
New tests of variability of the speed of light.

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Plan:

- 1. Main frameworks of varying constants theories.
- 2. Benefits and problems of varying c theories.
- 3. Redshift drift test of varying c models.
- 4. Measuring c with baryon acoustic oscillations (BAO).
- 5. Conclusions.

Collaborators:

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1. Main frameworks of varying constants theories.

Long story of varying constants theories:

H. Weyl (1919): electron radius/its gravitational radius $\sim 10^{40}$

A. Eddington (1935) discussed:

1. proton-to-electron mass $1/\beta = m_p/m_e \sim 1840$
2. an inverse of fine structure constant $1/\alpha = (hc)/(2\pi e^2) \sim 137$
3. electromagnetic to gravitational force between a proton and an electron
 $e^2/(4\pi\epsilon_0 G m_e m_p) \sim 10^{40}$
4. introduced “Eddington number” $N_{edd} \sim 10^{80}$

P.A.M. Dirac (1937) interesting remarks about the relations between atomic and cosmological quantities: If $G \propto H(t) = (da/dt)/a$, then $a(t) \propto t^{1/3}$ and $G(t) \propto 1/t$ - **fundamental constants must evolve in time.**

Conclusion: electromagnetic force is strong compared to gravitational since the universe is “old” i.e. $F_e/F_p \propto (e^2/m_e m_p)t \propto t$!!!

varying gravitational constant G theories

First fully quantitative framework: **Brans-Dicke** scalar-tensor gravity (1961)

The gravitational constant G is associated with an average gravitational potential (scalar field) ϕ surrounding a given particle:

$\langle \phi \rangle = GM/(c/H_0) \propto 1/G = 1.35 \times 10^{28} g/cm$. The **scalar field gives the strength of gravity**

$$G = \frac{1}{16\pi\Phi} \quad (1)$$

With the action

$$S = \int d^4x \sqrt{-g} \left(\Phi R - \frac{\omega}{\Phi} \partial_\mu \Phi \partial^\mu \Phi + \Lambda + L_m \right) \quad (2)$$

it relates to low-energy-effective **superstring** theory for $\omega = -1$

String coupling constant (running) $g_s = \exp(\phi/2)$ changes in time with ϕ - the **dilaton** and $\Phi = \exp(-\phi)$.

Varying speed of light c (VSL) theories

Attempts: Einstein (1907), Dicke (1957), J.-P. Petit (1988) (Einstein eqs remain same due to fine-tuned change of c and G), Moffat (1992).

Albrecht & Magueijo model (1998) (AM model) (Barrow 1999; Magueijo 2003):
Introduce a scalar field

$$c^4 = \psi(x^\mu) \quad (3)$$

and so the action is

$$S = \int d^4x \sqrt{-g} \left[\frac{\psi(R + 2\Lambda)}{16\pi G} + L_m + L_\psi \right] \quad (4)$$

AM model **breaks Lorentz invariance** (relativity principle and light principle) so that there is a preferred frame (cosmological or CMB) in which the field is minimally coupled to gravity. The Riemann tensor is computed in such a frame **for a constant $\psi = c^4$** and no additional terms $\partial_\mu \psi$ appear in this frame (though they do in other frames). **Einstein eqs remain the same except c now varies.**

Varying speed of light c (VSL) theories

Magueijo covariant (conformally) and **locally invariant** model (2000, 2001):

$$\psi = \ln \left(\frac{c}{c_0} \right) \quad \text{or} \quad c = c_0 e^\psi, \quad (5)$$

with the action

$$S = \int d^4x \sqrt{-g} \left[\frac{c_0^4 e^{\alpha\psi} (R + 2\Lambda + L_\psi)}{16\pi G} + e^{\beta\psi} L_m \right], \quad (6)$$

with

$$L_\psi = \kappa(\psi) \nabla_\mu \psi \nabla^\mu \psi. \quad (7)$$

Further assumption: $\alpha - \beta = 4$.

Interesting subcases:

$\alpha = 4; \beta = 0$ - Brans-Dicke with $\phi_{BD} = e^{4\psi}/G$ and $\kappa(\psi) = 16\omega_{BD}(\phi_{BD})$.

$\alpha = 0; \beta = -4$ - minimal VSL theory.

Varying fine structure constant α theories

Varying fine structure constant α (or charge $e = e_0\epsilon(x^\mu)$) theories (Webb et al. 1999, Sandvik 2002)

$$S = \int d^4x \sqrt{-g} \left(R - \frac{\omega}{2} \partial_\mu \psi \partial^\mu \psi - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} e^{-2\psi} + L_m \right) \quad (8)$$

with $\psi = \ln \epsilon$ and $f_{\mu\nu} = \epsilon F_{\mu\nu}$.

