Archimedes Force on Casimir Apparatus

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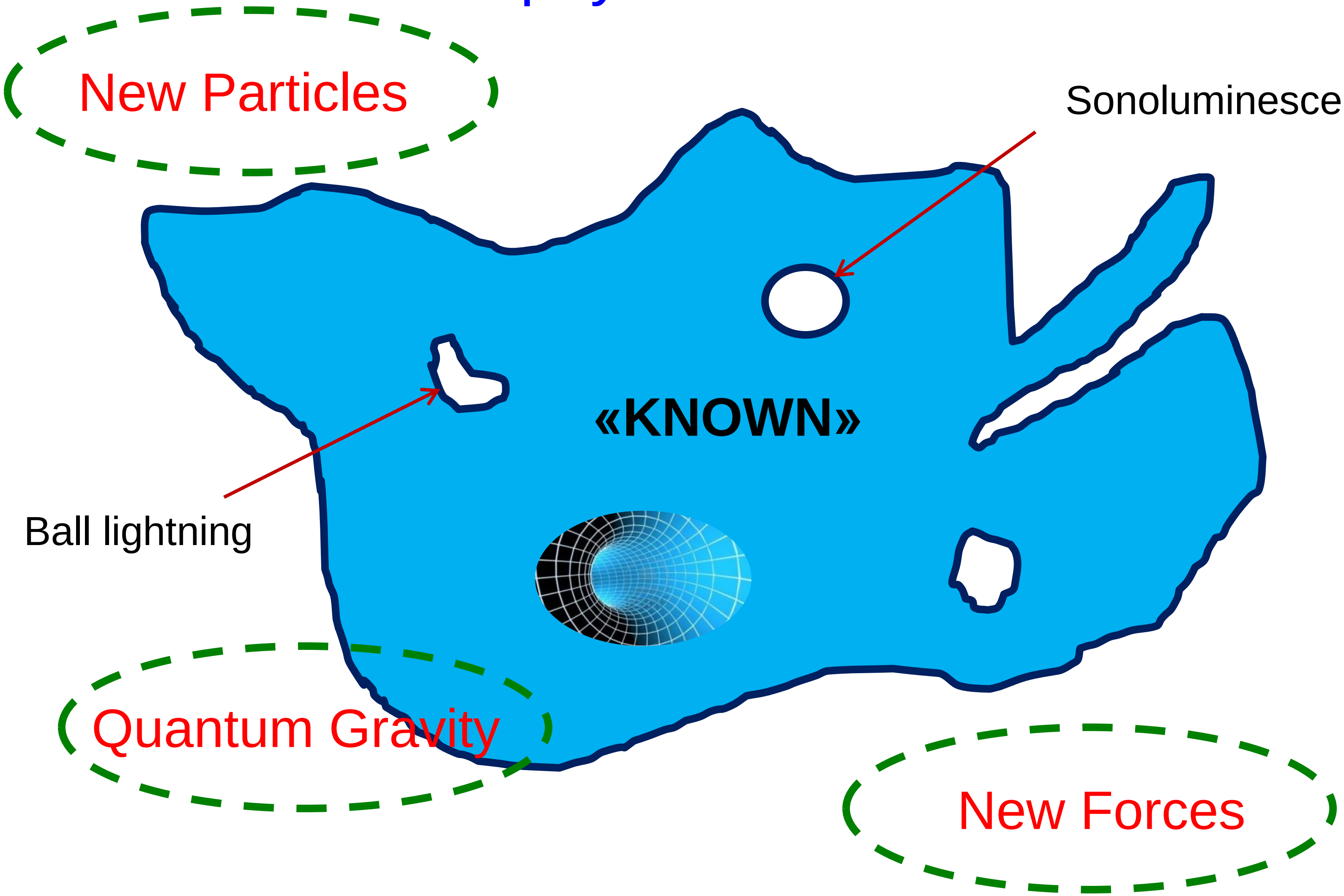
