Turbulent advection of a passive vector field: Effects of the anisotropy and finite correlation time

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Fully developed turbulence

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- Cascades of energy;
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The key parameters:

- W and L – power of the external source of energy and integral (external) scale;
- ν and l – viscosity coefficient and dissipation (internal) scale.

Fully developed turbulence: $\mathrm{Re} \gg 1 \Rightarrow L \gg l \Rightarrow$

Inertial range $l \ll r \ll L$

Kolmogorov–Obukhov theory

The equal-time structure functions

$$S_n(\mathbf{r}) = \left\langle \left[v_r(t, \mathbf{x}) - v_r(t, \mathbf{x}') \right]^n \right\rangle ,$$

where v_r is the component of the velocity field along the direction $\mathbf{r} = \mathbf{x} - \mathbf{x}'$.

From the two Kolmogorov hypotesis (independence of L for $L \gg r$ and independence of l for $l \ll r$) it follows, that in the inertial range $l \ll r \ll L$

$$S_n(\mathbf{r}) = C_n \, (Wr)^{n/3}$$

with exact exponents and universal amplitudes C_n.

Anomalous scaling

Due to the imtermittency statistical properties of the velocity are dominated by rare spatiotemporal configurations – the main contributions are given by infrequent, but strong events.

This phenomenon is connected with the strong fluctuations of the energy flux and leads to the violation of the classical K41 theory:

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The goal is to calculate γ_n within a regular expansion.

Outline

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- Renormalization of the operators $F_{N\,p}$, critical dimension matrix;
- Asymptotic behaviour of the pair correlation function, OPE.

The step with surprising result — diagonalization of the matrix of critical dimensions.

Definitions and aims

- The field $\boldsymbol{\theta}$ is a passive vector field, e. g. a field of impurity;
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$$F_{N\,p} = (\theta_i \theta_i)^p \; (n_s \theta_s)^{2m}, \quad N = 2(p+m);$$

- The measurable quantities are some correlators like

$$G_{12} = \langle F_1 \, F_2 \rangle \, .$$

Stochastic equation

The stochastic equation for advection of the passive field is

$$\partial_t\theta_i + \partial_k\left(v_k\theta_i - \mathcal{A}_0\ v_i\theta_k\right) + \partial\mathcal{P} = \nu_0\ \left(\partial_\perp^2 + f_0\partial_\parallel^2\right)\ \theta_i + f_i,$$

where $\mathcal{P}$ is pressure term and f_i is random foreign force (supplied the energy W in our system) with zero mean and preassigned correlator

$$D_f = \left\langle f_i(t,\ \mathbf{x})\ f_k(t',\ \mathbf{x}')\right\rangle = \delta(t-t')\ C_{ik}(r/L).$$

Gaussian ensembles

Passive field approximation — we neglect the feedback of the *passive* field θ to the velocity v $\Rightarrow$ instead of the stochastic Navier–Stokes equations the velocity field is modeled by Gaussian ensemble.

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Kraichnan's model (zero correlation time)

$$\langle v_i(x) v_j(x') \rangle = \delta(t - t')\, D_0 \int_{k>m} \frac{d\mathbf{k}}{(2\pi)^d}\, P_{ij}(\mathbf{k})\, \frac{1}{k^{d+\xi}}\, e^{\mathrm{i}\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}.$$

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Model with finite correlation time:

$$\mathcal{F}\left\{\langle v_i(x) v_j(x') \rangle\right\} = f(\omega, k;\ \xi, \eta),$$

where ξ and η are two UV regularizers and are characteristics of the environment.

Anisotropy and finite correlation time

The velocity field possesses defined direction $\mathbf{n}$:

$$\mathbf{v}(t, \mathbf{x}) = \mathbf{n} \cdot v(t, \mathbf{x}_\perp).$$

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$$\left\langle v_i(t,\, \mathbf{x})\, v_k(t',\, \mathbf{x}')\right\rangle = n_i n_k\ \left\langle v(t,\, \mathbf{x}_\perp)\ v(t',\, \mathbf{x}'_\perp)\right\rangle;$$

$$\left\langle v(t,\ \mathbf{x}_\perp)\ v(t',\ \mathbf{x}'_\perp)\right\rangle = \int_{k>m} \frac{d\mathbf{k}}{(2\pi)^d}\ e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{x}')}\ D_v(\omega, k).$$

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and for $D_v(\omega,k)$ we choose

$$D_v(\omega,k) = 2\pi\delta(k_{||})\ D_0\ \frac{k_\perp^{5-d-(\xi+\eta)}}{\omega^2+\left[\alpha_0\nu_0 k_\perp^{2-\eta}\right]^2}.$$

Action Functional

The standart problem of stochastic dynamics

$$\partial_t \theta = U(x,\theta) + f(x), \quad \langle f(x) f(x') \rangle = D(x,x'),$$

is equivalent to action functional

$$S(\theta,\theta') = \int\int dx dx' \theta'(x) D(x,x') \theta'(x')/2 + \\ + \int dx \theta'(x) \left[-\partial_t \theta(x) + U(\theta(x))\right]$$

Here

- $\theta(x) = \theta(t,\mathbf{x})$ is a random field,
- $U(x,\theta)$ is a given t-local functional,
- f is a random force with Gaussian distribution, zero mean and a given pair correlator D.

