Proton spin in leading order of the covariant approach

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Outline

- Introduction
- System of *non-interacting* fermions (J=1/2)
- Role of spins & orbital angular moments (relativistic and non-relativistic case)
- Generalization to the system of *quasi-free* fermions
- The use for description of the proton spin structure in DIS conditions & comparison with the DIS spin data
- Role of gluons in the proton spin
- Summary

Remark: *Since we work with the covariant representation, 3D description is obtained automatically.*

Introduction

Covariant approach has been discussed in the former studies, main results:

- Sum rules: Wanzura-Wilczek (WW), Burhardt-Cottinngham (BC) and Efremov-Leader-Teryaev (ELT)
- Relations between TMDs, PDFs and TMDs (giving predictions for TMDs)
- **Study and prediction of the role of OAM**

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The main aim of this talk is to further develop and extend the study of common role of the spin and OAM of quarks.

For details see P.Z. Phys. Rev. ***D*** *89, 014012 (2014).*

Non-interacting fermions

Eigenstates of angular momentum

Usual plane-wave spinors are replaced by spinor spherical harmonics (both in momentum representation):

$$u(\mathbf{p},\lambda\mathbf{n}) = \frac{1}{\sqrt{N}}\begin{pmatrix}\phi_{\lambda\mathbf{n}}\\ \frac{\mathbf{p}\sigma}{p_0+m}\phi_{\lambda\mathbf{n}}\end{pmatrix}$$

$$\frac{1}{2}\mathbf{n}\sigma\phi_{\lambda\mathbf{n}} = \lambda\phi_{\lambda\mathbf{n}}\,, \qquad N = \frac{2p_0}{p_0+m}$$

$$\Rightarrow$$

$$|j,j_z\rangle = \Phi_{jl_pj_z}(\omega) = \frac{1}{\sqrt{2\epsilon}}\begin{pmatrix}\sqrt{\epsilon+m}\,\Omega_{jl_pj_z}(\omega)\\ -\sqrt{\epsilon-m}\,\Omega_{j\lambda_pj_z}(\omega)\end{pmatrix}$$

$$\Omega_{jl_pj_z}(\omega) = \begin{pmatrix}\sqrt{\frac{j+j_z}{2j}}Y_{l_p,j_z-1/2}(\omega)\\ \sqrt{\frac{j-j_z}{2j}}Y_{l_p,j_z+1/2}(\omega)\end{pmatrix}; \quad l_p = j-\frac{1}{2},$$

$$\Omega_{jl_pj_z}(\omega) = \begin{pmatrix}-\sqrt{\frac{j-j_z+1}{2j+2}}Y_{l_p,j_z-1/2}(\omega)\\ \sqrt{\frac{j+j_z+1}{2j+2}}Y_{l_p,j_z+1/2}(\omega)\end{pmatrix}; \quad l_p = j+\frac{1}{2}$$

where ω represents the polar and azimuthal angles (θ,φ) of the momentum p with respect to the quantization axis, $l_p = j\pm 1/2$ and $\lambda_p = 2j - l_p$ (l_p defines parity).

New representation is convenient for general discussion about role of OAM.

Spinor spherical harmonics $|j,j_z>$

- SSH represent solutions of the free Dirac equation, which reflects the known QM rule:
In relativistic case spin and OAM are not separately conserved, but only sums j and $j_z = s_z + l_z$ are conserved.

- However, one can always calculate the mean values of corresponding operators:

$$s_z = \frac{1}{2}\begin{pmatrix} \sigma_z & 0 \\ 0 & \sigma_z \end{pmatrix}, \qquad l_z = -i\left(p_x \frac{\partial}{\partial p_y} - p_y \frac{\partial}{\partial p_x}\right)$$

and get the result

$$\langle s_z \rangle_{j,j_z} = \frac{1 + (2j+1)\,\mu}{4j\,(j+1)} j_z, \qquad \langle l_z \rangle_{j,j_z} = \left(1 - \frac{1 + (2j+1)\,\mu}{4j\,(j+1)}\right)$$

where $\mu = m/\varepsilon$.

