





## HEAVY EXOTICS STATES with Effective Field Theories



NORA BRAMBILLA

PHYSIK DEPARTMENT TUM T30F definition, importance, occurrence of heavy exotic states

#### the state of the art theory tools

#### Calculation of hybrids masses

#### QCD and strongly coupled gauge theories: challenges and perspectives

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We highlight the progress, current status, and open challenges of QCD-driven physics, in theory and in experiment. We discuss how the strong interaction is intimately connected to a broad sweep of physical problems, in settings ranging from astrophysics and cosmology to strongly-coupled, complex systems in particle and condensed-matter physics, as well as to searches for physics beyond the Standard Model. We also discuss how success in describing the strong interaction impacts other fields, and, in turn, how such subjects can impact studies of the strong interaction. In the course of the work we offer a perspective on the many research streams which flow into and out of QCD, as well as a vision for future developments.

State	M, MeV	$\Gamma$ , MeV	$J^{PC}$	Process (mode)	Experiment $(\#\sigma)$	Year	Statu
X(3872)	$3871.68 \pm 0.17$	< 1.2	$1^{++}$	$B \to K(\pi^+\pi^- J/\psi)$	Belle $[772, 992]$ (>10), BaBar $[993]$ (8.6)	2003	Ok
				$p\bar{p} \rightarrow (\pi^+\pi^- J/\psi) \dots$	CDF [994, 995] (11.6), D0 [996] (5.2)	2003	Ok
				$pp \rightarrow (\pi^+\pi^- J/\psi) \dots$	LHCb [997, 998] (np)	2012	Ok
				$B \to K(\pi^+\pi^-\pi^0 J/\psi)$	Belle [999] $(4.3)$ , BaBar [1000] $(4.0)$	2005	Ok
				$B \to K(\gamma J/\psi)$	Belle $[1001]$ (5.5), BaBar $[1002]$ (3.5)	2005	Ok
					LHCb $[1003] (> 10)$		
				$B \to K(\gamma  \psi(2S))$	BaBar $[1002]$ (3.6), Belle $[1001]$ (0.2)	2008	NC!
					LHCb [1003] (4.4)		
				$B \to K(D\bar{D}^*)$	Belle $[1004]$ (6.4), BaBar $[1005]$ (4.9)	2006	Ok
$Z_c(3885)^+$	$3883.9\pm4.5$	$25 \pm 12$	1+-	$Y(4260) \to \pi^- (D\bar{D}^*)^+$	BES III [1006] (np)	2013	NC!
$Z_c(3900)^+$	$3891.2 \pm 3.3$	$40\pm 8$	??-	$Y(4260) \to \pi^{-}(\pi^{+}J/\psi)$	BES III $[1007]$ (8), Belle $[1008]$ (5.2)	2013	Ok
					T. Xiao <i>et al.</i> [CLEO data] [1009] (>5)		
$Z_c(4020)^+$	$4022.9 \pm 2.8$	$7.9 \pm 3.7$	??-	$Y(4260, 4360) \to \pi^-(\pi^+ h_c)$	BES III [1010] (8.9)	2013	NC!
$Z_c(4025)^+$	$4026.3 \pm 4.5$	$24.8\pm9.5$	??-	$Y(4260) \to \pi^- (D^* \bar{D}^*)^+$	BES III [1011] (10)	2013	NC!
$Z_b(10610)^+$	$10607.2 \pm 2.0$	$18.4\pm2.4$	1+-	$\Upsilon(10860) \to \pi(\pi\Upsilon(1S, 2S, 3S))$	Belle $[1012-1014]$ (>10)	2011	Ok
				$\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$\Upsilon(10860) \to \pi^- (B\bar{B}^*)^+$	Belle $[1015]$ (8)	2012	NC!
$Z_b(10650)^+$	$10652.2 \pm 1.5$	$11.5\pm2.2$	1+-	$\Upsilon(10860) \to \pi^-(\pi^+\Upsilon(1S, 2S, 3S))$	Belle $[1012, 1013]$ (>10)	2011	Ok
				$\Upsilon(10860) \to \pi^-(\pi^+ h_b(1P, 2P))$	Belle [1013] (16)	2011	Ok
				$\Upsilon(10860) \to \pi^- (B^* \bar{B}^*)^+$	Belle $[1015]$ (6.8)	2012	NC!

TABLE 10: Quarkonium-like states at the open flavor thresholds. For charged states, the C-parity is given for the neutr members of the corresponding isotriplets.

#### arXiv:1404.3723v1

TABLE 12: Quarkonium-like states above the corresponding open flavor thresholds. For charged states, the C-parity is given for the neutral members of the corresponding isotriplets.

