Dual Condensates at Finite Isospin Chemical Potential

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♦ Abstract

Thermal properties of two dual condensates are investigated at finite isospin chemical potential in a Polyakov-loop enhanced chiral model of QCD. We demonstrate that the critical temperatures extracting from these dual observable have nothing to do with deconfinement transition. We thus argue that the dual observables are not appropriate order parameters for deconfinement if the quark mass is very small.

♦ Introduction

Two main phase transitions in hot and dense QCD:

- Chiral restoration (m ⇒0) Order parameter: $\langle\bar{\psi}\psi\rangle$
- Deconfinement (m ⇒ ∞)
  Order parameter : Polyakov loop $\langle L\rangle \propto \exp(-F_q/T)$.

Under the center transformation:

$$L = \mathrm{tr}P\left(\exp i\int_0^{1/T} A_0(\mathbf{x},\tau)d\tau\right) \qquad L \Longrightarrow zL$$

Recently, the dressed Polyakov loop is proposed as the order parameter for Z(3) center symmetry by adopting the twisted boundary condition :

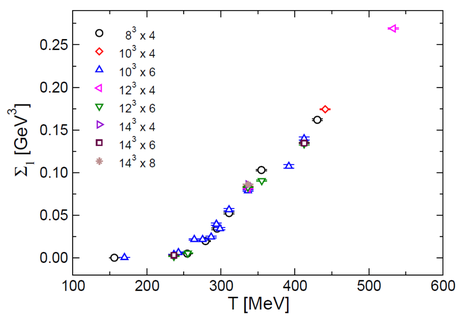
- twisted boundary condition $\psi(t+\beta,\vec{x}) = e^{i\varphi}\psi(t,\vec{x})$
- generalised quark condensate $\sigma(\varphi) = -\frac{1}{V}\langle \mathrm{Tr}[(m+D_\varphi)^{-1}]\rangle$
- dual quark condensates $\Sigma^{(n)} = -\int_0^{2\pi}\frac{d\varphi}{2\pi}e^{-in\varphi}\sigma(\varphi)$
- dressed Polyakov loop $\Sigma^{(1)} = -\int_0^{2\pi}\frac{d\varphi}{2\pi}e^{-i\varphi}\sigma(\varphi)$

New order parameter
for center symmetry

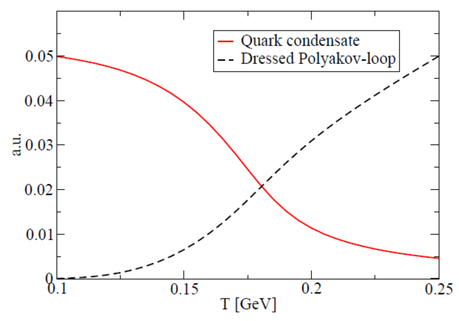
Under the center transformation:
$\Sigma^{(1)} \Longrightarrow z\Sigma^{(1)}$
in the heavy quark limit : $\Sigma^{(1)} \Longrightarrow L$

The advantage: the dPL can be calculated not only in LQCD but also in effective theories or models of QCD。

In principle, one can construct many dual observables which transform in the same way as the PL under the center transformation.



LQCD result



DSEs result

The calculations from the LQCD and effective theories of QCD, such as the DSEs, PNJL and NJL all suggest that the dressed Polyakov loop exhibits an order parameter like thermal behavior , just as the thin PL.

♦ Questions

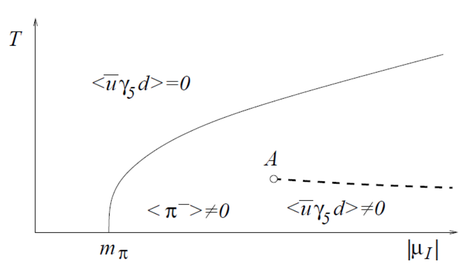
Note that the NJL model has no confinement.

- ***Why does the dPL obtained in a no confining model exhibit an order parameter like behavior for deconfinement ?***
- ***To which extent does the dPL to serve as an effective indicator for deconfinement transition in QCD ?***

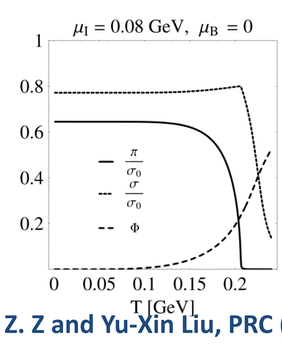
♦ Motivations

To further test the idea of dual observables as order Parameters for deconfinement transition, we extend the U(1)-valued boundary condition for fermions to dense QCD with finite isospin chemical potential $\mu_I > m_\pi/2$ ⇨

- **Pion condensation appears**
- **Construct new dual observables**
- **Test the dPL under the influence of the pion condensation**
- **Can be calculated in Lattice QCD**



$$\mu_I = \frac{\mu_u - \mu_d}{2} = \frac{\delta\mu}{2}$$



From **D. T. Son and M. Stephanov, PRL (2001)**

From **Z. Z and Yu-Xin Liu, PRC (2007)**

♦ Dual Pion condensate

$$\langle\bar{\psi}i\gamma_5\tau_1\psi\rangle = \pi \qquad \pi(\varphi) = -\frac{1}{V}\langle Tr[i\gamma_5\tau_1(m+D_\varphi)^{-1}]\rangle$$

$$\Sigma_\pi^{(n)} = -\int_0^{2\pi}\frac{d\varphi}{2\pi}e^{-in\varphi}\pi(\varphi) \qquad \Sigma_\pi^{(1)} = -\int_0^{2\pi}\frac{d\varphi}{2\pi}e^{-i\varphi}\pi(\varphi)$$