Can be related with the VSL theories due to the definition of the fine structure constant

$$\alpha = \frac{e^2}{\hbar c} \quad \text{i.e.} \quad \alpha(t) = \frac{e^2}{\hbar c(t)} \quad (9)$$

Assume linear expansion $e^\psi = 1 - 8\pi G \zeta (\psi - \psi_0) = 1 - \Delta\alpha/\alpha$ with the constraint on the local equivalence principle violation $|\zeta| \leq 10^{-3}$. **The relation to dark energy is** (e.g. Vielzeuf and Martins 2012):

$$\gamma = w + 1 = \frac{(8\pi G \frac{d\psi}{d \ln a})^2}{\Omega_\psi} . \quad (10)$$

Varying fine structure constant α theories

The field equations for Friedmann universes are (e.g. Barrow, Kimberly, Magueijo 2004)

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} (\rho_r + \rho_\psi) - \frac{kc^2}{a^2}, \quad (11)$$

$$\frac{\ddot{a}}{a} = -\frac{8\pi G}{3} (\rho_r + 2\rho_\psi), \quad (12)$$

$$\ddot{\psi} + 3\frac{\dot{a}}{a}\dot{\psi} = 0, \quad (13)$$

where $\rho_r \propto a^{-4}$ stands for the density of radiation while

$$\rho_\psi = \frac{p_\psi}{c^2} = \frac{\sigma}{2}\dot{\psi}^2 \quad (14)$$

stands for the density of the scalar field ψ (standard with $\sigma = +1$ and phantom with $\sigma = -1$) and

$$\alpha = \alpha_0 e^{2\psi}. \quad (15)$$

2. Benefits and problems of varying c and α theories.

Applying the simplest method one can obtain the generalized Einstein-Friedmann equations generalize in **varying speed of light (VSL)** theories and **varying gravitational constant G** theories as (ρ - mass density; $\varepsilon = \rho c^2(t)$ - energy density in $Jm^{-3} = Nm^{-2} = kgm^{-1}s^{-2}$)

$$\rho(t) = \frac{3}{8\pi G(t)} \left(\frac{\dot{a}^2}{a^2} + \frac{kc^2(t)}{a^2} \right), \quad (16)$$

$$p(t) = -\frac{c^2(t)}{8\pi G(t)} \left(2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{kc^2(t)}{a^2} \right), \quad (17)$$

and the generalized conservation law is obtained from (16) and (17)

$$\dot{\rho}(t) + 3\frac{\dot{a}}{a} \left(\rho(t) + \frac{p(t)}{c^2(t)} \right) = -\rho(t) \frac{\dot{G}(t)}{G(t)} + 3\frac{kc(t)\dot{c}(t)}{4\pi Ga^2}. \quad (18)$$

introducing Λ to varying c models

If one adds the Λ -term to the equations (16)-(17), and introduces the vacuum mass density

$$\varrho_{\Lambda}(t) = \frac{\Lambda c^2(t)}{8\pi G(t)} \quad (\Lambda = \text{const.}) \quad (19)$$

with

$$p_{\Lambda}(t) = -\varrho_{\Lambda}(t)c^2, \quad (20)$$

then one has to replace $\varrho \rightarrow \varrho + \varrho_{\Lambda}$, $p \rightarrow p + p_{\Lambda}$ in (16)-(17) and $\dot{\varrho} \rightarrow \dot{\varrho} + \dot{\varrho}_{\Lambda}$ in (18) to obtain

$$\dot{\varrho} + 3\frac{\dot{a}}{a} \left(\varrho + \frac{p}{c^2(t)} \right) + \varrho \frac{\dot{G}(t)}{G(t)} = \frac{(3k - \Lambda a^2)}{4\pi G(t)a^2} c(t)\dot{c}(t). \quad (21)$$

which is solved by (for $p = w\rho c^2$, $c(t) = c_0 a^n$, $G(t) = G_0 a^q$, $C = \text{const.}$)

$$\varrho(a) = \frac{C}{a^{3(w+1)+q}} + \frac{3c_0^2 n}{4\pi G_0} \left(\frac{k}{2n + 3w + 1} - \frac{\Lambda}{3} \frac{a^2}{2n + 3w + 3} \right) a^{2(n-1)-q}.$$

Benefits of varying c models

Solves basic problems of standard cosmology: flatness and horizon.

Flatness: inserting this into Friedmann (16) one gets

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G_0 C}{3} a^{-3(w+1)} + \frac{kc_0^2 a^{2n-2} (2n-1)}{2n+3w+1}, \quad (22)$$

and the density term (with C) will dominate the curvature term at large scale factor if

$$2 \geq 2n + 3(w + 1) \quad (23)$$

Horizon: For large scale factor the solution is $a(t) = t^{2/3(w+1)}$ and the proper distance to the horizon reads as

$$d_H = c(t)t = c_0 a^n(t)t = c_0 a_0^n t^{(3w+3+2n)/3(w+1)} \quad (24)$$

and the scale factor grows faster than d_H under the same condition as in (23).

varying c (and G) removing or changing singularities.

