Searching for the QCD critical point through power-law fluctuations of the proton density in A+A collisions at 158A GeV

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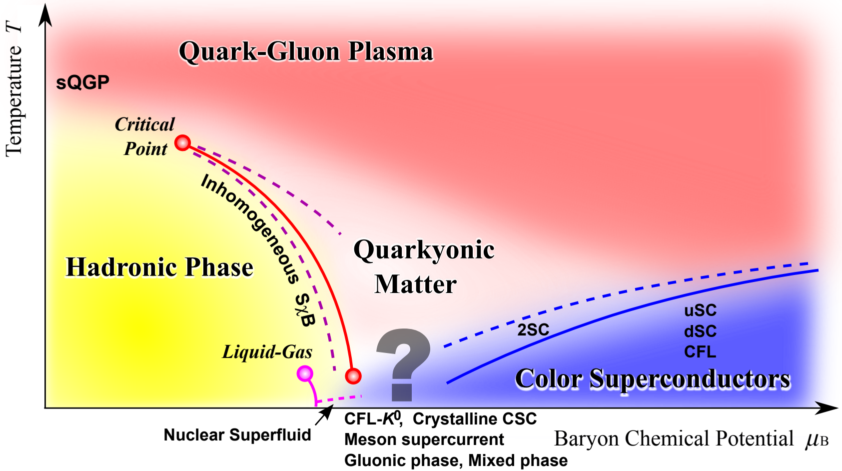
Phase diagram of QCD

- Objective: Detection / existence of the QCD Critical Point (CP)



K. Fukushima, T. Hatsuda, Rept. Prog. Phys. 74:014001 (2011)

Observables for critical fluctuations

- Detection of "chiral" critical point (CP) $\Rightarrow$ critical fluctuations of the order parameter
- Order parameter = "chiral" condensate

$$\sigma(x) = \langle \bar{q}(x) q(x) \rangle$$

($q(x)$ = quark field, sigma-field $\sigma(x)$=quantum state (wave function) describing the "chiral" condensate)

- In medium (finite baryon density) sigma-field mixes with net baryon density
- (Critical) fluctuations of the sigma field transferred to the net baryon density
- Look for observables tailored for CP search in ion collisions. Scan the phase diagram for the existence and location of the CP by varying the energy and size of the collision system.

Self-similar density fluctuations

- In our analysis, we use local observables ⇒ not sensitive to experimental acceptance, contrary to global observables.
- Local observable ⇒ self-similar density fluctuations of the order parameter in transverse configuration space (random fractal) ⇒

Power-law dependence (within scales) of the density-density correlation functions in transverse momentum space	⇔	Intermittency analysis (critical opalescence, correlation length vs. size)

[F.K. Diakonos, N.G. Antoniou and G. Mavromanolakis, PoS (CPOD2006) 010, Florence]

Critical exponents

- Power-law exponents are determined by universality class (critical exponents). For ideal, infinite size system in 3-D Ising class:

Sigmas

$$\langle n_\sigma(k) n_\sigma(k') \rangle \sim |k - k'|^{-4/3}$$

Baryons

$$\langle n_B(k) n_B(k') \rangle \sim |k - k'|^{-5/3}$$

where:
$n_\sigma(k) = \sigma^2(k)$,
$n_B =$ net baryon density at midrapidity,
k, k' are transverse momenta.

- The coupling of the (isospin zero) σ-field with protons transfers critical fluctuations to the net proton density

[Y. Hatta and M. A. Stephanov, PRL**91**, 102003 (2003).]

Observing power-law fluctuations

Experimental observation of local, power-law distributed fluctuations

$\Downarrow$

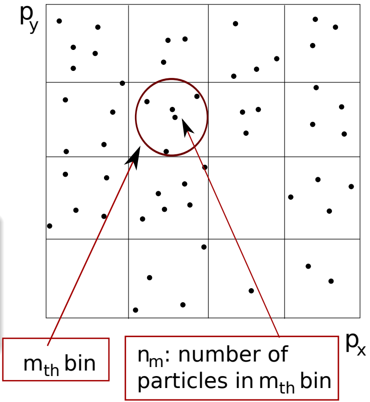
Intermittency in transverse momentum space (net protons at mid-rapidity)

(Critical opalescence in ion collisions)

- Transverse momentum space is partitioned into M^2 cells
- Calculate second factorial moments $F_2(M)$ as a function of cell size $\Leftrightarrow$ number of cells M:

$$F_2(M) \equiv \frac{\sum_m \langle n_m(n_m-1)\rangle}{\sum_m \langle n_m\rangle^2},$$

where $\langle\ldots\rangle$ denotes averaging over events.



