History

Low-energy QCD

RCQM

Spectroscopy

Multiplets

Structure

Nucleon E.m.

Baryon E.m.

Axial FFs

Gravitational FF

Strong πNN, $\pi N\Delta$ FFs

CC Model

Summary

The Constituent-Quark Model

–

Nowadays

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OAC Kolymbari, August 23rd, 2015

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Low-energy QCD

RCQM

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Summary

Hadrons known around 1960:

Name/Symbol	*Charge*	*Mass (proton as unit)*	*Strangeness*	*Stable*
Proton p	+1	1	0	Yes (?)
Neutron n	0	1	0	$\tau \sim$ 15 min free
Pion $\pi^{\pm}$	± 1	$\frac{1}{7}$	0	10^{-8} s
π^0	0	$\frac{1}{7}$	0	10^{-16} s
(1961) **Eta** η^0	0	$\frac{1}{2}$	0	10^{-19} s
Kaon $K^{\pm}$	± 1	$\frac{1}{2}$	± 1	10^{-8} s
$K^0, \overline{K^0}$	0	$\frac{1}{2}$	± 1	10^{-8} or 10^{-10} s
Sigma (1958) $\Sigma^{\pm}$	± 1	1.2	-1	10^{-10} s
Σ^0	0	1.2	-1	10^{-20} s
Lambda Λ^0	0	1.1	-1	10^{-10} s
Xi (1959) Ξ^0	0	1.3	-2	10^{-10} s
Ξ^-	-1	1.3	-2	10^{-10} s
Delta Δ^{++}	+2	1.2	0	10^{-23} s
Δ^+	+1	1.2	0	10^{-23} s
Δ^0	0	1.2	0	10^{-23} s
Δ^-	-1	1.2	0	10^{-23} s
(1961) Sigma-star				
$\Sigma^{\pm\star}$	± 1	1.4	-1	10^{-23} s
$\Sigma^{0\star}$	0	1.4	-1	10^{-23} s
(1962) Xi-star				
$\Xi^{0\star}$	0	1.5	-2	10^{-23} s
$\Xi^{-\star}$	-1	1.5	-2	10^{-23} s

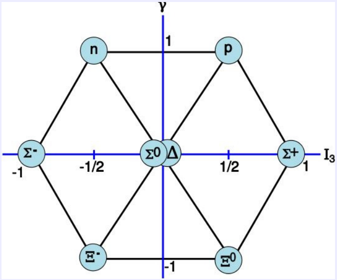
Yu. Ne'eman and M. Gell-Mann: 'Eightfold Way', i.e. $SU(3)$ symmetry

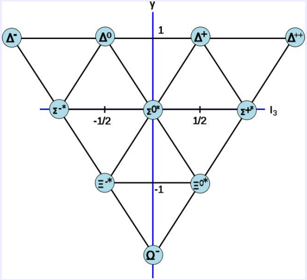
The Eightfold Way and $SU(3)_F$

Around **1964** one found $SU(3)$ as a (preliminary) order system for mesons and baryons
(independently by M. Gell-Mann and Y. Ne'eman and G. Zweig):

It implied multiplets of baryon ground states with flavors u, d, s:

octet **decuplet**



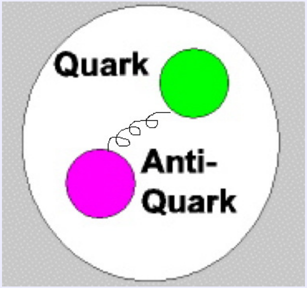


and in addition: **singlet** Λ^0

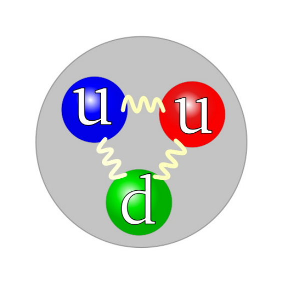
Valence-Quark Picture of Hadrons

M. Gell-Mann in 1964*):
Mesons and baryons can be constructed from quarks:

Mesons $\{Q\bar{Q}\}$



Baryons $\{QQQ\}$



Mesons and baryons are formed as <u>colorless objects</u> of quarks confined inside the hadrons.

*) M. Gell-Mann: Phys. Lett. **8**, 2014 (1964)

M. Gell-Mann: Quarks – G. Zweig: Aces

Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

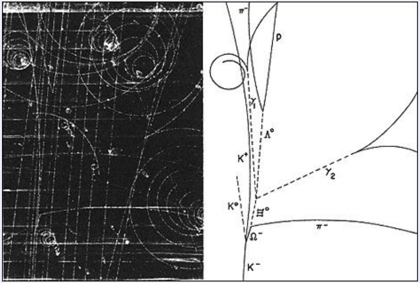
E S

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

G. Zweig *)
CERN – Geneva

Success of $SU(3)$ Symmetry of u, d, s

- The Ω^- was predicted (independently) by Ne'eman and Gell-Mann in 1962.
- It was discovered at Brookhaven Natl. Lab. by Nicholas Samios et al. in 1964.



The Ω^- earned M. Gell-Mann the Nobel Prize in 1969
"for his contributions and discoveries concerning the classification of elementary particles and their interactions".

Quark Interactions - Quantum Chromodynamics

1972/1973: Invention of **quantum chromodynamics** (QCD)

M. Gell-Mann and H. Fritzsch: Proceedings of the Int. Conf. on High-Energy Physics (Rochester Conf.), Chicago 1972

H. Fritzsch, M.Gell-Mann, and H. Leutwyler: Phys. Lett. **B47**, 365 (1973)

see also H. Fritzsch in: http://cerncourier.com/cws/article/cern/50796

Present methods for **non-perturbative hadrons** in QCD:

- QCD on a lattice (works but faces computational limitations)
- Chiral perturbation theory (works at low energies, as a systematic expansion but is limited to a few terms)
- Effective field theories or functional methods (depend on assumptions and regularizations)
- Effective models, e.g. constituent-quark models (depend on assumptions and input parameters)

Outline

Low-Energy QCD / Effective D.o.F.

Low-energy QCD of three flavors u, d, s is characterized by:

- spontaneous breaking of chiral symmetry ($SB\chi S$):

$$SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$$

$\rightarrow$ appearance of $(N_f^2 - 1) = 8$ **Goldstone bosons** $\vec{\phi}$
$\rightarrow$ generation of quasiparticles with dynamical mass, i.e. **constituent quarks** ψ

- thus (effective) interaction Lagrangian:

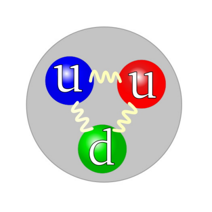
$$\mathcal{L}_{\text{int}} \sim ig\bar{\psi}\gamma_5\vec{\lambda}^F \cdot \vec{\phi}\psi$$

A. Manohar and H. Georgi: Nucl. Phys. B 234 (1984) 189
E.V. Shuryak: Phys. Rep. **115**, 151 (1984)
L.Ya. Glozman and D.O. Riska: Phys. Rep. **268**, 263 (1996)
see also:
S. Weinberg: Phys. Rev. Lett. **105**, 261601 (2010)

Baryons

Baryons are considered as colorless bound states of three constituent quarks.

Here the proton:



- 'Constituent' quarks are quasiparticles with dynamical mass, NOT the original QCD d.o.f. (i.e. 'current' quarks).
- 'Constituent' quarks are confined and interact via hyperfine interactions associated with $SB\chi S$, i.e. Goldstone-boson exchange.

