• definition, importance, occurrence of heavy exotic states

• the state of the art theory tools

• Calculation of hybrids masses
QCD and strongly coupled gauge theories: challenges and perspectives

We highlight the progress, current status, and open challenges of QCD-driven physics, in theory and in experiment. We discuss how the strong interaction is intimately connected to a broad sweep of physical problems, in settings ranging from astrophysics and cosmology to strongly-coupled, complex systems in particle and condensed-matter physics, as well as to searches for physics beyond the Standard Model. We also discuss how success in describing the strong interaction impacts other fields, and, in turn, how such subjects can impact studies of the strong interaction. In the course of the work we offer a perspective on the many research streams which flow into and out of QCD, as well as a vision for future developments.
TABLE 10: Quarkonium-like states at the open flavor thresholds. For charged states, the C-parity is given for the neutral members of the corresponding isotriplets.

<table>
<thead>
<tr>
<th>State</th>
<th>$M$, MeV</th>
<th>$\Gamma$, MeV</th>
<th>$J^{PC}$</th>
<th>Process (mode)</th>
<th>Experiment (#$\sigma$)</th>
<th>Year</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X(3872)$</td>
<td>3871.68 ± 0.17</td>
<td>&lt; 1.2</td>
<td>1++</td>
<td>$B \rightarrow K(\pi^+\pi^-J/\psi)$</td>
<td>Belle [772, 992] (&gt;10), BaBar [993] (8.6)</td>
<td>2003</td>
<td>Ok</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$p\bar{p} \rightarrow (\pi^+\pi^-J/\psi)$</td>
<td>CDF [994, 995] (11.6), D0 [996] (5.2)</td>
<td>2003</td>
<td>Ok</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$pp \rightarrow (\pi^+\pi^-J/\psi)$</td>
<td>LHCb [997, 998] (np)</td>
<td>2012</td>
<td>Ok</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$B \rightarrow K(\pi^+\pi^-\pi^0J/\psi)$</td>
<td>Belle [999] (4.3), BaBar [1000] (4.0)</td>
<td>2005</td>
<td>Ok</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$B \rightarrow K(\gamma J/\psi)$</td>
<td>Belle [1001] (5.5), BaBar [1002] (3.5)</td>
<td>2005</td>
<td>Ok</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$B \rightarrow K(\gamma \psi(2S))$</td>
<td>LHCb [1003] (&gt;10)</td>
<td>2012</td>
<td>Ok</td>
</tr>
<tr>
<td>$Z_c(3885)^+$</td>
<td>3883.9 ± 4.5</td>
<td>25 ± 12</td>
<td>1+-</td>
<td>$Y(4260) \rightarrow \pi^-(D\bar{D}^*)^+$</td>
<td>BaBar [1002] (3.6), Belle [1001] (0.2)</td>
<td>2008</td>
<td>NC</td>
</tr>
<tr>
<td>$Z_c(3900)^+$</td>
<td>3891.2 ± 3.3</td>
<td>40 ± 8</td>
<td>??-</td>
<td>$Y(4260) \rightarrow \pi^- (\pi^+J/\psi)$</td>
<td>LHCb [1003] (4.4)</td>
<td>2012</td>
<td>Ok</td>
</tr>
<tr>
<td>$Z_c(4020)^+$</td>
<td>4022.9 ± 2.8</td>
<td>7.9 ± 3.7</td>
<td>??-</td>
<td>$Y(4260, 4360) \rightarrow \pi^- (\pi^+h_c)$</td>
<td>Belle [1004] (6.4), BaBar [1005] (4.9)</td>
<td>2006</td>
<td>Ok</td>
</tr>
<tr>
<td>$Z_c(4025)^+$</td>
<td>4026.3 ± 4.5</td>
<td>24.8 ± 9.5</td>
<td>??-</td>
<td>$Y(4260) \rightarrow \pi^- (D^<em>\bar{D}^</em>)^+$</td>
<td>BES III [1006] (np)</td>
<td>2013</td>
<td>NC</td>
</tr>
<tr>
<td>$Z_b(10610)^+$</td>
<td>10607.2 ± 2.0</td>
<td>18.4 ± 2.4</td>
<td>1+-</td>
<td>$\Upsilon(10860) \rightarrow \pi^- (\pi^+\Upsilon(1S, 2S, 3S))$</td>
<td>BES III [1007] (8), Belle [1008] (5.2)</td>
<td>2013</td>
<td>Ok</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\Upsilon(10860) \rightarrow \pi^- (\pi^+h_b(1P, 2P))$</td>
<td>T. Xiao et al. [CLEO data] [1009] (&gt;)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Z_b(10650)^+$</td>
<td>10652.2 ± 1.5</td>
<td>11.5 ± 2.2</td>
<td>1+-</td>
<td>$\Upsilon(10860) \rightarrow \pi^- (B\bar{B}^*)^+$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### TABLE 12: Quarkonium-like states above the corresponding open flavor thresholds. For charged states, the $C$-parity is given for the neutral members of the corresponding isotopultipes.