Varying constants can **remove or change the nature of singularities** (MPD, Marosek 2013).

Type	Name	t sing.	$a(t_s)$	$\varrho(t_s)$	$p(t_s)$	$\dot{p}(t_s)$ etc.	$w(t_s)$	T	K
0	Big-Bang (BB)	0	0	∞	∞	∞	finite	strong	strong
I	Big-Rip (BR)	t_s	∞	∞	∞	∞	finite	strong	strong
I_l	Little-Rip (LR)	∞	∞	∞	∞	∞	finite	strong	strong
I_p	Pseudo-Rip (PR)	∞	∞	finite	finite	finite	finite	weak	weak
II	Sudden Future (SFS)	t_s	a_s	ϱ_s	∞	∞	finite	weak	weak
II_g	Gen. Sudden Future (GSFS)	t_s	a_s	ϱ_s	p_s	∞	finite	weak	weak
III	Finite Scale Factor (FSFS)	t_s	a_s	∞	∞	∞	finite	weak	strong
IV	Big-Separation (BS)	t_s	a_s	0	0	∞	∞	weak	weak
V	w-singularity (w)	t_s	a_s	0	0	0	∞	weak	weak

varying c (and G) removing or changing singularities.

Some of these can be regularized (removed by variable constants):

- In order to regularize an SFS or an FSF singularity by varying $c(t)$, the **light should slow and eventually stop propagating** at a singularity. (cf. loop quantum cosmology (LQC): **anti-newtonian limit** $c = c_0 \sqrt{1 - \varrho/\varrho_c} \rightarrow 0$ for $\varrho \rightarrow \varrho_c$ with ϱ_c being the critical density (Caialetta et al. 2012). The **low-energy limit** $\varrho \ll \varrho_0$ gives the standard limit $c \rightarrow c_0$.)
- To regularize an SFS, FSF by varying gravitational constant $G(t)$ - **the strength of gravity has to become infinite** at an initial (curvature) singularity. Effectively, a new singularity - **of strong coupling** for a physical field such as $G \propto 1/\Phi$ appears. Such problems were already dealt with in superstring and brane cosmology where both the curvature singularity and a strong coupling singularity show up (choice of coupling, quantum corrections).

Problems of varying c models

Main problem: to obtain the field equations out of **any** action (cf. also quantum cosmology)?

Equations (16)-(18) have just been obtained in a special frame - the one in which c is a constant and **does not lead to any extra boundary terms** (apart from standard ones). Einstein equations were simply generalized:

$$G_{\mu\nu} - g_{\mu\nu}\Lambda = \frac{8\pi G}{\psi} T_{\mu\nu} \quad (25)$$

while the action (4) varied in a **standard way** leads to different field equations

$$G_{\mu\nu} - g_{\mu\nu}\Lambda = \frac{8\pi G}{\psi} T_{\mu\nu} - \frac{1}{\psi} \psi_{;\nu;\mu} + \frac{1}{\psi} \square\psi. \quad (26)$$

The application of Bianchi identity to (25) gives a conservation equation with dynamical ψ

$$T_{;\mu}^{\mu\nu} = -T^{\mu\nu} \psi_{;\mu} \quad (27)$$

Problems of varying c quantum cosmology

If ψ was supposed to be a **dynamical matter field**, then one could get the evolution equation using the Lagrangian

$$L_\psi = -\frac{\omega}{16\pi G\psi}\dot{\psi}^2, \quad (28)$$

but working **only in a preferred frame** and with ψ not coupled to $\sqrt{-g}$.

Treating $\psi = c^4$ as constant in a preferred frame also requires special treatment of the boundary terms in c -varying quantum cosmology. As mentioned, we vary the action in the special frame where c is constant which means that **we drop c -induced boundary terms**, but recover the time dependence of c again to proceed towards WdW equation (V_3 is a 3-volume)

$$L = \frac{3V_3 c^3(x^0)}{8\pi G(x^0)} \left(ka - a_{,0}^2 a - \frac{\Lambda}{3} a^3 - \frac{8\pi G(x^0)}{3c^2} \rho a^3 \right) \quad (29)$$

Benefits of varying α cosmology

Since one does not brake Lorentz invariance in **varying fine structure constant α** theories, then there are **no such problems** in these models - the standard variational principle applies and the dynamical equation for the scalar field is given!