State	M,  MeV	$\Gamma$ , MeV	$J^{PC}$	Process (mode)	Experiment $(\#\sigma)$	Year	Status
Y(3915)	$3918.4 \pm 1.9$	$20\pm5$	$0/2^{?+}$	$B \to K(\omega J/\psi)$	Belle [1050] (8), BaBar [1000, 1051] (19)	2004	Ok
				$e^+e^- \rightarrow e^+e^-(\omega J/\psi)$	Belle [1052] (7.7), BaBar [1053] (7.6)	2009	Ok
$\chi_{c2}(2P)$	$3927.2\pm2.6$	$24\pm 6$	$2^{++}$	$e^+e^- \rightarrow e^+e^-(D\bar{D})$	Belle $[1054]$ (5.3), BaBar $[1055]$ (5.8)	2005	Ok
X(3940)	$3942^{+9}_{-8}$	$37^{+27}_{-17}$	??+	$e^+e^- \to J/\psi \left( D\bar{D}^* \right)$	Belle [1048, 1049] (6)	2005	NC!
Y(4008)	$3891 \pm 42$	$255\pm42$	1	$e^+e^- \to (\pi^+\pi^- J/\psi)$	Belle [1008, 1056] (7.4)	2007	NC!
$\psi(4040)$	$4039 \pm 1$	$80\pm10$	1	$e^+e^- \to (D^{(*)}\bar{D}^{(*)}(\pi))$	PDG [1]	1978	Ok
				$e^+e^- \to (\eta J/\psi)$	Belle $[1057]$ (6.0)	2013	NC!
$Z(4050)^+$	$4051^{+24}_{-43}$	$82^{+51}_{-55}$	??+	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle $[1058]$ (5.0), BaBar $[1059]$ (1.1)	2008	NC!
Y(4140)	$4145.8\pm2.6$	$18\pm8$	??+	$B^+ \to K^+(\phi J/\psi)$	CDF $[1060]$ (5.0), Belle $[1061]$ (1.9),	2009	NC!
					LHCb $[1062]$ $(1.4)$ , CMS $[1063]$ $(>5)$		
					D0 $[1064]$ $(3.1)$		
$\psi(4160)$	$4153\pm3$	$103\pm8$	1	$e^+e^- \to (D^{(*)}\bar{D}^{(*)})$	PDG [1]	1978	Ok
				$e^+e^- \to (\eta J/\psi)$	Belle $[1057]$ (6.5)	2013	NC!
X(4160)	$4156^{+29}_{-25}$	$139^{+113}_{-65}$	??+	$e^+e^- \to J/\psi \left(D^*\bar{D}^*\right)$	Belle $[1049]$ $(5.5)$	2007	NC!
$Z(4200)^+$	$4196_{-30}^{+35}$	$370^{+99}_{-110}$	1+-	$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle $[1065]$ $(7.2)$	2014	NC!
$Z(4250)^+$	$4248_{-45}^{+185}$	$177^{+321}_{-72}$	??+	$\bar{B}^0 \to K^-(\pi^+\chi_{c1})$	Belle $[1058]$ (5.0), BaBar $[1059]$ (2.0)	2008	NC!
Y(4260)	$4250\pm9$	$108\pm12$	1	$e^+e^- \to (\pi\pi J/\psi)$	BaBar [1066, 1067] (8), CLEO [1068, 1069] (11)	2005	Ok
					Belle $[1008, 1056]$ (15), BES III $[1007]$ (np)		
				$e^+e^- \to (f_0(980)J/\psi)$	BaBar $[1067]$ (np), Belle $[1008]$ (np)	2012	Ok
				$e^+e^- \to (\pi^- Z_c(3900)^+)$	BES III $[1007]$ (8), Belle $[1008]$ (5.2)	2013	Ok
				$e^+e^- \to (\gamma X(3872))$	BES III [1070] (5.3)	2013	NC!
Y(4274)	$4293\pm20$	$35\pm16$	??+	$B^+ \to K^+(\phi J/\psi)$	CDF [1060] (3.1), LHCb [1062] (1.0),	2011	NC!
					CMS [1063] (>3), D0 [1064] (np)		
X(4350)	$4350.6^{+4.6}_{-5.1}$	$13^{+18}_{-10}$	$0/2^{?+}$	$e^+e^- \rightarrow e^+e^-(\phi J/\psi)$	Belle $[1071]$ $(3.2)$	2009	NC!
Y(4360)	$4354 \pm 11$	$78\pm16$	1	$e^+e^- \to (\pi^+\pi^-\psi(2S))$	Belle $[1072]$ (8), BaBar $[1073]$ (np)	2007	Ok
$Z(4430)^+$	$4458 \pm 15$	$166^{+37}_{-32}$	1+-	$\bar{B}^0 \to K^-(\pi^+\psi(2S))$	Belle $[1074, 1075]$ (6.4), BaBar $[1076]$ (2.4)	2007	Ok
					LHCb $[1077]$ $(13.9)$		
				$\bar{B}^0 \to K^-(\pi^+ J/\psi)$	Belle $[1065]$ $(4.0)$	2014	NC!
X(4630)	$4634_{-11}^{+9}$	$92^{+41}_{-32}$	1	$e^+e^- \to (\Lambda_c^+ \bar{\Lambda}_c^-)$	Belle $[1078]$ $(8.2)$	2007	NC!
Y(4660)	$4665 \pm 10$	$53 \pm 14$	1	$e^+e^- \to (\pi^+\pi^-\psi(2S))$	Belle $[1072]$ (5.8), BaBar $[1073]$ (5)	2007	Ok
$\Upsilon(10860)$	$10876 \pm 11$	$55 \pm 28$	1	$e^+e^- \to (B^{(*)}_{(s)}\bar{B}^{(*)}_{(s)}(\pi))$	PDG [1]	1985	Ok
				$e^+e^- \to (\pi\pi\Upsilon(1S, 2S, 3S))$	Belle [1013, 1014, 1079] (>10)	2007	Ok
				$e^+e^- \rightarrow (f_0(980)\Upsilon(1S))$	Belle [1013, 1014] (>5)	2011	Ok
				$e^+e^- \to (\pi Z_b(10610, 10650))$	Belle [1013, 1014] (>10)	2011	Ok
				$e^+e^- \to (\eta \Upsilon(1S, 2S))$	Belle [948] (10)	2012	Ok
				$e^+e^- \to (\pi^+\pi^-\Upsilon(1D))$	Belle [948] (9)	2012	Ok
$Y_b(10888)$	$10888.4\pm3.0$	$30.7^{+8.9}_{-7.7}$	1	$e^+e^- \to (\pi^+\pi^-\Upsilon(nS))$	Belle [1080] (2.3)	2008	NC!





Quarkonium (=bound state of a heavy quark and a heavy antiquark) has been instrumental for the establishing of QCD, the theory of strong interaction, and the Standard Model of Particle



Aubert et al. BNL 74

## The November revolution in 1974: the $J/\psi$ discovery



Aubert et al. BNL 74

The November revolution in 1974: the  $J/\psi$  discovery

# Samuel Ting: "It is like to stumble on a village where people live 70000 years"



The November revolution in 1974: the  $J/\psi$  discovery

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it has been the confirmation of the charm quark prediction and of QCD (strong int theory) foundations

Aubert et al. BNL 74 narrow width and asymptotic freedom annihilation at large scale controlled by small  $\alpha_s$ first discovery of a quark of large mass moving "slowly"



#### stumble on a live 70000

nation of the ction and y) foundations

edom by small  $\alpha_s$ ioving "slowly"

#### The November revolution in the '70s: more quarkonia



Eichten et al . 75, 78, 80



bbar and ccbar energy levels in comparison to Coulomb and linear potential energy levels

Variety of potential models used Confinement and asymptotic freedom--> main properties of QCD

#### Heavy quarks offer a privileged access



#### Heavy quarks offer a privileged access



Heavy quarkonia are nonrelativistic bound systems: multiscale systems

#### many scales: a challenge and an opportunity



#### Quarkonium scales



#### Quarkonium scales



NR bound states have at least 3 scales  $m \gg mv \gg mv^2 \quad v \ll 1$   $mv \sim r^{-1}$ and Accd

The system is nonrelativistic(NR)  $\Delta E \sim mv^2, \Delta_{fs} E \sim mv^4$   $v_b^2 \sim 0.1, v_c^2 \sim 0.3$ 

S statesP statesNormalized with respect to  $\chi_b(1P)$  and  $\chi_c(1P)$ 



The mass scale is perturbative  $m_Q \gg \Lambda_{
m QCD}$  $m_b \simeq 5\,{
m GeV}; m_c \simeq 1.5\,{
m GeV}$ 

#### Quarkonium as a confinement and deconfinement probe

The rich structure of separated energy scales makes QQbar an ideal probe

#### At zero temperature

• The different quarkonium radii provide different measures of the transition from a Coulombic to a confined bound state.



quarkonia probe the perturbative (high energy) and non perturbative region (low energy) as well as the transition region in dependence of their radius r

#### Quarkonium as a confinement and deconfinement probe

At finite temperature T they are sensitive to the formation of a quark gluon plasma via color screening