<table>
<thead>
<tr>
<th>State</th>
<th>$M, \ MeV$</th>
<th>$\Gamma, \ MeV$</th>
<th>$J^{PC}$</th>
<th>Process (mode)</th>
<th>Experiment ($#\sigma$)</th>
<th>Year Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi(4160)$</td>
<td>4153 ± 3</td>
<td>103 ± 8</td>
<td>1−−</td>
<td>$e^+e^- \to (D^<em>\Xi_c^{(</em>)}c)$</td>
<td>PDG [1]</td>
<td>1978 Ok</td>
</tr>
<tr>
<td>$\chi_{c2}(2P)$</td>
<td>3927.2 ± 2.6</td>
<td>24 ± 6</td>
<td>2++</td>
<td>$e^+e^- \to e^+e^- (\omega J/\psi)$</td>
<td>BaBar [1052] (7.7), BaBar [1053] (7.6)</td>
<td>2009 Ok</td>
</tr>
<tr>
<td>$X(3940)$</td>
<td>3942.2 ± 2.3</td>
<td>37 ± 27</td>
<td>?+</td>
<td>$e^+e^- \to J/\psi (D^<em>\Xi_c^{(</em>)})$</td>
<td>Belle [1048, 1049] (6)</td>
<td>2005 NC!</td>
</tr>
<tr>
<td>$Y(4008)$</td>
<td>3891 ± 42</td>
<td>255 ± 42</td>
<td>1−−</td>
<td>$e^+e^- \to (\pi^+\pi^- J/\psi)$</td>
<td>Belle [1008, 1056] (7.4)</td>
<td>2007 NC!</td>
</tr>
<tr>
<td>$\psi(4040)$</td>
<td>4039 ± 1</td>
<td>80 ± 10</td>
<td>1−−</td>
<td>$e^+e^- \to (D^<em>\Xi_c^{(</em>)}c)$</td>
<td>PDG [1]</td>
<td>1978 Ok</td>
</tr>
<tr>
<td>$Z(4050)^+$</td>
<td>4051 $^{+24}_{-43}$</td>
<td>82 $^{+51}_{-35}$</td>
<td>?+</td>
<td>$\bar{B}^0 \to K^-(\pi^+\chi_c)$</td>
<td>Belle [1058] (5.0), BaBar [1059] (1.1)</td>
<td>2008 NC!</td>
</tr>
<tr>
<td>$Y(4140)$</td>
<td>4145 ± 26</td>
<td>18 ± 8</td>
<td>?+</td>
<td>$B^+ \to K^+(\phi J/\psi)$</td>
<td>CDF [1060] (5.0), Belle [1061] (1.9), LHCb [1062] (1.4), CMS [1063] (5)</td>
<td>2009 NC!</td>
</tr>
<tr>
<td>$Y(4274)$</td>
<td>4293 ± 20</td>
<td>35 ± 16</td>
<td>?+</td>
<td>$B^+ \to K^+(\phi J/\psi)$</td>
<td>CDF [1060] (3.1), LHCb [1062] (1.0), CMS [1063] (5), D0 [1064] (3.1)</td>
<td>2011 NC!</td>
</tr>
<tr>
<td>$X(4350)$</td>
<td>4350 $^{+6}_{-5.6}$</td>
<td>13 ± 18</td>
<td>0/2++</td>
<td>$e^+e^- \to e^+e^- (\phi J/\psi)$</td>
<td>BaBar [1067] (np), Belle [1008] (np)</td>
<td>2012 Ok</td>
</tr>
<tr>
<td>$Y(4360)$</td>
<td>4354 ± 11</td>
<td>78 ± 16</td>
<td>1−−</td>
<td>$e^+e^- \to (\pi^+\pi^- \psi(2S))$</td>
<td>BES III [1007] (8), Belle [1008] (5.2)</td>
<td>2013 Ok</td>
</tr>
<tr>
<td>$Z(4450)^+$</td>
<td>4458 ± 15</td>
<td>166 $^{+37}_{-32}$</td>
<td>1−−</td>
<td>$\bar{B}^0 \to K^-(\pi^+\psi(2S))$</td>
<td>Belle [1074, 1075] (6.4), BaBar [1076] (2.4)</td>
<td>2007 Ok</td>
</tr>
<tr>
<td>$X(4630)$</td>
<td>4634 $^{+9}_{-11}$</td>
<td>92 $^{+41}_{-32}$</td>
<td>1−−</td>
<td>$e^+e^- \to (\Lambda_c^+ \Lambda_c^-)$</td>
<td>Belle [1065] (4.0)</td>
<td>2014 NC!</td>
</tr>
<tr>
<td>$Y(4660)$</td>
<td>4665 ± 10</td>
<td>53 ± 14</td>
<td>1−−</td>
<td>$e^+e^- \to (\pi^+\pi^- \psi(2S))$</td>
<td>BaBar [1072] (5.8), BaBar [1073] (5)</td>
<td>2007 Ok</td>
</tr>
<tr>
<td>$Y(10860)$</td>
<td>10876 ± 11</td>
<td>55 ± 28</td>
<td>1−−</td>
<td>$e^+e^- \to (B_{s}^{(<em>)}\bar{B}_{s}^{(</em>)}(\pi))$</td>
<td>PDG [1]</td>
<td>1985 Ok</td>
</tr>
<tr>
<td>$Y_0(10888)$</td>
<td>10888.4 ± 3.0</td>
<td>30.7 $^{+8.9}_{-7.7}$</td>
<td>1−−</td>
<td>$e^+e^- \to (\pi^+\pi^- \psi(1S, 2S, 3S))$</td>
<td>Belle [1013, 1014, 1079] (5)</td>
<td>2007 Ok</td>
</tr>
</tbody>
</table>
Charmonium

the present revolution

\[ \begin{align*}
D_s \bar{D}_s & \quad \longrightarrow \quad \psi(4170) \quad \pi \pi J/\psi, \pi \pi h_c (\text{?}) \\
D_s \bar{D}_s & \quad \longrightarrow \quad \psi(4040) \quad \pi \pi h_c (\text{?}) \\
D_s \bar{D}_s & \quad \longrightarrow \quad \psi(3770) \quad \pi \pi J/\psi \\
D \bar{D}^* & \quad \longrightarrow \quad \psi' \quad \eta J/\psi \\
D \bar{D} & \quad \longrightarrow \quad \eta_c' \\
D \bar{D} & \quad \longrightarrow \quad h_c \\
D \bar{D}^* & \quad \longrightarrow \quad \eta_c \\
\end{align*} \]

\[ M \text{ GeV} \quad J^{PC} : 0^{--} \quad 1^{+-} \quad 1^{--} \quad 0^{++} \quad 1^{++} \quad 2^{++} \quad ? \]
bottomonium: the present revolution

$B_s \bar{B}_s$ $\ldots$
$B_s^* \bar{B}_s^*$ $\ldots$
$B_s B_s$ $\ldots$

$B^* \bar{B}^*$ $\ldots$ $Z'_b$ $\ldots$ $Z_b$ $\ldots$ $h_b(3P)$ $\ldots$ $\Upsilon(5S)$

$\eta_b(3S)$ $\ldots$ $\Upsilon(3S)$ $\ldots$ $h_b(2P)$ $\ldots$ $\chi_{bJ}(3P)$ $\ldots$ $\chi_{bJ}(2P)$ $\Upsilon(1^3D_J)$ $\ldots$ $1^1D_2$

$\eta_b(2S)$ $\ldots$ $\Upsilon(2S)$ $\ldots$ $h_b(1P)$ $\ldots$ $\chi_{bJ}(1P)$ $\ldots$

$\eta_b$ $\ldots$ $\Upsilon(1S)$ $\ldots$

$M$ GeV $J^{PC}$: $0^{-+}$ $1^{++}$ $1^{--}$ $0^{++}$ $1^{++}$ $2^{++}$ $1^{--}$, $2^{--}$, $3^{--}$ $2^{++}$

The Collider Detector at Fermilab

\[ \text{DØ} \]

\[ \text{Belle} \]

\[ \text{BaBar} \]

\[ \text{CLEO} \]

\[ \text{BES} \]

\[ \text{CLEO} \]
Quarkonium (=bound state of a heavy quark and a heavy antiquark) has been instrumental for the establishing of QCD, the theory of strong interaction, and the Standard Model of Particle
The November revolution in 1974: the $J/\psi$ discovery

$\Gamma \sim 90$ KeV

Aubert et al. BNL 74
The November revolution in 1974: the $J/\psi$ discovery

Samuel Ting: “It is like to stumble on a village where people live 70000 years”

$\Gamma \sim 90$ KeV
Samuel Ting: “It is like to stumble on a village where people live 70000 years”

The November revolution in 1974: the $J/\psi$ discovery

$\Gamma \sim 90$ KeV

it has been the confirmation of the charm quark prediction and of QCD (strong int theory) foundations

narrow width and asymptotic freedom

annihilation at large scale controlled by small $\alpha_s$

first discovery of a quark of large mass moving “slowly”
Samuel Ting: "It is like to stumble on a village where people live 70,000 years."