Relativistic quantum mechanics (RQM)

i.e. **quantum theory** respecting **Poincaré invariance**

(theory on a Hilbert space $\mathcal{H}$ corresponding to a finite number of particles, not a field theory)

Invariant mass operator

$$\hat{M} = \hat{M}_{free} + \hat{M}_{int}$$

Eigenvalue equations

$$\hat{M}\,|P,J,\Sigma\rangle = M\,|P,J,\Sigma\rangle \quad , \qquad \hat{M}^2 = \hat{P}^\mu \hat{P}_\mu$$

$$\hat{P}^\mu\,|P,J,\Sigma\rangle = P^\mu\,|P,J,\Sigma\rangle \quad , \qquad \hat{P}^\mu = \hat{M}\hat{V}^\mu$$

Interacting mass operator

$$\begin{aligned}
\hat{M} &= \hat{M}_{free} + \hat{M}_{int} \\
\hat{M}_{free} &= \sqrt{\hat{H}^2_{free} - \hat{\vec{P}}^2_{free}} \\
\hat{M}^{rest\ frame}_{int} &= \sum_{i<j}^{3} \hat{V}_{ij} = \sum_{i<j}^{3} [\hat{V}^{conf}_{ij} + \hat{V}^{hf}_{ij}]
\end{aligned}$$

fulfilling the **Poincaré algebra**

$$\begin{array}{lll}
[\hat{P}_i, \hat{P}_j] = 0, & [\hat{J}_i, \hat{H}] = 0, & [\hat{P}_i, \hat{H}] = 0, \\
[\hat{K}_i, \hat{H}] = -i\hat{P}_i & [\hat{J}_i, \hat{J}_j] = i\epsilon_{ijk}\hat{J}_k & [\hat{J}_i, \hat{K}_j] = i\epsilon_{ijk}\hat{K}_k, \\
[\hat{J}_i, \hat{P}_j] = i\epsilon_{ijk}\hat{P}_k, & [\hat{K}_i, \hat{K}_j] = -i\epsilon_{ijk}\hat{J}_k, & [\hat{K}_i, \hat{P}_j] = -i\delta_{ij}\hat{H}
\end{array}$$

$\hat{H}, \hat{P}_i$ ... time and space translations,
$\hat{J}_i$ ... rotations, $\hat{K}_i$ ... Lorentz boosts

Universal GBE RCQM

Phenomenologically, baryons with 5 flavors: u, d, s, c, b

$$\Rightarrow \quad H_{free} = \sum_{i=1}^{3} \sqrt{m_i^2 + \vec{k}_i^2}$$

$$V^{conf}(\vec{r}_{ij}) = B + C\, r_{ij}$$

$$V^{hf}(\vec{r}_{ij}) = \left[V_{24}(\vec{r}_{ij}) \sum_{f=1}^{24} \lambda_i^f \lambda_j^f + V_0(\vec{r}_{ij}) \lambda_i^0 \lambda_j^0 \right] \vec{\sigma}_i \cdot \vec{\sigma}_j$$

- i.e., for $N_f = 5$, we have the exchange of a **24-plet** plus a **singlet** of Goldstone bosons.

L.Ya. Glozman and D.O. Riska: Nucl. Phys. A **603**, 326 (1996)
J.P. Day, K.-S. Choi, and W. Plessas: arXiv:1205.6918
J.P. Day, K.-S. Choi, and W. Plessas: Few-Body Syst. **54**, 329 (2013)

Universal GBE RCQM Parametrization

$$V^{conf}(\vec{r}_{ij}) = B + C\, r_{ij}$$

$$V_\beta(\vec{r}_{ij}) = \frac{g_\beta^2}{4\pi}\frac{1}{12 m_i m_j}\left\{\mu_\beta^2 \frac{e^{-\mu_\beta r_{ij}}}{r_{ij}} - 4\pi\delta(\vec{r}_{ij})\right\}$$

$$= \frac{g_\beta^2}{4\pi}\frac{1}{12 m_i m_j}\left\{\mu_\beta^2 \frac{e^{-\mu_\beta r_{ij}}}{r_{ij}} - \Lambda_\beta^2 \frac{e^{-\Lambda_\beta r_{ij}}}{r_{ij}}\right\}$$

$B = -402$ MeV, $C = 2.33$ fm^{-2}

$\beta = 24:$ $\frac{g_{24}^2}{4\pi} = 0.7,$ $\mu_{24} = \mu_\pi = 139$ MeV, $\Lambda_{24} = 700.5$ MeV

$\beta = 0:$ $\left(\frac{g_0}{g_{24}}\right)^2 = 1.5,$ $\mu_0 = \mu_{\eta'} = 958$ MeV, $\Lambda_0 = 1484$ MeV

$m_u = m_d = 340$ MeV, $m_s = 480$ MeV,

$m_c = 1675$ MeV, $m_b = 5055$ MeV

Remarks/Caveats on the Setting of the RCQM

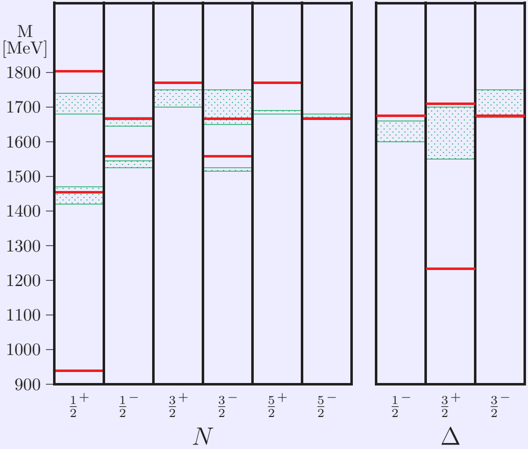
- **Relativistic** (i.e. Poincaré-invariant) **framework**
  - **Hamiltonian** theory with a **finite number** of d.o.f.
  - based on relativistically **invariant mass operator**
  - solvable with advanced **few-body methods** (differential and / or integral equations)
- Observes eminent **low-energy QCD dynamics**:
  - **confinement**
  - **spontaneous chiral-symmetry breaking** ($SB\chi S$)
- The **R**elativistic **C**onstituent **Q**uark **M**odel (RCQM)
  - is a **model** of a given dynamics, here QCD
  - thus, is not itself a (fundamental) **dynamical theory**
  - has a **limited range** of validity ($\sim$ non-perturbative regime of QCD)

Excitation Spectra

of Baryons with All Flavors

u, *d*, *s*, *c*, *b*

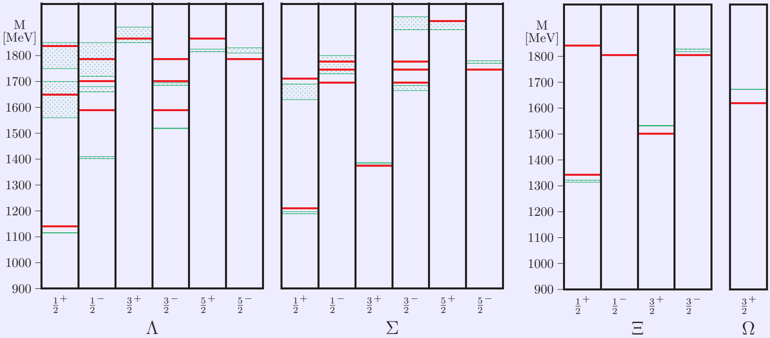
Light Baryon Spectra



red Universal GBE RCQM

green PDG 2013 (experiment)

Strange Baryon Spectra



red Universal GBE RCQM

green PDG 2013 (experiment)

Charm Baryon Spectra

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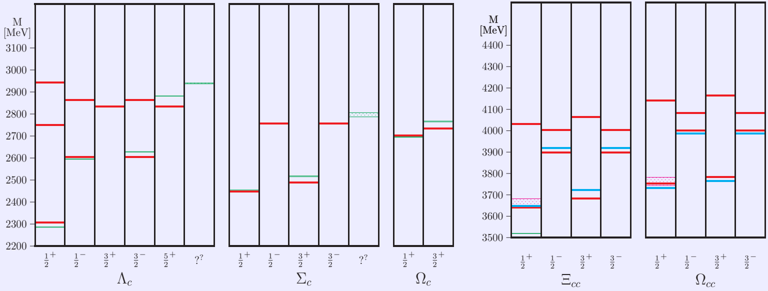


Left panels – single charm:

red Universal GBE RCQM prediction

green PDG 2013 (experiment)

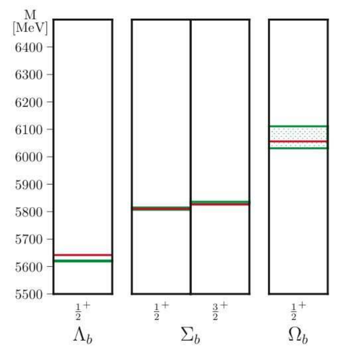
Right panel – double charm:

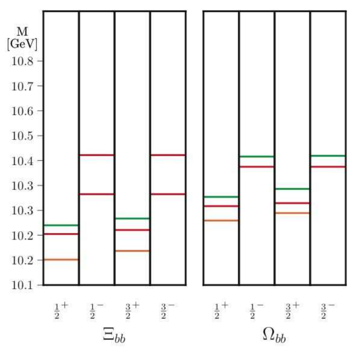
green M. Mattson et al.: Phys. Rev. Lett. 89 (2002) 112001 (SELEX experiment)

cyan S. Migura, D. Merten, B. Metsch, and H.-R. Petry: Eur. Phys. J. A 28 (2006) 41 (Bonn RCQM)

magenta L. Liu et al.: Phys. Rev. D 81 (2010) 094505 (Lattice QCD)