Debye charge screening 
$$m_D \sim gT$$
  
 $V(r) \sim -\alpha_s \frac{e^{-m_D r}}{r}$   
 $r \sim \frac{1}{m_D} \longrightarrow \begin{array}{c} \text{Bound state} \\ \text{dissolves} \end{array}$   
Matsui Satz 1986

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Matsui Satz 1986

quarkonia dissociate at different temperature in dependence of their radius: they are a Quark Gluon Plasma thermometer



#### QCD theory of Quarkonium: a very hard problem

#### Close to the bound state $\, lpha_{ m s} \sim v \,$



1  $\sum \frac{1}{E - \left(\frac{p^2}{m} + V\right)}$ 

#### QCD theory of Quarkonium: a very hard problem

#### Close to the bound state $\, lpha_{ m s} \sim v \,$



#### QCD theory of Quarkonium: a very hard problem

#### Close to the bound state $\alpha_{\rm s}\sim v$



 $E \sim mv^2$  multiscale diagrams have a complicate power counting and contribute to all orders in the coupling



#### Quarkonium with Non relativistic Effective Field Theories



#### Color degrees of freedom 3X3=1+8 singlet and octet QQbar

Hard

Soft (relative momentum)

Ultrasoft (binding energy)

 $\langle O_n \rangle \sim E_\lambda^n$ 

 $\mathcal{L}_{\rm EFT} = \sum c_n (E_{\Lambda}/\mu) \frac{O_n(\mu, \lambda)}{E_{\Lambda}}$ 

#### Quarkonium with Non relativistic Effective Field Theories

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r

#### Quarkonium with NR EFT: Non Relativistic QCD (NRQCD)



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#### Quarkonium with NR EFT: Non Relativistic QCD (NRQCD)



 $\mathcal{L}_{\text{NRQCD}} = \sum c(\alpha_{s}(m/\mu)) \times \frac{O_{n}(\mu, \lambda)}{m^{n}}$ 

n







 $\mathcal{L}_{\text{pNRQCD}} = \sum_{k} \sum_{n} \frac{1}{m^{k}} c_{k}(\alpha_{s}(m/\mu)) \times V(r\mu', r\mu) \times O_{n}(\mu', \lambda) r^{n}$ 



 $\mathcal{L}_{\text{pNRQCD}} = \sum_{k} \sum_{n} \frac{1}{m^{k}} c_{k}(\alpha_{s}(m/\mu)) \times V(r\mu', r\mu) \times O_{n}(\mu', \lambda) r^{n}$ 



N.B., Pineda, Soto, Vairo Review of Modern Physis 77(2005) 1423

#### weakly coupled pNRQCD

Singlet static potential

$$\mathcal{L} = -\frac{1}{4} F^{a}_{\mu\nu} F^{\mu\nu\,a} + \operatorname{Tr} \left\{ \mathbf{S}^{\dagger} \left( i\partial_{0} - \frac{\mathbf{p}^{2}}{m} - V_{s} \right) \mathbf{S} + \mathbf{O}^{\dagger} \left( iD_{0} - \frac{\mathbf{p}^{2}}{m} - V_{o} \right) \mathbf{O} \right\}$$
LO in  $r$ 

#### Octet static potential

$$+V_{A}\operatorname{Tr}\left\{ \mathbf{O}^{\dagger}\mathbf{r} \cdot g\mathbf{E}\,\mathbf{S} + \mathbf{S}^{\dagger}\mathbf{r} \cdot g\mathbf{E}\,\mathbf{O} \right\}$$
$$+\frac{V_{B}}{2}\operatorname{Tr}\left\{ \mathbf{O}^{\dagger}\mathbf{r} \cdot g\mathbf{E}\,\mathbf{O} + \mathbf{O}^{\dagger}\mathbf{O}\mathbf{r} \cdot g\mathbf{E} \right\}$$
$$+\cdots$$

 $r \ll \Lambda_{\rm QCD}^{-1}$ 

NLO in r

S singlet field O octet field

singlet propagator octet propagator

Pineda, Soto 97; Brambilla, Pineda, Soto, Vairo 99-
## strongly coupled pNRQCD $r \sim \Lambda_{QCD}^{-1}$

⇒ The singlet quarkonium field S of energy mv<sup>2</sup> is the only the degree of freedom of pNRQCD (up to ultrasoft light quarks, e.g. pions).

$$\mathcal{L} = \operatorname{Tr} \left\{ \mathbf{S}^{\dagger} \left( i \partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) \mathbf{S} \right\}$$

Brambilla Pineda Soto Vairo 00

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Brambilla Pineda Soto Vairo 00

- A potential description emerges from the EFT
- The potentials  $V = \operatorname{Re}V + ImV$  from QCD in the matching: get spectra and decays
- V to be calculated on the lattice or in QCD vacuum models

## Quarkonium singlet static potential

$$V = V_0 + \frac{1}{m}V_1 + \frac{1}{m^2}(V_{SD} + V_{VD})$$

$$V_s^{(0)} = \lim_{T \to \infty} \frac{i}{T} \ln \langle W(r \times T) \rangle = \lim_{T \to \infty} \frac{i}{T} \ln \langle \Box \rangle$$

$$W = \langle \exp\{ig \oint A^{\mu} dx_{\mu}\} \rangle$$



• Koma Koma NPB 769(07)79

## many experimental data and opportunities

# Quarkonium today is a golden system to study strong interactions

new theoretical tools: Effective Field Theories (EFTs) of QCD and progress in lattice QCD pNRQCD and quarkonium (away from the stong decay threshold)

### The EFT has been constructed

\*Work at calculating higher order perturbative corrections in v and alpha\_s

- \*Resumming the log
- \*Calculating/extracting nonperturbatively the low energy quantities
- \*Extending the theory (electromagnetic effect, 3 bodies)

The issue here is precision physics and the study of confinement

Precise and systematic high order calculations allow the extraction of precise determinations of standard model parameters like the quark masses and alpha\_s

 The eft has allowed to systematically factorize and to study the low energy nonperturbative contributions **pNRQCD and quarkonium** (at finite temperature T) **The EFT is being constructed** (at small coupling) Laine et al, 2007, Escobedo, Sot 2007 N. B. et al. 2008
\*Results on the static potential hint at a new physical picture of dissociation
\*Mass and width of quarkonium at m alpha^5(Y(1S) bbar at LHC), N. B. Escobedo, Ghiglieri, Vairo Soto, N. B., Ghiglieri, Petreczky, Vairo 2010
2010-2014

\*Polyakov loop calculation

The eft allows us to discover new, unexpected and important facts:

• The potential is neither the color singlet free energy nor the internal energy

 The quarkonium dissociation is a consequence of the apparence of a thermal decay width rather than being due to the color screening of the real part of the potential

We have now a coherent and systematical setup to calculate masses and width of quarkonium at finite T for small coupling

pNRQCD and quarkonium (close or above the strong decay threshold

The EFT has not yet been constructed (Exotics close to threshold) \*Degrees of freedom still to be identified

Near theshold heavy-light mesons have to be included

No systematic treatment is available; lattice calculations are also challenging and in the infancy state in this case

pNRQCD and quarkonium (close or above the strong decay threshold

The EFT has not yet been constructed (Exotics close to threshold) \*Degrees of freedom still to be identified

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### The QCD spectrum with light quarks

 We still have states just made of heavy quarks and gluons. They may develop a width because of the decay through pion emission. If new states made with heavy and light quarks develop a mass gap of order Λ<sub>QCD</sub> with respect to the former ones, then these new states may be asborbed into the definition of the potentials or of the (local or non-local) condensates.