Non-relativistic limit ($\mu=1$):

$\mu = m/\varepsilon$

$j \geq 1/2$

$l_p = j - 1/2$

$$\langle s_z \rangle_{j,j_z} = \frac{j_z}{2j}, \qquad \langle l_z \rangle_{j,j_z} = \left(1 - \frac{1}{2j}\right) j_z$$

Relativistic case ($\mu \to 0$):

$$\langle s_z \rangle_{j,j_z} = \frac{j_z}{4j\,(j+1)}, \qquad \langle l_z \rangle_{j,j_z} = \left(1 - \frac{1}{4j\,(j+1)}\right) j_z$$

$$\left|\langle s_z \rangle_{j,j_z}\right| \leq \frac{1}{4\,(j+1)} \leq \frac{1}{6}, \qquad \frac{\left|\langle s_z \rangle_{j,j_z}\right|}{\left|\langle l_z \rangle_{j,j_z}\right|} \leq \frac{1}{4j^2 + 4j - 1} \leq \frac{1}{2}$$

… and for $j=1/2$:

$$\left|\langle s_z \rangle_{j,j_z}\right| = \frac{1}{6} \qquad \frac{\langle s_z \rangle_{j,j_z}}{\langle l_z \rangle_{j,j_z}} = \frac{1}{2}$$

Many-fermion states

Composition of one-particle states (SSH) representing composed particle with spin $J=J_z=1/2$:

$$\left|(j_1, j_2, \ldots j_n)_c J, J_z\right\rangle = \sum_{j_{z1}=-j_1}^{j_1} \sum_{j_{z2}=-j_2}^{j_2} \ldots \sum_{j_{zn}=-j_n}^{j_n} c_j \left|j_1, j_{z1}\right\rangle \left|j_2, j_{z2}\right\rangle \ldots \left|j_n, j_{zn}\right\rangle$$

where c_j's consist of Clebsch-Gordan coefficients:

$$c_j = \left\langle j_1, j_{z1}, j_2, j_{z2} \left| J_3, J_{3z} \right.\right\rangle \left\langle J_3, J_{z3}, j_3, j_{z3} \left| J_4, J_{z4} \right.\right\rangle \ldots \left\langle J_n, J_{zn}, j_n, j_{zn} \left| J, J_z \right.\right\rangle$$

$$\langle \mathbb{S}_z \rangle_{c,1/2,1/2} = \langle s_{z1} + s_{z2} + \ldots + s_{zn} \rangle_c \,, \qquad \langle \mathbb{L}_z \rangle_{c,1/2,1/2} = \langle l_{z1} + l_{z2} + \ldots + l_{zn} \rangle_c$$

$$\langle \mathbb{S}_z \rangle_{c,1/2,1/2} + \langle \mathbb{L}_z \rangle_{c,1/2,1/2} = \frac{1}{2},$$

$$|\langle \mathbb{S}_z \rangle| \leq \frac{1}{6}, \qquad \frac{|\langle \mathbb{S}_z \rangle|}{|\langle \mathbb{L}_z \rangle|} \leq \frac{1}{2} \qquad J_z = \langle \mathbb{L}_z \rangle + \langle \mathbb{S}_z \rangle = \frac{1}{2}$$

for $\mu \rightarrow 0$

Structure functions:

Total spin, and other details of many-fermion (quark) states, are measured via spin structure functions. They are invariants by definition. Its measuring gives <u>invariant</u> representation of DIS data and/or state of the target in terms of parameters x_B, Q^2, ***S*.**
Calculation of SF corresponding to a many fermion state is a delicate technical task, here only results. We assume sufficiently large Q^2, then the even bound fermions (quarks) can be considered as quasi-free in any reference frame.

Spin structure functions: explicit form

For $Q^2 \gg 4M^2x^2$ we get (in terms of rest frame variables)

$$x = Q^2/2Pq$$

$$g_1(x) = \frac{1}{2}\int\left(\mathrm{u}(\epsilon)\left(p_1 + m + \frac{p_1^2}{\epsilon+m}\right) + \mathrm{v}(\epsilon)\left(p_1 - m + \frac{p_1^2}{\epsilon-m}\right)\right)\delta\left(\frac{\epsilon+p_1}{M} - x\right)\frac{d^3p}{\epsilon},$$

$$g_2(x) = -\frac{1}{2}\int\left(\mathrm{u}(\epsilon)\left(p_1 + \frac{p_1^2 - p_T^2/2}{\epsilon+m}\right) + \mathrm{v}(\epsilon)\left(p_1 + \frac{p_1^2 - p_T^2/2}{\epsilon-m}\right)\right)\delta\left(\frac{\epsilon+p_1}{M} - x\right)\frac{d^3p}{\epsilon}.$$

where u, v are functions defined by the initial state $\Psi_{1/2}$

This result is exact for SFs generated by (free) many-fermion state **J=1/2** represented by the spin spherical harmonics. For given state $\Psi_{1/2}$ we have checked calculation:

$$\langle \mathbb{S}_z \rangle = \left\langle \Psi_{1/2} \left| \mathbb{S}_z \right| \Psi_{1/2} \right\rangle = \langle s_{z1} + s_{z2} + \ldots + s_{zn} \rangle$$

$$\Gamma_1 = \int_0^1 g_1(x)\,dx$$

give equivalent results! ✓

Proton spin structure

- We assume this approximation is valid for effectively free quarks at a limited space-time domain corresponding to DIS. We do not aim to describe the complete nucleon dynamic structure, but only a very short time interval $\Delta\tau$ during DIS.

