New tests of variability of the speed of light.

Mariusz P. Dąbrowski

Institute of Physics, University of Szczecin, Poland;
National Center for Nuclear Research, Otwock, Poland;
Copernicus Center for Interdisciplinary Studies, Kraków, Poland

4th International Conference on New Frontiers in Physics, OAC, Crete - 25 August 2015
Plan:

1. Main frameworks of varying constants theories.
2. Benefits and problems of varying $c$ theories.
3. Redshift drift test of varying $c$ models.
4. Measuring $c$ with baryon acoustic oscillations (BAO).
5. Conclusions.
Collaborators:

- MPD, K. Marosek, JCAP 02 (2013), 012.
- V. Salzano, A. Balcerzak, MPD - in progress
1. Main frameworks of varying constants theories.

Long story of varying constants theories:

H. Weyl (1919): electron radius/its gravitational radius $\sim 10^{40}$

A. Eddington (1935) discussed:

1. proton-to-electron mass $1/\beta = m_p/m_e \sim 1840$

2. an inverse of fine structure constant $1/\alpha = (hc)/(2\pi e^2) \sim 137$

3. electromagnetic to gravitational force between a proton and an electron $e^2/(4\pi\varepsilon_0 G m_e m_p) \sim 10^{40}$

4. introduced “Eddington number” $N_{edd} \sim 10^{80}$

P.A.M. Dirac (1937) interesting remarks about the relations between atomic and cosmological quantities: If $G \propto H(t) = (da/dt)/a$, then $a(t) \propto t^{1/3}$ and $G(t) \propto 1/t$ - fundamental constants must evolve in time.

Conclusion: electromagnetic force is strong compared to gravitational since the universe is “old” i.e. $F_e/F_p \propto (e^2/m_e m_p)t \propto t$ !!!

New tests of variability of the speed of light. – p. 4/38
varying gravitational constant $G$ theories

First fully quantitative framework: Brans-Dicke scalar-tensor gravity (1961)

The gravitational constant $G$ is associated with an average gravitational potential (scalar field) $\phi$ surrounding a given particle:

$$<\phi> = GM/(c/H_0) \propto 1/G = 1.35 \times 10^{28} \text{g/cm}.$$ The scalar field gives the strength of gravity

$$G = \frac{1}{16\pi\Phi} \quad (1)$$

With the action

$$S = \int d^4x \sqrt{-g} \left( \Phi R - \frac{\omega}{\Phi} \partial_\mu \Phi \partial^\mu \Phi + \Lambda + L_m \right) \quad (2)$$

it relates to low-energy-effective superstring theory for $\omega = -1$

String coupling constant (running) $g_s = \exp(\phi/2)$ changes in time with $\phi$ - the dilaton and $\Phi = \exp(-\phi)$. 

New tests of variability of the speed of light. – p. 5/38
Varying speed of light $c$ (VSL) theories


Introduce a scalar field

$$c^4 = \psi(x^\mu)$$

and so the action is

$$S = \int d^4x \sqrt{-g} \left[ \frac{\psi(R + 2\Lambda)}{16\pi G} + L_m + L_\psi \right]$$

AM model breaks Lorentz invariance (relativity principle and light principle) so that there is a preferred frame (cosmological or CMB) in which the field is minimally coupled to gravity. The Riemann tensor is computed in such a frame for a constant $\psi = c^4$ and no additional terms $\partial_\mu \psi$ appear in this frame (though they do in other frames). Einstein eqs remain the same except $c$ now varies.
Magueijo covariant (conformally) and locally invariant model (2000, 2001):

\[ \psi = \ln \left( \frac{c}{c_0} \right) \quad \text{or} \quad c = c_0 e^\psi, \quad (5) \]

with the action

\[ S = \int d^4x \sqrt{-g} \left[ \frac{c_0^4 e^{\alpha \psi} (R + 2\Lambda + L_\psi)}{16\pi G} + e^{\beta \psi} L_m \right], \quad (6) \]

with

\[ L_\psi = \kappa(\psi) \nabla_\mu \psi \nabla^\mu \psi. \quad (7) \]

Further assumption: \( \alpha - \beta = 4 \).