Bottom Baryon Spectra





Left panel – single bottom:

red Universal GBE RCQM prediction

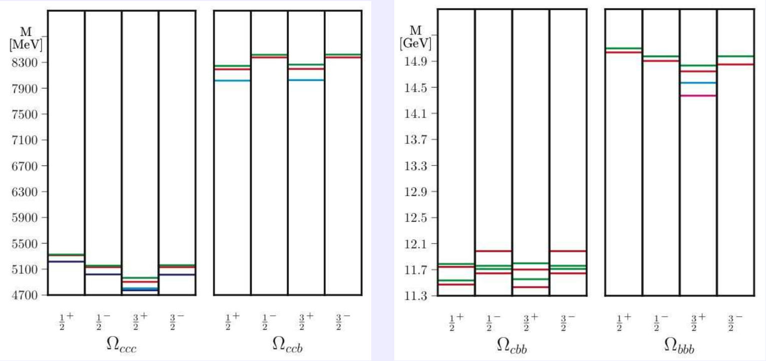
green PDG 2013 (experiment)

Right panel – double bottom:

green W. Roberts and M. Pervin: Int. J. Mod. Phys. A 23 (2008) 2817 (nonrel. one-gluon-exchange CQM)

orange D. Ebert, R.N. Faustov, V.O. Galkin, and A.P. Martynenko: Phys. Rev. D 66 (2002) 014008 (RCQM)

Triple-Heavy Baryon Spectra



red Universal GBE RCQM

green W. Roberts and M. Pervin: Int. J. Mod. Phys. A 23 (2008) 2817 (nonrelativistic one-gluon-exchange CQM)

blue S. Migura, D. Merten, B. Metsch, and H.-R. Petry: Eur. Phys. J. A 28 (2006) 41 (Bonn RCQM)

cyan A.P. Martynenko: Phys. Lett. B 663 (2008) 317 (RCQM)

magenta S. Meinel: Phys. Rev. D 82 (2010) 114502 (lattice QCD)

Mass operator eigenstates

$$\hat{M}\,|P,J,\Sigma,T,M_T\rangle = M\,|P,J,\Sigma,T,M_T\rangle$$

represented in configuration space

$$\left\langle \vec{\xi},\vec{\eta}\,\middle|\,P,J,\Sigma,T,M_T\right\rangle = \Psi_{PJ\Sigma TM_T}(\vec{\xi},\vec{\eta})$$

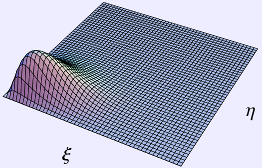
with $\vec{\xi}$ and $\vec{\eta}$ the usual Jacobi coordinates.

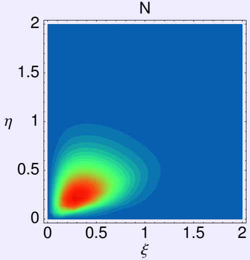
Picture the baryon wave functions through
spatial probability density distributions

$$\rho(\xi,\eta) = \xi^2\eta^2 \int d\Omega_\xi d\Omega_\eta \; \Psi^\star_{PJ\Sigma TM_T}(\xi,\Omega_\xi,\eta,\Omega_\eta)\Psi_{PJ\Sigma TM_T}(\xi,\Omega_\xi,\eta,\Omega_\eta)$$

Pictures of Baryons (rest frame)

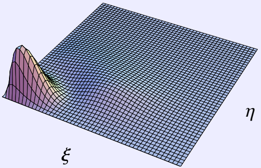
N GBE CQM

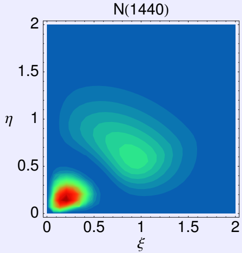




units are [*fm*]

N(1440) GBE CQM



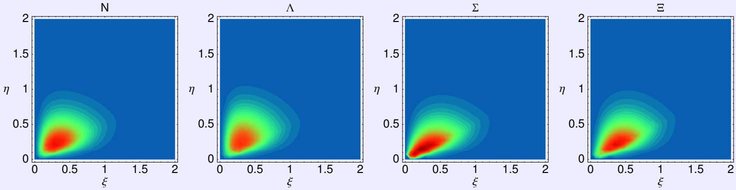


units are [*fm*]

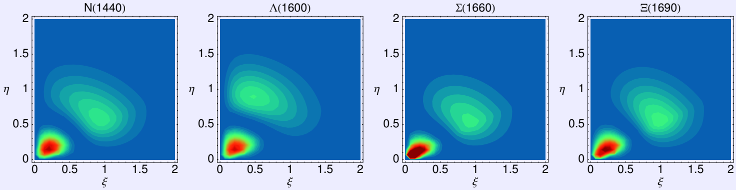
T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77** (2008) 114002

Spatial Probability Density Distributions

$\rho(\xi,\eta)$ for the $\frac{1}{2}^+$ **octet** baryon ground states $N(939)$, $\Lambda(1116)$, $\Sigma(1193)$, $\Xi(1318)$:



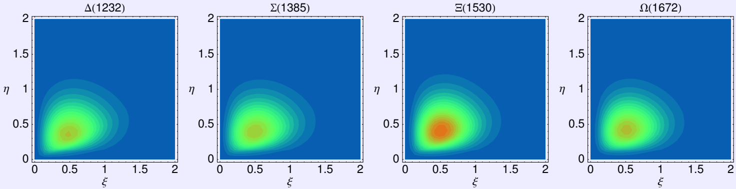
$\rho(\xi,\eta)$ for the $\frac{1}{2}^+$ **octet** baryon states $N(1440)$, $\Lambda(1600)$, $\Sigma(1660)$, $\Xi(1690)$:



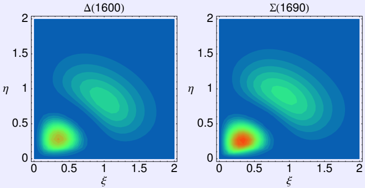
T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77** (2008) 114002

Spatial Probability Density Distributions

$\rho(\xi, \eta)$ for the $\frac{3}{2}^+$ **decuplet** baryon states **$\Delta(1232)$, $\Sigma(1385)$, $\Xi(1530)$, $\Omega(1672)$**:



$\rho(\xi, \eta)$ for the $\frac{3}{2}^+$ **decuplet** baryon states $\Delta(1600)$, $\Sigma(1690)$:

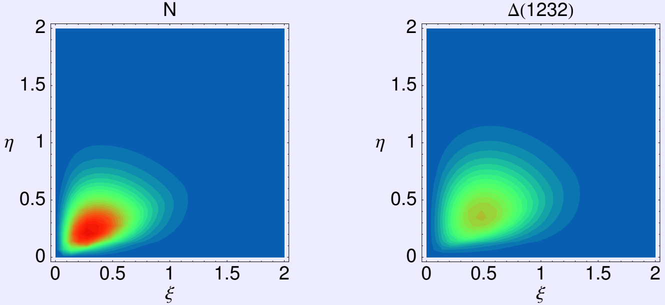


T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77** (2008) 114002

N and Δ Rest-Frame Wave Functions

Rest-frame **spatial distribution of constituent quarks** in terms of 3-body Jacobi coordinates $\vec{\xi}$ and $\vec{\eta}$:

$$\rho(\xi,\eta) = \xi^2\eta^2 \int d\Omega_\xi d\Omega_\eta \; \Psi^{\star}_{PJ\Sigma TM_T}(\xi,\Omega_\xi,\eta,\Omega_\eta)\Psi_{PJ\Sigma TM_T}(\xi,\Omega_\xi,\eta,\Omega_\eta)$$



Units on abscissa and ordinates are [fm]

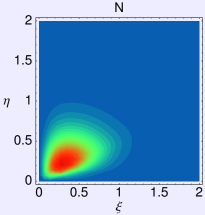
Root-Mean-Square Radii

The **root-mean-square radius** (in the rest frame):

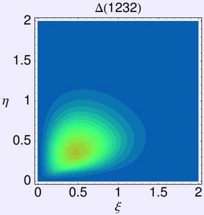
$$r_{\mathrm{rms}} = \sqrt{\langle r_i^2 \rangle} = \left(\int d^3 r_i \left\langle P=0, J, \Sigma \left| \hat{r}_i^{\,2} \right| P=0, J, \Sigma \right\rangle \right)^{\frac{1}{2}}$$

Is NOT an **observable**! Is NOT **relativistically invariant**!