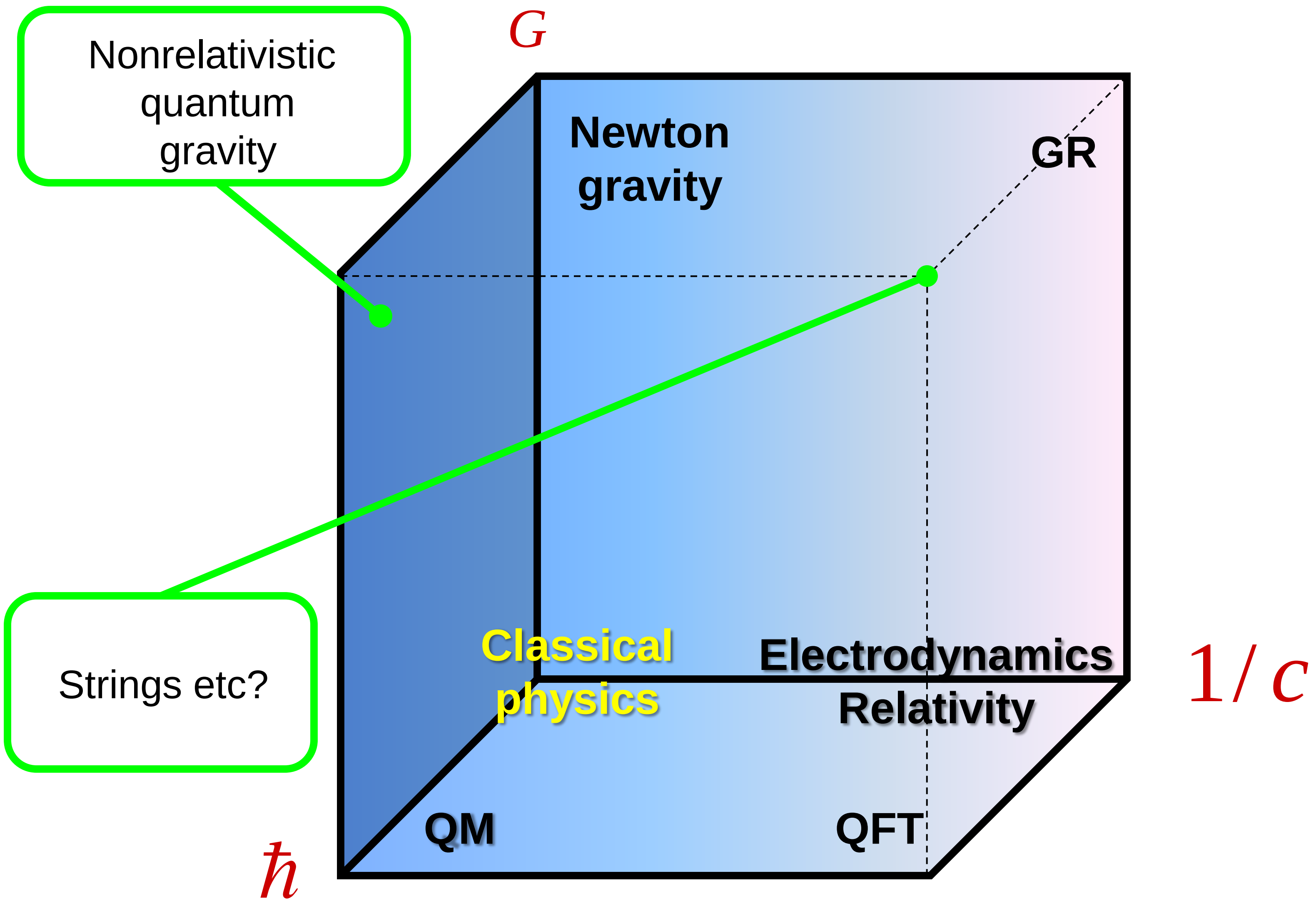
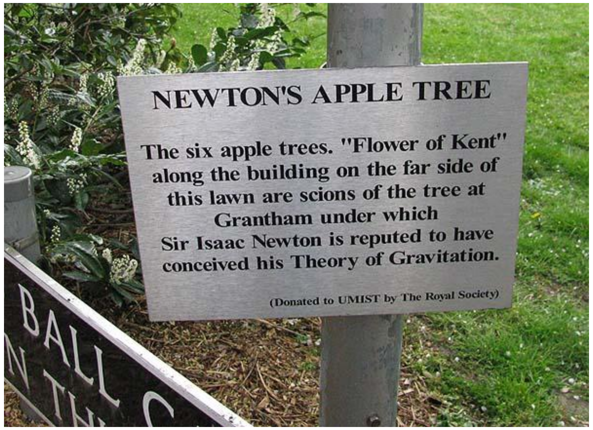
Swiss cheese – like topology of physics frontiers