Interesting subcases:
\( \alpha = 4; \beta = 0 \) - Brans-Dicke with \( \phi_{BD} = e^{4\psi}/G \) and \( \kappa(\psi) = 16\omega_{BD}(\phi_{BD}) \).
\( \alpha = 0; \beta = -4 \) - minimal VSL theory.
Varying fine structure constant $\alpha$ theories

Varying fine structure constant $\alpha$ (or charge $e = e_0 \epsilon(x^\mu)$) theories (Webb et al. 1999, Sandvik 2002)

$$S = \int d^4x \sqrt{-g} \left( R - \frac{\omega}{2} \partial_\mu \psi \partial^\mu \psi - \frac{1}{4} f_{\mu\nu} f^{\mu\nu} e^{-2\psi} + L_m \right)$$  \hspace{1cm} (8)

with $\psi = \ln \epsilon$ and $f_{\mu\nu} = \epsilon F_{\mu\nu}$.

Can be related with the VSL theories due to the definition of the fine structure constant

$$\alpha = \frac{e^2}{\hbar c} \quad \text{i.e.} \quad \alpha(t) = \frac{e^2}{\hbar c(t)}$$  \hspace{1cm} (9)

Assume linear expansion $e^{\psi} = 1 - 8\pi G \zeta (\psi - \psi_0) = 1 - \Delta \alpha / \alpha$ with the constraint on the local equivalence principle violence $|\zeta| \leq 10^{-3}$. The relation to dark energy is (e.g. Vielzeuf and Martins 2012):

$$\gamma = w + 1 = \frac{(8\pi G \frac{d\psi}{d \ln a})^2}{\Omega_\psi}$$  \hspace{1cm} (10)
The field equations for Friedmann universes are (e.g. Barrow, Kimberly, Magueijo 2004)

\[
\begin{align*}
\frac{\dot{a}^2}{a^2} &= \frac{8\pi G}{3} (\varrho_r + \varrho_\psi) - \frac{kc^2}{a^2}, \\
\ddot{a} &= -\frac{8\pi G}{3} (\varrho_r + 2\varrho_\psi), \\
\ddot{\psi} + 3\frac{\dot{a}}{a}\dot{\psi} &= 0,
\end{align*}
\]

where \(\varrho_r \propto a^{-4}\) stands for the density of radiation while

\[
\varrho_\psi = \frac{p_\psi}{c^2} = \frac{\sigma}{2} \dot{\psi}^2
\]

stands for the density of the scalar field \(\psi\) (standard with \(\sigma = +1\) and phantom with \(\sigma = -1\)) and

\[
\alpha = \alpha_0 e^{2\psi}.
\]
2. Benefits and problems of varying $c$ and $\alpha$ theories.

Applying the simplest method one can obtain the generalized Einstein-Friedmann equations generalize in varying speed of light (VSL) theories and varying gravitational constant $G$ theories as ($\varrho$ - mass density; $\varepsilon = \varrho c^2(t)$ - energy density in $Jm^{-3} = Nm^{-2} = kgm^{-1}s^{-2}$)

$$\varrho(t) = \frac{3}{8\pi G(t)} \left( \frac{\ddot{a}}{a^2} + \frac{k c^2(t)}{a^2} \right),$$

$$p(t) = -\frac{c^2(t)}{8\pi G(t)} \left( 2 \frac{\dot{a}}{a} + \frac{\ddot{a}}{a^2} + \frac{k c^2(t)}{a^2} \right),$$

and the generalized conservation law is obtained from (16) and (17)

$$\dot{\varrho}(t) + 3 \frac{\dot{a}}{a} \left( \varrho(t) + \frac{p(t)}{c^2(t)} \right) = -\varrho(t) \frac{\dot{G}(t)}{G(t)} + 3 \frac{k c(t) \dot{c}(t)}{4\pi G a^2}. \quad (18)$$
introducing $\Lambda$ to varying $c$ models