According to the definition, any variability of c (e , \hbar) is **related** to the variability of α :

$$\frac{\Delta\alpha}{\alpha} = -\frac{\Delta c}{c}. \quad (30)$$

The best constraints on $\Delta\alpha$ are: Oklo natural nuclear reactor:

$$\Delta\alpha/\alpha = (0.15 \pm 1.05) \cdot 10^{-7} \text{ at } z = 0.14$$

$$\text{VLT/UVES quasars: } \Delta\alpha/\alpha = (0.15 \pm 0.43) \cdot 10^{-5} \text{ at } 1.59 < z < 2.92$$

$$\text{SDSS quasars: } \Delta\alpha/\alpha = (1.2 \pm 0.7) \cdot 10^{-4} \text{ at } 0.16 < z < 0.8 .$$

More bounds on variation of α .

By Webb et al. (PRL 107, 191101 (2011)) (α -dipole $R.A.17.4 \pm 0.9h$, $\delta = -58 \pm 9$: Keck ($\Delta\alpha < 0$) and VLT) as well as other specific measurements of α given in the table below (in parts per million):

Object	z	$\Delta\alpha/\alpha$	Spectrograph	Ref.
HE0515–4414	1.15	-0.1 ± 1.8	UVES	Molaro et al. (2008)
HE0515–4414	1.15	0.5 ± 2.4	HARPS/UVES	Chand et al. (2006)
HE0001–2340	1.58	-1.5 ± 2.6	UVES	Agafonowa et al. (2011)
HE2217–2818	1.69	1.3 ± 2.6	UVES–LP	Molaro et al. (2013)
Q1101–264	1.84	5.7 ± 2.7	UVES	Molaro et al. (2008)

UVES - Ultraviolet and Visual Echelle Telescope

HARPS - High Accuracy Radial velocity Planet Searcher

LP - Large Program measurement

Strongest – atomic clock Rosenband bound at $z = 0$

Rosenband (2008) measurement gives the following bound at $z = 0$

$$\left(\frac{\dot{\alpha}}{\alpha}\right)_0 = (-1.6 \pm 2.3) \times 10^{-17} \text{yr}^{-1}. \quad (31)$$

which can be transformed onto the bound for the scalar field coupling ξ :

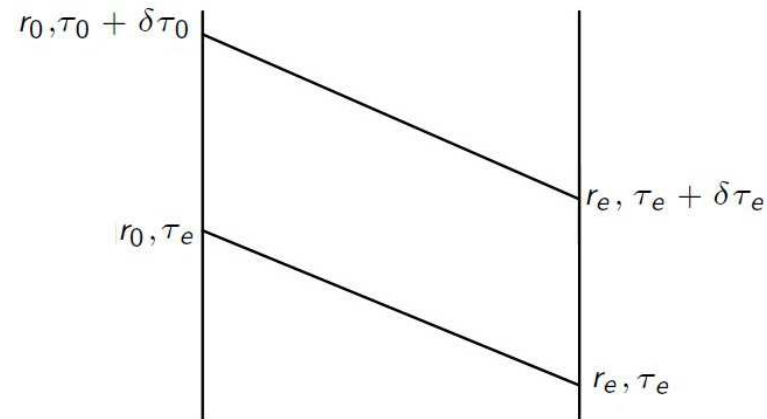
$$\left|\frac{\dot{\alpha}}{\alpha}\right|_0 = |\xi| H_0 \sqrt{3\Omega_{\Phi 0} |1 + w_{\Phi 0}|}, \quad (32)$$

which translates for $H_0 = (67.4 \pm 1.4) \text{km.s}^{-1}\text{Mpc}^{-1}$ (Planck value) into the conservative (3σ) bound

$$|\xi| \sqrt{3\Omega_{\Phi 0} |1 + w_{\Phi 0}|} < 10^{-6}. \quad (33)$$

3. Redshift drift test of varying c models.

Redshift drift (Sandage 1962, Loeb 1998) - the idea is to collect data from two light cones separated by 10-20 years to look for a change in redshift of a source as a function of time.



There is a relation between the times of emission of light by the source τ_e and $\tau_e + \Delta\tau_e$ and times of their observation at τ_o and $\tau_o + \Delta\tau_o$:

$$\int_{\tau_e}^{\tau_o} \frac{d\tau}{a(\tau)} = \int_{\tau_e + \Delta\tau_e}^{\tau_o + \Delta\tau_o} \frac{d\tau}{a(\tau)}, \quad (34)$$

which for small $\Delta\tau_e$ and $\Delta\tau_o$ reads as $\frac{\Delta\tau_e}{a(\tau_e)} = \frac{\Delta\tau_o}{a(\tau_o)}$.

Redshift drift test.