«Cube of Theories»



Matter in the Earth's gravitational field



Quantum matter in classical (weak) gravitational field

- Light deflection by the Sun's gravity `1919
- Pound-Rebka experiment `1959
- Shapiro delay `1964
- Quantum states of neutrons `2002
- ALPHA, AEGIS, GBAR @ CERN

photons

neutrons

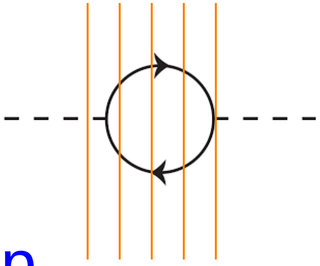
antihydrogen

Even less trivial quantum field theoretical effects..

- Black hole evaporation by Hawking radiation (1975)
- Drummond-Hathrell (1979)

Photon propagates in gravitational field faster than in the vacuum

$$\frac{v}{c} = 1 + \frac{11}{45}\frac{\alpha G}{m_e^2}(\rho + p)$$



One-loop phenomenon, analogous to two-loop Scharnhorst effect in Casimir vacuum .

.. and no knowledge of how genuine quantum objects (for example, entangled ones) interact gravitationally with themselves.

Semiclassical gravity

(but [*D.Page, C.Geiliker (1981)*])

The basic fact governing non-relativistic motion of a test classical body in weak gravitational field is well known from school textbooks:

$$\mathbf{f} = m\mathbf{g} = \rho V \mathbf{g}$$

Simplicity of this formula should not camouflage a highly nontrivial physical fact, that the force depends on the only parameter of the body - its mass (and not, for example, on its chemical composition, entropy etc).

The situation gets more complex if the test body is immersed into gas or fluid. Then

$$\mathbf{f} = (\rho - \rho_f) V \mathbf{g}$$

and the second term is known as Archimedes force.

What is behind?

Inertial mass is assumed to be independent on the medium properties, therefore bodies of equal gravitational masses but different volumes accelerate differently (in dense medium).

On the other hand, for relativistic vacuum medium such as gluon condensate

$$\langle T^{\mu\nu} \rangle = -\epsilon \cdot \eta^{\mu\nu} \qquad \langle T^{\mu}_{\mu} \rangle = -4\epsilon = \frac{\beta(g)}{2g} \langle \mathrm{Tr} G_{\mu\nu} G^{\mu\nu} \rangle$$

so mass of the proton (both inertial and gravitational) is

$$m_p = \frac{1}{c^2} \int d^3\mathbf{x} \, \left[\langle T^{00} \rangle_{3q}(x) - \epsilon \right]$$

The reason for these two contributions (~ 98% of the mass of ordinary matter) to be indistinguishable in QCD is that there is one and the only nonperturbative dimensionful scale in this theory.

- Universal dependence on the body’s volume

small ratio of gas/fluid molecules size to the body size (holes in the body's surface etc), which makes continuous medium approximation applicable.