If one adds the $\Lambda$-term to the equations (16)-(17), and introduces the vacuum mass density

$$\rho_\Lambda(t) = \frac{\Lambda c^2(t)}{8\pi G(t)} \quad (\Lambda = \text{const.}) \quad (19)$$

with

$$p_\Lambda(t) = -\rho_\Lambda(t)c^2, \quad (20)$$

then one has to replace $\rho \rightarrow \rho + \rho_\Lambda, p \rightarrow p + p_\Lambda$ in (16)-(17) and $\dot{\rho} \rightarrow \dot{\rho} + \dot{\rho}_\Lambda$ in (18) to obtain

$$\dot{\rho} + \frac{3\dot{\rho}}{\dot{a}} \left( \rho + \frac{p}{c^2(t)} \right) + \rho \frac{\dot{G}(t)}{G(t)} = \frac{(3k - \Lambda a^2)}{4\pi G(t)a^2} c(t) \dot{c}(t). \quad (21)$$

which is solved by (for $p = w \rho c^2, c(t) = c_0 a^n, G(t) = G_0 a^q, C = \text{const.}$)

$$\rho(a) = \frac{C}{a^{3(w+1)+q}} + \frac{3c_0^2 n}{4\pi G_0} \left( \frac{k}{2n + 3w + 1} - \frac{\Lambda}{3} \frac{a^2}{2n + 3w + 3} \right) a^{2(n-1)-q}.$$
Benefits of varying $c$ models

Solves basic problems of standard cosmology: flatness and horizon.

**Flatness:** inserting this into Friedmann (16) one gets

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G_0 C}{3} a^{-3(w+1)} + \frac{k c_0^2 a^{2n-2}(2n-1)}{2n + 3w + 1}, \quad (22)$$

and the density term (with $C$) will dominate the curvature term at large scale factor if

$$2 \geq 2n + 3(w + 1) \quad (23)$$

**Horizon:** For large scale factor the solution is $a(t) = t^{2/3(w+1)}$ and the proper distance to the horizon reads as

$$d_H = c(t)t = c_0 a^n(t)t = c_0 a_0^n t^{(3w+3+2n)/3(w+1)} \quad (24)$$

and the scale factor grows faster than $d_H$ under the same condition as in (23).
Varying constants can **remove or change the nature of singularities** (MPD, Marosek 2013).

<table>
<thead>
<tr>
<th>Type</th>
<th>Name</th>
<th>$t_{\text{sing.}}$</th>
<th>$a(t_s)$</th>
<th>$\varrho(t_s)$</th>
<th>$p(t_s)$</th>
<th>$\dot{p}(t_s)$ etc.</th>
<th>$w(t_s)$</th>
<th>$T$</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Big-Bang (BB)</td>
<td>0</td>
<td>0</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>finite</td>
<td>strong</td>
<td>strong</td>
</tr>
<tr>
<td>I</td>
<td>Big-Rip (BR)</td>
<td>$t_s$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>finite</td>
<td>strong</td>
<td>strong</td>
</tr>
<tr>
<td>$I_l$</td>
<td>Little-Rip (LR)</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>finite</td>
<td>strong</td>
<td>strong</td>
</tr>
<tr>
<td>$I_p$</td>
<td>Pseudo-Rip (PR)</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>finite</td>
<td>finite</td>
<td>finite</td>
<td>finite</td>
<td>weak</td>
<td>weak</td>
</tr>
<tr>
<td>II</td>
<td>Sudden Future (SFS)</td>
<td>$t_s$</td>
<td>$a_s$</td>
<td>$\varrho_s$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>finite</td>
<td>weak</td>
<td>weak</td>
</tr>
<tr>
<td>$I_{g}$</td>
<td>Gen. Sudden Future (GSFS)</td>
<td>$t_s$</td>
<td>$a_s$</td>
<td>$\varrho_s$</td>
<td>$p_s$</td>
<td>$\infty$</td>
<td>finite</td>
<td>weak</td>
<td>weak</td>
</tr>
<tr>
<td>III</td>
<td>Finite Scale Factor (FSFS)</td>
<td>$t_s$</td>
<td>$a_s$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>finite</td>
<td>weak</td>
<td>strong</td>
</tr>
<tr>
<td>IV</td>
<td>Big-Separation (BS)</td>
<td>$t_s$</td>
<td>$a_s$</td>
<td>0</td>
<td>0</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>weak</td>
<td>weak</td>
</tr>
<tr>
<td>V</td>
<td>w-singularity (w)</td>
<td>$t_s$</td>
<td>$a_s$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\infty$</td>
<td>weak</td>
<td>weak</td>
</tr>
</tbody>
</table>
varying $c$ (and $G$) removing or changing singularities.