The redshift drift is defined as ($\tau \rightarrow t$ here)

$$\Delta z = z_e - z_0 = \frac{a(t_0 + \Delta t_0)}{a(t_e + \Delta t_e)} - \frac{a(t_0)}{a(t_e)}, \quad (35)$$

which can be expanded in series and to first order in Δt as

$$\Delta z = \frac{a(t_0) + \dot{a}(t_0)\Delta t_0}{a(t_e) + \dot{a}(t_e)\Delta t_e} - \frac{a(t_0)}{a(t_e)} \approx \frac{a(t_0)}{a(t_e)} \left[\frac{\dot{a}(t_0)}{a(t_0)}\Delta t_0 - \frac{\dot{a}(t_e)}{a(t_e)}\Delta t_e \right]. \quad (36)$$

Using above relations we have

$$\Delta z = \Delta t_0 [H_0(1 + z) - H(t(z))] = (1 + z) \frac{\Delta v}{c}, \quad (37)$$

where Δv is the velocity shift and $H(t(z))$ is given in a standard way.

Redshift drift in varying c theory.

In VSL theory the relation (34) generalizes into

$$\int_{t_e}^{t_o} \frac{c(t)dt}{a(t)} = \int_{t_e+\Delta t_e}^{t_o+\Delta t_o} \frac{c(t)dt}{a(t)}, \quad (38)$$

which for small Δt_e and Δt_o transforms into

$$\frac{c(t_e)\Delta t_e}{a(t_e)} = \frac{c(t_o)\Delta t_o}{a(t_o)}. \quad (39)$$

The definition of redshift in VSL theories remains [the same](#) as in standard Einstein relativity and reads as (Barrow, Magueijo 1999)

$$1 + z = \frac{a(t_o)}{a(t_e)}. \quad (40)$$

Redshift drift - varying c

Using (39) we have

$$\Delta z = \Delta t_0 \left[H_0(1+z) - H(t_e) \frac{c(t_0)}{c(t_e)} \right] , \quad (41)$$

which after applying the ansatz

$$c(t) = c_0 a^n(t) \quad (42)$$

gives

$$\frac{\Delta z}{\Delta t_0} = \frac{\Delta z}{\Delta t_0}(z, n) = H_0(1+z) - H(z)(1+z)^n . \quad (43)$$

Redshift drift - varying c

In the limit $n \rightarrow 0$ the formula (43) reduces to (37) for standard Friedmann universe. Bearing in mind definitions Ω 's, and assuming $K = 0$ we have

$$H^2(z) = H_0^2 [\Omega_{m0}(1+z)^3 + \Omega_\Lambda] \quad (44)$$

and so (43) gives

$$\begin{aligned} \frac{\Delta z}{\Delta t_0} &= H_0 \left[1 + z - (1+z)^n \sqrt{\Omega_{m0}(1+z)^3 + \Omega_\Lambda} \right] \\ &= H_0 \left[1 + z - \sqrt{\Omega_{m0}(1+z)^{3+2n} + \Omega_\Lambda(1+z)^{2n}} \right] \end{aligned} \quad (45)$$