- No quantum and relativistic corrections

small ratio of quantum correlation length/time of the medium to the body size

- Weak field approximation

no genuine GR physics

- Invariance with respect to constant shifts $\rho \to \rho + \mathrm{const}$

self-renormalizable, no cosmological constant problem

piece of vacuum with the «mass» $\frac{1}{c^2}\int dV\ \langle T^{00}\rangle_{vac}$ *does not fall*

A remark about measurement

Defining quantum field theory means to define action and integration measure.

$$Z = \int \mathcal{D}\phi \, e^{iS[\phi]}$$

Dynamics can be shifted from action to measure and back.

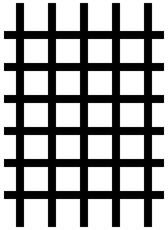
UV-regularization: $\int \mathcal{D}\phi(k) \to \int_{k<\Lambda} \mathcal{D}\phi(k)$

Measure can encode some a priori existing or assumed knowledge.

Example: Casimir boundary conditions: $\int \mathcal{D}A_\mu \to \int \mathcal{D}A_\mu \, \delta(n^\alpha \tilde{F}_{\alpha\beta}(z = a_i))$

What information about our theory at $k > \Lambda$ we need to be able to work at low energy? Just a few numbers – coefficients of marginal operators, like 1/137.

Measuring local condensates



For quantum field theory defined on a lattice with link **a** :

$$\phi \sim \frac{1}{a}$$

Actual dynamics IS fine tuned:

$$\Lambda_{QCD} \sim \frac{1}{a} e^{-b/g^2(a)} \underset{a \to 0}{\sim} \frac{\infty}{\infty}$$

$$\langle T^{\mu}_{\mu} \rangle \sim \frac{1}{a^4} + \ldots + 10^{-29} \mathrm{g/cm}^3$$

$$\langle G^2_{\mu\nu} \rangle \sim \frac{1}{a^4} + 0.012\ \mathrm{GeV}^4$$

$$M_H^2 \sim \frac{1}{a^2} + (125\ \mathrm{GeV})^2$$

Lattice here can be seen as a «detector», a measuring device which brings its own story to the theory.

We know from experiment in all these cases that 1/a **terms are irrelevant.**

How to disentangle correctly «physics of the detector» from «physics of the physics»? Not (yet) deep enough understanding of measurement procedure in QFT.

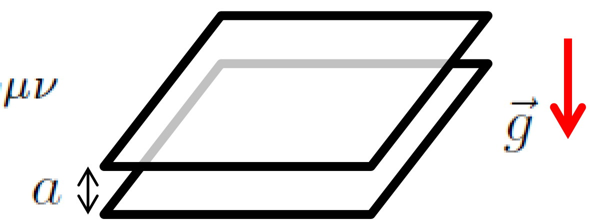
How does Casimir cavity fall?

There used to be some controversy in the literature…

M.Karim, A.Bokhari, B.Ahmedov, 2000; R.Caldwell, 2002; F.Sorge, 2005; E.Calloni, L. Di Fiore, G.Esposito, L.Milano, L.Rosa, 2001 – .. ; S.Fulling, K.Milton, P.Parashar, A.Romeo, K.Shajesh, J.Wagner, 2007; E.Shevrin, V.Sh., 2015.

For weak fields we parameterize $g_{\mu\nu} = \eta_{\mu\nu} + 2h_{\mu\nu}$

The gravitational energy
(for static case) is $\delta E_g = -\int d^3x\, h_{\mu\nu} T^{\mu\nu}$



Leaving aside material contribution from the plates, there is a part coming from Casimir energy-momentum tensor

$$\langle T^{\mu\nu} \rangle = \frac{E_C}{a} \mathrm{diag}(1, -1, -1, 3) \qquad E_C = -\frac{\pi^2 \hbar c}{720 a^3}$$

All components could contribute, contrary to nonrelativistic case where $|T^{00}| \gg |T^{ij}|$

The result would depend on the choice of the metric $h_{\mu\nu}$

For example, one obtains three different answers for

Fermi coordinate choice $h_{00} = -\frac{gz}{c^2}$; $h_{ij} = 0$

expanded Schwarzschild metric $h_{00} = h_{ii} = \frac{GM}{c^2 R} - \frac{gz}{c^2}$

and $h_{00} = h_{33} = -\frac{gz}{c^2}$; $h_{11} = h_{22} = 0$

But they all correspond to uniform static field at this order!