$\rightarrow$ Idea about the spatial distribution of constituent quarks.



$r^N_{\mathrm{rms}} = 0.304$ fm



$r^\Delta_{\mathrm{rms}} = 0.390$ fm

Exp.: $r_E^p \sim 0.88$ fm

$(r_E^n)^2 \sim -0.12$ fm^2

$r_E^{\Delta^{++}} = r_E^{\Delta^+} = r_E^{\Delta^-} = 0.656$ fm

$r_E^{\Delta^0} = 0$ fm

New Quark-Model Classification

multiplet	$(LS)J^P$				
octet	$(0\frac{1}{2})\frac{1}{2}^+$	$N(939)^{100}$	$\Lambda(1116)^{100}$	$\Sigma(1193)^{100}$	$\Xi(1318)^{100}$
octet	$(0\frac{1}{2})\frac{1}{2}^+$	$N(1440)^{100}$	$\Lambda(1600)^{96}$	$\Sigma(1660)^{100}$	$\Xi(1690)^{100}$
octet	$(0\frac{1}{2})\frac{1}{2}^+$	$N(1710)^{100}$		$\Sigma(1880)^{99}$	
octet	$(1\frac{1}{2})\frac{1}{2}^-$	$N(1535)^{100}$	$\Lambda(1670)^{72}$	$\Sigma(1560)^{94}$	
octet	$(1\frac{3}{2})\frac{1}{2}^-$	$N(1650)^{100}$	$\Lambda(1800)^{100}$	$\Sigma(1620)^{100}$	
octet	$(1\frac{1}{2})\frac{3}{2}^-$	$N(1520)^{100}$	$\Lambda(1690)^{72}$	$\Sigma(1670)^{94}$	$\Xi(1820)^{97}$
octet	$(1\frac{3}{2})\frac{3}{2}^-$	$N(1700)^{100}$		$\Sigma(1940)^{100}$	
octet	$(1\frac{3}{2})\frac{5}{2}^-$	$N(1675)^{100}$	$\Lambda(1830)^{100}$	$\Sigma(1775)^{100}$	$\Xi(1950)^{100}$
decuplet	$(0\frac{3}{2})\frac{3}{2}^+$	$\Delta(1232)^{100}$	$\Sigma(1385)^{100}$	$\Xi(1530)^{100}$	$\Omega(1672)^{100}$
decuplet	$(0\frac{3}{2})\frac{3}{2}^+$	$\Delta(1600)^{100}$	$\Sigma(1690)^{99}$		
decuplet	$(1\frac{1}{2})\frac{1}{2}^-$	$\Delta(1620)^{100}$	$\Sigma(1750)^{94}$		
decuplet	$(1\frac{1}{2})\frac{3}{2}^-$	$\Delta(1700)^{100}$			
singlet	$(1\frac{1}{2})\frac{1}{2}^-$	$\Lambda(1405)^{71}$			
singlet	$(1\frac{1}{2})\frac{3}{2}^-$	$\Lambda(1520)^{71}$			
singlet	$(0\frac{1}{2})\frac{1}{2}^+$	$\Lambda(1810)^{92}$			

T. Melde, W. Plessas, and B. Sengl: Phys. Rev. D **77**, 114002 (2008)

See also the PDG: Phys. Rev. D **86**, 010001 (2012)

$SU(3)$ Flavor Multiplets – New

Classification of baryon resonances by the PDG since 2010

(results from the GBE relativistic CQM marked by asterisks)

J^P	$(D, L_N^P)\,S$		Octet members			Singlets
$1/2^+$	$(56,0_0^+)$ $1/2$	$N(939)$	$\Lambda(1116)$	$\Sigma(1193)$	$\Xi(1318)$	
$1/2^+$	$(56,0_2^+)$ $1/2$	$N(1440)$	$\Lambda(1600)$	$\Sigma(1660)$	$\Xi(1690)^\dagger$	
$1/2^-$	$(70,1_1^-)$ $1/2$	$N(1535)$	$\Lambda(1670)$	$\Sigma(1620)$ $\Sigma(1560)^\dagger$	$\Xi(?)$	$\Lambda(1405)$
$3/2^-$	$(70,1_1^-)$ $1/2$	$N(1520)$	$\Lambda(1690)$	$\Sigma(1670)$	$\Xi(1820)$	$\Lambda(1520)$
$1/2^-$	$(70,1_1^-)$ $3/2$	$N(1650)$	$\Lambda(1800)$	$\Sigma(1750)$ $\Sigma(1620)^\dagger$	$\Xi(?)$	
$3/2^-$	$(70,1_1^-)$ $3/2$	$N(1700)$	$\Lambda(?)$	$\Sigma(1940)^\dagger$	$\Xi(?)$	
$5/2^-$	$(70,1_1^-)$ $3/2$	$N(1675)$	$\Lambda(1830)$	$\Sigma(1775)$	$\Xi(1950)^\dagger$	
$1/2^+$	$(70,0_2^+)$ $1/2$	$N(1710)$	$\Lambda(1810)$	$\Sigma(1880)$	$\Xi(?)$	$\Lambda(1810)^\dagger$
$3/2^+$	$(56,2_2^+)$ $1/2$	$N(1720)$	$\Lambda(1890)$	$\Sigma(?)$	$\Xi(?)$	
$5/2^+$	$(56,2_2^+)$ $1/2$	$N(1680)$	$\Lambda(1820)$	$\Sigma(1915)$	$\Xi(2030)$	
$7/2^-$	$(70,3_3^-)$ $1/2$	$N(2190)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	$\Lambda(2100)$
$9/2^-$	$(70,3_3^-)$ $3/2$	$N(2250)$	$\Lambda(?)$	$\Sigma(?)$	$\Xi(?)$	
$9/2^+$	$(56,4_4^+)$ $1/2$	$N(2220)$	$\Lambda(2350)$	$\Sigma(?)$	$\Xi(?)$	

PDG: J. Phys. G **37**, 075021 (2010); Phys. Rev. D **86**, 010001 (2012);
Chin. Phys. C **38**, 090001 (2014)

Baryon Reactions

Applications of the RCQM

RCQM studies of various baryon reactions:

- Nucleon electromagnetic form factors
  (including analysis of flavor content of the nucleons)
- Nucleon axial form factors
- Δ and hyperon electroweak structures (FFs)
- Nucleon gravitational form factors
- $NN\pi$ and $N\Delta\pi$ strong vertex form factors
- Strong resonance decays

Various Baryon Reactions

Matrix elements of a transition operator $\hat{O}$ between baryon eigenstates $|P, J, \Sigma, T, T_3, Y\rangle$

$$\langle P', J', \Sigma', T', T_3', Y'|\hat{O}|P, J, \Sigma, T, T_3, Y\rangle$$

$\hat{O}$ ... $\hat{J}^{\mu}_{\mathrm{em}}$ $\rightarrow$ electromagnetic FF's

... $\hat{A}^{\mu}_{\mathrm{axial}}$ $\rightarrow$ axial FF's

... $\hat{S}$ $\rightarrow$ scalar FF

... $\hat{\Theta}^{\mu\nu}$ $\rightarrow$ gravitational/tensor FF's

... $\hat{D}^{\mu}_{\lambda}$ $\rightarrow$ hadronic decays

To be calculated from microscopic three-quark ME's

$$\langle p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3'; f_{i_1'}, f_{i_2'}, f_{i_3'}|\hat{O}|p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3; f_{i_1}, f_{i_2}, f_{i_3}\rangle$$

$\Uparrow$ boosted 3-body states $\qquad$ $\Uparrow$ boosted 3-body states

Electroweak Structure

of

the Nucleons / Baryons

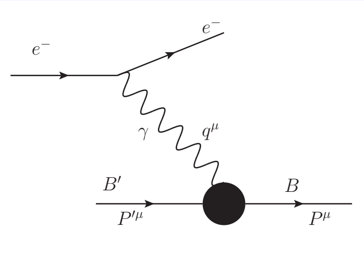
Electromagnetic Nucleon Observables

Covariant predictions for:

- **Electromagnetic** nucleon form factors $G_E^p(Q^2)$, $G_M^p(Q^2)$; $G_E^n(Q^2)$, $G_M^n(Q^2)$
- **Electric radii** and **magnetic moments** r_E^p, μ^p; r_E^n, μ^n

$\rightarrow$ Comparison to experiment

Elastic electron scattering:



Invariant form factors:

$$F^{\nu}_{\Sigma'\Sigma}(Q^2) = \langle P', J, \Sigma', T, M_T | \hat{J}^{\nu}_{\rm em} | P, J, \Sigma, T, M_T \rangle$$

$$\text{with } Q^2 = -q^2 \, ; \qquad q^\mu = P^\mu - P'^\mu$$

Spin-$\frac{1}{2}$ baryons:

$$G_E^B(Q^2) = \frac{1}{2M} F^{\nu=0}_{\frac{1}{2}\frac{1}{2}}(Q^2)$$

$$G_M^B(Q^2) = \frac{1}{Q} F^{\nu=1}_{\frac{1}{2}-\frac{1}{2}}(Q^2)$$

Spin-$\frac{3}{2}$ baryons:

$$G_E^B(Q^2) = \frac{1}{4M}[F^{\nu=0}_{\frac{1}{2}\frac{1}{2}}(Q^2) + F^{\nu=0}_{\frac{3}{2}\frac{3}{2}}(Q^2)]$$

$$G_M^B(Q^2) = \frac{3}{5Q}[F^{\nu=1}_{\frac{1}{2}-\frac{1}{2}}(Q^2) + \sqrt{3}F^{\nu=1}_{\frac{3}{2}\frac{1}{2}}(Q^2)]$$

Electric/charge radius r_E:

$$r_E^2 = -6\frac{d}{d\,Q^2}G_E(Q^2)\Big|_{Q^2=0}$$

Transition Matrix Elements in Point Form

Incoming baryon state: $|V, M, J, \Sigma\rangle \qquad \hat{=} |P, J, \Sigma\rangle$
Outgoing baryon state: $|V', M', J', \Sigma'\rangle \qquad \hat{=} |P', J', \Sigma'\rangle$
Transition operator: $\hat{O} = \hat{J}^{\mu}_{\rm em}$

$$\langle V', M', J', \Sigma' | \hat{J}^{\mu}_{\rm em} | V, M, J, \Sigma\rangle =$$

$$= \frac{2}{MM'} \sum_{\sigma_i \sigma_i'} \sum_{\mu_i \mu_i'} \int d^3\vec{k}_2 d^3\vec{k}_3 d^3\vec{k}_2' d^3\vec{k}_3'$$

$$\times \sqrt{\frac{\left(\sum_i \omega_i'\right)^3}{\prod_i 2\omega_i'}} \prod_{\sigma_i'} D^{\star \frac{1}{2}}_{\sigma_i' \mu_i'} \left\{R_W \left[k_i'; B\left(V'\right)\right]\right\} \Psi^{\star}_{M'J'\Sigma'} \left(\vec{k}_1', \vec{k}_2', \vec{k}_3'; \mu_1', \mu_2', \mu_3'\right)$$

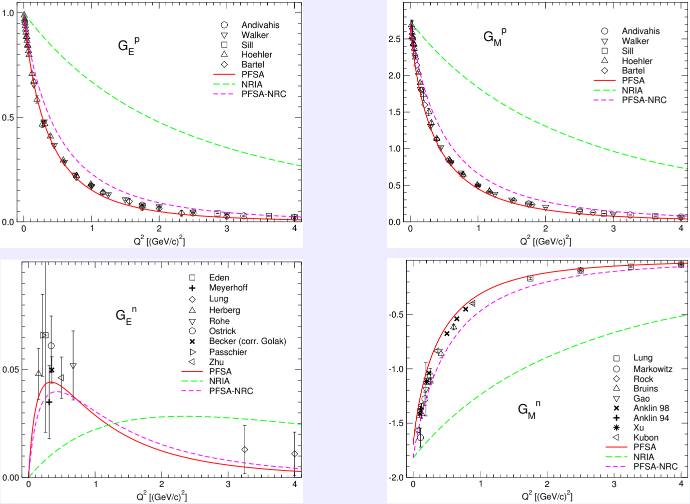
$$\times \langle p_1', p_2', p_3'; \sigma_1', \sigma_2', \sigma_3' | \hat{J}^{\mu}_{rd} | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3\rangle$$

$$\times \sqrt{\frac{\left(\sum_i \omega_i\right)^3}{\prod_i 2\omega_i}} \prod_{\sigma_i} D^{\frac{1}{2}}_{\sigma_i \mu_i} \left\{R_W \left[k_i; B\left(V\right)\right]\right\} \Psi_{MJ\Sigma} \left(\vec{k}_1, \vec{k}_2, \vec{k}_3; \mu_1, \mu_2, \mu_3\right)$$

$$\times 2MV_0 \delta^3 \left(M\vec{V} - M'\vec{V}' - \vec{q}\right)$$

where $p_i = B_c(V) k_i$, $p_i' = B_c(V') k_i'$, and $\omega_i = \sqrt{\vec{k}_i^2 + m_i^2}$

Covariant predictions of the GBE RCQM:



R.F. Wagenbrunn, S. Boffi, W. Klink, W. Plessas, and M. Radici: Phys. Lett. **B511** (2001) 33

Nucleon Electric Radii and Magnetic Moments

Electric radii r_E^2 [fm^2]

Baryon	GBE RCQM	Experiment
p	0.82	0.7692 ± 0.0123 [1)]
		0.70870 ± 0.00113 [2)]
n	-0.13	-0.1161 ± 0.0022

1) CODATA value (PDG)

2) Pohl et al.: Nature **466** (2010) 213

Magnetic moments μ [n.m.]

Baryon	GBE RCQM	Experiment
p	2.70	2.792847356
n	-1.70	-1.9130427

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Nucleon r_E^2 and μ – Nonrelativistic !!!

Electric radii r_E^2 [fm^2]

Baryon	GBE RCQM	GBE NRIA	Experiment
p	0.82	0.10	0.7692 ± 0.0123 [1)]
			0.70870 ± 0.00113 [2)]
n	-0.13	-0.01	-0.1161 ± 0.0022

[1)] CODATA value (PDG) [2)] Pohl et al.: Nature **466** (2010) 213

Magnetic moments μ [n.m.]

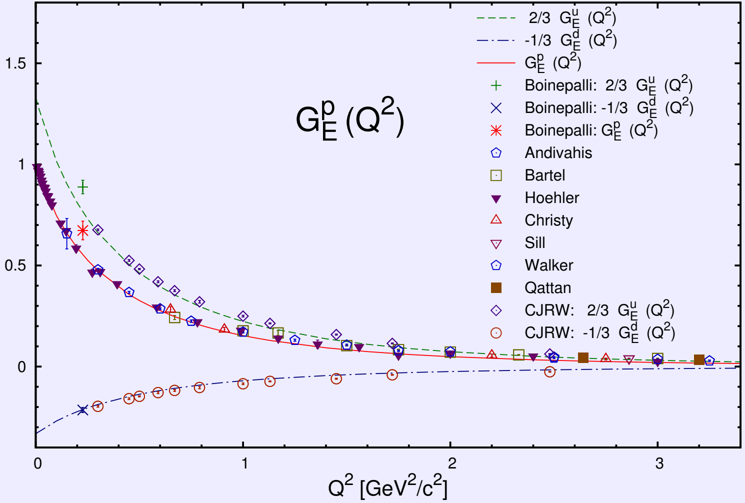
Baryon	GBE RCQM	GBE NRIA	Experiment
p	2.70	2.74	2.792847356
n	-1.70	-1.82	-1.9130427

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

Nucleons N

Proton Electric Form Factor

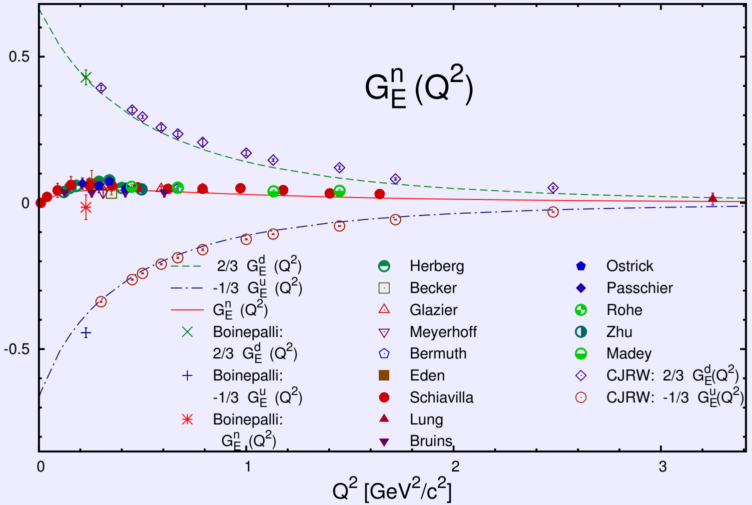
$$G_E^p = \frac{2}{3} G_E^u - \frac{1}{3} G_E^d$$