Dual pion condensates **A new type of dressed Polyakov-loop is introduced**

♦ Method

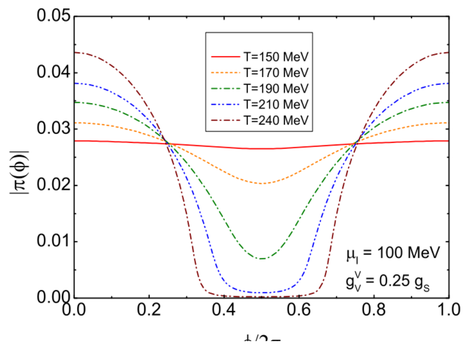
We use the 2-flavor Polyakov-loop enhanced NJL model

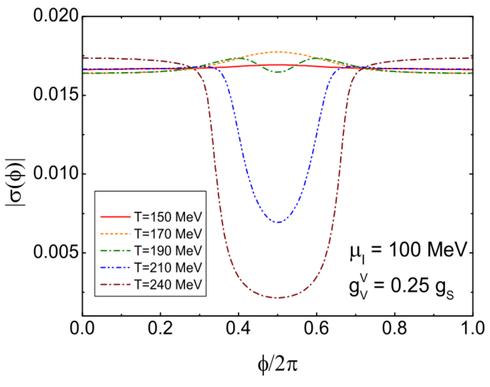
$$\begin{aligned}\mathcal{L} = & \bar{\psi}\left(i\gamma_\mu D^\mu + \gamma_0\hat{\mu} - \hat{m}_0 - i\hat{\lambda}\gamma_5\tau_1\right)\psi \\ & + g_s\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right] \\ & - g_v^s\left(\bar{\psi}\gamma_\mu\psi\right)^2 - g_v^v\left(\bar{\psi}\vec{\tau}\gamma_\mu\psi\right)^2 - \mathcal{U}\left(\Phi,\bar{\Phi},T\right)\end{aligned}$$

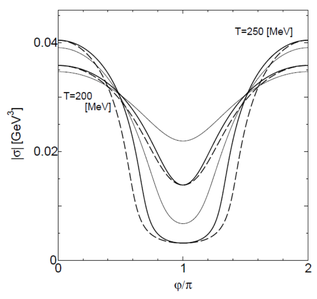
Pl potential with Z(3) symmetry

♦ Results (for ui=100 MeV)

φ-dependence of the generalized pion (up) and quark (down) Condensates for different temperatures



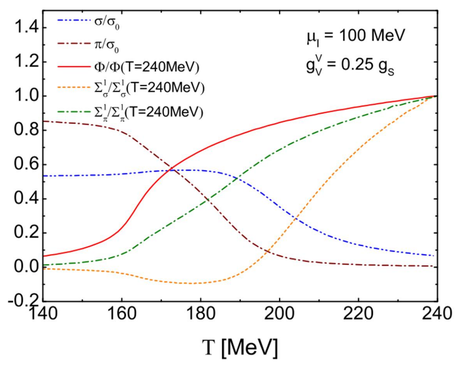




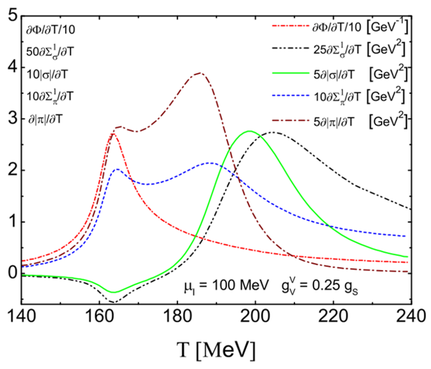
φ- dependence of the generalized quark condensate for different temperatures at vanishing isospin chemical Potential

From K. Kashiwa, PRD (2009)

T-dependences of the (pseudo-) order parameters



T-dependences of the susceptibilities



- **The dual pion condensate behaves like an order parameter**
- **The dPL exhibits abnormal thermal behavior : it even decreases with T at low temperatures !**
  A bad indicator for deconfinement transition !

- **The rapid rises of the dPC and dPL are driven by the melting the pion condensation and the chiral restoration, respectively. The later has been pointed out by Benic.**

♦ Discussion and Conclusion

- **Dual pion condensate exhibits an order-parameter behavior for the center symmetry.**
- **However, the rapid rise of dPC with T is driven by the restoration of isospin symmetry rather than the decofinement transition.**
- **The dressed Polyakov-loop exhibits abnormal thermal behavior compared to the thin Polyakov-loop.**
- **The dual condensates are not appropriate order parameters for deconfinement if the quark mass is very small.**
- **The so called coincidence of the chiral and deconfinement phase transitions obtained from the thermal properties of dual observables may be just an artifact, at least in the model studies.**
- **Is this also true in QCD ? In QCD, the center symmetry is badly violated by light quarks. Our calculation can be checked in Lattice study with physical quark masses.**
- **The fate of $T_\chi \simeq T_{\rm conf}$ even using the PL as the order parameter ?**

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