Some of these can be regularized (removed by variable constants):

- In order to regularize an SFS or an FSF singularity by varying $c(t)$, the light should slow and eventually stop propagating at a singularity. (cf. loop quantum cosmology (LQC): anti-newtonian limit $c = c_0 \sqrt{1 - \varrho/\varrho_c} \rightarrow 0$ for $\varrho \rightarrow \varrho_c$ with $\varrho_c$ being the critical density (Cailettau et al. 2012). The low-energy limit $\varrho \ll \varrho_0$ gives the standard limit $c \rightarrow c_0$.)

- To regularize an SFS, FSF by varying gravitational constant $G(t)$ - the strength of gravity has to become infinite at an initial (curvature) singularity. Effectively, a new singularity - of strong coupling for a physical field such as $G \propto 1/\Phi$ appears. Such problems were already dealt with in superstring and brane cosmology where both the curvature singularity and a strong coupling singularity show up (choice of coupling, quantum corrections).
Main problem: to obtain the field equations out of any action (cf. also quantum cosmology)?

Equations (16)-(18) have just been obtained in a special frame - the one in which \( c \) is a constant and does not lead to any extra boundary terms (apart from standard ones). Einstein equations were simply generalized:

\[
G_{\mu\nu} - g_{\mu\nu}\Lambda = \frac{8\pi G}{\psi} T_{\mu\nu}
\]  
(25)

while the action (4) varied in a standard way leads to different field equations

\[
G_{\mu\nu} - g_{\mu\nu}\Lambda = \frac{8\pi G}{\psi} T_{\mu\nu} - \frac{1}{\psi} \psi_{\nu;\mu} + \frac{1}{\psi} \Box \psi.
\]  
(26)

The application of Bianchi identity to (25) gives a conservation equation with dynamical \( \psi \)

\[
T;_{\mu\nu} = -T^{\mu\nu} \psi_{;\mu}
\]  
(27)
Problems of varying $c$ quantum cosmology

If $\psi$ was supposed to be a dynamical matter field, then one could get the evolution equation using the Lagrangian

$$L_\psi = -\frac{\omega}{16\pi G\psi}\dot{\psi}^2,$$

but working only in a preferred frame and with $\psi$ not coupled to $\sqrt{-g}$.

Treating $\psi = c^4$ as constant in a preferred frame also requires special treatment of the boundary terms in $c$-varying quantum cosmology. As mentioned, we vary the action in the special frame where $c$ is constant which means that we drop $c$-induced boundary terms, but recover the time dependence of $c$ again to proceed towards WdW equation ($V_3$ is a 3-volume)

$$L = \frac{3V_3 c^3(x^0)}{8\pi G(x^0)} \left( ka - a,\dot{a}^2 a - \frac{\Lambda}{3} a^3 - \frac{8\pi G(x^0)}{3c^2} \rho a^3 \right).$$

(29)
Benefits of varying $\alpha$ cosmology

Since one does not brake Lorentz invariance in varying fine structure constant $\alpha$ theories, then there are no such problems in these models - the standard variational principle applies and the dynamical equation for the scalar field is given! According to the definition, any variability of $c\ (e, \hbar)$ is related to the variability of $\alpha$:

$$\frac{\Delta \alpha}{\alpha} = - \frac{\Delta c}{c}.$$  \hspace{1cm} (30)

The best constraints on $\Delta \alpha$ are: Oklo natural nuclear reactor:

$$\Delta \alpha/\alpha = (0.15 \pm 1.05) \cdot 10^{-7} \text{ at } z = 0.14$$

VLT/UVES quasars: $\Delta \alpha/\alpha = (0.15 \pm 0.43) \cdot 10^{-5}$ at $1.59 < z < 2.92$

SDSS quasars: $\Delta \alpha/\alpha = (1.2 \pm 0.7) \cdot 10^{-4}$ at $0.16 < z < 0.8$.
More bounds on variation of $\alpha$.