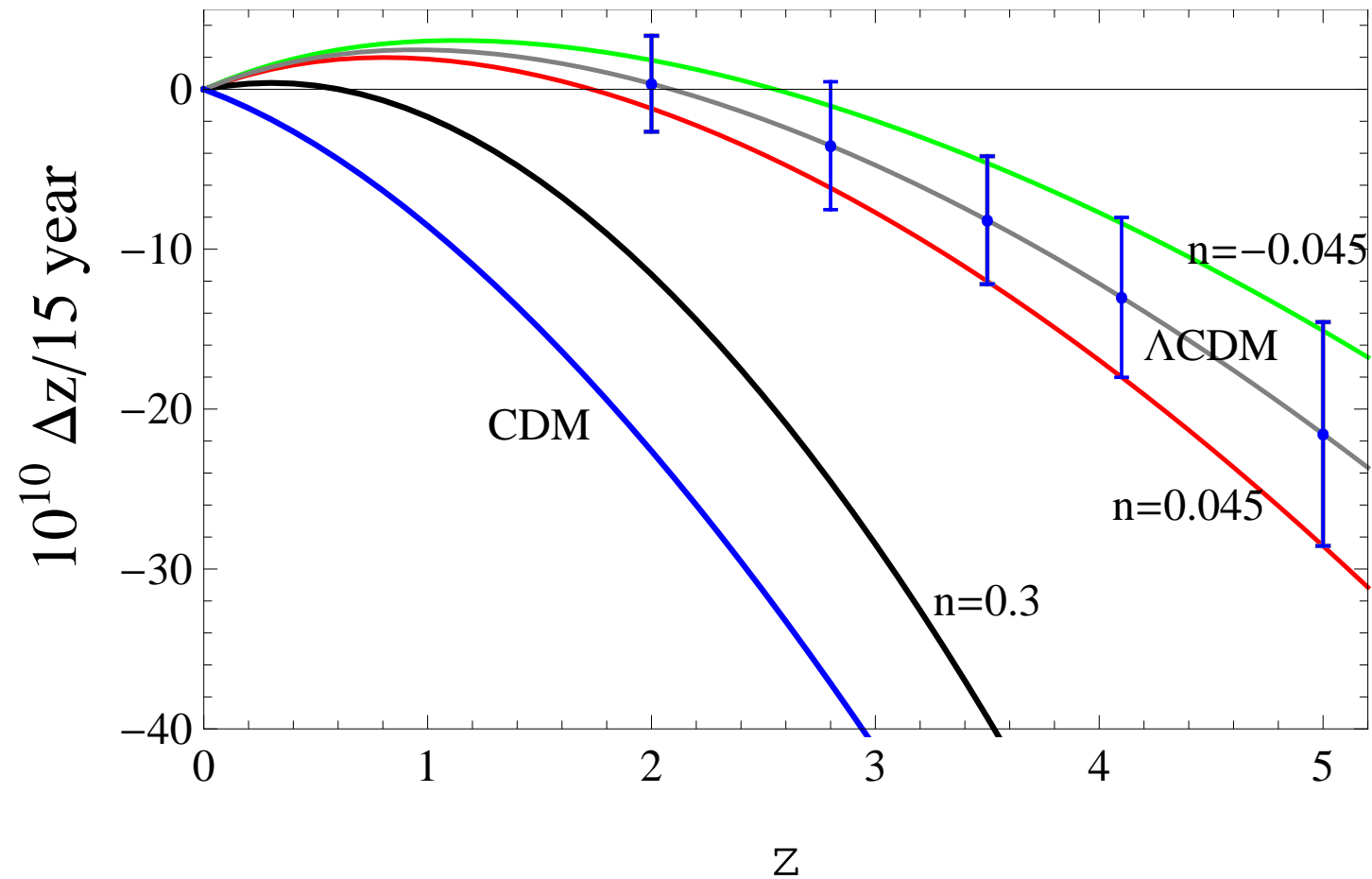
which can further be rewritten to define new redshift function

$$\tilde{H}(z) \equiv (1+z)^n H(z) = H_0 \sqrt{\sum_{i=1}^{i=k} \Omega_{wi} (1+z)^{3(w_{eff}+1)}} \quad , \quad (46)$$

where $w_{eff} = w_i + \frac{2}{3}n$.

Redshift drift test - varying c

The VSL redshift drift effect for 15 year period of observations.



Redshift drift test - varying c

- If $n < 0$ (c decreases) then dust matter becomes little negative pressure matter and the cosmological constant became phantom. Both components **can mimic dark energy**.
- If $n > 0$ then (growing $c(t)$) VSL model becomes **more like** Cold Dark Matter (CDM) model.
- Theoretical error bars are taken from Quercellini et al. 2012 and presumably show that for $|n| < 0.045$ – **one cannot distinguish between VSL models and Λ CDM models**.
- In other words, by measuring redshift drift, **bounds on the variability of c will be given**.

redshift drift - future experiments

- European Extremely Large Telescope (EELT) (with its spectrograph CODEX (COsmic Dynamics EXperiment))
- Thirty Meter Telescope (TMT), the Giant Magellan Telescope (GMT)
- Problems to measure Lyman- α lines of the number of quasars for $z < 1.7$ from the ground.
- [gravitational wave interferometers](#) DECIGO/BBO (DECI-hertz Interferometer Gravitational Wave Observatory/Big Bang Observer).
Detection even at $z \sim 0.2$.

4. Measuring c with baryon acoustic oscillations (BAO)

Speed of light c appears in **many** observational quantities.

Among them in the **angular diameter distance**

$$D_A = \frac{D_L}{(1+z)^2} = \frac{a_0}{1+z} \int_{t_1}^{t_2} \frac{c(t)dt}{a(t)} \quad (47)$$

where D_L is the luminosity distance, a_0 present value of the scale factor (normalized to $a_0 = 1$ later), and we have taken the spatial curvature $k = 0$ (otherwise there would be \sin or \sinh in front of the integral). Using the definition of redshift and the dimensionless parameters Ω_i we have

$$D_A = \frac{1}{1+z} \int_0^z \frac{c(z)dz}{H(z)}, \quad (48)$$

where

$$H(z) = \sqrt{\Omega_{r0}(1+z)^4 + \Omega_{m0}(1+z)^3 + \Omega_{\Lambda}}. \quad (49)$$

Angular diameter distance maximum.