Action Functional

Our stochastic problem is equivalent to the field theoretic model of the extended set of three felds $\Phi = \{\boldsymbol{\theta}', \boldsymbol{\theta}, \boldsymbol{v}\}$ with action functional

$$
\begin{aligned}
S(\Phi) &= \frac{1}{2}\theta'_i D_f \theta'_k - \frac{1}{2} v_i D_v^{-1} v_k + \\
&+ \theta'_k \left[-\partial_t \theta_k - (v_i \partial_i)\theta_k + \mathcal{A}_0(\theta_i \partial_i) v_k + \nu_0(\partial_\perp^2 + f_0 \partial_\parallel^2)\theta_k \right],
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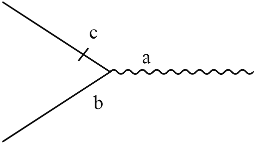
$$

triple vertex (nonlinearity in stochastic equation; interaction part)

$V_{cab} =$



$= i\delta_{bc}\ p_a^\theta - i\mathcal{A}\delta_{ac}\ p_b^\theta$

Propagators

and propagators (quadratic in fields part of action; free action S_0)

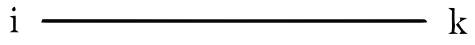
$\langle v_i v_j \rangle_0 =$

$$= n_i n_j \cdot \delta(k_{\|}) \cdot D_0 \cdot \frac{k_{\perp}^{5-d-(\xi+\eta)}}{\omega^2 + \left[\alpha_0 \nu_0 k_{\perp}^{2-\eta}\right]};$$

$\left\langle \theta_i \theta'_j \right\rangle_0 =$

$$= \frac{P_{ij}(\mathbf{k})}{-i\omega + \nu_0 \left(k_{\perp}^2 + f_0 k_{\|}^2\right)};$$

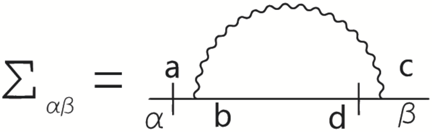
$\langle \theta_i \theta_j \rangle_0 =$



$$= \frac{C_{ik}(\mathbf{k})}{\omega^2 + \left[\nu_0(\mathbf{k}_{\perp}^2 + f_0 k_{\|}^2)\right]^2}.$$

Renormalization constants

The only divergent diagram is the Self-Energy operator:

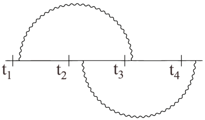


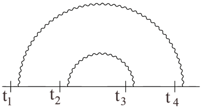
From Dyson equation it follows that:

- one *new* dimensionless constant u is needed;
- $\nu_0 = \nu,\ \mathcal{A}_0 = \mathcal{A},\ \alpha_0 = \alpha$ — no renormalization required;
- $f_0 = f \cdot Z_f$ with nontrivial constant Z_f;
- $u_0 = u \cdot Z_u$ with nontrivial constant Z_u.

Multiloop diagrams

Some examples of the multiloop diagrams:





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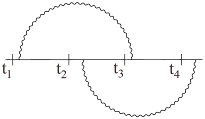

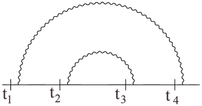

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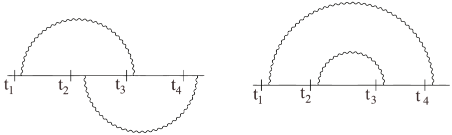
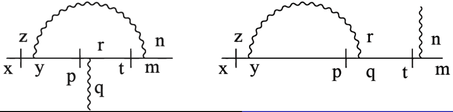


In Kraichnan's model (zero correlation time) they are equal to zero due to the closed cycles of the retarded propagators.

Finite correlation time destroys this simple construction, but all these diagrams still are equal to zero:



Fixed points and asymptotic

From remormalization group (RG) it follows, that in the case of one charge the asymptotic behaviour of the invariant charge $\bar{g}$ is

$$\bar{g}\,(s) \cong g^{*} + const \cdot s^{\omega},$$

where $s = 1/\mu r$, μ is the renormalization mass, g^{*} is fixed point:

$$\beta_{g}(g^{*}) = 0.$$

IR asymptotic behaviour $(s \to 0 \Leftrightarrow r \to \infty)$: $\omega = \beta'(g^{*}) > 0$.