By Webb et al. (PRL 107, 191101 (2011)) ($\alpha$-dipole $R.A.17.4 \pm 0.9h$, $\delta = -58 \pm 9$: Keck ($\Delta\alpha < 0$) and VLT) as well as other specific measurements of $\alpha$ given in the table below (in parts per million):

<table>
<thead>
<tr>
<th>Object</th>
<th>$z$</th>
<th>$\Delta\alpha/\alpha$</th>
<th>Spectrograph</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>HE0515−4414</td>
<td>1.15</td>
<td>$-0.1 \pm 1.8$</td>
<td>UVES</td>
<td>Molaro et al. (2008)</td>
</tr>
<tr>
<td>HE0515−4414</td>
<td>1.15</td>
<td>$0.5 \pm 2.4$</td>
<td>HARPS/UVES</td>
<td>Chand et al. (2006)</td>
</tr>
<tr>
<td>HE0001−2340</td>
<td>1.58</td>
<td>$-1.5 \pm 2.6$</td>
<td>UVES</td>
<td>Agafonowa et al. (2011)</td>
</tr>
<tr>
<td>HE2217−2818</td>
<td>1.69</td>
<td>$1.3 \pm 2.6$</td>
<td>UVES−LP</td>
<td>Molaro et al. (2013)</td>
</tr>
<tr>
<td>Q1101−264</td>
<td>1.84</td>
<td>$5.7 \pm 2.7$</td>
<td>UVES</td>
<td>Molaro et al. (2008)</td>
</tr>
</tbody>
</table>

UVES - Ultraviolet and Visual Echelle Telescope  
HARPS - High Accuracy Radial velocity Planet Searcher  
LP - Large Program measurement
Strongest – atomic clock Rosenband bound at $z = 0$

Rosenband (2008) measurement gives the following bound at $z = 0$

$$\left( \frac{\dot{\alpha}}{\alpha} \right)_0 = (-1.6 \pm 2.3) \times 10^{-17} \text{yr}^{-1}. \quad (31)$$

which can be transformed onto the bound for the scalar field coupling $\xi$: 

$$\left| \frac{\dot{\alpha}}{\alpha} \right|_0 = |\xi| H_0 \sqrt{3\Omega_{\Phi_0}} \left| 1 + w_{\Phi_0} \right|, \quad (32)$$

which translates for $H_0 = (67.4 \pm 1.4) \text{km.s}^{-1}\text{Mpc}^{-1}$ Planck value) into the conservative ($3\sigma$) bound

$$|\xi| \sqrt{3\Omega_{\Phi_0}} \left| 1 + w_{\Phi_0} \right| < 10^{-6}. \quad (33)$$
3. Redshift drift test of varying $c$ models.

Redshift drift (Sandage 1962, Loeb 1998) - the idea is to collect data from two light cones separated by 10-20 years to look for a change in redshift of a source as a function of time.

There is a relation between the times of emission of light by the source $\tau_e$ and $\tau_e + \Delta\tau_e$ and times of their observation at $\tau_o$ and $\tau_o + \Delta\tau_o$:

$$
\int_{\tau_e}^{\tau_o} \frac{d\tau}{a(\tau)} = \int_{\tau_e + \Delta\tau_e}^{\tau_o + \Delta\tau_o} \frac{d\tau}{a(\tau)}, \quad (34)
$$

which for small $\Delta\tau_e$ and $\Delta\tau_o$ reads as

$$
\frac{\Delta\tau_e}{a(\tau_e)} = \frac{\Delta\tau_o}{a(\tau_o)}.
$$
Redshift drift test.