Aubert et al. BNL 74

$\Gamma \sim 90$ KeV

The November revolution in 1974: J/\(\psi\) discovery

...it has been the confirmation of the charm quark prediction and of QCD (strong interaction) foundations... narrow width and asymptotic freedom... has been the confirmation of the first discovery of a quark of large mass moving “slowly”...
The November revolution in the ’70s: more quarkonia

Variety of potential models used
Confinement and asymptotic freedom--> main properties of QCD
Heavy quarks offer a privileged access

A large scale $m_Q \gg \Lambda_{QCD}$, $\alpha_s(m_Q) \ll 1$

with $Q, \bar{Q} = c, b, t$

$m_c \sim 1.5\text{GeV}$

$m_b \sim 5\text{GeV}$

$m_t \sim 170\text{GeV}$
Heavy quarks offer a privileged access to the strong sector of the Standard Model. Multiscale systems are accessible, with $Q, \bar{Q} = c, b, t$.

- $m_c \sim 1.5\text{GeV}$
- $m_b \sim 5\text{GeV}$
- $m_t \sim 170\text{GeV}$

A large scale $m_Q \gg \Lambda_{\text{QCD}}$, $\alpha_s(m_Q) \ll 1$.

Heavy quarkonia are nonrelativistic bound systems: multiscale systems.

Many scales: a challenge and an opportunity.
Quarkonium scales

<table>
<thead>
<tr>
<th>MeV</th>
<th>Y(4S)</th>
<th>(\psi(3S))</th>
<th>X_b(2P)</th>
<th>(\psi(3770))</th>
<th>(\psi(2S))</th>
<th>(\eta_c(2S))</th>
<th>X_b(1P)</th>
<th>X_c(1P)</th>
<th>h_c(1P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>Y(3S)</td>
<td>(\psi(3S))</td>
<td></td>
<td>(\psi(3770))</td>
<td>(\psi(2S))</td>
<td>(\eta_c(2S))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-500)</td>
<td>Y(1S)</td>
<td>(J/\psi)</td>
<td>(\eta_c(1S))</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

S states

Normalized with respect to \(\chi_b(1P)\) and \(\chi_c(1P)\)

\[2S+1 L_J\]
### Quarkonium scales

<table>
<thead>
<tr>
<th>MeV</th>
<th>States</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>Y(4S)</td>
</tr>
<tr>
<td>0</td>
<td>Y(3S)</td>
</tr>
<tr>
<td>0</td>
<td>Y(2S)</td>
</tr>
<tr>
<td>-500</td>
<td>Y(1S)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>S states</th>
<th>P states</th>
</tr>
</thead>
</table>

Normalized with respect to $\chi_b(1P)$ and $\chi_c(1P)$

**NR bound states have at least 3 scales**

$m \gg mv \gg mv^2 \quad v \ll 1$

$m v \sim r^{-1}$

**and $\Lambda_{QCD}$**

**The system is nonrelativistic (NR)**

$\Delta E \sim mv^2, \Delta_{fs} E \sim mv^4$

$v_b^2 \sim 0.1, v_c^2 \sim 0.3$

**The mass scale is perturbative**

$m_Q \gg \Lambda_{QCD}$

$m_b \sim 5 \text{ GeV}; m_c \sim 1.5 \text{ GeV}$
Quarkonium as a confinement and deconfinement probe

The rich structure of separated energy scales makes QQbar an ideal probe

At zero temperature

- The different quarkonium radii provide different measures of the transition from a Coulombic to a confined bound state.

\[ V^m(r) \quad (\text{GeV}) \]

\[ \Lambda_{\text{QCD}} \]

Low lying \( Q \bar{Q} \)

High lying \( Q \bar{Q} \)

\[ Y \quad Y' \quad Y'' \quad Y''' \]

\[ \eta, \chi, \psi, \psi' \]

\[ r(\text{fm}) \]

Godfrey Isgur PRD 32(85)189

Quarkonia probe the perturbative (high energy) and non-perturbative region (low energy) as well as the transition region in dependence of their radius \( r \)
Quarkonium as a confinement and deconfinement probe

At finite temperature \( T \) they are sensitive to the formation of a quark gluon plasma via color screening

Debye charge screening \( m_D \sim gT \)

\[
V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}
\]

\[
r \sim \frac{1}{m_D} \quad \text{Bound state dissolves}
\]

Matsui Satz 1986
At finite temperature $T$ they are sensitive to the formation of a quark gluon plasma via color screening.

Debye charge screening

$$m_D \sim g T$$

$$V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$$

$$r \sim \frac{1}{m_D} \quad \text{Bound state dissolves}$$

Matsui Satz 1986

Quarkonia dissociate at different temperature in dependence of their radius: they are a Quark Gluon Plasma thermometer.
QCD theory of Quarkonium: a very hard problem

Close to the bound state $\alpha_s \sim \nu$

$$p \sim m\alpha_s + \cdots$$

$$\frac{g^2}{p^2} (1 + \frac{m\alpha_s}{p})$$

$$\sim \frac{1}{E - \left(\frac{p^2}{m} + V\right)}$$
QCD theory of Quarkonium: a very hard problem

Close to the bound state $\alpha_s \sim \nu$

\[
p \sim m\alpha_s \quad + \quad \frac{g^2}{p^2} \left( 1 + \frac{m\alpha_s}{p} \right) \quad + \quad \ldots
\]

\[
\sim \frac{1}{E - \left( \frac{p^2}{m} + V \right)}
\]

- From $(\frac{p^2}{m} + V)\phi = E\phi \rightarrow p \sim mv$ and $E = \frac{p^2}{m} + V \sim mv^2$. 
QCD theory of Quarkonium: a very hard problem

Close to the bound state \( \alpha_s \sim \nu \)

\[
p \sim m\alpha_s + \ldots
\]

\[
\frac{g^2}{p^2} (1 + \frac{m\alpha_s}{p})
\]

\[
g^2
\]

\[
\begin{array}{c}
p \sim m\nu
\end{array}
\]

\[
E \sim m\nu^2
\]

multiscale diagrams have a complicate power counting and contribute to all orders in the coupling

\[
L^{-1} \ll \lambda \ll \Lambda \ll a^{-1}
\]

