eta\eta'\eta'} / \Gamma_{\tilde{G}}^{tot}$	0.00013	0
$\Gamma_{\tilde{G} \rightarrow KK\pi} / \Gamma_{\tilde{G}}^{tot}$	0.47	0.47
$\Gamma_{\tilde{G} \rightarrow \eta\pi\pi} / \Gamma_{\tilde{G}}^{tot}$	0.16	0.17
$\Gamma_{\tilde{G} \rightarrow \eta'\pi\pi} / \Gamma_{\tilde{G}}^{tot}$	0.095	0.090

W. I. Eshraim; S. Janowski; F. Giacosa; D. H. Rischke. Phys.Rev. D87 (2013) 054036 [arXiv: 1208.6474 [hep-ph]].

**PANDA/FAIR will be able to scan the energy above 2.5 GeV**

BESIII has measured a candidate: X(2370)

The decay of the pseudoscalar glueball into three pions vanishes:

$$\Gamma_{\tilde{G} \rightarrow \pi\pi\pi} = 0$$

## Predict branching ratios for decays into a scalar and a pseudoscalar meson

$$\tilde{G} \rightarrow PS$$

Quantity	Case (i): $M_{\tilde{G}} = 2.6 \text{ GeV}$	Case (ii): $M_{\tilde{G}} = 2.37 \text{ GeV}$
$\Gamma_{\tilde{G} \rightarrow KK_S} / \Gamma_{\tilde{G}}^{tot}$	0.060	0.070
$\Gamma_{\tilde{G} \rightarrow a_0 \pi} / \Gamma_{\tilde{G}}^{tot}$	0.083	0.10
$\Gamma_{\tilde{G} \rightarrow \eta \sigma_N} / \Gamma_{\tilde{G}}^{tot}$	0.0000026	0.0000030
$\Gamma_{\tilde{G} \rightarrow \eta' \sigma_N} / \Gamma_{\tilde{G}}^{tot}$	0.039	0.026
$\Gamma_{\tilde{G} \rightarrow \eta \sigma_S} / \Gamma_{\tilde{G}}^{tot}$	0.012 (0.015)	0.0094 (0.017)
$\Gamma_{\tilde{G} \rightarrow \eta' \sigma_S} / \Gamma_{\tilde{G}}^{tot}$	0 (0.0082)	0 (0)

whereas

Could be measured by



$$K_s = K_0^*(1430), a_0 = a_0(1450), \sigma_N \approx f_0(1370), \sigma_S \approx f_0(1710)$$

The full width of the pseudoscalar glueball is expected to be small

W. I. Eshraim; S. Janowski; F. Giacosa; D. H. Rischke. Phys.Rev. D87 (2013) 054036 [arXiv: 1208.6474 [hep-ph]; W.I. Eshraim and S. Janowski, PoS ConfinementX 118, (2012) [arXiv:1301.3345 [hep-ph]]; W.I. Eshraim and S. Janowski, J. Phys.Conf. Ser. 426, 012018 (2013) [arXiv:1211.7323 [hep-ph]].

*Charmed mesons  
in the extended Linear Sigma Model*

## The extension to the four-flavor case

- Additional mass term is required to reproduce charmed meson masses

$$- 2 \text{Tr}[\varepsilon \Phi^\dagger \Phi]$$

with

$$\varepsilon = \begin{pmatrix} \varepsilon_U & 0 & 0 & 0 \\ 0 & \varepsilon_D & 0 & 0 \\ 0 & 0 & \varepsilon_S & 0 \\ 0 & 0 & 0 & \varepsilon_C \end{pmatrix}$$

where  $\varepsilon \propto m_i^2$ ,  $\varepsilon_U = \varepsilon_D = \varepsilon_S = 0$ , while  $\varepsilon_C$  is nonzero

# Charmed mesons in the model

- The number of mesons in the model

$$4N_f^2 + 2 \text{ mesons}$$

- For  $N_f = 4$  there are 66 mesons: 64 quark-antiquark fields + one pseudoscalar glueball  $\tilde{G}$  + one scalar glueball  $G$

Quantum numbers ( $J^{PC}$ )