The redshift drift is defined as \((\tau \rightarrow t \text{ here})\)

\[
\Delta z = z_e - z_0 = \frac{a(t_0 + \Delta t_0)}{a(t_e + \Delta t_e)} - \frac{a(t_0)}{a(t_e)},
\]

which can be expanded in series and to first order in \(\Delta t\) as

\[
\Delta z = \frac{a(t_0) + \dot{a}(t_0) \Delta t_0}{a(t_e) + \dot{a}(t_e) \Delta t_e} - \frac{a(t_0)}{a(t_e)} \approx \frac{a(t_0)}{a(t_e)} \left[ \dot{a}(t_0) \Delta t_0 - \dot{a}(t_e) \Delta t_e \right].
\]

Using above relations we have

\[
\Delta z = \Delta t_0 \left[ H_0 (1 + z) - H(t(z)) \right] = (1 + z) \frac{\Delta v}{c},
\]

where \(\Delta v\) is the velocity shift and \(H(t(z))\) is given in a standard way.
Redshift drift in varying $c$ theory.

In VSL theory the relation (34) generalizes into
\[ \int_{t_e}^{t_o} \frac{c(t)dt}{a(t)} = \int_{t_e+\Delta t_e}^{t_o+\Delta t_o} \frac{c(t)dt}{a(t)}, \]  
(38)

which for small $\Delta t_e$ and $\Delta t_o$ transforms into
\[ \frac{c(t_e)\Delta t_e}{a(t_e)} = \frac{c(t_0)\Delta t_o}{a(t_o)}. \]  
(39)

The definition of redshift in VSL theories remains the same as in standard Einstein relativity and reads as (Barrow, Magueijo 1999)
\[ 1 + z = \frac{a(t_0)}{a(t_e)}. \]  
(40)
Redshift drift - varying $c$

Using (39) we have

$$\Delta z = \Delta t_0 \left[ H_0 (1 + z) - H(t_e) \frac{c(t_0)}{c(t_e)} \right], \quad (41)$$

which after applying the ansatz

$$c(t) = c_0 a^n(t) \quad (42)$$

gives

$$\frac{\Delta z}{\Delta t_0} = \frac{\Delta z}{\Delta t_0} (z, n) = H_0 (1 + z) - H(z)(1 + z)^n. \quad (43)$$
Redshift drift - varying $c$

In the limit $n \to 0$ the formula (43) reduces to (37) for standard Friedmann universe. Bearing in mind definitions $\Omega$’s, and assuming $K = 0$ we have

$$H^2(z) = H_0^2 \left[ \Omega_m(1 + z)^3 + \Omega_\Lambda \right] \quad (44)$$

and so (43) gives

$$\frac{\Delta z}{\Delta t_0} = H_0 \left[ 1 + z - (1 + z)^n \sqrt{\Omega_m(1 + z)^3 + \Omega_\Lambda} \right]$$

$$= H_0 \left[ 1 + z - \sqrt{\Omega_m(1 + z)^{3+2n} + \Omega_\Lambda(1 + z)^{2n}} \right] \quad (45)$$

which can further be rewritten to define new redshift function

$$\tilde{H}(z) \equiv (1 + z)^n H(z) = H_0 \sqrt{\sum_{i=k}^{i=k} \Omega_i (1 + z)^{3(w_{eff}+1)}} \quad (46)$$

where $w_{eff} = w_i + \frac{2}{3} n$. 

New tests of variability of the speed of light. – p. 24/38
Redshift drift test - varying $c$

The VSL redshift drift effect for 15 year period of observations.
Redshift drift test - varying $c$

- If $n < 0$ ($c$ decreases) then dust matter becomes little negative pressure matter and the cosmological constant became phantom. Both components can mimic dark energy.

- If $n > 0$ then (growing $c(t)$) VSL model becomes more like Cold Dark Matter (CDM) model.

- Theoretical error bars are taken from Quercellini et al. 2012 and presumably show that for $|n| < 0.045$ – one cannot distinguish between VSL models and $\Lambda$CDM models.