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In the case of many charges $\beta_i(g_i^{*}) = 0$ and $\Omega_{ik} = \partial\beta_i/\partial g_k$ at the point $g_j = g_j^{*}$ has to be positive.

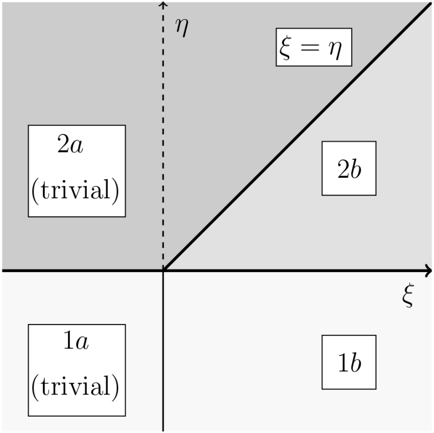
Fixed points and asymptotic

Depending of the exponents ξ and η the model possesses 4 fixed points:

- 1*a*, 2*a* are trivial (Gaussian fixed point)
- 1*b* corresponds to the limit of "frozen" velocity field
- 2*b* corresponds to the limit of "rapid-change" (Kraichnan's) model

Fixed points and asymptotic

Picture of the fixed points on the $\xi - \eta$ plane



Operator diagram

Our aim is to obtain asymptotic behaviour of the correlation function

$$G = \langle F_{N_1\, p_1}\, F_{N_2\, p_2} \rangle$$

of two composite operators, built solely from the fields itself:

$$F_{N\, p} = (\theta_i \theta_i)^p\ (n_s \theta_s)^{2m}, \quad N = 2(p+m).$$

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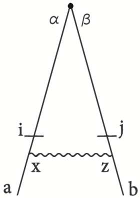
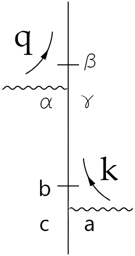
Therefore we have to calculate one one-loop diagram



and then contract it with external fields and operator vertex.

Multiloop diagram

Any multiloop diagram contains as a part the structure



and therefore all of them are equal to zero.

This property is closely connected with the presence of the anisotropy.

Operator diagram

The (exact!) result is

$$\frac{\delta^2 F_{N,p,m}}{\delta\theta_1 \cdot \delta\theta_2} \cdot I_{12}^{ab} \cdot \theta_a \theta_b =$$

$$= 2m(2m-1) \cdot F_{N,p+1,m-1} \; + \; (2p + 8pm - 2m(2m-1)) \cdot F_{N,p,m} \; +$$

$$+ (4p(p-1) - 2p - 8pm) \cdot F_{N,p-1,m+1} \; - \; 4p(p-1) \cdot F_{N,p-2,m+2}.$$

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There is mixing of operators $\Longrightarrow$ renormalization constant Z_F is a matrix!

The matrix Z_F: renormalization

Therefore the renormalozation matrix Z_F and takes the form

$$Z_F = \begin{pmatrix} a_{11} & a_{12} & a_{13} & 0 & \dots & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & & \vdots \\ 0 & a_{32} & a_{33} & a_{34} & \ddots & 0 \\ \vdots & 0 & a_{43} & \ddots & \ddots & a_{n-2n} \\ \vdots & & & \ddots & \ddots & a_{n-1n} \\ 0 & \dots & \dots & 0 & a_{nn-1} & a_{nn} \end{pmatrix}.$$

Critical dimension matrix

And we can calculate anomalous dimension matrix $\gamma^*_{Np,\,Np'}$:

$$
\begin{aligned}
\gamma^*_{N,\ p+1} &= \mathcal{B}\cdot 2m(2m-1)\cdot\xi;\\
\gamma^*_{N,\ p} &= \mathcal{B}\cdot(2p+8pm-2m(2m-1))\cdot\xi;\\
\gamma^*_{N,\ p-1} &= \mathcal{B}\cdot(4p(p-1)-2p-8pm)\cdot\xi;\\
\gamma^*_{N,\ p-2} &= \mathcal{B}\cdot(-4p(p-1))\cdot\xi,
\end{aligned}
$$

and critical dimension matrix $\Delta_{Np,\ Np'}$:

$$\Delta_{Np,\ Np'} = -N + \gamma^*_{Np,\ Np'},$$

where N is its canonical dimension.

RG equation

The basic RG statement is the following: if the quantity is multiplicatively renormalizable, we can use RG equation:

$$[\mathcal{D}_{RG} + \gamma_F]\, F_R = 0,$$

where $\mathcal{D}_{RG}$ is some differential operator, γ_F – anomalous dimension of F.