M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

Neutron Electric Form Factor

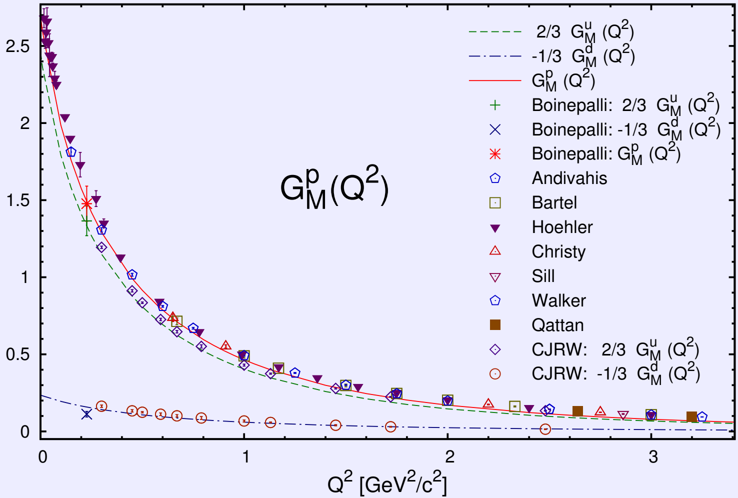
$$G_E^n = \frac{2}{3} G_E^d - \frac{1}{3} G_E^u$$



M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

Proton Magnetic Form Factor

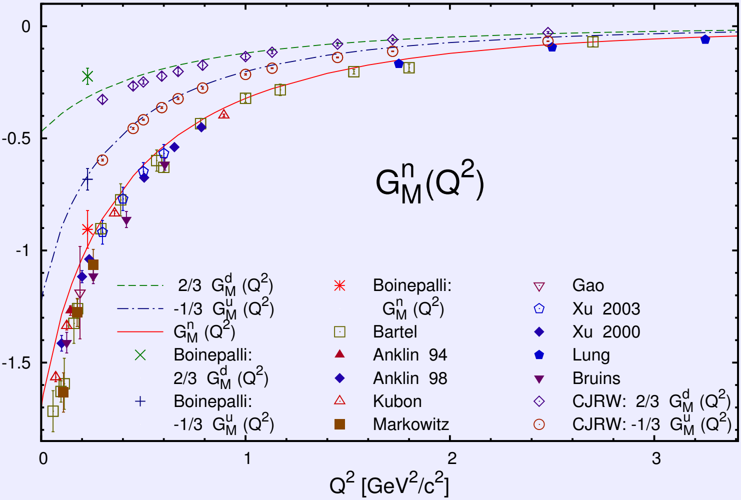
$$G_M^p = \frac{2}{3} G_M^u - \frac{1}{3} G_M^d$$



M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

Neutron Magnetic Form Factor

$$G_M^n = \frac{2}{3} G_M^d - \frac{1}{3} G_M^u$$



M. Rohrmoser, Ki-Seok Choi, and W. Plessas: arXiv:1110.3665

Δ and Hyperon E.m. Form Factors

Δ

Λ, Σ, Ξ

Σ^*, Ξ^*, Ω

Baryon Electric Radii and Magnetic Moments

Electric radii r_E^2 [fm^2]

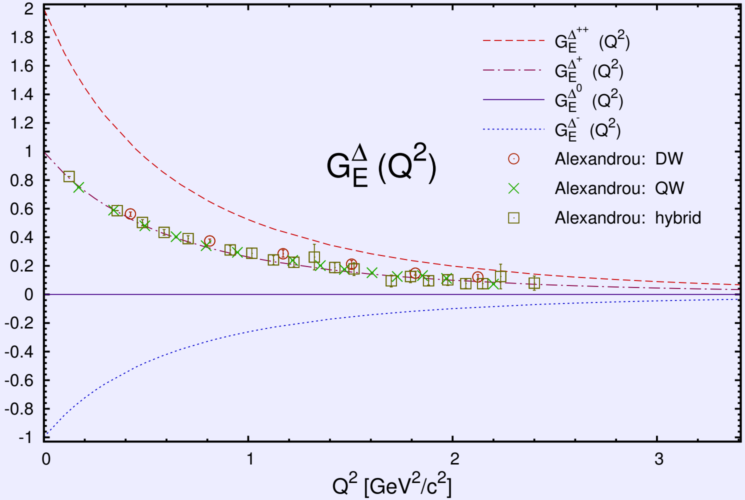
Baryon	GBE RCQM	Experiment
p	0.82	0.7692 ± 0.0123
n	-0.13	-0.1161 ± 0.0022
Σ^-	0.72	$0.61 \pm 0.12 \pm 0.09$

Magnetic moments μ [n.m.]

Baryon	GBE RCQM	Experiment
p	2.70	2.792847356
n	-1.70	-1.9130427
Λ	-0.64	-0.613 ± 0.004
Σ^+	2.38	2.458 ± 0.010
Σ^-	-0.93	-1.160 ± 0.025
Ξ^0	-1.25	-1.250 ± 0.014
Ξ^-	-0.70	-0.6507 ± 0.0025
Δ^+	2.08	$2.7^{+1.0}_{-1.3} \pm 1.5 \pm 3$
Δ^{++}	4.17	$3.7 - 7.5$
Ω^-	-1.59	-2.020 ± 0.05

K. Berger, R.F. Wagenbrunn, and W. Plessas: Phys. Rev. D **70**, 094027 (2004)

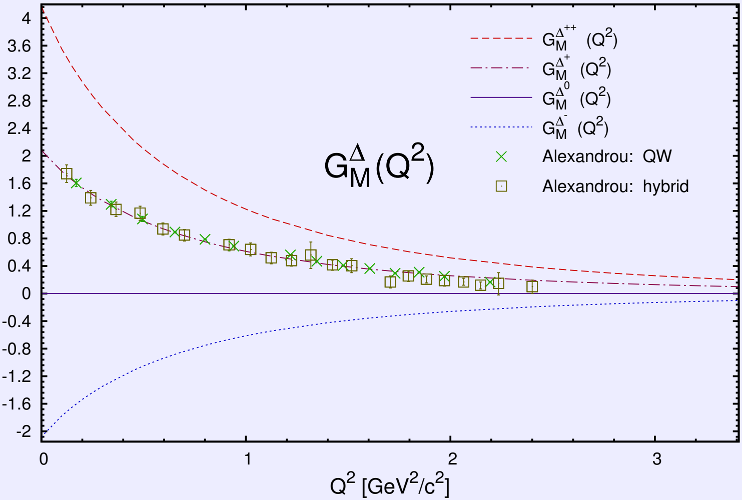
Electric Δ Form Factors



GBE RCQM: Ki-Seok Choi: PhD Thesis, Univ. Graz, 2011

Lattice QCD: C. Alexandrou et al. Phys. Rev. D **79** (2009) 014507

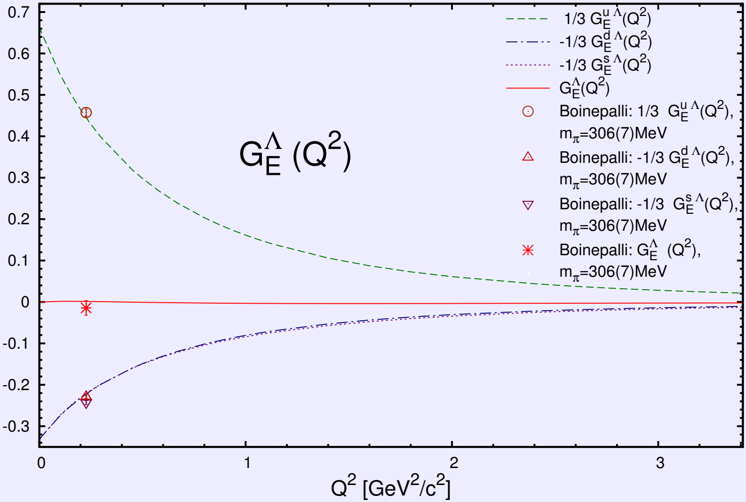
Magnetic Δ Form Factors



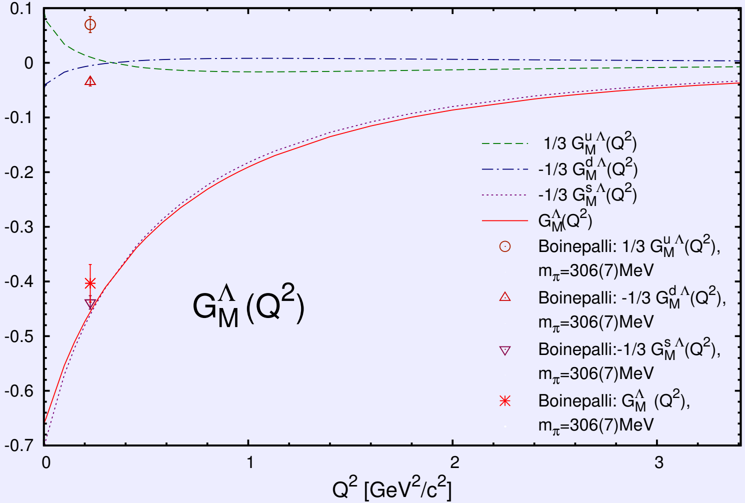
GBE RCQM: Ki-Seok Choi: PhD Thesis, Univ. Graz, 2011