Pseudoscalar mesons:  $0^{-+}$

$$D^0$$

$$D^\pm$$

$$D_s^\pm$$

$$\eta_c \Rightarrow \eta_c(1S)$$

Scalar mesons:  $0^{++}$

$$D_0^{*0} \Rightarrow D_0^*(2400)^0$$

$$D_0^{*\pm} \Rightarrow D_0^*(2400)^\pm$$

$$D_{S0}^{*\pm} \Rightarrow D_{S0}^*(2317)^\pm$$

$$\chi_{c0} \Rightarrow \chi_{c0}(1P)$$

Vector mesons:  $1^{--}$

$$D^{*0} \Rightarrow D^*(2007)^0$$

$$D^{*\pm} \Rightarrow D^*(2010)^\pm$$

$$D_s^\pm$$

$$J/\psi \Rightarrow J/\psi(1S)$$

Axial-vector mesons:  $1^{++}$

$$D_1^0 \Rightarrow D_1(2420)^0$$

$$D_1^\pm \Rightarrow D_1(2420)^\pm$$

$$D_{S1}^\pm \Rightarrow D_{S1}(2536)^\pm$$

$$\chi_{c1} \Rightarrow \chi_{c1}(1P)$$

# Including charm degree of freedom

## 1) Pseudoscalar fields:

$$P = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}(\eta_N + \pi^0) & \pi^+ & K^+ & D^0 \\ \pi^- & \frac{1}{\sqrt{2}}(\eta_N - \pi^0) & K^0 & D^- \\ K^- & \bar{K}^0 & \eta_S & D_S^- \\ \bar{D}^0 & D^+ & D_S^+ & \eta_c \end{pmatrix}$$

## 2) Scalar fields:

$$S = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}(\sigma_N + a_0^0) & a_0^+ & K_0^{*+} & D_0^{*0} \\ a_0^- & \frac{1}{\sqrt{2}}(\sigma_N - a_0^0) & K_0^{*0} & D_0^{*-} \\ K_0^{*-} & \bar{K}_0^{*0} & \sigma_S & D_{S0}^{*-} \\ \bar{D}_0^{*0} & D_0^{*+} & D_{S0}^{*+} & \chi_{c0} \end{pmatrix}$$

$\bar{n}n \propto \bar{u}u + \bar{d}d$   
 $\bar{s}s$   
 $\bar{c}c$

The multiplet of the scalar and pseudoscalar quark-antiquark states:  $\Phi = S + iP$

## 3) Vector fields:

$$V^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}(\omega_N + \rho^0) & \rho^+ & K^*(892)^+ & D^{*0} \\ \rho^- & \frac{1}{\sqrt{2}}(\omega_N - \rho^0) & K^*(892)^0 & D^{*-} \\ K^*(892)^- & \bar{K}^*(892)^0 & \omega_S & D_S^{*-} \\ \bar{D}^{*0} & D^{*+} & D_S^{*+} & J/\psi \end{pmatrix}^\mu$$

## 4) Axial vector fields:

$$A^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}}(f_{1,N} + a_1^0) & a_1^+ & K_1^+ & D_1^0 \\ a_1^- & \frac{1}{\sqrt{2}}(f_{1,N} - a_1^0) & K_1^0 & D_1^- \\ K_1^- & \bar{K}_1^0 & f_{1,S} & D_{S1}^- \\ \bar{D}_1^0 & D_1^+ & D_{S1}^+ & \chi_{c,1} \end{pmatrix}^\mu$$

The left-handed matrix:  $L^\mu = V^\mu + A^\mu$  and the right-handed matrix:  $R^\mu = V^\mu - A^\mu$

W. I. Eshraim, PoS QCD -TNT-III (2014) 049 [arXiv:1401.3260 [hep-ph]]; W. I. Eshraim; Giacosa; and D. H. Rischke; [arXiv:1405.5861 [hep-ph]]

## Perform Spontaneous Symmetry Breaking (SSB)

$$G \rightarrow G + G_0, \quad \sigma_N \rightarrow \sigma_N + \phi_N$$

$$\sigma_S \rightarrow \sigma_S + \phi_S, \quad \chi_{C0} \rightarrow \chi_{C0} + \phi_C$$

where

$$\phi_N = Z_N f_N, \quad \phi_S = \frac{2Z_K f_K - \phi_N}{\sqrt{2}},$$

and

$$\phi_S = \frac{Z_{\eta_c} f_{\eta_c}}{\sqrt{2}}$$

**There are 29 eqs. of square masses of mesons with 15 unknown parameters.**

# Parameters

Parameter	Value	Parameter	Value
$m_1^2$	$0.413 \times 10^6 \text{ MeV}^2$	$m_0^2$	$-0.918 \times 10^6 \text{ MeV}^2$
$\phi_C^2 c/2$	$450 \cdot 10^{-6} \text{ MeV}^{-2}$	$\delta_S$	$0.151 \times 10^6 \text{ MeV}^2$
$g_1$	5.84	$h_1$	0
$h_2$	9.88	$h_3$	3.87
$\phi_N$	164.6 MeV	$\phi_S$	126.2 MeV
$\lambda_1$	0	$\lambda_2$	68.3

[D. Parganlija, P. Kovacs, G. Wolf, F. Giacosa and D. H. Rischke, Phys. Rev. D 87 (2013) 014011 [arXiv:1208.0585 [hep-ph]].