Subtracting the background from factorial moments

- Experimental data is noisy $\Rightarrow$ a background of uncorrelated/non-critical pairs must be subtracted at the level of factorial moments.
- Intermittency will be revealed at the level of subtracted moments $\Delta F_2(M)$.

Partitioning of pairs into critical/background

$$\langle n(n-1)\rangle = \underbrace{\langle n_c(n_c-1)\rangle}_{\text{critical}} + \underbrace{\langle n_b(n_b-1)\rangle}_{\text{background}} + \underbrace{2\langle n_b n_c\rangle}_{\text{mixed term}}$$

$$\underbrace{\Delta F_2(M)}_{\text{correlator}} = \underbrace{F_2^{(d)}(M)}_{\text{data}} - \lambda(M)^2 \underbrace{F_2^{(b)}(M)}_{\text{background}} - 2 \underbrace{\lambda(M)}_{\text{ratio } \frac{<n>_b}{<n>_d}} (1-\lambda(M))\, f_{bc}$$

- The mixed term can be neglected for dominant background (non-trivial! Justified by CMC simulations)

Scaling of factorial moments – Subtracting mixed events

For $\lambda \lesssim 1$ (background domination), $\Delta F_2(M)$ can be approximated by:

$$\Delta F_2^{(e)}(M) = F_2^{\text{data}}(M) - F_2^{\text{mix}}(M)$$

For a critical system, ΔF_2 scales with cell size (number of cells, M) as:

$$\Delta F_2(M) \sim \left(M^2\right)^{\varphi_2}$$

where φ_2 is the intermittency index.

Theoretical predictions for φ_2

universality class, effective actions

$$\varphi_{2,cr}^{(\sigma)} = \frac{2}{3} \ (0.66\ldots)$$

sigmas (neutral isoscalar dipions)

[N. G. Antoniou **et al**, Nucl. Phys. A **693**, 799 (2001)]

$$\varphi_{2,cr}^{(p)} = \frac{5}{6} \ (0.833\ldots)$$

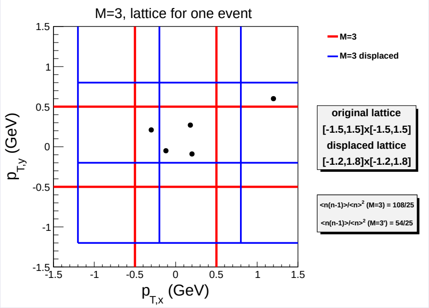
net baryons (protons)

[N. G. Antoniou, F. K. Diakonos, A. S. Kapoyannis, K. S. Kousouris, Phys. Rev. Lett. **97**, 032002 (2006)]

Improving calculation of $F_2(M)$ via lattice averaging

- Problem: With low statistics/multiplicity, lattice boundaries may split pairs of neighboring points, affecting $F_2(M)$ values (see example below).
- Solution: Calculate moments several times on different, slightly displaced lattices (see example)
- Average corresponding $F_2(M)$ over all lattices. Errors can be estimated by variance over lattice positions.
- Lattice displacement is larger than experimental resolution, yet maximum displacement must be of the order of the finer binnings, so as to stay in the correct p_T range.

Displaced lattice — a simple example



Improved confidence intervals for ϕ_2 via resampling

- In order to estimate the statistical errors of $\Delta F_2(M)$, we need to produce variations of the original event sample. This, we can achieve by using the statistical method of resampling (bootstrapping) $\Rightarrow$
  - Sample original events with replacement, producing new sets of the same statistics (# of events)
  - Calculate $\Delta F_2(M)$ for each bootstrap sample in the same manner as for the original.
  - The variance of sample values provides the statistical error of $\Delta F_2(M)$.

[W.J. Metzger, *"Estimating the Uncertainties of Factorial Moments"*, HEN-455 (2004).]