In particular, the force acting on the cavity got dependence on its orientation, which would clearly violate equivalence principle.

So, what is going on?

The key point (*S.Fulling et al, `2007*) is gauge non-invariance of the coupling: $h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$
since Casimir energy-momentum tensor alone (without «material» parts coming from the plates, robes, springs etc) is not conserved: $\partial_\mu T^{\mu\nu} \neq 0$

Either careful work with full covariantly conserved energy-momentum tensor *or* arguments in favor of one coordinate choice to be «more physical» than others.

Both paths have been followed, and then done correctly, the answer is in accord with equivalence principle – the cavity feels small upward push:

$$\mathbf{f} = \mathbf{g} \, \frac{E_C}{c^2} \, S$$

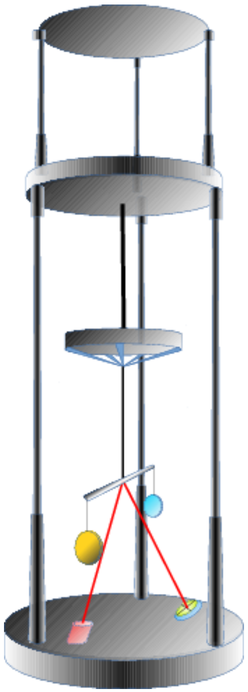
Is this answer universal?

Archimedes project [*E.Calloni et al (2014)*]: a feasibility study for weighting the vacuum energy.

Idea: to modulate energy change forcing normal-superconducting state transition by external conditions (temperature or magnetic field) and use advanced techniques from gravitational wave searches to extract the signal over various noises.

The scale: $\Delta U_{\text{cas}} \approx \eta(a) \frac{N\pi^2}{720} \frac{\hbar c}{a^3}$

where the reduction factor $\eta(a) = 4 \cdot 10^{-4} \times \sqrt{\frac{a}{1\ \text{nm}}}$ is estimated for high-T_c superconductors, and $N \approx 10^5$ corresponds to $F \approx 10^{-15}$ Newton



Active noise reduction and methods of data analysis are crucial

Internal energy weights, is it possible to check that free energy and entropy do not gravitate separately?

$$U = F + TS$$

For Casimir plates at finite temperature (in geometric approximation) [*L.Brown, G.Maclay (1969)*]:

$$\langle T^{\mu\nu}\rangle_T = (n^\mu n^\nu + \hat{z}^\mu \hat{z}^\nu)\left(k_B T/a^3\right) s(\xi) + (4\hat{z}^\mu \hat{z}^\nu - \eta^{\mu\nu})\left(\hbar c/a^4\right) f(\xi)$$

$$\langle T^{00}\rangle_T = \frac{\hbar c}{a^4}(f(\xi) + \xi s(\xi)) \qquad\qquad \xi = \frac{k_B T a}{\hbar c}$$