- In other words, by measuring redshift drift, bounds on the variability of $c$ will be given.
redshift drift - future experiments

- European Extremely Large Telescope (EELT) (with its spectrograph CODEX (COsmic Dynamics EXperiment))
- Thirty Meter Telescope (TMT), the Giant Magellan Telescope (GMT)
- Problems to measure Lyman-α lines of the number of quasars for $z < 1.7$ from the ground.
- **gravitational wave interferometers** DECIGO/BBO (DECi-hertz Interferometer Gravitational Wave Observatory/Big Bang Observer). Detection even at $z \sim 0.2$. 
4. Measuring $c$ with baryon acoustic oscillations (BAO)

Speed of light $c$ appears in many observational quantities. Among them in the angular diameter distance

$$D_A = \frac{D_L}{(1 + z)^2} = \frac{a_0}{1 + z} \int_{t_1}^{t_2} \frac{c(t) dt}{a(t)}$$

(47)

where $D_L$ is the luminosity distance, $a_0$ present value of the scale factor (normalized to $a_0 = 1$ later), and we have taken the spatial curvature $k = 0$ (otherwise there would be $\sin$ or $\sinh$ in front of the integral). Using the definition of redshift and the dimensionless parameters $\Omega_i$ we have

$$D_A = \frac{1}{1 + z} \int_0^z \frac{c(z) dz}{H(z)},$$

(48)

where

$$H(z) = \sqrt{\Omega_r(1 + z)^4 + \Omega_m(1 + z)^3 + \Omega_\Lambda},$$

(49)
Angular diameter distance maximum.

Due to the expansion of the universe, there is a maximum of the distance at

\[ D_A(z_m) = \frac{c(z_m)}{H(z_m)}. \] (50)

which can be obtained by simple differentiating (48) with respect to \( z \):

\[ \frac{\partial D_A}{\partial z} = -\frac{1}{(1+z)^2} \int_0^z \frac{c(z)dz}{H(z)} + \frac{1}{1+z} \frac{c(z)}{H(z)} = 0 \] (51)

In a flat \( k = 0 \) cold dark matter CDM model

\[ z_m = 1.25 \quad \text{and} \quad D_A \approx 1230 \, \text{Mpc} \] (52)

For standard \( \Lambda \)CDM model of our interest:

\[ 1.4 < z_m < 1.8. \] (53)
**$D_A$ versus $H(z)$**

**The point:** The product of $D_A$ and $H$ gives **exactly** the speed of light $c$ at maximum (the curves intersect at $z_m$):

$$D_A(z_m)H(z_m) = c_0 \equiv 299792.458 \text{ kms}^{-1}$$  \hspace{1cm} (54)

if we believe it is constant! (defined officially www.bipm.org; a relative error $10^{-9}$ by Evenson et al. 1972)
**Measuring \( z_m \)**

Measuring \( z_m \) problematic if one uses \( D_A \) only (large plateau around \( z_m \) makes it difficult to avoid errors from small sample of data – besides, one has binned data, observational errors, and intrinsic dispersion).

However, one can appeal to an independent measurement of \( c_0/H(z) \) which is the radial (line-of-sight) mode of the baryon acoustic oscillations surveys (BAO) for which \( D_A(z) \) is the tangential mode (e.g. Nesseris et al. 2006). In other words, we have both tangential and horizontal modes as

\[
y_t = \frac{D_A}{r_s} \quad y_r = \frac{c}{Hr_s},
\]

where

\[
r_s = \int_{z_{dec}}^{\infty} \frac{c c_s(z) dz}{H(z)}
\]

is the sound horizon size at decoupling and \( c_s \) the speed of sound.
Baryon acoustic oscillations.

From BOSS DR11 CMASS (Samushia et al. 2014)

\[ \frac{D_V}{r_s(z_d)} = 13.85 \pm 0.17 \quad \text{at} \quad \bar{z} = 0.57, \quad (57) \]

where the volume-averaged distance is

\[ D_V = \left[ (1 + z)^2 cz \frac{D_A^2}{H} \right]^{\frac{1}{3}}, \quad (58) \]

while from BOSS DR11 LOWZ (Tojeiro et al. 2014)

\[ D_V = (1264 \pm 25) \left( \frac{r_s(z_d)}{r_{s,fid}(z_d)} \right) \quad \text{at} \quad \bar{z} = 0.32. \quad (59) \]
The method to measure $c$.

(Salzano, MPD, Lazkoz 2015)

- Measure independently $D_A(z)$ and $H(z)$.
- Calculate $z_m$.
- The product $D_A(z_m)H(z_m) = c(z_m)$.
- But $c(z_m)$ may not be equal to $c_0$, so that we can measure $\Delta c = c(z_m) - c_0$.
- This would determine possible variability of $c$. 