- The proton state can be formally represented by a superposition of the Fock states:

$$\Psi = \sum_{q,g} a_{qg} \left|\varphi_1, \ldots \varphi_{n_q}\right\rangle \left|\psi_1, \ldots \psi_{n_g}\right\rangle$$

- In a first step we ignore possible contribution of gluons:

$$\Psi = \sum_{q} a_{q} \left|\varphi_1, \ldots \varphi_{n_q}\right\rangle$$

where the quark states $\left|\varphi_1, \ldots \varphi_{n_q}\right\rangle$ are represented by eigenstates:

$$J = J_z = \langle \mathbb{L}_z \rangle + \langle \mathbb{S}_z \rangle = \frac{1}{2}$$

Comparison with polarized DIS data

Burkhardt-Cottingham sum rule can be easily obtained:

$$\Gamma_2 = \int_0^1 g_2(x)\,dx = 0$$

cf. experiments [25,26,27]

To simplify discussion, in the next we assume *m* ⟶ *0* :

$$g_1(x) = \frac{1}{2}\int (\mathrm{u}(\epsilon) + \mathrm{v}(\epsilon))\left(p_1 + \frac{p_1^2}{\epsilon}\right)\delta\left(\frac{\epsilon + p_1}{M} - x\right)\frac{d^3p}{\epsilon},$$

$$g_2(x) = -\frac{1}{2}\int (\mathrm{u}(\epsilon) + \mathrm{v}(\epsilon))\left(p_1 + \frac{p_1^2 - p_T^2/2}{\epsilon}\right)\delta\left(\frac{\epsilon + p_1}{M} - x\right)\frac{d^3p}{\epsilon}.$$

The functions satisfy the **Wanzura-Wilczek (WW), Efremov-Leader-Teryaev (ELT)** and other rules that we proved for massless quarks. Cf. experiments [25,26,27].

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Proton spin content

We have shown the system **J=1/2** composed of (quasi) free fermions $\mu \to 0$ satisfies:

$$|\langle \mathbb{S}_z \rangle| \leq \frac{1}{6},$$

(or the same in terms of Γ_1)

Reduced spin is compensated by OAM

$$\langle \mathbb{L}_z \rangle + \langle \mathbb{S}_z \rangle = \frac{1}{2}$$

and equality takes place for a simplest configuration:

$$\jmath_1 = \jmath_2 = \jmath_3 = \ldots = \jmath_{n_q} = \frac{1}{2}$$

Conditions of this system fit to our simplified proton.
If we change notation

$$|\langle \mathbb{S}_z \rangle| \leq \frac{1}{6},$$

$\Rightarrow$

$$\Delta\Sigma \lesssim 1/3$$

this result is well compatible with the data
(cf. experiments [30-32]):

✓

$$\Delta\Sigma = 0.32 \pm 0.03(stat.)$$

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Role of gluons in proton spin

(for details see P.Z. arXiv:1503.07924)

- Until now we assumed the simplest scenario: $\mu = m/\varepsilon \to 0$ and $\mathbf{J_g} = 0$, which gave $\Delta\Sigma \approx 1/3$. This complies with the data very well, for both, quarks and gluons.
- However, the recent data from RHIC could suggest $\mathbf{J_g} > 0$. Such value does not contradict our approach. If one admits also $\mu = m/\varepsilon > 0$, then instead of

$$|\langle \mathbb{S}_z^q \rangle| = \frac{1}{6} \qquad \frac{\langle \mathbb{S}_z^q \rangle}{\langle \mathbb{L}_z^q \rangle} = \frac{1}{2}$$

we have

$$|\langle \mathbb{S}_z^q \rangle| = \frac{1+2\tilde{\mu}}{6} \qquad \frac{\langle \mathbb{S}_z^q \rangle}{\langle \mathbb{L}_z^q \rangle} = \frac{1+2\tilde{\mu}}{2-2\tilde{\mu}} \qquad J^q = \langle \mathbb{S}_z^q \rangle + \langle \mathbb{L}_z^q \rangle \qquad \tilde{\mu} = \left\langle \frac{m}{\epsilon} \right\rangle$$