For typical high-T_c superconductors critical field is ~ **1 T**

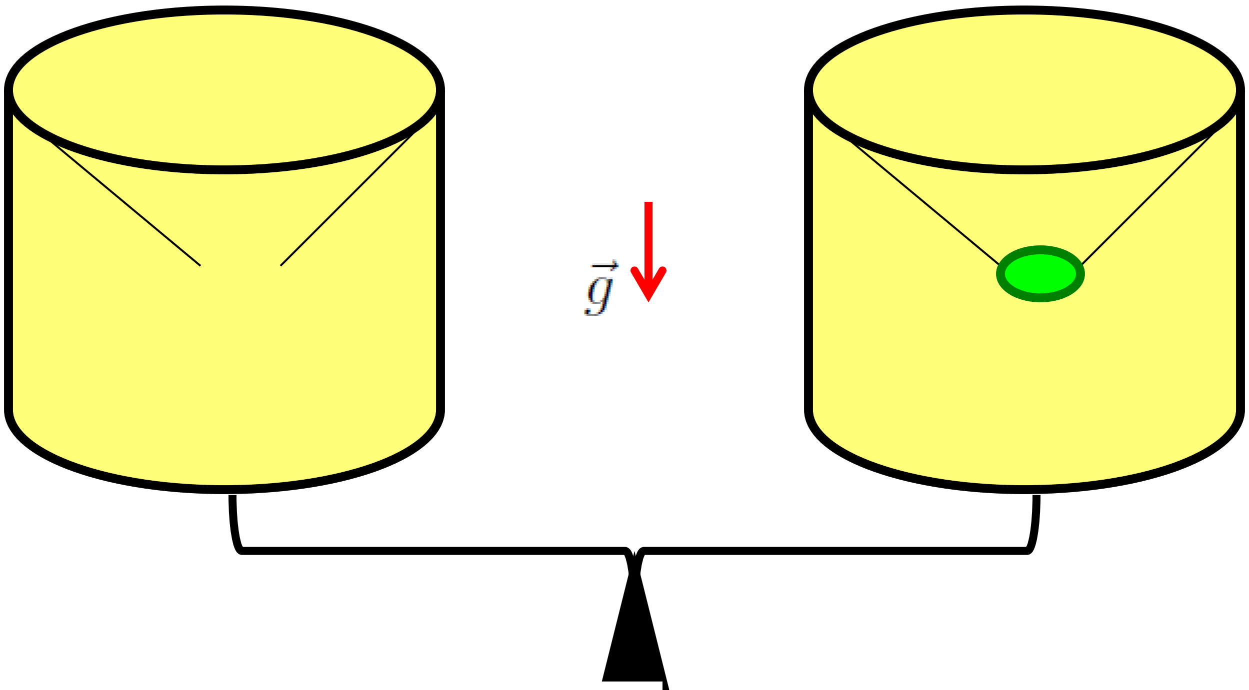
so free energy density variation $\Delta F(T) = \dfrac{B_c(T)^2}{2}$ ~ **10^5 J / m^3**

and composition strongly varies with the temperature.

Gravitation of low entropic states

Weight is integral of pressure at fixed z [*T.Padmanabhan*]

Compare weight of two identical boxes when one has small Casimir apparatus inside:



$$w_0(z) = \int_{S_1} d\mathbf{x}_\perp\, g_{33}(z) \langle T^{33} \rangle_0(z) \qquad w(z) = \int_{S_2} d\mathbf{x}_\perp\, g_{33}(z) \langle T^{33} \rangle(z)$$

Choice of the metric $ds^2 = g_{00}(z)c^2dt^2 + g_{33}(z)dz^2 + d\mathbf{x}_\perp^2$
with $g_{00}(0) = -1$

Then from covariant conservation of energy momentum tensor inside each box

$$\nabla_\mu T^{\mu\nu} = 0$$

it can be shown that net force defined as the difference

$$f = w(0) - w_0(0)$$

is equal to

$$f = \frac{1}{2}\int d^3\mathbf{x}\,\sqrt{-g_{00}(z)}\left(\frac{\partial g_{00}(z)}{\partial z}\right)\left[\langle T^{00}\rangle(z) - \langle T^{00}\rangle_0(z)\right]$$

with additional assumption that $\lim_{z\to\infty}\frac{\langle T^{33}\rangle(z)}{\langle T^{33}\rangle_0(z)} = 1$

No weak field approximation!