Due to the expansion of the universe, there is a maximum of the distance at

$$D_A(z_m) = \frac{c(z_m)}{H(z_m)}. \quad (50)$$

which can be obtained by simple differentiating (48) with respect to z :

$$\frac{\partial D_A}{\partial z} = -\frac{1}{(1+z)^2} \int_0^z \frac{c(z)dz}{H(z)} + \frac{1}{1+z} \frac{c(z)}{H(z)} = 0 \quad (51)$$

In a flat $k = 0$ cold dark matter CDM model

$$z_m = 1.25 \quad \text{and} \quad D_A \approx 1230 \text{ Mpc} \quad (52)$$

For standard Λ CDM model of our interest:

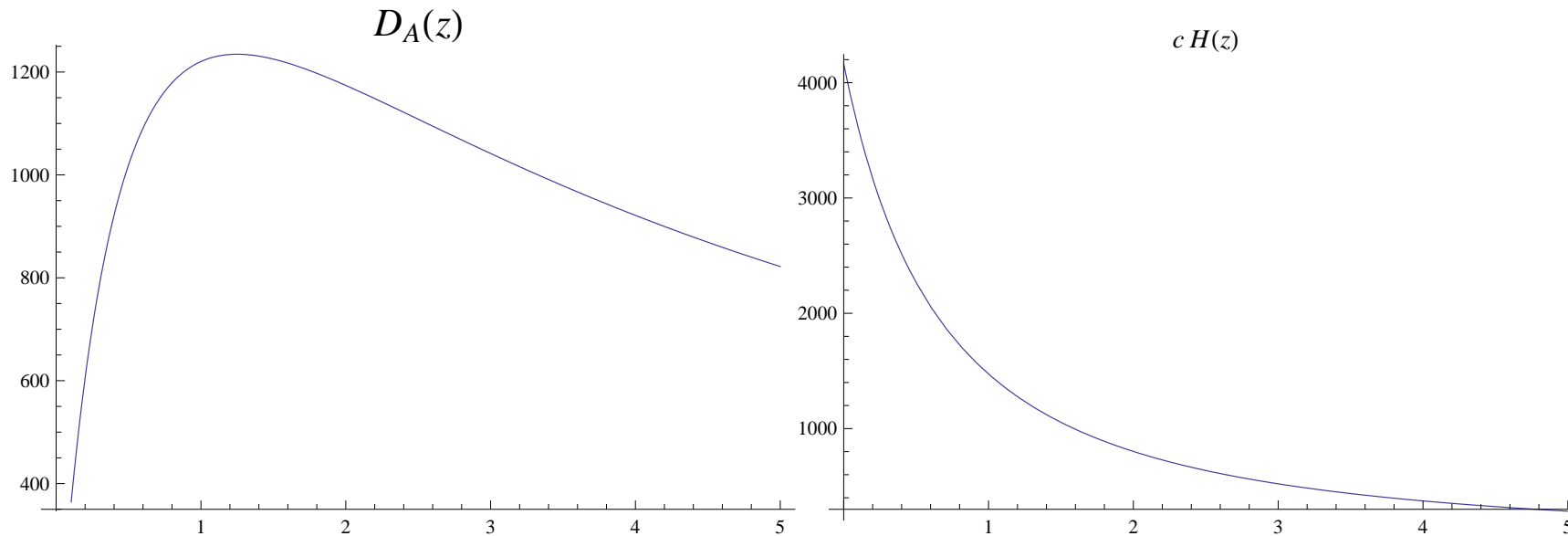
$$1.4 < z_m < 1.8. \quad (53)$$

D_A versus $H(z)$

The point: The product of D_A and H gives **exactly** the speed of light c at maximum (the curves intersect at z_m):

$$D_A(z_m)H(z_m) = c_0 \equiv 299792.458 \text{ kms}^{-1} \quad (54)$$

if we believe it is constant! (defined officially www.bipm.org; a relative error 10^{-9} by Evenson et al. 1972)



Measuring z_m

Measuring z_m problematic if one uses D_A only (large plateau around z_m makes it difficult to avoid errors from small sample of data – besides, one has binned data, observational errors, and intrinsic dispersion).

However, one can appeal to an independent measurement of $c_0/H(z)$ which is the radial (line-of-sight) mode of the baryon acoustic oscillations surveys (BAO) for which $D_A(z)$ is the tangential mode (e.g. Nesseris et al. 2006). In other words, we have both tangential and horizontal modes as

$$y_t = \frac{D_A}{r_s} \quad y_r = \frac{c}{H r_s}, \quad (55)$$

where

$$r_s = \int_{z_{dec}}^{\infty} \frac{c c_s(z) dz}{H(z)} \quad (56)$$

is the sound horizon size at decoupling and c_s the speed of sound.

Baryon acoustic oscillations.