Difficult also for the lattice!
Quarkonium with Nonrelativistic Effective Field Theories

\[ \mathcal{L}_{EFT} = \sum_n c_n \left( \frac{E_\Lambda}{\mu} \right) \frac{O_n(\mu, \lambda)}{E_\Lambda} \]

\[ \langle O_n \rangle \sim E_\lambda^n \]

Color degrees of freedom
3X3 = 1 + 8
singlet and octet QQbar

Hard

Soft
(relative momentum)

Ultrasoft
(binding energy)

m
perturbative matching
perturbative matching

m \nu
NRQCD

m \nu^2

nonperturbative matching
(long-range quarkonium)

perturbative matching
(short-range quarkonium)

pNRQCD

\[ \mu \]

\[ \mu' \]
Quarkonium with Non-relativistic Effective Field Theories

\[ \mathcal{L}_{\text{EFT}} = \sum_n c_n \left( \frac{E_\Lambda}{\mu} \right) \frac{O_n(\mu, \lambda)}{E_\Lambda} \]

\[ \langle O_n \rangle \sim E_\lambda^n \]

Color degrees of freedom
3X3=1+8
singlet and octet QQbar

Hard

Soft
(relative momentum)

Ultrasoft
(binding energy)

QCD

perturbative matching

\[ \frac{E_\lambda}{E_\Lambda} = \frac{m v}{m} \]

NRQCD

perturbative matching

pNRQCD

nonperturbative matching
(long-range quarkonium)

perturbative matching
(short-range quarkonium)
Quarkonium with Non relativistic Effective Field Theories

\[ \mathcal{L}_{\text{EFT}} = \sum_n c_n \frac{E_\Lambda}{\mu} \frac{O_n(\mu, \lambda)}{E_\Lambda} \]

\[ \langle O_n \rangle \sim E_\lambda^n \]

Color degrees of freedom

3X3=1+8

singlet and octet QQbar

Hard

Soft (relative momentum)

Ultrasoft (binding energy)

\[ \frac{E_\lambda}{E_\Lambda} = \frac{mv}{m} \]

\[ \frac{E_\lambda}{E_\Lambda} = \frac{mv^2}{mv} \]
\[
\mathcal{L}_{\text{EFT}} = \sum_n c_n \left( \frac{E_{\Lambda}}{\mu} \right) \mathcal{O}_n(\mu, \lambda)
\]

\[
\langle \mathcal{O}_n \rangle \sim E_{\Lambda}
\]

Quarkonium with Nonrelativistic Effective Field Theories

Peaking to QCD

\[
\begin{align*}
E_{\Lambda} &= \frac{m v^2}{\mu} \\
\mathcal{O}_n(\mu, \lambda) &= \frac{E_{\Lambda}}{\mu} = \frac{m v}{m}
\end{align*}
\]

Singlet and octet \( Q\bar{Q} \) bar

Color degrees of freedom

3X3 = 1 + 8

Perturbative matching (short-range quarkonium)

Nonperturbative matching (long-range quarkonium)

PNRQCD

CONSTRUCTED to be EQUIVALENT

QCD

Hard

Soft

Ultrasoft

(relative energy)

(binding energy)

(relative momentum)
Quarkonium with NR EFT: Non Relativistic QCD (NRQCD)

QCD

perturbative matching

mv

NRQCD

perturbative matching

mv²

nonperturbative matching (long-range quarkonium)

perturbative matching (short-range quarkonium)

pNRQCD

E \sim mv²

\sim m

p \sim mv
Quarkonium with NR EFT: Non Relativistic QCD (NRQCD)

- QCD
  - Perturbative matching
  - $m$

- NRQCD
  - Perturbative matching
  - $\mu$
  - Nonperturbative matching
  - $mv^2$
  - (long-range quarkonium)

- pNRQCD
  - Perturbative matching
  - $\mu'$
  - (short-range quarkonium)

- $E \sim mv^2$
- $p \sim mv$
- $\sim m$
Quarkonium with NR EFT: Non Relativistic QCD (NRQCD)

\[ \mathcal{L}_{\text{NRQCD}} = \sum_n c(\alpha_s(m/\mu)) \times \frac{O_n(\mu, \lambda)}{m^n} \]
Quarkonium with NR EFT: potential NonRelativistic QCD (pNRQCD)

- QCD
  - perturbative matching
  - \( \mu \)

- NRQCD
  - nonperturbative matching (long-range quarkonium)
  - \( \mu' \)
  - perturbative matching (short-range quarkonium)

- pNRQCD
  - \( E \sim mv^2 \)
  - \( \sim m \)
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Quarkonium with NR EFT: potential Non-Relativistic QCD (pNRQCD)

- **QCD**
  - Perturbative matching

- **NRQCD**
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  - Perturbative matching (short-range quarkonium)

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- $p \sim m v$

- $\sim m$

- Perturbative matching
Quarkonium with NR EFT: potential NonRelativistic QCD (pNRQCD)

\[ \mathcal{L}_{\text{pNRQCD}} = \sum_k \sum_n \frac{1}{m^k} c_k(\alpha_s(m/\mu)) \times V(r\mu', r\mu) \times O_n(\mu', \lambda) r^n \]
Quarkonium with NR EFT: potential NonRelativistic QCD (pNRQCD)

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In QCD another scale is relevant \( \Lambda_{QCD} \)

Quarkonium with NR EFT: pNRQCD

A potential picture arises at the level of pNRQCD:
- the potential is perturbative if \( mv \gg \Lambda_{QCD} \)
- the potential is non-perturbative if \( mv \sim \Lambda_{QCD} \)

Pineda, Soto 97, N.B., Pineda, Soto, Vairo 99
N.B. Vairo, Pineda, Soto 00--014
weakly coupled pNRQCD $r \ll \Lambda_{QCD}^{-1}$

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu} a + \text{Tr} \left\{ S^\dagger \left( i\partial_0 - \frac{p^2}{m} \right) - V_s \right\} S + \text{O}^\dagger \left( iD_0 - \frac{p^2}{m} - V_o \right) \text{O} \]

Singlet static potential

LO in $r$

Octet static potential

\[ + V_A \text{Tr} \left\{ O^\dagger r \cdot gE S + S^\dagger r \cdot gE O \right\} \]
\[ + \frac{V_B}{2} \text{Tr} \left\{ O^\dagger r \cdot gE O + O^\dagger O r \cdot gE \right\} + \cdots \]

NLO in $r$

S singlet field
O octet field

Pineda, Soto 97; Brambilla, Pineda, Soto, Vairo 99-
strongly coupled pNRQCD \[ r \sim \Lambda_{QCD}^{-1} \]