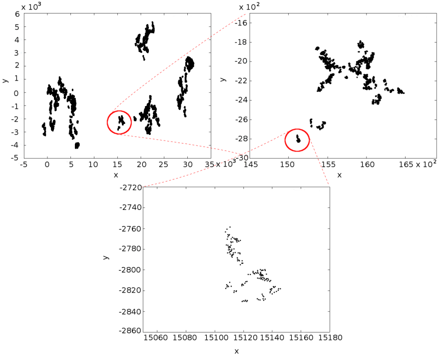
- Furthermore, we can obtain a distribution $P(\varphi_2)$ of φ_2 values. Each bootstrap sample of $\Delta F_2(M)$ is fit with a power-law:

$$\Delta F_2(M; \mathcal{C}, \varphi_2) = e^{\mathcal{C}} \cdot (M^2)^{\varphi_2}$$

and we can extract a confidence interval for φ_2 from the distribution of values. [B. Efron, *The Annals of Statistics* **7**,1 (1979)]

Critical Monte Carlo (CMC) algorithm for baryons

- Simplified version of CMC* code:
  - Fractal clusters of protons produced by random Lévy walk in transverse momentum space
  - One cluster per event, product of two 1D Lévy walks of $\tilde{d}_F^{(B,1)} = 1/6$.
  - Cluster center uniform in p_T.
  - Adjustable step & multiplicity.



Input parameters

Parameter	p_{min} (MeV)	p_{max} (MeV)	$\langle p \rangle$	$\Delta\langle p \rangle$	Δp_0 (MeV)
Value	0.5	500	3.1	1.6	800

* *[Antoniou, Diakonos, Kapoyannis and Kousouris, Phys. Rev. Lett. 97, 032002 (2006).]*

Analysed data sets & cuts

A	"C"+C*	"Si"+Si*	Pb+Pb (01I)	Pb+Pb (00B)
# Bootstrap Samples	1000			
Rapidity range	$-0.75 \leq y_{CM} \leq 0.75$			
# lattice positions	11 (2×5 + central)			
Lattice range (GeV)	$[-1.529, 1.471] \rightarrow [-1.471, 1.529]$			
Beam Energy	158 A GeV			
$\sqrt{s_{NN}}$	17.3 GeV			
Centrality range	$0 \rightarrow 12\%$			$0 \rightarrow 10\%$
Proton purity	$> 80\%$		$> 90\%$	
# events	148 060	165 941	200 758	329 789
$\langle p_{data} \rangle$ (after cuts)	1.6 ± 0.9	3.1 ± 1.7	10.8 ± 3.7	9.12 ± 3.15

** Beam Components: "C" = C,N, "Si" = Si,Al,P*

- Standard NA49 event/track cuts [T. Anticic *et al*, PR**C 81**, 149 (2010)].
- Mid-rapidity selected because of approximately constant proton density in rapidity in this region (also avoids nucleons in the corona).

[N.G. Antoniou, F.K. Diakonos, A.S. Kapoyannis and K.S. Kousouris, PRL.**97**, 032002 (2006)]

Split tracks & the q_{inv} cut

- Events may contain split tracks: sections of the same track erroneously identified as a pair of tracks that are close in momentum space.
- Intermittency analysis is based on pairs distribution $\Rightarrow$ split tracks can create a false positive, and so must be reduced or removed.
- Standard cuts remove part of split tracks. In order to estimate the residual contamination, we check the q_{inv} distribution of track pairs:

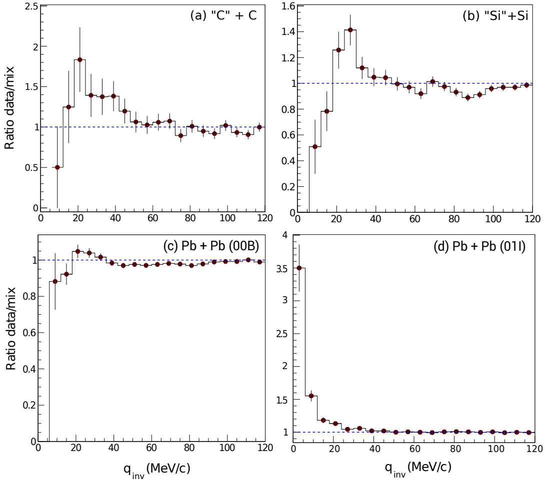
$$q_{inv}(p_i, p_j) \equiv \frac{1}{2}\sqrt{-(p_i - p_j)^2},$$

p_i : 4-momentum of i^{th} track.

- We calculate the ratio of $q_{inv}^{data} / q_{inv}^{mixed}$. A peak at low q_{inv} (below 20 MeV/c) indicates a possible split track contamination that must be removed.

q_{inv} test – Analysed datasets

- High-intensity Pb+Pb (01I), 158A GeV exhibits possible split track contamination ⇒ intermittency analysis considered unreliable.