• Brambilla et al. PRD 67(03)034018

In addition new states built using the light quark quantum numbers may form.

Soto NP PS 185(08)107

### States made of two heavy and light quarks

Pairs of heavy-light mesons: DD, BB, ...

Pairs of heavy-light baryons. • Qiao PLB 639 (2006) 263

Molecular states, i.e. states built on the pair of heavy-light mesons.
 o Torngvist PRL 67 (91) 556

Tetraquark states.

MAIANI, PICCININI, POLOSA ET AL. 2005--

Jaffe PRD 15(77)267
Ebert Faustov Galkin PLB 634(06)214

(hadro-quarkonium). Voloshin

Having the spectrum of tetraquark potentials, like we have for the gluonic excitations, would allow us to build a plethora of states on each of the tetraquark potentials, many of them developing a width due to decays through pion (or other light hadrons) emission. Diquarks have been recently investigated on the lattice.

Alexandrou et al. PRL 97(06)222002
 Fodor et al. PoS LAT2005(06)310



### States made of two heavy and light quarks

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choosing one of these degrees of freedom and an interaction originates a model for exotics.

## X(3872): interpretations



### 4-quark state with $J^{PC} = 1^{++}$

## X(3872): interpretations



Høgassen et al 05

$$X \sim (c\bar{c})_{S=1}^8 \otimes (q\bar{q})_{S=1}^8 \\ \sim (c\bar{q})_{S=0}^1 \otimes (q\bar{c})_{S=1}^1 + (c\bar{q})_{S=1}^1 \otimes (q\bar{c})_{S=0}^1$$

ą

### Molecular model



$$X \sim (c\bar{q})_{S=0}^{1} \otimes (q\bar{c})_{S=1}^{1} + (c\bar{q})_{S=1}^{1} \otimes (q\bar{c})_{S=0}^{1} \\ \sim D\,\bar{D}^{*} + D^{*}\,\bar{D}$$

This is assumed to be the dominant long-range Fock component; short-range components of the type  $(c\bar{c})_{S=1}^1 \otimes (q\bar{q})_{S=1}^1 \sim J/\psi \,\rho, \omega$  are assumed as well.



 $X \sim (cq)_{S=1}^{\bar{3}} \otimes (\bar{c}\bar{q})_{S=0}^{3} + (cq)_{S=0}^{\bar{3}} \otimes (\bar{c}\bar{q})_{S=1}^{3}$ 

the dynamical assumption (there is no scale separation like in the doubly heavy baryons) is that quark pair cluster in tightly bound color triplet diquarks (see 1-gluon exchange); the difficulty in breaking the system explains the narrow width.

### **Tetraquark model**

Predictions based on the phenomenological Hamiltonian:  $H = \sum_{i,j} \kappa_{ij} \sigma \otimes \sigma$ ; the

Predictions based on the phenomenological  $H = -\sum_{ij} C_{ij} T^a \otimes T^a \boldsymbol{\sigma} \otimes \boldsymbol{\sigma};$ 





## In some cases it is possible to develop an EFT owing to special dynamical condition



• An example is the X(3872) intepreted as a  $D^0 \bar{D}^{*0}$  or  $\bar{D}^0 D^{*0}$  molecule. In this case, one may take advantage of the hierarchy of scales:  $\Lambda_{\rm QCD} \gg m_\pi \gg m_\pi^2/M_{D^0} \approx 10 \text{ MeV} \gg E_{\rm binding}$ 

 $\approx M_X - (M_{D^{*0}} + M_{D^0}) = (0.1 \pm 1.0) \text{ MeV}$ 

Systems with a short-range interaction and a large scattering length have universal properties that may be exploited: in particular, production and decay amplitudes factorize in a short-range and a long-range part, where the latter depends only on one single parameter, the scattering length. An universal property that fits well with the observed large branching fraction of the X(3872) decaying into  $D^0 \bar{D}^0 \pi^0$  is  $\mathcal{B}(X \to D^0 \bar{D}^0 \pi^0) \approx \mathcal{B}(D^{*\,0} \to D^0 \pi^0) \approx 60\%$ . Pakvasa Suzuki 03, Voloshin 03, Braaten Kusunoki 03

### **Gluonic excitations**

A plethora of states built on each of the hybrid potentials is expected. These states typically develop a width also without including light quarks, since they may decay into lower states, e.g. like hybrid  $\rightarrow$  glueball + quark-antiquark.

We may integrate out modes scaling like 1/r and  $\Lambda_{QCD}$  and describe hybrids as heavy quark-antiquark states bound by potentials that are the energies of the corresponding gluonic excitations between static sources  $\rightarrow$  Born–Oppenheimer approximation.

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If more states are nearly degenerate, then all of these need to be considered as effective low-energy degrees of freedom and mix.

We have obtained an EFT description of hybrids matching the NRQCD static energies to pNRQCD potential in the short range

### even the case without light quark is difficult

## Lattice energies



### Symmetries

Static states classified by symmetry group  $D_{\infty h}$ Representations labeled  $\Lambda_n^{\sigma}$ 

- $\Lambda$  rotational quantum number  $|\hat{\mathbf{n}} \cdot \mathbf{K}| = 0, 1, 2...$  corresponds to  $\Lambda = \Sigma, \Pi, \Delta ...$
- $\eta$  eigenvalue of *CP*: g = +1 (gerade), u = -1 (ungerade)
- σ eigenvalue of reflections
- σ label only displayed on Σ states (others are degenerate)
  - The static energies correspond to the irreducible representations of  $D_{\infty}$
  - In general it can be more than one state for each irreducible representat
     D<sub>∞ h</sub>, usually denoted by primes, e.g. Π<sub>u</sub>, Π'<sub>u</sub>, Π''<sub>u</sub>...

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### even the case without light quark is difficult

### static Lattice energies



- Σ<sup>+</sup><sub>g</sub> is the ground state potential that generates the standard quarkonium states.
- The rest of the static energies correspond to excited gluonic states that generate hybrids.
- The two lowest hybrid static energies are Π<sub>u</sub> and Σ<sub>u</sub><sup>-</sup>, they are nearly degenerate at short distances.
- The static energies have been computed in quenched lattice QCD, the most recent data by Juge, Kuti, Morningstar, 2002 and Bali and Pineda 2003.
- Quenched and unquenched calculations for Σ<sup>+</sup><sub>g</sub> and Π<sub>u</sub> were compared in Bali et al 2000 and good agreement was found below string breaking distance.

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### Gluonic excitations in pNRQCD:more symmetry!