⇒ The singlet quarkonium field \( S \) of energy \( mv^2 \) is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

\[
\mathcal{L} = \text{Tr} \left\{ S^\dagger \left( i\partial_0 - \frac{p^2}{m} - V_s \right) S \right\}
\]
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\[
\mathcal{L} = \text{Tr} \left\{ S^\dagger \left( i \partial_0 - \frac{p^2}{m} - V_s \right) S \right\}
\]

• A potential description emerges from the EFT

• The potentials \( V = \text{Re}V + \text{Im}V \) from QCD in the matching: get spectra and decays

• \( V \) to be calculated on the lattice or in QCD vacuum models

Brambilla Pineda Soto Vairo 00
Quarkonium singlet static potential

\[ V = V_0 + \frac{1}{m} V_1 + \frac{1}{m^2} (V_{SD} + V_{VD}) \]

\[ V_s^{(0)} = \lim_{T \to \infty} \frac{i}{T} \ln \langle W(r \times T) \rangle = \lim_{T \to \infty} \frac{i}{T} \ln \langle \square \rangle \]

\[ W = \langle \exp \left\{ ig \oint A^\mu dx_\mu \right\} \rangle \]
many experimental data and opportunities

Quarkonium today is a golden system to study strong interactions

new theoretical tools:
Effective Field Theories (EFTs) of QCD
and progress in lattice QCD
The EFT has been constructed

- Work at calculating higher order perturbative corrections in \( v \) and \( \alpha_s \)
- Resumming the log
- Calculating/extracting nonperturbatively the low energy quantities
- Extending the theory (electromagnetic effect, 3 bodies)

The issue here is precision physics and the study of confinement

- Precise and systematic high order calculations allow the extraction of precise determinations of standard model parameters like the quark masses and \( \alpha_s \)
- The EFT has allowed to systematically factorize and to study the low energy nonperturbative contributions
pNRQCD and quarkonium

The EFT is being constructed \((\text{at small coupling})\) \(\text{Laine et al, 2007, Escobedo, Soto, 2007 N. B. et al. 2008}\)

*Results on the static potential hint at a new physical picture of dissociation

*Mass and width of quarkonium at \(m \alpha^5(Y(1S) b\bar{b} \text{bar at LHC})\) \(\text{N. B. Escobedo, Ghiglieri, Vairo Soto, 2010-2014}\)

*Polyakov loop calculation  \(\text{N. B., Ghiglieri, Petreczky, Vairo 2010}\)

The eft allows us to discover new, unexpected and important facts:

• The potential is neither the color singlet free energy nor the internal energy

• The quarkonium dissociation is a consequence of the appearance of a thermal decay width rather than being due to the color screening of the real part of the potential.

We have now a coherent and systematical setup to calculate masses and width of quarkonium at finite \(T\) for small coupling
pNRQCD and quarkonium (close or above the strong decay threshold)

The EFT has not yet been constructed (Exotics close to threshold)

*Degrees of freedom still to be identified

Near threshold heavy-light mesons have to be included

No systematic treatment is available; lattice calculations are also challenging and in the infancy state in this case
pNRQCD and quarkonium (close or above the strong decay threshold)

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---

The QCD spectrum with light quarks

- We still have states just made of heavy quarks and gluons. They may develop a width because of the decay through pion emission. If new states made with heavy and light quarks develop a mass gap of order $\Lambda_{QCD}$ with respect to the former ones, then these new states may be absorbed into the definition of the potentials or of the (local or non-local) condensates.
  - Brambilla et al. PRD 67(03)034018
- In addition new states built using the light quark quantum numbers may form.
  - Soto NP PS 185(08)107
States made of two heavy and light quarks

- **Pairs of heavy-light mesons**: $D\bar{D}$, $B\bar{B}$, ...

- **Pairs of heavy-light baryons**.
  - Qiao PLB 639 (2006) 263

- **Molecular states**, i.e. states built on the pair of heavy-light mesons.
  - Tornqvist PRL 67(91)556

- **Tetraquark states**.
  - Maiani, Piccinini, Polosa et al. 2005--
    - Jaffe PRD 15(77)267
    - Ebert Faustov Galkin PLB 634(06)214

Having the spectrum of tetraquark potentials, like we have for the gluonic excitations, would allow us to build a plethora of states on each of the tetraquark potentials, many of them developing a width due to decays through pion (or other light hadrons) emission. Diquarks have been recently investigated on the lattice.
  - Alexandrou et al. PRL 97(06)222002
  - Fodor et al. PoS LAT2005(06)310

(hadro-quarkonium). Voloshin
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Choosing one of these degrees of freedom and an interaction originates a model for exotics.
$X(3872)$: interpretations

4-quark state with $J^{PC} = 1^{++}$
**X(3872): interpretations**

- **Molecular model**

  \[ X \sim (c\bar{c})^S_{S=1} \otimes (q\bar{q})^S_{S=1} \]
  \[ \sim (c\bar{q})^I_{I=0} \otimes (q\bar{c})^I_{I=1} + (c\bar{q})^I_{I=1} \otimes (q\bar{c})^I_{I=0} \]

- **Törnqvist 93, Swanson 04**

  \[ X \sim (c\bar{q})^I_{I=0} \otimes (q\bar{c})^I_{I=1} + (c\bar{q})^I_{I=1} \otimes (q\bar{c})^I_{I=0} \]
  \[ \sim D \overline{D}^* + D^* \overline{D} \]

This is assumed to be the dominant long-range Fock component; short-range components of the type \((c\bar{c})^I_{I=1} \otimes (q\bar{q})^I_{I=1}\)

- **Tetraquark model**

  \[ X \sim (c\bar{c})^3_{I=0} \otimes (\bar{q}q)^3_{I=1} + (c\bar{q})^I_{I=0} \otimes (q\bar{c})^I_{I=1} \]

  The dynamical assumption (there is no scale separation like in the doubly heavy baryons) is that quark pair cluster in tightly bound color triplet diquarks (see 1-gluon exchange); the difficulty in breaking the system explains the narrow width.