→  $\chi^2 / d.o.f = 1.23$

There are only three new parameters for  $N_f = 4$  :

$$\begin{aligned} \phi_C &= (176 \pm 28) \text{ MeV}, & \delta_C &= (3.91 \pm 0.36) \times 10^6 \text{ MeV}^2, \\ \varepsilon_C &= (2.23 \pm 0.71) \times 10^6 \text{ MeV}^2. & & \text{with } \chi^2 / d.o.f = 1 \end{aligned}$$

# Masses of (open and hidden) charmed mesons:

Resonance	Quark content	$J^P$	Our Value [MeV]	Experimental Value [MeV]
$D^0$	$u\bar{c}, \bar{u}c$	$0^-$	$1981 \pm 73$	$1864.86 \pm 0.13$
$D_S^\pm$	$s\bar{c}, \bar{s}c$	$0^-$	$2004 \pm 74$	$1968.50 \pm 0.32$
$\eta_c(1S)$	$c\bar{c}$	$0^-$	$2673 \pm 118$	$2983.7 \pm 0.7$
$D_0^*(2400)^0$	$u\bar{c}, \bar{u}c$	$0^+$	$2414 \pm 77$	$2318 \pm 29$
$D_{S0}^*(2317)^\pm$	$s\bar{c}, \bar{s}c$	$0^+$	$2467 \pm 76$	$2317.8 \pm 0.6$
$\chi_{c0}(1P)$	$c\bar{c}$	$0^+$	$3144 \pm 128$	$3414.75 \pm 0.31$
$D^*(2007)^0$	$u\bar{c}, \bar{u}c$	$1^-$	$2168 \pm 70$	$2006.99 \pm 0.15$
$D_s^*$	$s\bar{c}, \bar{s}c$	$1^-$	$2203 \pm 69$	$2112.3 \pm 0.5$
$J/\psi(1S)$	$c\bar{c}$	$1^-$	$2947 \pm 109$	$3096.916 \pm 0.011$
$D_1(2420)^0$	$u\bar{c}, \bar{u}c$	$1^+$	$2429 \pm 63$	$2421.4 \pm 0.6$
$D_{S1}(2536)^\pm$	$s\bar{c}, \bar{s}c$	$1^+$	$2480 \pm 63$	$2535.12 \pm 0.13$
$\chi_{c1}(1P)$	$c\bar{c}$	$1^+$	$3239 \pm 101$	$3510.66 \pm 0.07$

W. I. Eshraim, F. Giacosa and D. H. Rischke, [arXiv:1405.5861 [hep-ph]];

W. I. Eshraim, PoS QCD -TNT-III (2014) 049 [arXiv:1401.3260 [hep-ph]].



# Decay widths of open charmed mesons:

Decay Channel	Theoretical result [MeV]	Experimental result [MeV]
$D_0^*(2400)^0 \rightarrow D\pi = D^+\pi^- + D^0\pi^0$	$139^{+243}_{-114}$	$D^+\pi^-$ seen; full width $\Gamma = 267 \pm 40$
$D_0^*(2400)^+ \rightarrow D\pi = D^+\pi^0 + D^0\pi^+$	$51^{+182}_{-51}$	$D^+\pi^0$ seen; full width: $\Gamma = 283 \pm 24 \pm 34$
$D^*(2007)^0 \rightarrow D^0\pi^0$	$0.025 \pm 0.003$	seen; $< 1.3$
$D^*(2007)^0 \rightarrow D^+\pi^-$	0	not seen
$D^*(2010)^+ \rightarrow D^+\pi^0$	$0.018^{+0.002}_{-0.003}$	$0.029 \pm 0.008$
$D^*(2010)^+ \rightarrow D^0\pi^+$	$0.038^{+0.005}_{-0.004}$	$0.065 \pm 0.017$
$D_1(2420)^0 \rightarrow D^*\pi = D^{*+}\pi^- + D^{*0}\pi^0$	$65^{+51}_{-37}$	$D^{*+}\pi^-$ seen; full width: $\Gamma = 27.4 \pm 2.5$
$D_1(2420)^0 \rightarrow D^0\pi\pi = D^0\pi^+\pi^- + D^0\pi^0\pi^0$	$0.59 \pm 0.02$	seen
$D_1(2420)^0 \rightarrow D^+\pi^-\pi^0$	$0.21^{+0.01}_{-0.015}$	seen
$D_1(2420)^0 \rightarrow D^+\pi^-$	0	not seen; $\Gamma(D^+\pi^-)/\Gamma(D^{*+}\pi^-) < 0.24$
$D_1(2420)^+ \rightarrow D^*\pi = D^{*+}\pi^0 + D^{*0}\pi^+$	$65^{+51}_{-36}$	$D^{*0}\pi^+$ seen; full width: $\Gamma = 25 \pm 6$
$D_1(2420)^+ \rightarrow D^+\pi\pi = D^+\pi^+\pi^- + D^+\pi^0\pi^0$	$0.56 \pm 0.02$	seen
$D_1(2420)^+ \rightarrow D^0\pi^0\pi^+$	$0.22 \pm 0.01$	seen
$D_1(2420)^+ \rightarrow D^0\pi^+$	0	not seen; $\Gamma(D^0\pi^+)/\Gamma(D^{*0}\pi^+) < 0.18$
$D_{S1}(2536)^+ \rightarrow D^*K = D^{*0}K^+ + D^{*+}K^0$	$25^{+22}_{-15}$	seen; full width $\Gamma = 0.92 \pm 0.03 \pm 0.04$
$D_{S1}(2536)^+ \rightarrow D^+K^0$	0	not seen
$D_{S1}(2536)^+ \rightarrow D^0K^+$	0	not seen