### In the limit $r \to 0$ more symmetry: $D_{\infty h} \to O(3) \times C$

- Several  $\Lambda_{\eta}^{\sigma}$  representations contained in one  $J^{PC}$  representation:
- Static energies in these multiplets have same  $r \rightarrow 0$  limit.



	L = 1	L = 2
$\Sigma_g^{+\prime}$	$\mathbf{r} \cdot (\mathbf{E})$	
$\Sigma_g^-$		$(\mathbf{r} \cdot \mathbf{D})(\mathbf{r} \cdot \mathbf{B})$
$\Pi_{g}$	$\mathbf{r}  imes (\mathbf{E})$	
$\Pi'_{\boldsymbol{g}}$		$\mathbf{r} \times ((\mathbf{r} \cdot \mathbf{D})\mathbf{B} + \mathbf{D}(\mathbf{r} \cdot \mathbf{B}))$
$\Delta_g$		$(\mathbf{r}  imes \mathbf{D})^i (\mathbf{r}  imes \mathbf{B})^j +$
		$+(\mathbf{r} imes \mathbf{D})^{j}(\mathbf{r} imes \mathbf{B})^{i}$
$\Sigma_u^+$		$(\mathbf{r} \cdot \mathbf{D})(\mathbf{r} \cdot \mathbf{E})$
$\Sigma_u^-$	$\mathbf{r} \cdot \mathbf{B}$	
$\Pi_u$	$\mathbf{r}  imes \mathbf{B}$	
$\Pi'_{u}$		$\mathbf{r} \times ((\mathbf{r} \cdot \mathbf{D})\mathbf{E} + \mathbf{D}(\mathbf{r} \cdot \mathbf{E}))$
$\Delta_u$		$(\mathbf{r}  imes \mathbf{D})^i (\mathbf{r}  imes \mathbf{E})^j +$
		$+({f r} imes {f D})^j ({f r} imes {f E})^i$

pNRQCD predicts the structure of multiplets at short distance and the ordering

Brambilla Pineda Soto Vairo 0

## Gluonic excitations in pNRQCD: one can determine the form of the potential

• At lowest order in the multipole expansion, the singlet decouples

while the octet is still coupled to aluons.

Static hybrids at short distance are called gluelumps and are described by a static adjoint source (O) in the presence of a gluonic field (H):

$$\mathbf{H}(R, r, t) = \mathrm{Tr}\{\mathbf{O}H\}$$



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### **Hybrid Static energies**

### $\Lambda_H$

- It is a non-perturbative quantity.
- It depends on the particular operator H<sup>a</sup>, however it is the same for operators corresponding to different projections of the same gluonic operators.
- The gluelump masses have been determined in the lattice. Foster et all 1999; Bali, Pineda 2004; Marsh Lewis 2014

$$V_H = V_o + \Lambda_H + b_H r^2 \,,$$

### b<sub>H</sub>

It is a non-perturbative quantity.



- Proportional to  $r^2$  due to rotational invariance and the multipole expansion.
- We are going to fix it through a fit to the static energies lattice data.
- Breaks the degeneracy of the potentials.

## Hybrids masses

calculated by using the potential  $V_H = V_o + \Lambda_H + b_H r^2$ in the Schrodinger equation for heavy quarks obtained from the matching between NRQCD and pNRQCD

with  $V_0$  calculated in perturbation theory,

from the lattice=gluelump mass, in  $\Lambda_H$  RS scheme =0.87\pm 0.15 GeV most of the uncertainty  $b_H$  fit from the lattice data comes from this error

and the mixing inside the multiplet taken into account with the coupled Schroedinger equations obtained in the Berwein,N.B., matching

Tarrus, Vairo 2015, see also E. Braaten et al 2013, 2014 the spin is not included at this order of the matching

### **Lowest energy multiplet** $\Sigma_u^- - \Pi_u$

- The two lowest laying hybrid static energies are  $\Pi_u$  and  $\Sigma_u^-$ .
- They are generated by a gluelump with quantum numbers 1<sup>+-</sup> and thus are degenerate at short distances.
- The kinetic operator mixes them but not with other multiplets.
- Well separated by a gap of  $\sim 1$  GeV from the next multiplet with the same CP.

$$V_H = V_o + \Lambda_H + b_H r^2$$

$$\Lambda_H$$
 and  $b_H$ 

## are nonperturbative and should be obtained from lattice calculations



#### $V^{(0.25)}$

- ▶  $r \leq 0.25$  fm: pNRQCD potential.
  - Lattice data fitted for the r = 0 0.25 fm range with the same energy offsets as in  $V^{(0.5)}$ .

$$b_{\Sigma}^{(0.25)} = 1.246 \,\mathrm{GeV/fm}^2, \quad b_{\Pi}^{(0.25)} = 0.000 \,\mathrm{GeV/fm}^2$$

ightarrow r > 0.25 fm: phenomenological potential.

• 
$$\mathcal{V}'(r) = \frac{a_1}{r} + \sqrt{a_2r^2 + a_3} + a_4$$

- Same energy offsets as in  $V^{(0.25)}$ .
- Constraint: Continuity up to first derivatives.

## Hybrid state masses from $V^{(0.25)}$

GeV	$c\overline{c}$			bē				bb				
	m <sub>H</sub>	$\langle 1/r \rangle$	E <sub>kin</sub>	$P_{\Pi}$	m <sub>H</sub>	$\langle 1/r \rangle$	E <sub>kin</sub>	$P_{\Pi}$	m <sub>H</sub>	$\langle 1/r \rangle$	E <sub>kin</sub>	$P_{\Pi}$
$H_1$	4.15	0.42	0.16	0.82	7.48	0.46	0.13	0.83	10.79	0.53	0.09	0.86
$H_1'$	4.51	0.34	0.34	0.87	7.76	0.38	0.27	0.87	10.98	0.47	0.19	0.87
$H_2$	4.28	0.28	0.24	1.00	7.58	0.31	0.19	1.00	10.84	0.37	0.13	1.00
$H_2'$	4.67	0.25	0.42	1.00	7.89	0.28	0.34	1.00	11.06	0.34	0.23	1.00
$H_3$	4.59	0.32	0.32	0.00	7.85	0.37	0.27	0.00	11.06	0.46	0.19	0.00
$H_4$	4.37	0.28	0.27	0.83	7.65	0.31	0.22	0.84	10.90	0.37	0.15	0.87
$H_5$	4.48	0.23	0.33	1.00	7.73	0.25	0.27	1.00	10.95	0.30	0.18	1.00
$H_6$	4.57	0.22	0.37	0.85	7.82	0.25	0.30	0.87	11.01	0.30	0.20	0.89
$H_7$	4.67	0.19	0.43	1.00	7.89	0.22	0.35	1.00	11.05	0.26	0.24	1.00

#### Solving the coupled Schrödinger equations we obtain

### *Consistency test:*

- 1. The multipole expansion requires  $\langle 1/r \rangle > E_{kin}$ .
- As expected the our approach works better in bottomonium than charmonium