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$$[\mathcal{D}_{RG} + \gamma_F]\, F_R = 0,$$

where $\mathcal{D}_{RG}$ is some differential operator, γ_F – anomalous dimension of F. This equation provides us an exponential law, and the leading term of IR asymptotic behaviour is at fixed point $g = g^*$:

$$F \cong const \cdot x^{\Delta_F}.$$

The necessity of the diagonalization

If Z is a matrix Z_{ik}, i.e., if there is a mixing of operators,

$$F_i = Z_{ik} F_k^R,$$

to solve the RG equations we need to diagonalize our system. So our aim is to find the eigenvalues of the matrix

$$\Delta_{Np,\ Np'} = -2(p+m) + \gamma^*_{Np,\ Np'},$$

or, equivalently, to find the diagonal matrix

$$\widetilde{\Delta}_F = U_F^{-1} \Delta_F U_F.$$

Jordan Form

It is proved, that for any dimension N

$$\lambda_1 = \ \ldots \ = \lambda_{N/2+1} = -2(p+m) = -N,$$

and the critical dimension marix is not diagonalizable, but have a Jordan form!

Jordan Form

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$$\widetilde{\Delta}_F = \begin{pmatrix} -2(p+m) & 1 & 0 & \dots & 0 \\ 0 & -2(p+m) & 1 & & \vdots \\ \vdots & 0 & \ddots & \ddots & 0 \\ \vdots & & & \ddots & 1 \\ 0 & \dots & & 0 & -2(p+m) \end{pmatrix}.$$

Asymptotic behaviour of the operator $F_{N\,p}$

Therefore the RG equations mixes up with each other and asymptotic behaviour of the operators $\widetilde{F}^R$ contains the logarithmic corrections:

$$\widetilde{F}^R_1 \propto (M/\mu)^{\lambda} \cdot P_{N/2}\,(\ln M/\mu)\,,$$
$$\widetilde{F}^R_2 \propto (M/\mu)^{\lambda} \cdot P_{N/2-1}\,(\ln M/\mu)\,,$$
$$\vdots$$
$$\widetilde{F}^R_{N/2+1} \propto (M/\mu)^{\lambda},$$

where $P_X\,(\ln M/\mu)$ is the X degree polynomial with $\ln M/\mu$ as an argument.

RG equation for correlation function G

The main aim is to find the asymptotic behaviour of the pair correlation function

$$G = \langle F_{N_1\, p_1}\, F_{N_2\, p_2} \rangle$$

of two composite operators with arbitrary N_1 and N_2.

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RG equations in this case take form

$$\mathcal{D}_{RG}\ G_{ik} = \Delta_{is} G_{sk} + \Delta_{ks} G_{is},$$

where $G_{ij} = \langle F_i F_j \rangle$, Δ_{ij} is the critical dimension of G_{ij}.

Solution of the RG equation for correlator G

Solution of the RG equation gives us asymptotic behaviour for $r \gg l$:

$$G^R_{ik} \propto (\Lambda r)^{N_1+N_2} \cdot P_{(N_1+N_2)/2}\left[\ln \Lambda r\right] \cdot \Phi\left(Mr,\ mr,\ fr^{\xi}\right) \quad \forall i, k,$$

it also contains logarithms.

Operator Product Expansion

The inertial range $l \ll r \ll L$ corresponds to the additional condition, which is studied using OPE. According to the OPE

$$F_1(x')F_2(x'') = \sum_{\widetilde{F}} C_{\tilde{F}}(\mathbf{r})\widetilde{F}(t,\mathbf{x}),$$

where $\widetilde{F}$ are all possible operators. After averaging with the weight $\exp S_R$ the sought-for asymptotic takes the form

$$\langle \widetilde{F}_\alpha \rangle \propto (Mr)^{\widetilde{\Delta}_\alpha},$$

where $\widetilde{\Delta}_\alpha$ is a Jordan matrix.

Asymptotic behaviour of the correlator G

Taking into account the nilpotency of the matrix Z_{ik}, using RG and OPE together one can obtain the desired asymptotic behaviour:

$$G(r) = \langle F_{N_1\, p_1}\ F_{N_2\, p_2} \rangle \propto [\ln \Lambda r]^{(N_1+N_2)/2} \cdot [\ln Mr]^{(N_1+N_2)/2} \cdot \widetilde{\Phi}\left(fr^{\xi} \right).$$

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No power dependence, only logarithmic corrections!

Conclusion

- We applied *the field theoretic renormalization group* and *the operator product expansion* to the analysis of the inertial-range asymptotic behavior of a divergence-free vector field, passively advected by the anisotropic turbulent random flow with finite correlation time;

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Thank you for your attention!