New tests of variability of the speed of light. – p. 33/38
The scenarios.

Take background ΛCDM model with an ansatz (Magueijo 2003)

\[ c(a) \propto c_0 \left(1 + \frac{a}{a_c}\right)^n \]  \hspace{1cm} (60)

where \( a_c \) is the scale factor at the transition epoch from some \( c(a) \neq c_0 \) (at early times) to \( c(a) \rightarrow c_0 \) (at late times to now).

Three scenarios (Salzano, MPD, Lazkoz 2015):

1) standard case \( c = c_0 \);

2) \( a_c = 0.005, \ n = -0.01 \rightarrow \Delta c/c \approx 1\% \) at \( z \propto 1.5 \);

3) \( a_c = 0.005, \ n = -0.001 \rightarrow \Delta c/c \approx 0.1\% \) at \( z \propto 1.5 \).
The results.

Based on $10^3$ Euclid project (Laureijs et al. 0912.0914) mock data simulations (Font-Ribeira et al. 2014):

1) $z_m = 1.592^{+0.043}_{-0.039}$ (fiducial model input $z_m = 1.596$) and $c/c_0 = 1 \pm 0.009$

2) $z_m = 1.528^{+0.038}_{-0.036}$ (fiducial $z_m = 1.532$) and $c(z_m)/c_0 = 1.00925 \pm 0.00831$

and

\[
< c(z_m)/c_0 - 1\sigma_{c(z_m)/c_0} > = 1.00094^{+0.00014}_{-0.00033}
\]  \(\text{(61)}\)

so that **a detection by Euclid of 1% variation at 1\(\sigma\)-level will be possible.**

3) $z_m = 1.584^{+0.042}_{-0.039}$ (fiducial $z_m = 1.589$) and $c(z_m)/c_0 = 1.00095 \pm 0.00852$

and

\[
< c(z_m)/c_0 - 1\sigma_{c(z_m)/c_0} > = 0.99243^{+0.00016}_{-0.00013}
\]  \(\text{(62)}\)

so that **a detection by Euclid of 1% variation at 1\(\sigma\)-level will not be possible.**
Euclid will have 1/10 of the errors of the current missions like WiggleZ Dark Energy Survey (e.g. Blake et al. 2011, 2012).

Other missions which will be competitive to Euclid and useful for our task will be:

- Dark Energy Spectroscopic Instrument (DESI) (Levi et al. 1308.0847)
- Square Kilometer Array (SKA) (Bull et al. 1405.1452)
- Wide-Field Infrared Survey Telescope (WFIRST) (Spergel et al. 1305.5425) (esp. having largest sensitivity at potential $z_m$ region i.e. $1.5 < z < 1.6$).
5. Conclusions

- Varying constants theories (and especially varying \( c \) and \( \alpha \)) have their advantages as well as problems. The firmest seems to be varying \( G \) theories.

- The advantages of varying \( c \) theories are: solution of the flatness and horizon problems; singularity problem.

- Violation of Lorentz invariance in \( c \)-varying theories leads to a choice of a preferred frame and a drop of standard variational principle.

- \( \alpha \)-varying theories have better formulation - variability of \( \alpha \) is related to variability of \( c \).

- New tests to check variability of \( c \) in future telescope/space missions have been proposed.
1. **Redshift drift test** which give clear prediction for redshift drift effect which can potentially be measured by future telescopes (E-ELT, TMT, GMT, DECIGO/BBO).

2. **Baryon acoustic oscillations** test to independently measure the radial $D_A$ and tangential mode $c/H$ of the volume distance at the angular diameter distance maximum $z_m$.

In simple terms we have a “cosmic” measurement of the speed of light $c$ with $D_A$ giving the dimension of length being a “cosmic ruler” and $1/H$ giving the dimension of time being a “cosmic clock” i.e.

$$c = \frac{D_A}{\left(\frac{1}{H}\right)}.$$  \hspace{1cm} (63)

We have proven that **1% variability of $c$ can be tested at $1\sigma$ level by EUCLID mission.** Likely that they can be tested in DESI, SKA, WFIRST.