Spin symmetry multiplets

$$\begin{array}{c|cccc} H_1 & \{1^{--}, (0, 1, 2)^{-+}\} & \Sigma_u^-, \Pi_u \\ H_2 & \{1^{++}, (0, 1, 2)^{+-}\} & \Pi_u \\ H_3 & \{0^{++}, 1^{+-}\} & \Sigma_u^- \\ H_4 & \{2^{++}, (1, 2, 3)^{+-}\} & \Sigma_u^-, \Pi_u \\ H_5 & \{2^{--}, (1, 2, 3)^{-+}\} & \Pi_u \\ H_6 & \{3^{--}, (2, 3, 4)^{-+}\} & \Sigma_u^-, \Pi_u \\ H_7 & \{3^{++}, (2, 3, 4)^{+-}\} & \Pi_u \end{array}$$

Experimental
candidates for
hybrids

State	$M \ ({\rm MeV})$	$\Gamma$ (MeV)	$J^{PC}$	Decay modes	1 <sup>st</sup> observation
X(3823)	$3823.1 \pm 1.9$	< 24	??-	$\chi_{c1}\gamma$	Belle 2013
X(3872)	$3871.68 \pm 0.17$	< 1.2	1++	$J/\psi\pi^+\pi^-,J/\psi\pi^+\pi^-\pi^0$	Belle 2003
				$D^0\bar{D}^0\pi^0,D^0\bar{D}^0\gamma$	
				$J/\psi\gamma,\psi(2S)\gamma$	
X(3915)	$3917.5 \pm 1.9$	$20\pm5$	0++	$J/\psi\omega,$	Belle 2004
$\chi_{c2}(2P)$	$3927.2\pm2.6$	$24\pm 6$	$2^{++}$	$D\bar{D},$	Belle 2005
X(3940)	$3942^{+9}_{-8}$	$37^{+27}_{-17}$	??+	$D^*\bar{D}, D\bar{D}^*$	Belle 2007
G(3900)	$3943\pm21$	$52 \pm 11$	1	$D\bar{D},$	Babar 2007
Y(4008)	$4008^{+121}_{-49}$	$226\pm97$	1	$J/\psi  \pi^+\pi^-,$	Belle 2007
Y(4140)	$4144.5\pm2.6$	$15^{+11}_{-7}$	??+	$J/\psi\phi$	CDF 2009
X(4160)	$4156^{+29}_{-25}$	$139^{+113}_{-65}$	??+	$D^*\bar{D}^*$	Belle 2007
Y(4220)	$4216\pm7$	$39 \pm 17$	1	$h_c(1P)\pi^+\pi^-,$	BESIII 2013
Y(4230)	$4230 \pm 14$	$38 \pm 14$	1	$\chi_{c0}\omega,$	BESIII 2014
Y(4260)	$4263_{-9}^{+8}$	$95 \pm 14$	1	$J/\psi  \pi^+ \pi^-,  J/\psi  \pi^0 \pi^0$	Babar 2005
				$Z_c(3900) \pi,$	
Y(4274)	$4293\pm20$	$35 \pm 16$	??+	$J/\psi\phi$	CDF 2010
X(4350)	$4350.6_{-5.1}^{+4.6}$	$13.3^{+18.4}_{-10.0}$	$0/2^{++}$	$J/\psi\phi,$	Belle 2009
Y(4360)	$4354 \pm 11$	$78\pm16$	1	$\psi(2S)\pi^+\pi^-,$	Babar 2007
X(4630)	$4634^{+\ 9}_{-11}$	$92^{+41}_{-32}$	1	$\Lambda_c^+\Lambda_c^-,$	Belle 2007
Y(4660)	$4665 \pm 10$	$53 \pm 14$	1	$\psi(2S)  \pi^+ \pi^-,$	Belle 2007
$Y_b(10890)$	$10888.4 \pm 3.0$	$30.7^{+8.9}_{-7.7}$	1	$\Upsilon(nS)\pi^+\pi^-$	Belle 2010

TABLE V: Neutral mesons above open flavor threshold excluding isospin partners of charged states.

### Identification with experimental states

Most of the candidates have  $1^{--}$  or  $0^{++}/2^{++}$  since the main observation channels are production by  $e^+e^-$  or  $\gamma\gamma$  annihilation respectively.



Charmonium states (Belle, CDF, BESIII, Babar):

Bottomonium states:  $Y_b(10890)[1^{--}]$ ,  $m = 10.8884 \pm 3.0$  (Belle). Possible  $H_1$  candidate,  $m_{H_1} = 10.79 \pm 0.15$ .

◆□▶ ◆□▶ ◆□▶ ◆□▶

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However, except for Y(4220), all other candidates observed decay modes violate Heavy Quark Spin Symmetry.

## Conclusions

Quarkonium is a golden system to study strong interactions

Nonrelativistic Effective Field Theories provide a systematic tool to investigate a wide range of heavy quarkonium observables in the realm of QCD

At T=0, away from threshold, EFTs allow us to make calculations with unprecented precision, where high order perturbative calculations are possible and to systematically factorize short from long range contributions where observables are sentitive to the nonperturbative dynamics of QCD.

Some lattice calculations are still needed (glue correlators, quenched and unquenched Wilson loops with field insertions).

At finite T allow us to give the appropriate definition and define a calculational scheme for quantities of huge phenomenological interest like the qqbar potential and energies at finite T

In the EFT framework heavy quark bound states become a unique laboratory for the study of strong interaction from the high energy to the low energy scales

### Conclusions

For states close or above the strong decay threshold the situation is much more complicated.

Many degrees of freedom show up and the absence of a clear systematic is an obstacle to a universal picture

We have presented results obtained for the hybrid masses in pNRQCD that show a very rich structure of multiplets.

These results are promising but need to be complemented by decay and transitions calculations. A version of strongly coupled pNRQCD including hybrids should be eventually obtained in this framework and the inclusion of the operators carring the synamics light quark degrees of freedom should be realized.

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 Fundamental experimental input (like confirmation, quantum numbers, widths and masses) is still crucially missing for some of these states. These theory tools can match some of the intense experimental progress of the last few years and of the near future These theory tools can match some of the intense experimental progress of the last few years and of the near future

the near future In this direction go the list of 65 production given at the end of the QWG (Quarkonium Working Group) doc treatment for all magnetic and electric transitions tic corrections contributing to the E1 transitions In particular, a rigorous treatment of the relativis-and a nonperturbative analysis of the E1 transitions M1 transit tic corrections contributing to the E1 transitions is missing. The first is relevant for transitions 7. CONCLUSIONS AND PRIORITIES and a nonperturbative analysis of the M1 transitions is missing. The first is relevant for transitions Below we present a summary of the most crucial developments in each of the major topics and sugrested tions is missing. The first is relevant for transitions the ground state. Below we present a summary of the most crucial directions for further advancement.

developments in each of the major of directions for further advancement.

