Interacting relativistic quantum dynamics for multi-time wave functions

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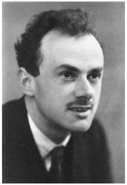
Idea of a multi-time wave function

Setting: N particles, coordinates $x_k = (t_k, \mathbf{x}_k) \in \mathbb{R}^{1+d}$



Non-relativistic (single-time) Schrödinger picture: $\varphi(\mathbf{x}_1, ..., \mathbf{x}_N; t)$

Multi-time Schrödinger picture: $\psi(t_1, \mathbf{x}_1, ..., t_N, \mathbf{x}_N)$

Relation: $\varphi(\mathbf{x}_1, ..., \mathbf{x}_N; t) = \psi(t, \mathbf{x}_1, ..., t, \mathbf{x}_N)$

Domain of ψ: space-like configurations $\Omega \subset \underbrace{\mathbb{R}^{1+d} \times \cdots \times \mathbb{R}^{1+d}}_{N \text{ times}}$

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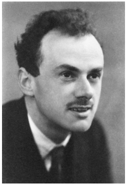

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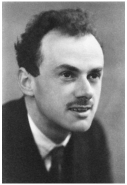

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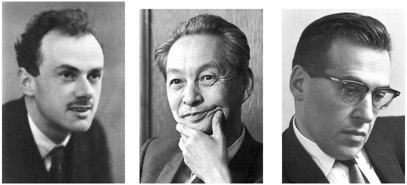

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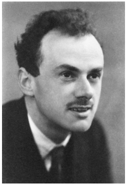

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Hamiltonian multi-time equations

Evolution equations:

$$i\frac{\partial}{\partial t_1}\psi = H_1\psi$$
$$\vdots$$
$$i\frac{\partial}{\partial t_N}\psi = H_N\psi$$

Consistency condition: $[i\partial_{t_1} - H_1, i\partial_{t_2} - H_2] \stackrel{!}{=} 0$

No-go theorem (Petrat/Tumulka 2015)

Interaction potentials excluded, i.e. if

$$H_i = H_{0,i}^{\text{Dirac}} + V_i(x_1, x_2)$$

then this has to be gauge-equivalent to $V_i \equiv V_i(x_i)$.

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Interaction via boundary conditions: a $(1+1)$-dim. model

Assumptions: $N = 2, \quad d = 1, \quad m_1 = m_2 = 0, \quad \psi = (\psi_1, \psi_2, \psi_3, \psi_4)$

Multi-time Dirac equations on Ω:

$$
\begin{aligned}
i \frac{\partial}{\partial t_1} \psi(t_1, z_1, t_2, z_2) &= -i\, \sigma_3 \otimes 1 \frac{\partial}{\partial z_1} \psi(t_1, z_1, t_2, z_2) \\
i \frac{\partial}{\partial t_2} \psi(t_1, z_1, t_2, z_2) &= -i\, 1 \otimes \sigma_3 \frac{\partial}{\partial z_2} \psi(t_1, z_1, t_2, z_2)
\end{aligned}
$$

Initial conditions at $t_1 = t_2 = 0$

Boundary conditions at $\mathcal{C} = \{(t_1, z_1, t_2, z_2) : t_1 = t_2,\ z_1 = z_2\}$

Basic idea: multi-time characteristics

Write out the two-time system in matrix-vector form:

$$i\frac{\partial}{\partial t_1}\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{pmatrix} = -i\begin{pmatrix}1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1\end{pmatrix}\frac{\partial}{\partial z_1}\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\\ \psi_4\end{pmatrix},$$

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$$\text{E.g. for } \psi_1: \quad \left(\frac{\partial}{\partial t_1}+\frac{\partial}{\partial z_1}\right)\psi_1 = 0, \quad \left(\frac{\partial}{\partial t_2}+\frac{\partial}{\partial z_2}\right)\psi_1 = 0$$

$$\Rightarrow \quad \psi_1(t_1,z_1,t_2,z_2) = f_1(z_1-t_1, z_2-t_2).$$

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Existence and uniqueness result

Theorem

There exists a unique C^k solution of the following initial boundary value problem on Ω_1 (i.e. Ω with $z_1 < z_2$):

$$\begin{aligned}
\psi_i(0, z_1, 0, z_2) &\stackrel{!}{=} g_i(z_1, z_2), \quad z_1 < z_2, \quad i = 1, 2, 3, 4 \\
\psi_2(t, z-0, t, z+0) &\stackrel{!}{=} h_2(t, z), \quad t < 0, \quad z \in \mathbb{R} \\
\psi_3(t, z-0, t, z+0) &\stackrel{!}{=} h_3(t, z), \quad t > 0, \quad z \in \mathbb{R}
\end{aligned}$$

where g_i, h_j are C^k functions and compatible, i.e. the transitions between initial values and boundary conditions is also C^k.

Relativistic probability conservation

Conserved tensor current: $j^{\mu\nu}(x_1, x_2) = \overline{\psi}(x_1, x_2)\gamma^\mu \otimes \gamma^\nu \psi(x_1, x_2)$

$d = 1$: $\gamma^0 = \sigma_1$, $\gamma^1 = \sigma_1\sigma_3$, $\partial_{1,\mu} j^{\mu\nu} = \partial_{2,\nu} j^{\mu\nu} = 0$, $j^{00} = \psi^\dagger\psi$

Probability conservation on space-like hypersurfaces Σ:

$$\int_{(\Sigma\times\Sigma)\cap\Omega} d\sigma_\mu(x_1) d\sigma_\nu(x_2)\, j^{\mu\nu}(x_1, x_2) \;=\; 1 \;\; \forall\, \Sigma$$

Theorem

Probability conservation as well as existence and uniqueness are ensured for boundary conditions
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Lorentz invariance

Apart from the boundary conditions, LI is manifest.

Representation of the proper Lorentz group $\mathcal{L}_+^\uparrow$ in $d = 1$:

- Just one generator (boosts in z-direction).
- $\psi(x_1, x_2) \stackrel{\Lambda}{\longmapsto} S_1[\Lambda]S_2[\Lambda]\,\psi(\Lambda^{-1}x_1, \Lambda^{-1}x_2)$
- $S_i[\Lambda] = \exp(\frac{\beta}{2}\gamma_i^0\gamma_i^1), \quad \beta \in \mathbb{R}$
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Usually: interaction defined by potential term in Hamiltonian. More general criterion needed here.

Criterion

A model is **interacting** if there exist initial product wave functions that become entangled during time evolution.

Calculate time evolution of initial product states $\psi = \phi \otimes \chi$:

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Summary

We have obtained the first **multi-time** model with the following properties:

- Rigorous (existence and uniqueness of solutions)
- Manifestly Lorentz invariant
- Relativistic probability conservation
- Interacting

Questions?

Paper: M. Lienert, J. Math. Phys. 56, 042301 (2015), arXiv:1411.2833