From BOSS DR11 CMASS (Samushia et al. 2014)

$$\frac{D_V}{r_s(z_d)} = 13.85 \pm 0.17 \quad \text{at} \quad \bar{z} = 0.57, \quad (57)$$

where the volume-averaged distance is

$$D_V = \left[(1+z)^2 cz \frac{D_A^2}{H} \right]^{\frac{1}{3}}, \quad (58)$$

while from BOSS DR11 LOWZ (Tojeiro et al. 2014)

$$D_V = (1264 \pm 25) \left(\frac{r_s(z_d)}{r_{s, fid}(z_d)} \right) \quad \text{at} \quad \bar{z} = 0.32. \quad (59)$$

The method to measure c .

(Salzano, MPD, Lazkoz 2015)

- Measure independently $D_A(z)$ and $H(z)$.
- Calculate z_m .
- The product $D_A(z_m)H(z_m) = c(z_m)$.
- But $c(z_m)$ may not be equal to c_0 , so that we can measure $\Delta c = c(z_m) - c_0$.
- This would determine possible variability of c .

The scenarios.

Take background Λ CDM model with an ansatz (Magueijo 2003)

$$c(a) \propto c_0 \left(1 + \frac{a}{a_c} \right)^n \quad (60)$$

where a_c is the scale factor at the transition epoch from some $c(a) \neq c_0$ (at early times) to $c(a) \rightarrow c_0$ (at late times to now).

Three scenarios (Salzano, MPD, Lazkoz 2015):

- 1) standard case $c = c_0$;
- 2) $a_c = 0.005$, $n = -0.01 \rightarrow \Delta c/c \approx 1\%$ at $z \propto 1.5$;
- 3) $a_c = 0.005$, $n = -0.001 \rightarrow \Delta c/c \approx 0.1\%$ at $z \propto 1.5$.

The results.

Based on 10^3 Euclid project (Laureijs et al. 0912.0914) mock data simulations (Font-Ribeira et al. 2014):

- 1) $z_m = 1.592_{-0.039}^{+0.043}$ (fiducial model input $z_m = 1.596$) and $c/c_0 = 1 \pm 0.009$
 - 2) $z_m = 1.528_{-0.036}^{+0.038}$ (fiducial $z_m = 1.532$) and $c(z_m)/c_0 = 1.00925 \pm 0.00831$
- and

$$\langle c(z_m)/c_0 - 1\sigma_{c(z_m)/c_0} \rangle = 1.00094_{-0.00033}^{+0.00014} \quad (61)$$

so that **a detection by Euclid of 1% variation at 1σ -level will be possible.**

- 3) $z_m = 1.584_{-0.039}^{+0.042}$ (fiducial $z_m = 1.589$) and $c(z_m)/c_0 = 1.00095 \pm 0.00852$
- and

$$\langle c(z_m)/c_0 - 1\sigma_{c(z_m)/c_0} \rangle = 0.99243_{-0.00013}^{+0.00016} \quad (62)$$

so that **a detection by Euclid of 1% variation at 1σ -level will not be possible.**

Perspectives.

- Euclid will have **1/10 of the errors** of the current missions like WiggleZ Dark Energy Survey (e.g. Blake et al. 2011, 2012).
- Other missions which will be competitive to Euclid and useful for our task will be:
- Dark Energy Spectroscopic Instrument (DESI) (Levi et al. 1308.0847)
- Square Kilometer Array (SKA) (Bull et al. 1405.1452)
- Wide-Field Infrared Survey Telescope (WFIRST) (Spergel et al. 1305.5425) (esp. having **largest sensitivity** at potential z_m region i.e. $1.5 < z < 1.6$).

5. Conclusions

- Varying constants theories (and especially varying c and α) have their advantages as well as problems. The **firmest** seems to be varying G theories.
- The **advantages of varying c** theories are: solution of the flatness and horizon problems; singularity problem.
- Violation of Lorentz invariance in c -varying theories leads to a choice of **a preferred frame** and a drop of standard variational principle.
- α -varying theories have **better formulation** - variability of α is related to variability of c .
- **New tests** to check variability of c in future telescope/space missions have been proposed.

Results and conclusions contd.

- 1. **Redshift drift test** which give clear prediction for redshift drift effect which can potentially be measured by future telescopes (E-ELT, TMT, GMT, DECIGO/BBO).
- 2. Baryon acoustic oscillations test to independently measure the radial D_A and tangential mode c/H of the volume distance at the **angular diameter distance maximum z_m** .
- In simple terms we have a “cosmic” measurement of the speed of light c with D_A giving the dimension of length being a “cosmic ruler” and $1/H$ giving the dimension of time being a “cosmic clock” i.e.

$$c = \frac{D_A}{\left(\frac{1}{H}\right)}. \quad (63)$$

- We have proven that **1% variability of c can be tested at 1σ level by EUCLID mission**. Likely that they can be tested in DESI, SKA, WFIRST.