Spectroscopy: An overview of the last decade's progress in heavy anarkonium spectroscopy was given in Sect. 2

Spectroscopy: An overview of the last decade's progress in heavy quarkonium spectroscopy was given in Sect. 2 With regard to experimental progress. we conclude:

n heavy quarkonium spectroscopy was given in Sect. 2 With regard to experimental progress, we conclude:

1. New measurements of inclusive hadronic cross sections (i.e. R) for  $e^+e^-$  collisions inst above

New measurements of inclusive hadronic cross open  $c\bar{c}$  and  $b\bar{b}$  flavor thresholds have enabled in.

sections (i.e., R) for ere collisions Just above of some resonance variable in open cc and oo havor thresholds have enabled in proved determinations of some resonance parallel in ters hut more precision and fine-grained studies are

proved determinations of some resonance parameters but more precision and fine-grained studies and ambieutities. Like

ters but more precision and me-grained studies are hear made studies and ambiguities. Like

needed to resolve puzzles and ambiguities. wise, progress has been made studying exclusive open-flavor two-body and multibody composition

Wise, progress has been made studying exclusive in these regions, but further data are needed to

open-havor two-body and multibody composition in these regions, but further data are neoded to clarify the details. Theory has not vert heeded to

in these regions, but further data are needed to exclusive two-body cross

clamy the details. Theory has not yet been able sections been able

2. Successful observations were made (Table 4) of 6 hew conventional heavy ouarkonium states (4 cc 2)

Successing observations were made (Table 4) of  $\delta \delta$ ; of these, only the  $\eta_{k}(1S)$  lacks a second index

new conventional heavy quarkonium states (4 cc, 2 bb); of these, only the 7%(15) lacks a second, inde pendent 50 confirmation. Improved measurement

b); of these, only the  $\mathcal{H}(1S)$  lacks a second, inde of  $n_{n}(1S)$  and  $n_{n}(2S)$  masses and widths would be

pendent 5 $\sigma$  continuation. Improved measurement of  $\eta_c(1S)$  and  $\eta_c(2S)$  masses and widths would be quite valuable. Unambiguous observations and be

of  $\eta_c(1S)$  and  $\eta_c(2S)$  masses and widths would be guite valuable. Unambiguous observations and width measurements are needed for

Quite valuable. Unambiguous observations and pre-cise mass and width measurements are needed for  $n_{\lambda}(2S)$ .  $h_{\lambda}(^{1}P_{1})$ .  $\Upsilon(^{13}D_{1})$ , and  $\Upsilon(^{13}D_{3})$  in order to cise mass and width measurements are needed to:  $\eta_b(2S)$ ,  $h_b(^1P_1)$ ,  $\Upsilon(^{13}D_1)$ , and  $\Upsilon(^{13}D_3)$  in order to: constrain theoretical descriptions.

Experimental evidence has been gathered (Table 9)

up to 17 unconventional heavy quarkonium-like  $k_{a}$ , All but  $Y_{b}(10888)$  are in the charmonium-like  $k_{a}$ ,  $k_{a}$ ,  $k_{b}$ ,

es. All but Y6(10888) are in the chamonium

region, and an but o remain uncommed at y level. Confirmation or refutation of the re-

ical interpretations for the unconventional

tai uuerpretauous tor tue uucouventoua able 20) range from coupled-channel ef

tone 20) tange trom coupled-cuanties et hark-gluon hybrids, mesonic molecules,

arks. More measurements and theorets

and international and internat particular, high-resolution measures

and  $\gamma J/\psi$  three times less. The X(3872) quantum numbers have been narrowed to  $1^{++}$  or  $2^{-+}$ .

invaluable clues to the nature of these states. 10. The complete set of Wilson loop field strength aver. 38. Further light could be shed on the nonperturbative ity expansion and its invaluable on the NRQCD veloc.

and  $\gamma J/\psi$  three times less. The X(3872) qua numbers have been narrowed to  $1+\psi$  or  $2-\psi$ .

6. The charged Z states observed in  $Z^{-}$ and  $\pi^{-} Y_{-1}$  would be if confirmed.  $manifest I_{V e_{X}}$ 

. The charged Z states observed in  $Z^{-1}$ and  $\pi^{-1}\chi_{cl}$  would be, if confirmed in  $Z^{-1}$ otic. Hence their confirmed, manifestly  $\psi(2S)$ the utmost importance.  $\psi(2S)$ 

With regard to lattice QCD calculations:

7. Lattice QCD technology has progressed to the accurate calculations

Lattice QCD point that it technology has progressed to of the energies of provide accurate calculations open flavor threshold, and also provide information

of the energies of quarkonium states below the about higher states.

8. Precise and definitive calculations of the cc and bi meson spectra below threshold are needed. Un

Precise and definitive calculations of the cc and below threshold are needed. Un-

neson spectra below threshold are needed. Quenching effects, valence quark annihilation chan-nels and spin contributions should be fully in-

quenching effects, valence quark annihilation chan-hels and spin contributions should be fully in-

9. Unquenched calculations of states above the sholds are needed. These would provide

. Unquenched calculations of states above the sholds are needed. These would provide invaluable chues to the nature of these states.

havor thresholds are needed. These would pro

11. Calculations of local and nonlocal gluon conden-sates on the lattice are needed as inputs to weakly.

Calculations of local and nonlocal gluon counded nNROCD spectra and decav calculations.

sates on the lattice are needed as inputs to weakly.

12. NRQCD matching coefficients in the lattice scheme at one loon (or more) are needed.

13. Higher-order calculations of all the relevant

14. Lattice calculation

from above the ground state.

32. New resummation schemes for the perturbative ex-pressions of the quarkonium decay widths should be

pressions of the quarkonium decay widths should be stacle to precise theoretical determinations of the developed. At the moment, this is the major ob stacle to precise theoretical determinations of the  $\chi_{(IS)}$  and  $m_{(IS)}$  inclusive and electromagnetic determinations of the

 $\begin{array}{c} stacle \ to \ precise \ theoretical \ determinations \ of \ the cays \ (Sect. \ 3.2.1). \end{array}$ 

33. More rigorous techniques to describe above and transitious,

I. More rigorous techniques to describe above descriptions still rely upon und transitions, should 3.4).

Production: The theoretical and experimental status of production of heavy quarkonia was given in Sect. 4.

Production: The theoretical and experimental status and priorities are as follows:

34. It is very important either to establish that the NRQCD factorization formula is valid to all orders

It is very important either to establish that in perturbation theory or to demonstrate that it NRQCD factorization formula is valid to all orders breaks down at some fixed order.

or production of heavy quarkonia was go Conclusions and priorities are as follows:

35. A more accurate treatment of higher-order contributions at the

A more accurate treatment of higher-order contributions and the LHC is urgently needed. The

rections to the color-singlet contributions and the LHC is urgently needed. The berturbation series that is

Tevatron and the LHC is urgently needed. The fragmentation for the perturbation series that is approach

Provided by the fragmentation series that is may be an important tool.

provided by the tragmentation-function (Sect. 4.1.5) may be an important tool.