# Decay widths of hidden charmed mesons:

1) Decay widths of (axial-)vector charmonium states:

$$\Gamma_{J/\psi} = 0 \quad \text{and} \quad \Gamma_{\chi_{c1}} = 0$$

2) Decay widths of a pseudoscalar charmonium state ( $\chi_{c0}$ ):

Decay Channel	theoretical result [MeV]	Experimental result [MeV]
$\Gamma_{\chi_{c0} \rightarrow \bar{K}_0^{*0} K_0^{*0}}$	0.01	$0.01 \pm 0.0047$
$\Gamma_{\chi_{c0} \rightarrow K^- K^+}$	0.059	$0.061 \pm 0.007$
$\Gamma_{\chi_{c0} \rightarrow \pi\pi}$	0.089	$0.088 \pm 0.0092$
$\Gamma_{\chi_{c0} \rightarrow \bar{K}^{*0} K^{*0}}$	0.0175	$0.017 \pm 0.0072$
$\Gamma_{\chi_{c0} \rightarrow w w}$	0.01	$0.0099 \pm 0.0017$
$\Gamma_{\chi_{c0} \rightarrow \phi\phi}$	0.004	$0.0081 \pm 0.0013$
$\Gamma_{\chi_{c0} \rightarrow k_1^+ K^-}$	0.005	$0.063 \pm 0.0233$

W. I. Eshraim and D. H. Rischke, in preparation preliminary!

## Decay widths of a pseudoscalar charmonium state ( $\chi_{c0}$ ):

Decay Channel	theoretical result [MeV]	Experimental result [MeV]
$\Gamma_{\chi_{c0} \rightarrow a_0 a_0}$	0.004	-
$\Gamma_{\chi_{c0} \rightarrow k_1 \bar{K}_1}$	0.005	-
$\Gamma_{\chi_{c0} \rightarrow \eta \eta}$	0.022	$0.031 \pm 0.0039$
$\Gamma_{\chi_{c0} \rightarrow \eta' \eta'}$	0.02	$0.02 \pm 0.0035$
$\Gamma_{\chi_{c0} \rightarrow \eta \eta'}$	0.004	$< 0.0024$
$\Gamma_{\chi_{c0} \rightarrow K^* K_0^*}$	0.00007	-
$\Gamma_{\chi_{c0} \rightarrow \rho \rho}$	0.01	-
$\Gamma_{\chi_{c0} \rightarrow f_0(1370) f_0(1370)}$	0.005	$< 0.003$
$\Gamma_{\chi_{c0} \rightarrow f_0(1500) f_0(1500)}$	0.004	$< 0.0005$
$\Gamma_{\chi_{c0} \rightarrow f_0(1370) f_0(1500)}$	0.000004	$< 0.001$
$\Gamma_{\chi_{c0} \rightarrow f_0(1370) f_0(1710)}$	0.0003	$0.0069 \pm 0.004$
$\Gamma_{\chi_{c0} \rightarrow f_0(1500) f_0(1710)}$	0.00004	$< 0.0007$
$\Gamma_{\chi_{c0} \rightarrow K_0^* K \eta}$	0.008	-
$\Gamma_{\chi_{c0} \rightarrow K_0^* K \eta'}$	0.004	-
$\Gamma_{\chi_{c0} \rightarrow f_0(1370) \eta \eta}$	0.0004	-
$\Gamma_{\chi_{c0} \rightarrow f_0(1500) \eta \eta}$	0.003	-
$\Gamma_{\chi_{c0} \rightarrow f_0(1370) \eta' \eta'}$	0.0027	-
$\Gamma_{\chi_{c0} \rightarrow f_0(1370) \eta \eta'}$	0.000089	-
$\Gamma_{\chi_{c0} \rightarrow f_0(1500) \eta \eta'}$	0.011	-
$\Gamma_{\chi_{c0} \rightarrow f_0(1710) \eta \eta}$	0.00008	-
$\Gamma_{\chi_{c0} \rightarrow f_0(1710) \eta \eta'}$	0.00003	-