At the same time:

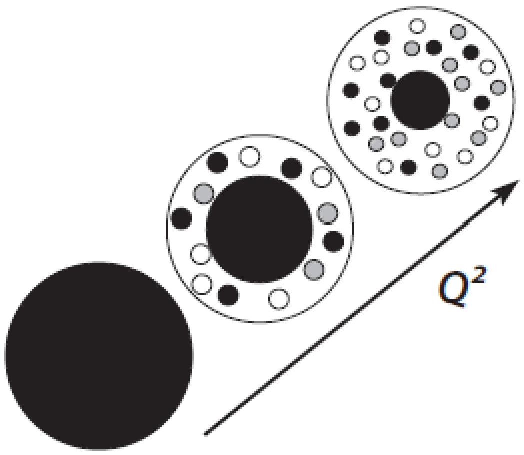
$$\frac{1}{2} = J^q + J^g \quad \Rightarrow \quad \Delta\Sigma = \frac{1}{3}\left(1 - 2J^g\right)\left(1 + 2\tilde{\mu}\right)$$

SPIN OF THE PARTICLE IN ITS SCALE DEPENDENT PICTURE

Two questions:

- ❑ How much do the virtual particles surrounding bare particle contribute to the spin of corresponding real, dressed particle?

- ❑ How much do the virtual particles mediating binding of the constituents of a composite particle contribute to its spin?



The **electron**, as a Dirac particle, in its rest frame has AM defined by its spin, $s = 1/2$. This value is the same for the dressed electron (as proved experimentally) and for the bare one (as defined by the QED Lagrangian).

So, can the OAM contribution of virtual cloud $J^{\gamma}(Q^2)$ differ from zero and how much?

Semiclassical calculation for stationary electromagnetic field:

$$\Phi_{jl_pj_z}(\mathbf{r}) = \frac{1}{\sqrt{2\epsilon}} \begin{pmatrix} \sqrt{\epsilon + m} R_{kl_p} \Omega_{jl_pj_z}(\omega) \\ -\sqrt{\epsilon - m} R_{k\lambda_p} \Omega_{j\lambda_pj_z}(\omega) \end{pmatrix}$$

$$I_\mu = (I_0, \mathbf{I}) = \Phi^\dagger_{jl_pj_z}(\mathbf{r}) \gamma^0 \gamma_\mu \Phi_{jl_pj_z}(\mathbf{r})$$

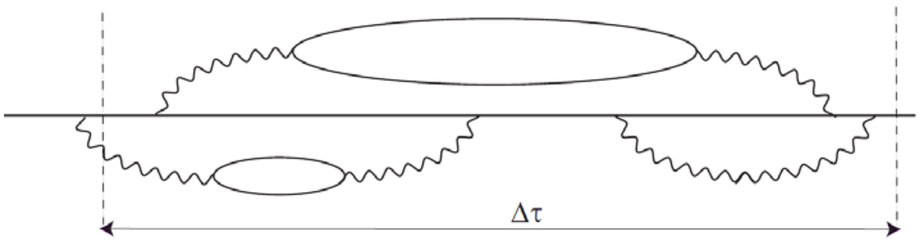
$$\mathbf{E}(\mathbf{r}) = \int I_0(\mathbf{r}') \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^{3/2}} d^3\mathbf{r}'$$

$$\mathbf{H}(\mathbf{r}) = \int \mathbf{I}(\mathbf{r}') \times \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^{3/2}} d^3\mathbf{r}'$$

$$\mathbf{J}^\gamma = \int \mathbf{r} \times (\mathbf{E} \times \mathbf{H}) d^3\mathbf{r},$$

$$\mathbf{J}^\gamma = 0$$

This result represents a mean value, which is not influenced by the fluctuations generated by single γ.



Can a similar calculation be done for the color field ?

Summary

- In the framework of the covariant QPM (spin spherical harmonics representation) we have studied the interplay between the spins and OAMs of the quarks, which collectively generate the proton spin.
- We have shown the ratio $\mu = m/\varepsilon$ plays a crucial role, since it controls a "contraction" of the spin component which is compensated by the OAM. It is an QM effect of relativistic kinematics.
- A very good agreement with the data, particularly as for $\Delta\Sigma$ is a strong argument in favor of this approach.
- The role of gluons was discussed in the context of recent RHIC data.

Thank you for your attention!