36. An outstanding theoretical challenge is the devel opment of methods to compute color-octet level

An outstanding theoretical challenge is the devel opment of methods to compute color octet long distance NROCD production matrix elements on

opment of methods to compute color-octet long the lattice. Droduction matrix elements on

37. If NRQCD factorization is valid, it likely holds only for values of pr that are much greater than the

If NRQCD factorization is valid, it likely holds only beavy-quark mass. Therefore, it is important for

for values of pr that are much greater than the experiments to make measurements of quarkonium

heavy-quark mass. Therefore, it is important for orduction. differentially in  $v_r$ . at the highest bos. experiments to make measurements of quarkonium sible values of  $p_r$ .

<sup>1</sup> New resummation schemes for the perturbative expected eveloped. At the moment, this is the maior ob

experiment

tify direct at

direct product would both be

40. It is important to

between the CDF

, ep, pp, and

Polarization, which

Pidity Panges, /g/ < 0

A useful first step we

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41. It would be advantageous

Au mouton GUAIKODIUM DOLATION commentation time to an formation info comments of the second info

Spin-quantization frames and t

Spunguanus and the second and the se Invariant quanture out of the falles [722, 723, 1031]

anterent trauco las interest pola

Vaken in companies interview of the that dependences of t

Inclus to more active active to the kinematic ranges of the

have been taken into account.

42. Measurements of inclusive cross section and bolaria

Measurements of ucuus ve charmonium states we

num anguar austroutous raneters for p-wave charmonium states wo vide forther innortant information alout

nium production mechanisms.

43. Studies of quarkonium production at different  $\sqrt{s}$  at the Tevatron and the LHC. studies

Studies of quarkonum production at unset of Vs at the Tevatron and the UHC, studies hadronic energy hear to and away from the quarko

ues of Vs at the levalion and the low and any of the field of the leval of the leva

hadronic energy hear to and away from the production of heavy-flavor mesons in

hium direction at the Tevatron and the LHC, association with a quarkonium at  $e^+e^-$ , en point and the LHC, and the LHC, and the LHC, and the term of te

 $\begin{array}{l} association \ with \ a \ quarkonium \ at \ e^+e^-, \\ mentary \ to \ that \ provided \ b_V \ traditional \ observa. \end{array}$ 

Pp machines could give information that is tions of quarkonium provided by traditional observa-broduction rates and observa-

mentary to that provided by traditional observa-tions.

 $s_{uutes} or the production or heavy have$  $association with a quarkonium at <math>e^+e^-$ 

44. Theoretical uncertainties in the marie

expansions in how

duction.

45. In



## QUARK CONFINEMENT AND THE HADRON SPECTRUM XII

## from 29 August 2016 to 2 September 2016 Ioannis Vellidis conference centre, THESSALONIKI





### **Comparison with direct lattice computations**

### **Charmonium sector**

- Calculations done by the Hadron Spectrum Collaboration using unquenched lattice QCD with a pion mass of 400 MeV. Liu et all 2012
- They worked in the constituent gluon picture, which consider the multiplets H<sub>2</sub>, H<sub>3</sub>, H<sub>4</sub> as part of the same multiplet.
- Their results are given with the  $\eta_c$  mass subtracted.



Error bands take into account the uncertainty on the gluelump mass  $\pm 0.15~\text{GeV}$ 

Split (GeV)	Liu	$V^{(0.25)}$
$\delta m_{H_2-H_1}$	0.10	0.13
$\delta m_{H_4-H_1}$	0.24	0.22
$\delta m_{H_4-H_2}$	0.13	0.09
$\delta m_{H_3-H_1}$	0.20	0.44
$\delta m_{H_3-H_2}$	0.09	0.31

- Our masses are 0.1 − 0.14 GeV lower except the for the H<sub>3</sub> multiplet, which is the only one dominated by Σ<sup>−</sup><sub>u</sub>.
- Good agreement with the mass gaps between multiplets, in particular the Λ-doubling effect (δm<sub>H2</sub>-H1).

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## **Comparison with direct lattice computations**

## **Bottomonium sector**

- Calculations done by Juge, Kuti, Morningstar 1999 and Liao, Manke 2002 using quenched lattice QCD.
- ► Juge, Kuti, Morningstar 1999 included no spin or relativistic effects.
- Liao, Manke 2002 calculations are fully relativistic.



Error bands take into account the uncertainty on the gluelump mass  $\pm 0.15~\text{GeV}$ 

Split (GeV)	JKM	$V^{(0.25)}$
$\delta m_{H_2-H_1}$	0.04	0.05
$\delta m_{H_3-H_1}$	0.33	0.27
$\delta m_{H_3-H_2}$	0.30	0.22
$\delta m_{H_1'-H_1}$	0.42	0.19

- Our masses are 0.15 0.25 GeV lower except the for the H<sup>'</sup><sub>1</sub> multiplet, which is larger by 0.36 GeV.
- Good agreement with the mass gaps between multiplets, in particular the Λ-doubling effect (δm<sub>H2</sub>-H1).

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- We have computed the heavy hybrid masses using a QCD analog of the Born-Oppenheimer approximation including the Λ-doubling terms by using coupled Schröringer equations.
- The static energies have been obtained combining pNRQCD for short distances and lattice data for long distances.
- A large set of masses for spin symmetry multiplets for cc̄, bc̄ and bb̄ has been obtained.
- Λ-doubling effect lowers the mass of the multiplets generated by a mix of static energies, the same pattern is observed in direct lattice calculations and QCD sum rules.
- Mass gaps between multiplets in good agreement with direct lattice computations, but the absolute values are shifted.
- Several experimental candidates for Charmonium hybrids belonging to the H<sub>1</sub>, H<sub>2</sub>, H<sub>4</sub> and H'<sub>1</sub> multiplets.
- One experimental candidate to the bottomonium  $H_1$  multiplet.

**Coupled radial equations for**  $\Sigma_u^- - \Pi_u$ 

$$\begin{bmatrix} -\frac{\partial_r^2}{m} + \frac{1}{mr^2} \begin{pmatrix} l(l+1)+2 & 2\sqrt{l(l+1)} \\ 2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_{\Sigma}^{(0)} & 0 \\ 0 & E_{\Pi}^{(0)} \end{pmatrix} \end{bmatrix} \begin{pmatrix} \Psi_{\epsilon,\Sigma^N} \\ \psi_{\epsilon,\Pi}^N \end{pmatrix} = \mathcal{E}_N \begin{pmatrix} \Psi_{\epsilon,\Sigma^N} \\ \Psi_{\epsilon,\Pi}^N \end{pmatrix}$$
$$\begin{bmatrix} -\frac{\partial_r^2}{m} + \frac{l(l+1)}{mr^2} + E_{\Pi}^{(0)} \end{bmatrix} \psi_{-\epsilon,\Pi}^{(N)} = \mathcal{E}_N \psi_{-\epsilon,\Pi}^{(N)}.$$

The coupled Schrödinger equations can be solved numerically.