Classically for weak field $g_{00} = -1 + 2\mathbf{g}\mathbf{z}/c^2 + \mathcal{O}(1/c^4)$

we have inside the body

$$\int_{V_{body}} d^3\mathbf{x}\,\langle T^{00}\rangle(x) = mc^2 \qquad \int_{V_{body}} d^3\mathbf{x}\,\langle T^{00}\rangle_0(x) = \rho_f c^2 V_{body}$$

and $\langle T^{00}\rangle(x) = \langle T^{00}\rangle_0(x)$ outside the body, coming back to

$$\mathbf{f} = (\rho - \rho_f)V\,\mathbf{g}$$

In quantum case

a) energy-momentum of a body is not localized inside it

b) there are quantum fluctuations

At the next order in weak field expansion

$$f = f_0 - \int d^3\mathbf{x}\, h_{00}(z) \left(\frac{\partial h_{00}(z)}{\partial z}\right) \left[\langle T^{00}\rangle_\eta(z) - \langle T^{00}\rangle_{0,\eta}(z)\right] +$$

$$+\frac{1}{2}\int d^3\mathbf{x} \left(\frac{\partial h_{00}(z)}{\partial z}\right) \left[\langle t^{00}\rangle_g(x) - \langle t^{00}\rangle_{0,g}(x)\right]$$

where $t^{\mu\nu}(x) = T^{\mu\nu}(x) - \langle T^{\mu\nu}\rangle_\eta(x)$

$$\langle t^{\mu\nu}(x)\rangle_g = \int d^4x'\, h_{\alpha\beta}(x') \cdot i\theta(x_0 - x'_0)\langle[t^{\mu\nu}(x), t^{\alpha\beta}(x')]\rangle_\eta$$

Weight of fluctuations

Notice that $\int d^4x' \langle t^{\mu\nu}(x) t^{\alpha\beta}(x')\rangle$ is typically not small

in Casimir systems [*L.Ford (1981)*].

Casimir apparatus in thermal bath

It is convenient to start from the free energy

$$\beta F_\beta = -\log \int \mathcal{D}\Phi \, e^{-S[\Phi]}$$

where $S = \frac{1}{2}\int_0^\beta d\tau \int d^d\mathbf{x} \; \Phi \hat{F} \Phi$

and make use of heat kernel technique

$$\beta F_\beta = -\frac{1}{2}\int\limits_0^\infty \frac{ds}{s} \left(\mathrm{Tr}\hat{K}^\beta(s) - \mathrm{Tr}\hat{K}(s)\right)$$

where $$\left(\frac{d}{ds} + \hat{F}\right) \hat{K}^\beta(s|x,y) = \hat{1}\cdot\delta(s)\delta(x,y)$$

$$\mathrm{Tr}\hat{K}^\beta(s) = \theta_3\left(0, e^{-\frac{\beta^2}{4s}}\right) \mathrm{Tr}\hat{K}(s)$$

The main result [*S.Minakshisundaram, A.Pleijel (1949); F.Brownell (1957); B.DeWitt (1965); H.McKean, I.Singer (1967); P.Greiner (1971); T.Sakai (1971); P.Gilkey (1975); J.Dowker, G.Kennedy (1978); I.Avramidi (1993); Yu.Gusev, V.Zelnikov (2001)*]

$$\mathrm{Tr}\hat{K}(s) = \frac{1}{(4\pi s)^{\frac{d+1}{2}}} \sum_{n=0}^{\infty} \left(s^n A_n + s^{n/2} B_{n/2} \right)$$

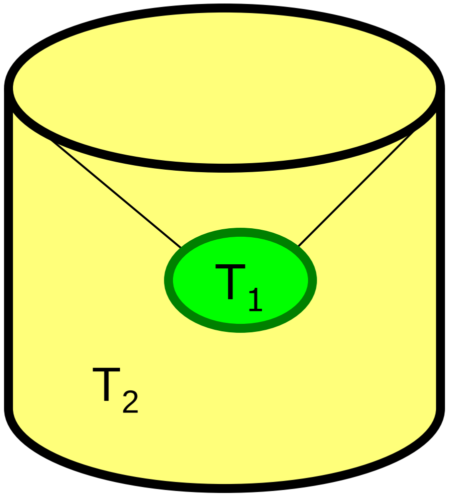
with the bulk and boundary contributions

$$A_n = \int_{\mathcal{M}} d^{d+1}x \, \sqrt{-g(x)} \, a_n(x) \qquad B_{n/2} = \int_{\partial\mathcal{M}} d^d x \, \sqrt{\gamma(x)} \, b_{n/2}(x)$$

gives high-temperature expansion of the form

$$F_\beta = -\frac{\pi^2}{90}\frac{1}{\beta^4} V^{(3)} + b\frac{\zeta(3)}{8\pi}\frac{1}{\beta^3} S^{(2)} + \mathcal{O}(\beta^{-2})$$