# Decay width of $\eta_C$ into a pseudoscalar glueball

**BESIII:**  $m_{\tilde{G}} = 2370 \text{ MeV}$

$$\Gamma_{\eta_C \rightarrow \tilde{G} \pi \pi}$$

=

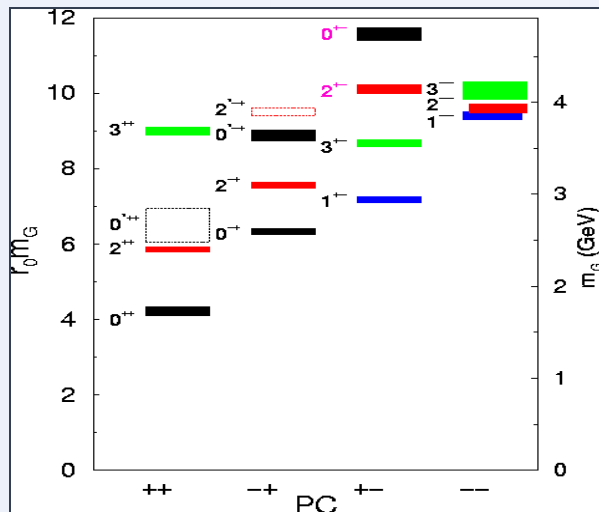
=  $0.16 \text{ MeV}$

Could be measured by



=  $0.124 \text{ MeV}$

Lattice QCD calculations:



$m_{\tilde{G}} = 2600 \text{ MeV}$

W. I. Eshraim and D. H. Rischke, in preparation

# Decay width of $\eta_c$

## 3) Decay widths of scalar charmonium state ( $\eta_c$ ):

Decay Channel	theoretical result [MeV]	Experimental result [MeV]
$\Gamma_{\eta_c \rightarrow \bar{K}_0^* K}$	0.01	-
$\Gamma_{\eta_c \rightarrow a_0 \pi}$	0.01	-
$\Gamma_{\eta_c \rightarrow f_0(1370) \eta}$	0.00018	-
$\Gamma_{\eta_c \rightarrow f_0(1500) \eta}$	0.006	-
$\Gamma_{\eta_c \rightarrow f_0(1710) \eta}$	0.000032	-
$\Gamma_{\eta_c \rightarrow f_0(1370) \eta'}$	0.027	-
$\Gamma_{\eta_c \rightarrow f_0(1500) \eta'}$	0.024	-
$\Gamma_{\eta_c \rightarrow f_0(1710) \eta'}$	0.0006	-
$\Gamma_{\eta_c \rightarrow \eta \eta \eta}$	0.052	-
$\Gamma_{\eta_c \rightarrow \eta' \eta' \eta'}$	0.0023	-
$\Gamma_{\eta_c \rightarrow \eta' \eta \eta}$	0.44	-
$\Gamma_{\eta_c \rightarrow \eta' \eta' \eta}$	0.0034	-
$\Gamma_{\eta_c \rightarrow \eta K \bar{K}}$	0.15	$0.32 \pm 0.17$
$\Gamma_{\eta_c \rightarrow \eta' K K}$	0.41	-
$\Gamma_{\eta_c \rightarrow \eta \pi \pi}$	0.12	$0.54 \pm 0.18$
$\Gamma_{\eta_c \rightarrow \eta' \pi \pi}$	0.08	$1.3 \pm 0.0.6$
$\Gamma_{\eta_c \rightarrow K K \pi}$	0.095	-

W. I. Eshraim and D. H. Rischke, in preparation, preliminary!

# Conclusions

1. In the case of  $N_f = 3$  : Decay of a pseudoscalar glueball with a mass above of 2 GeV.
2. Linear sigma model with  $N_f = 4$  and vector and axial-vector mesons.
3. Masses of (open and hidden) charmed mesons.
4. Decay widths of (open and hidden) charmed mesons.

Thank you!