**Predictions based on the phenomenological Hamiltonian:**

\[ H = - \sum_{ij} \kappa_{ij} \sigma \otimes \sigma; \]

\[ \kappa_{ij} \sim \frac{1}{2} \delta_{ij} + \frac{3}{2} \lambda_i \sigma \]

4-quark state with \( J^{PC} = 1^{++} \)

Høgassen et al 05

**Predictions based on the phenomenological Hamiltonian:**

\[ H = \sum_{ij} C_{ij} T^a \otimes T^a \sigma \otimes \sigma; \]

\[ X \sim (c\bar{c})^S_{S=1} \otimes (q\bar{q})^S_{S=1} \]

\[ \sim (c\bar{q})^I_{I=0} \otimes (q\bar{c})^I_{I=1} + (c\bar{q})^I_{I=1} \otimes (q\bar{c})^I_{I=0} \]
In some cases it is possible to develop an EFT owing to special dynamical condition

\[ \Lambda_{QCD} \gg m_\pi \gg \frac{m_\pi^2}{M_{D^0}} \approx 10 \text{ MeV} \gg E_{\text{binding}} \]

\[ \approx M_X - (M_{D^{*0}} + M_{D^0}) = (0.1 \pm 1.0) \text{ MeV} \]

Systems with a short-range interaction and a large scattering length have universal properties that may be exploited: in particular, production and decay amplitudes factorize in a short-range and a long-range part, where the latter depends only on one single parameter, the scattering length. An universal property that fits well with the observed large branching fraction of the \( X(3872) \) decaying into \( D^0 \bar{D}^0 \pi^0 \) is \( \mathcal{B}(X \to D^0 \bar{D}^0 \pi^0) \approx \mathcal{B}(D^{*0} \to D^0 \pi^0) \approx 60\% \).

Pakvasa Suzuki 03, Voloshin 03, Braaten Kusunoki 03
even the case without light quark is difficult:

**Gluonic excitations**

A plethora of states built on each of the hybrid potentials is expected. These states typically develop a width also without including light quarks, since they may decay into lower states, e.g. like $\text{hybrid} \rightarrow \text{glueball} + \text{quark-antiquark}$.

We may integrate out modes scaling like $1/r$ and $\Lambda_{\text{QCD}}$ and describe hybrids as heavy quark-antiquark states bound by potentials that are the energies of the corresponding gluonic excitations between static sources $\rightarrow$ Born–Oppenheimer approximation.

If more states are nearly degenerate, then all of these need to be considered as effective low-energy degrees of freedom and mix.
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We have obtained an EFT description of hybrids matching the NRQCD static energies to pNRQCD potential in the short range.
even the case without light quark is difficult

**Lattice energies**

Static states classified by symmetry group $D_{\infty h}$

Representations labeled $\Lambda_\eta$

- $\Lambda$ rotational quantum number
  - $|\hat{n} \cdot \mathbf{K}| = 0, 1, 2 \ldots$ corresponds to
  - $\Lambda = \Sigma, \Pi, \Delta \ldots$

- $\eta$ eigenvalue of $CP$:
  - $g \triangleq +1$ (gerade), $u \triangleq -1$ (ungerade)

- $\sigma$ eigenvalue of reflections

- $\sigma$ label only displayed on $\Sigma$ states (others are degenerate)

- The static energies correspond to the irreducible representations of $D_{\infty h}$

- In general it can be more than one state for each irreducible representation of $D_{\infty h}$, usually denoted by primes, e.g. $\Pi_u, \Pi'_u, \Pi''_u$...

even the case without light quark is difficult

static Lattice energies

- $\Sigma^+_g$ is the ground state potential that generates the standard quarkonium states.
- The rest of the static energies correspond to excited gluonic states that generate hybrids.
- The two lowest hybrid static energies are $\Pi_u$ and $\Sigma^-_u$, they are nearly degenerate at short distances.
- The static energies have been computed in quenched lattice QCD, the most recent data by Juge, Kuti, Morningstar, 2002 and Bali and Pineda 2003.
- Quenched and unquenched calculations for $\Sigma^+_g$ and $\Pi_u$ were compared in Bali et al 2000 and good agreement was found below string breaking distance.

even the case without light quark is difficult

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- $\Sigma_g^+$ is the ground state potential that generates the standard quarkonium states.
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Gluonic excitations in pNRQCD: more symmetry!

In the limit $r \to 0$ more symmetry: $D_{\infty_h} \to O(3) \times C$

- Several $\Lambda^\sigma_{\eta}$ representations contained in one $J^{PC}$ representation:
- Static energies in these multiplets have same $r \to 0$ limit.

### pNRQCD predicts the structure of multiplets at short distance and the ordering

<table>
<thead>
<tr>
<th>Multiplet</th>
<th>$L = 1$</th>
<th>$L = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Sigma_{g}'$</td>
<td>$r \cdot (E)$</td>
<td></td>
</tr>
<tr>
<td>$\Sigma_{g}$</td>
<td>$(r \cdot D)(r \cdot B)$</td>
<td>$(r \cdot D)(r \cdot B)$</td>
</tr>
<tr>
<td>$\Pi_{g}$</td>
<td>$r \times (r \cdot D)(r \cdot B)$</td>
<td>$r \times ((r \cdot D)B + D(r \cdot B))$</td>
</tr>
<tr>
<td>$\Delta_{g}$</td>
<td>$((r \cdot DB + D(r \cdot B))$</td>
<td>$(r \times D)^i(r \times B)^j + (r \times D)^j(r \times B)^i$</td>
</tr>
<tr>
<td>$\Sigma_{u}$</td>
<td></td>
<td>$(r \cdot D)(r \cdot E)$</td>
</tr>
<tr>
<td>$\Sigma_{\bar{u}}$</td>
<td>$r \cdot B$</td>
<td>$r \cdot B$</td>
</tr>
<tr>
<td>$\Pi_{u}$</td>
<td>$r \times B$</td>
<td>$r \times ((r \cdot D)E + D(r \cdot E))$</td>
</tr>
<tr>
<td>$\Pi'_{u}$</td>
<td>$r \times ((r \cdot D)E + D(r \cdot E))$</td>
<td>$(r \times D)^i(r \times E)^j + (r \times D)^j(r \times E)^i$</td>
</tr>
<tr>
<td>$\Delta_{u}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Brambilla Pineda Soto Vairo 00
Gluonic excitations in pNRQCD: one can determine the form of the potential

- **At lowest order in the multipole expansion, the singlet decouples while the octet is still coupled to gluons.**

- **Static hybrids at short distance are called gluelumps and are described by a static adjoint source \((O)\) in the presence of a gluonic field \((H)\):

  \[
  H(R, r, t) = \text{Tr}\{OH\}
  \]

\[
E_H = V_o + \frac{i}{T} \ln \langle H^a \left( \frac{T}{2} \right) \phi_{ab}^{\text{adj}} H^b \left( -\frac{T}{2} \right) \rangle
\]

\[
\langle H^a \left( \frac{T}{2} \right) \phi_{ab}^{\text{adj}} H^b \left( -\frac{T}{2} \right) \rangle_{np} \sim h e^{-i T \Lambda_H}
\]

\[
E_H(r) = V_o(r) + \Lambda_H + \mathcal{O}(r^2)
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E_H(r) = V_o(r) + \Lambda_H + \mathcal{O}(r^2)
\]

octet potential  gluelump mass  correction softly breaking the symmetry
Hybrid Static energies