$b = 1$ Dirichlet $\qquad$ $b = -1$ Neumann



$\vec{g}$ ↓

$$\mathcal{B}[\phi(x)] = 0 \quad x \in \partial V$$

size of the apparatus is large $\quad r \gg \dfrac{\hbar c}{k_B T}$

$$U = F + TS$$

$$\mathbf{f} - m\mathbf{g} = \mathbf{g}\frac{\hbar}{c}\left(\frac{\pi^2}{30}\left(T_1^4 - T_2^4\right)\left(\frac{k_B}{\hbar c}\right)^4 V^{(3)} - b\frac{\zeta(3)}{4\pi}\left(T_1^3 + T_2^3\right)\left(\frac{k_B}{\hbar c}\right)^3 S^{(2)}\right)$$

The second term depends on the boundary conditions.

Can be important for metrology, thermometers calibration etc. [*Yu.Gusev, (2014)*]

If chameleon wishes to change color at nonzero temperature, it would cost him energy and change his weight

Conclusion

- Nontrivial interplay of gravity and quantum takes place not only at energies **10^{19} GeV**, but also at normal Earth-like conditions.

- The price to pay is extreme weakness.

- We have seen a few examples in the history of physics then multiplicity saves the case (e.g. expected lifetime of the proton vs $\mathbf{N_{Avogadro}}$, collider physics etc).

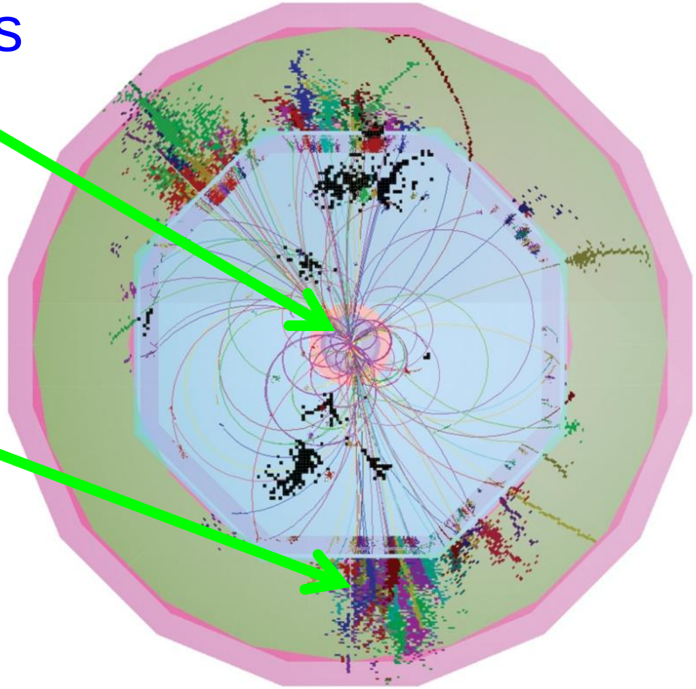
- To find proper (and experimentally reasonable) «multiplication» factor for weak gravity of quantum states/energies does not look hopeless.

- There can be surprises prepared by the Lord for us here.

Thank you for attention

In most cases in particle physics we assume that physics here («action») is uncorrelated with physics here («measure»).
Asymptotic states, plane waves basis etc.

«Beautiful» field theoretic part and «ugly» detector part...



(picture from http://www.linearcollider.org)

But is it correlated or not is a quantitative physical question.