Lattice QCD: C. Alexandrou et al. Phys. Rev. D **79** (2009) 014507

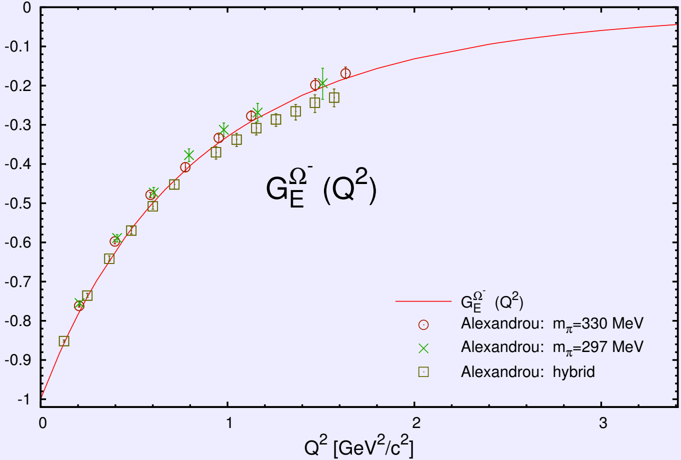
Octet $\Lambda(uds)$ Electric Form Factor



Octet $\Lambda(uds)$ Magnetic Form Factor

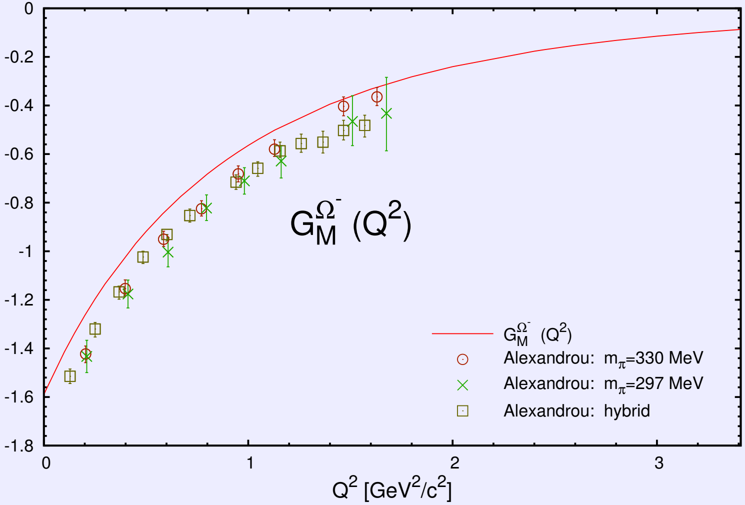


Decuplet $\Omega^-(sss)$ Electric Form Factor



Lattice-QCD: C. Alexandrou et al.: Phys. Rev. D**82** (2010) 034504

Decuplet $\Omega^-(sss)$ Magnetic Form Factor



Lattice-QCD: C. Alexandrou et al.: Phys. Rev. D**82** (2010) 034504

Axial **Charges** and Axial **Form Factors**

of

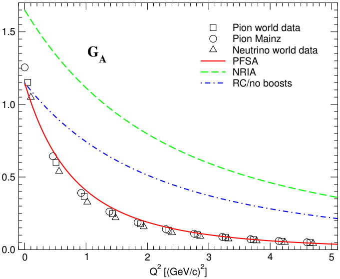
N Ground State and N^* Resonances

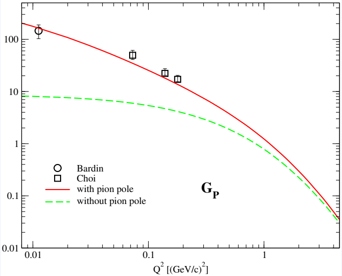
as well as

$\Delta, \Sigma, \Xi, \Sigma^*, \Xi^*$

Axial Nucleon Form Factors

Covariant predictions of the GBE RCQM:





$$g_A^{GBE} = 1.15 \quad \text{vs.} \quad g_A^{exp} = 1.2695 \pm 0.0029$$

L.Ya. Glozman, M. Radici, R.F. Wagenbrunn, S. Boffi, W. Klink, and W. Plessas: Phys. Lett. B **516**, 183 (2001)

Axial Charges of N and N^* Resonances

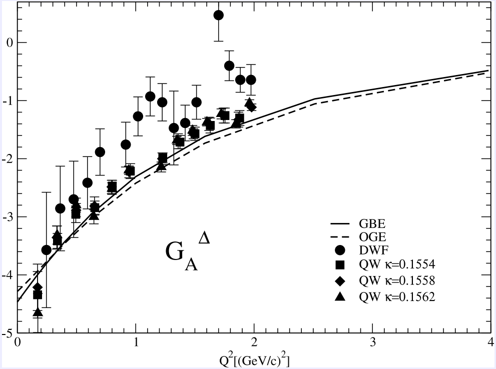
State	J^P	EGBE	Lattice QCD	GN	NR
N(939)	$\frac{1}{2}^+$	1.15	1.18∼1.31	1.66	1.65
N(1440)	$\frac{1}{2}^+$	1.16	?	1.66	1.61
N(1535)	$\frac{1}{2}^-$	0.02	∼0.00	-0.11	-0.20
N(1710)	$\frac{1}{2}^+$	0.35	?	0.33	0.42
N(1650)	$\frac{1}{2}^-$	0.51	∼0.55	0.55	0.64

EGBE	**E**xtended **GBE** RCQM covariant result
Lattice	**Lattice QCD** calculations by LHPC Collaboration, Takahashi-Kunihiro (Kyoto) etc.
GN	**G**lozman-**N**efediev $SU(6) \times O(3)$ nonrelativistic QM
NR	**N**on-**R**elativistic EGBE result

K.-S. Choi, W. Plessas, and R.F. Wagenbrunn: Phys. Rev. C **81**, 028201 (2010)

Axial Form Factor of the Δ

Covariant predictions of the GBE and OGE RCQMs:



K.-S. Choi and W. Plessas: Few-Body Syst. **54**, 1055 (2013)

(Lattice QCD data from C. Alexandrou et al., PoS LATTICE2010, 141 (2010))

Axial Charges of $\Delta, \Sigma, \Xi, \Sigma^*, \Xi^*$

	J^P	Exp	EGBE	LO	EOT	JT	NR
N	$\frac{1}{2}^+$	1.2695	1.15	1.18	1.314	1.18	1.65
Σ	$\frac{1}{2}^+$	-	0.65	0.636	0.686	0.73	0.93
Ξ	$\frac{1}{2}^+$	-	-0.21	-0.277	-0.299	-0.23	-0.32
Δ	$\frac{3}{2}^+$	-	-4.48	-	-	$\sim$-4.5	-6.00
Σ^*	$\frac{3}{2}^+$	-	-1.06	-	-	-	-1.41
Ξ^*	$\frac{3}{2}^+$	-	-0.75	-	-	-	-1.00

EGBE — **E**xtended **GBE** RCQM covariant result
LO — **L**in and **O**rginos lattice-QCD calculation
EOT — **E**rkol, **O**ka, and **T**akahashi lattice-QCD calculation
JT — **J**iang and **T**iburzi χPT calculation
NR — **N**on-**R**elativistic EGBE result

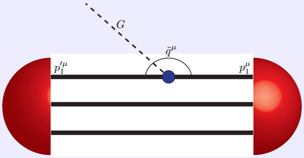
K.-S. Choi, W. Plessas, and R.F. Wagenbrunn: Phys. Rev. D **82**, 014007 (2010)