$\Lambda_H$

- It is a non-perturbative quantity.
- It depends on the particular operator $H^a$, however it is the same for operators corresponding to different projections of the same gluonic operators.
- The gluelump masses have been determined in the lattice. Foster et al 1999; Bali, Pineda 2004; Marsh Lewis 2014

$$V_H = V_o + \Lambda_H + b_H r^2,$$

$b_H$

- It is a non-perturbative quantity.
- Proportional to $r^2$ due to rotational invariance and the multipole expansion.
- We are going to fix it through a fit to the static energies lattice data.
- Breaks the degeneracy of the potentials.
Hybrids masses

calculated by using the potential in the Schrodinger equation for heavy quarks obtained from the matching between NRQCD and pNRQCD

\[ V_H = V_0 + \Lambda_H + b_H r^2 \]

with \( V_0 \) calculated in perturbation theory,

\[ \Lambda_H \quad \text{RS scheme} = 0.87 \pm 0.15 \text{GeV} \]

\( b_H \) fit from the lattice data

and the mixing inside the multiplet taken into account with the coupled Schrödinger equations obtained in the matching

Berwein, N.B., Tarrus, Vairo 2015, see also E. Braaten et al 2013, 2014

the spin is not included at this order of the matching
Lowest energy multiplet $\Sigma_u^− - \Pi_u$

- The two lowest laying hybrid static energies are $\Pi_u$ and $\Sigma_u^−$.
- They are generated by a gluelump with quantum numbers $1^{++}$ and thus are degenerate at short distances.
- The kinetic operator mixes them but not with other multiplets.
- Well separated by a gap of $\sim 1$ GeV from the next multiplet with the same CP.

$$V_H = V_o + \Lambda_H + b_H r^2$$

$\Lambda_H$ and $b_H$ are nonperturbative and should be obtained from lattice calculations.
Lattice data: Bali, Pineda 2004; Juge, Kuti, Morningstar 2003, dashed line $V^{(0.5)}$, solid line $V^{(0.25)}$

$V^{(0.25)}$

- $r \leq 0.25$ fm: pNRQCD potential.
  - Lattice data fitted for the $r = 0 - 0.25$ fm range with the same energy offsets as in $V^{(0.5)}$.
  - $b^{(0.25)}_{\Sigma} = 1.246$ GeV/fm$^2$, $b^{(0.25)}_{\Pi} = 0.000$ GeV/fm$^2$.

- $r > 0.25$ fm: phenomenological potential.
  - $V'(r) = \frac{a_1}{r} + \sqrt{a_2 r^2 + a_3} + a_4$.
  - Same energy offsets as in $V^{(0.25)}$.
  - Constraint: Continuity up to first derivatives.
Hybrid state masses from $V^{(0.25)}$

Solving the coupled Schrödinger equations we obtain

<table>
<thead>
<tr>
<th>GeV</th>
<th>$m_H$</th>
<th>$\langle 1/r \rangle$</th>
<th>$E_{kin}$</th>
<th>$P_\Pi$</th>
<th>$m_H$</th>
<th>$\langle 1/r \rangle$</th>
<th>$E_{kin}$</th>
<th>$P_\Pi$</th>
<th>$m_H$</th>
<th>$\langle 1/r \rangle$</th>
<th>$E_{kin}$</th>
<th>$P_\Pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
<td>4.15</td>
<td>0.42</td>
<td>0.16</td>
<td>0.82</td>
<td>7.48</td>
<td>0.46</td>
<td>0.13</td>
<td>0.83</td>
<td>10.79</td>
<td>0.53</td>
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<td>0.37</td>
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<tr>
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<td>4.67</td>
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<td>0.42</td>
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<td>7.89</td>
<td>0.28</td>
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<td>0.46</td>
<td>0.19</td>
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<tr>
<td>$H_7$</td>
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<td>0.19</td>
<td>0.43</td>
<td>1.00</td>
<td>7.89</td>
<td>0.22</td>
<td>0.35</td>
<td>1.00</td>
<td>11.05</td>
<td>0.26</td>
<td>0.24</td>
<td>1.00</td>
</tr>
</tbody>
</table>

**Consistency test:**

1. The multipole expansion requires $\langle 1/r \rangle > E_{kin}$.

As expected the our approach works better in bottomonium than charmonium

<table>
<thead>
<tr>
<th>Spin symmetry multiplets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
</tr>
<tr>
<td>$H_2$</td>
</tr>
<tr>
<td>$H_3$</td>
</tr>
<tr>
<td>$H_4$</td>
</tr>
<tr>
<td>$H_5$</td>
</tr>
<tr>
<td>$H_6$</td>
</tr>
<tr>
<td>$H_7$</td>
</tr>
<tr>
<td>State</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>X(3823)</td>
</tr>
<tr>
<td>X(3872)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>X(3915)</td>
</tr>
<tr>
<td>$\chi_c(2P)$</td>
</tr>
<tr>
<td>X(3940)</td>
</tr>
<tr>
<td>G(3900)</td>
</tr>
<tr>
<td>Y(4008)</td>
</tr>
<tr>
<td>Y(4140)</td>
</tr>
<tr>
<td>X(4160)</td>
</tr>
<tr>
<td>Y(4220)</td>
</tr>
<tr>
<td>Y(4230)</td>
</tr>
<tr>
<td>Y(4260)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Y(4274)</td>
</tr>
<tr>
<td>X(4350)</td>
</tr>
<tr>
<td>Y(4360)</td>
</tr>
<tr>
<td>X(4630)</td>
</tr>
<tr>
<td>Y(4660)</td>
</tr>
<tr>
<td>$Y_b(10890)$</td>
</tr>
</tbody>
</table>

**TABLE V**: Neutral mesons above open flavor threshold excluding isospin partners of charged states.
Identification with experimental states

Most of the candidates have $1^{--}$ or $0^{++}/2^{++}$ since the main observation channels are production by $e^+e^-$ or $\gamma\gamma$ annihilation respectively.

▶ Charmonium states (Belle, CDF, BESIII, Babar):

▶ Bottomonium states: $Y_b(10890)[1^{--}]$, $m = 10.8884 \pm 3.0$ (Belle). Possible $H_1$ candidate, $m_{H_1} = 10.79 \pm 0.15$.

However, except for $Y(4220)$, all other candidates observed decay modes violate Heavy Quark Spin Symmetry.
Conclusions

Quarkonium is a golden system to study strong interactions

Nonrelativistic Effective Field Theories provide a systematic tool to investigate a wide range of heavy quarkonium observables in the realm of QCD.