Gravitational Form Factors

of

the Nucleon

Gravitational Form Factors

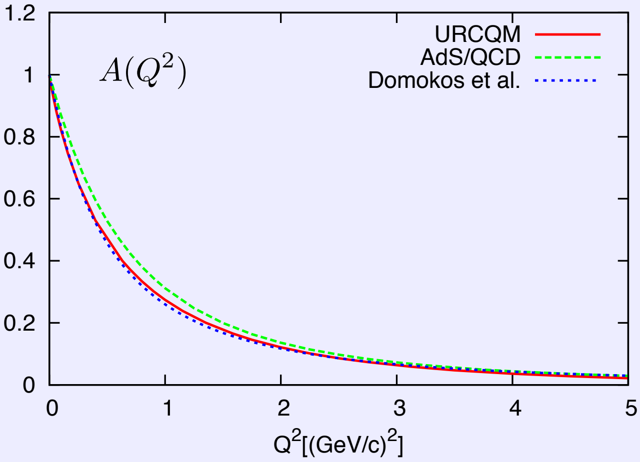


Invariant ME of **energy-momentum tensor** $\hat{\Theta}^{\mu\nu}$:

$$\langle P'J\Sigma'|\hat{\Theta}^{\mu\nu}|PJ\Sigma\rangle = \bar{U}(P')\left[\gamma^{(\mu}\bar{P}^{\nu)}A(Q^2) + \frac{i}{2M}\bar{P}^{(\mu}\sigma^{\nu)}B(Q^2) + \frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M}C(Q^2)\right]U(P)$$

$$A(Q^2) \sim \langle P'J\Sigma'|\Theta^{00}|PJ\Sigma\rangle$$

Nucleon Gravitational Form Factor $A(Q^2)$



J.P. Day: PhD Thesis, Univ. Graz, August 2013

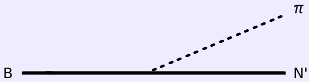
Microscopic Description

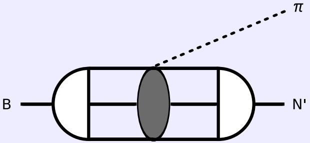
of

Meson-Baryon Interaction Vertices

Meson-Baryon Interaction Vertices

Interaction vertices





$$F_{i\to f} = (2\pi)^4 \langle f|\, \mathcal{L}_I\,(0)\,|i\rangle \qquad \equiv \qquad \langle V', M', J', \Sigma'|\, \hat{D}^{\pi}_{\rm rd}\,| V, M, J, \Sigma\rangle$$

where

$$\left\langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 \left| \hat{D}^{\pi}_{\rm rd} \right| p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \right\rangle =$$

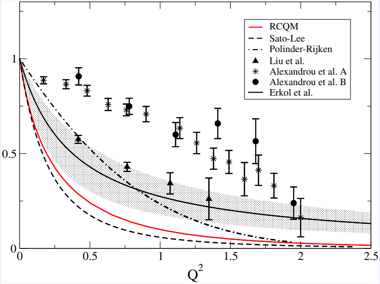
$$3\mathcal{N}_{\rm S}\,\frac{i g_{qqm}}{2m_1\,(2\pi)^{\frac{3}{2}}}\,\bar{u}(p'_1,\sigma'_1)\gamma_5\gamma_\mu\lambda_m u(p_1,\sigma_1)\tilde{q}^{\mu}\, 2p_{20}\,\delta(\vec{p}_2-\vec{p}'_2)\,2p_{30}\,\delta(\vec{p}_3-\vec{p}'_3)\,\delta_{\sigma_2\sigma'_2}\delta_{\sigma_3\sigma'_3}$$

and

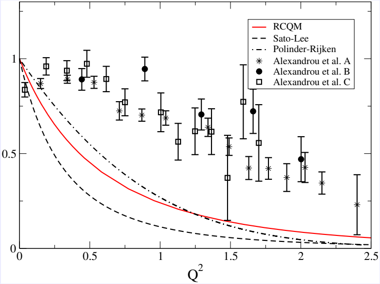
$$G_{\pi NN}\left(Q^2\right) = \frac{1}{f_{\pi NN}}\,\frac{m_\pi\sqrt{2\pi}}{\sqrt{2M_N}}\,\frac{\sqrt{E'_N+M'_N}}{E'_N+M'_N+\omega}\,\frac{F_{i\to f}}{Q_z}$$

$$G_{\pi N\Delta}\left(Q^2\right) = -\frac{1}{f_{\pi N\Delta}}\,\frac{3\sqrt{2\pi}}{2}\,\frac{m_\pi}{\sqrt{E'_N+M'_N}\sqrt{2M_\Delta}}\,\frac{F_{i\to f}}{Q_z}$$

πNN and $\pi N\Delta$ Interaction Vertices



$G_{\pi NN}$



$G_{\pi N\Delta}$

T. Melde, L. Canton, and W. Plessas: Phys. Rev. Lett. **102**, 132002 (2009)

Form-Factor Parametrizations

$$G\left(\vec{q}^{\,2}\right) = \frac{1}{1+\left(\frac{\vec{q}}{\Lambda_1}\right)^2+\left(\frac{\vec{q}}{\Lambda_2}\right)^4} \qquad G\left(Q^2\right) = \frac{1}{1+\left(\frac{Q}{\Lambda}\right)^2}$$

		RCQM	SL	PR		LIU	ERK	ALX
	$\frac{f_N^2}{4\pi}$	0.0691	0.08	0.075		0.0649	0.0481	0.0412
N	Λ_1	0.451	0.453	0.940	Λ	0.747	0.614	1.65
	Λ_2	0.931	0.641	1.102		-	-	-
	$\frac{f_\Delta^2}{4\pi}$	0.188	0.334	0.478				
Δ	Λ_1	0.594	0.458	0.853				
	Λ_2	0.998	0.648	1.014				

T. Melde, L. Canton, and W. Plessas: Phys. Rev. Lett. **102**, 132002 (2009)

Way to a Coupled-Channels Quark Model

Introducing

explicit mesonic degrees of freedom

(ongoing work with Regina Kleinhappel)

Coupled-Channels Approach

- **Coupled-channel mass operator eigenvalue equation:**
  - E.g., excited hadron can decay into GS and additional particle

$$\begin{pmatrix} M_i & K^\dagger \\ K & M_{i+1} \end{pmatrix} \begin{pmatrix} |\psi_i\rangle \\ |\psi_{i+1}\rangle \end{pmatrix} = m \begin{pmatrix} |\psi_i\rangle \\ |\psi_{i+1}\rangle \end{pmatrix}$$

- i=1: Allows, e.g., dressing of bare hadrons by meson loops
- i=2: Allows, e.g., dressing of pure $q\bar{q}$ states by meson loops
- i=3: Allows, e.g., dressing of pure qqq states by meson loops

Summary and Conclusions

- **Baryon spectroscopy of all flavors** consistently described in a universal RCQM based on GBE dynamics
- The **covariant structures** of the ground states (N, Δ, Λ, ...) in good agreement with experiment (wherever such data are available)
- Predictions by the GBE RCQM reasonable consistent with (reliable) **lattice-QCD** results.
- **Disturbing shortcomings** of the $\{QQQ\}$ quark model for hadronic decays (π, η, K modes, ...)

Achievements and Future Potentials of RCQMs

To be able to describe/understand

- in a consistent manner
- on the microscopic level
- in accordance with the properties of low-energy QCD such phenomena like

- **hadron spectra**:
  baryon ground states & excitations ✓ ☺
- **baryon structure**: r_E, μ, g_A ; G_E, G_M, G_A, G_P, ...
  i.e. electroweak form factors etc. ✓ ☺
- **resonance decays**:
  $N^* \to N\pi$, $\Delta \to N\pi$, $\Lambda^* \to KN$, ... ✓ ☹
- **resonance excitations**: $\gamma N \to N^*$, $e^- N \to N^*$, ...
- **meson-baryon interactions**: $\pi - N$, $K - N$, ...
- **hyperon-hyperon interactions**: $N - N$, $N - Y$, ...

etc. etc.

Constituent-Quark-Model Review

W. Plessas:
Int. J. Mod. Phys. A30 (2015) 02, 1530013

also in:

"50 Years of Quarks"

ed. by M. Gell-Mann and H. Fritzsch

(World Scientific, Singapore, 2015)

Collaborators

History

Low-energy QCD

RCQM

Spectroscopy

Multiplets

Structure

Nucleon E.m.

Baryon E.m.

Axial FFs

Gravitational FF

Strong πNN, $\pi N\Delta$ FFs

CC Model

Summary

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Thank you very much

for

your attention!