At \( T=0 \), away from threshold, EFTs allow us to make calculations with unprecedented precision, where high order perturbative calculations are possible and to systematically factorize short from long range contributions where observables are sensitive to the nonperturbative dynamics of QCD.

Some lattice calculations are still needed (glue correlators, quenched and unquenched Wilson loops with field insertions).

At finite \( T \) allow us to give the appropriate definition and define a calculational scheme for quantities of huge phenomenological interest like the q\( q \)bar potential and energies at finite \( T \).

In the EFT framework heavy quark bound states become a unique laboratory for the study of strong interaction from the high energy to the low energy scales.
Conclusions

For states close or above the strong decay threshold the situation is much more complicated.

Many degrees of freedom show up and the absence of a clear systematic is an obstacle to a universal picture.

We have presented results obtained for the hybrid masses in pNRQCD that show a very rich structure of multiplets.

These results are promising but need to be complemented by decay and transitions calculations. A version of strongly coupled pNRQCD including hybrids should be eventually obtained in this framework and the inclusion of the operators carrying the dynamics light quark degrees of freedom should be realized.
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These results are promising but need to be complemented by decay and transitions calculations. A version of strongly coupled pNRQCD including hybrids should be eventually obtained in this framework and the inclusion of the operators carrying the dynamics light quark degrees of freedom should be realized.

- Fundamental experimental input (like confirmation, quantum numbers, widths and masses) is still crucially missing for some of these states.
These theory tools can match some of the intense experimental progress of the last few years and of the near future.
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\textbf{arXiv:1010.5827}
QUARK CONFINEMENT AND THE HADRON SPECTRUM XII

from 29 August 2016 to 2 September 2016
Ioannis Vellidis conference centre, THESSALONIKI

ORGANIZED BY YIOTA FOKA
BACKUP
Comparison with direct lattice computations
Charmonium sector

- Calculations done by the Hadron Spectrum Collaboration using unquenched lattice QCD with a pion mass of 400 MeV. 
  Liu et al. 2012
- They worked in the constituent gluon picture, which consider the multiplets $H_2$, $H_3$, $H_4$ as part of the same multiplet.
- Their results are given with the $\eta_c$ mass subtracted.

Error bands take into account the uncertainty on the gluelump mass ±0.15 GeV

<table>
<thead>
<tr>
<th>Split (GeV)</th>
<th>Liu</th>
<th>$V^{(0.25)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta m_{H_2-H_1}$</td>
<td>0.10</td>
<td>0.13</td>
</tr>
<tr>
<td>$\delta m_{H_4-H_1}$</td>
<td>0.24</td>
<td>0.22</td>
</tr>
<tr>
<td>$\delta m_{H_4-H_2}$</td>
<td>0.13</td>
<td>0.09</td>
</tr>
<tr>
<td>$\delta m_{H_3-H_1}$</td>
<td>0.20</td>
<td>0.44</td>
</tr>
<tr>
<td>$\delta m_{H_3-H_2}$</td>
<td>0.09</td>
<td>0.31</td>
</tr>
</tbody>
</table>

- Our masses are 0.1 – 0.14 GeV lower except the for the $H_3$ multiplet, which is the only one dominated by $\Sigma_u^-$. 
- Good agreement with the mass gaps between multiplets, in particular the $\Lambda$-doubling effect ($\delta m_{H_2-H_1}$).
Comparison with direct lattice computations

**Bottomonium sector**

- Calculations done by Juge, Kuti, Morningstar 1999 and Liao, Manke 2002 using quenched lattice QCD.
- Juge, Kuti, Morningstar 1999 included no spin or relativistic effects.
- Liao, Manke 2002 calculations are fully relativistic.

![Graph showing mass splits and mass gaps between multiplets with error bands and confidence intervals.](image)

- Error bands take into account the uncertainty on the gluelump mass ±0.15 GeV.

<table>
<thead>
<tr>
<th>Split (GeV)</th>
<th>JKM</th>
<th>$V^{(0.25)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\delta m_{H_2-H_1}$</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>$\delta m_{H_3-H_1}$</td>
<td>0.33</td>
<td>0.27</td>
</tr>
<tr>
<td>$\delta m_{H_3-H_2}$</td>
<td>0.30</td>
<td>0.22</td>
</tr>
<tr>
<td>$\delta m_{H'_1-H_1}$</td>
<td>0.42</td>
<td>0.19</td>
</tr>
</tbody>
</table>

- Our masses are 0.15 – 0.25 GeV lower except for the $H'_1$ multiplet, which is larger by 0.36 GeV.
- Good agreement with the mass gaps between multiplets, in particular the $\Lambda$-doubling effect ($\delta m_{H_2-H_1}$).

Berwein, N.B., Tarrus, Vairo 2014,
We have computed the heavy hybrid masses using a QCD analog of the Born-Oppenheimer approximation including the $\Lambda$–doubling terms by using coupled Schröringer equations.

The static energies have been obtained combining pNRQCD for short distances and lattice data for long distances.

A large set of masses for spin symmetry multiplets for $c\bar{c}$, $b\bar{c}$ and $b\bar{b}$ has been obtained.

$\Lambda$–doubling effect lowers the mass of the multiplets generated by a mix of static energies, the same pattern is observed in direct lattice calculations and QCD sum rules.

Mass gaps between multiplets in good agreement with direct lattice computations, but the absolute values are shifted.

Several experimental candidates for Charmonium hybrids belonging to the $H_1$, $H_2$, $H_4$ and $H'_1$ multiplets.

One experimental candidate to the bottomonium $H_1$ multiplet.
The two lowest laying hybrid static energies are $\Pi_u$ and $\Sigma_{-u}$. They are generated by a gluelump with quantum numbers $1^+_{-}$ and thus are degenerate at short distances. The kinetic operator mixes them but not with other multiplets. Well separated by a gap of $\sim 1$ GeV from the next multiplet with the same CP.

Coupled radial equations for $\Sigma_{-u} - \Pi_u$

\[
\begin{bmatrix}
-\frac{\partial^2}{m} + \frac{1}{mr^2} \left( \frac{I(I+1)}{2} + \frac{2}{2\sqrt{I(I+1)}} \right) \\
\frac{I(I+1)}{l(l+1)} \\
\end{bmatrix}
+ \begin{pmatrix}
E_{\Sigma}^{(0)} & 0 \\
0 & E_{\Pi}^{(0)} \\
\end{pmatrix}
\begin{pmatrix}
\Psi_{\epsilon, \Sigma}^N \\
\Psi_{\epsilon, \Pi}^N \\
\end{pmatrix}
= E_N \begin{pmatrix}
\Psi_{\epsilon, \Sigma}^N \\
\Psi_{\epsilon, \Pi}^N \\
\end{pmatrix}
\]

The coupled Schrödinger equations can be solved numerically.