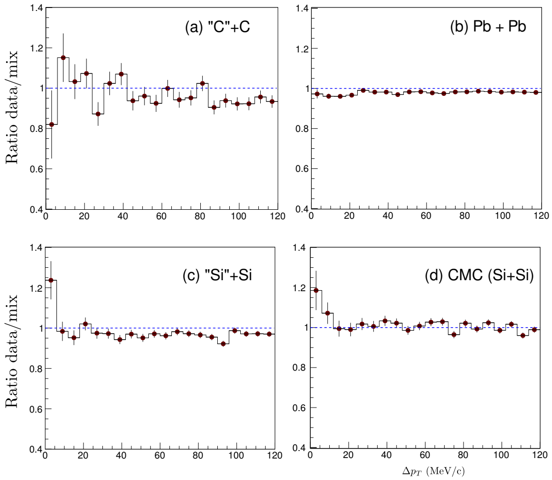
- Anti-correlations due to F-D effects and Coulomb repulsion must be removed before intermittency analysis ⇒ "dip" in low q_{inv}, peak predicted around 20 MeV/c

  [Koonin, PLB 70, 43-47 (1977)]

- Universal cutoff of $q_{inv} > 25$ MeV/c applied to all sets before analysis.

NA49 analysis – Δp_T distributions

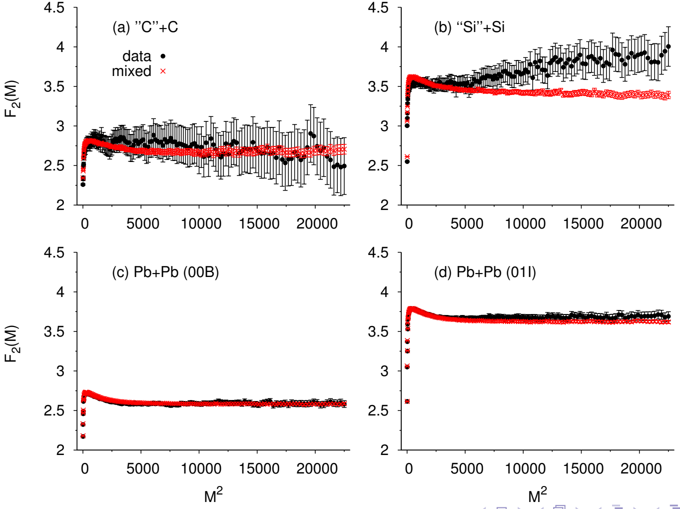
- We measure correlations in relative p_T of protons via $\Delta p_T = 1/2\sqrt{(p_{X_1} - p_{X_2})^2 + (p_{Y_1} - p_{Y_2})^2}$



- Strong correlations for $\Delta p_T \rightarrow 0$ indicate power-law scaling of the density-density correlation function $\Rightarrow$ intermittency presence
- We find a strong peak in the "Si"+Si dataset
- A similar peak is seen in the Δp_T profile of simulated CMC protons with the characteristics of "Si"+Si.

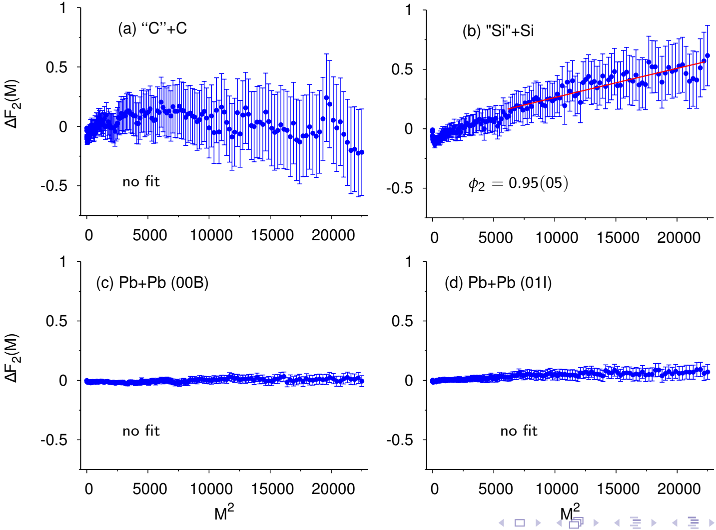
Analysis results - $F_2(M)$ for protons

- Evidence for intermittent behaviour in “Si”+Si – but large statistical errors.



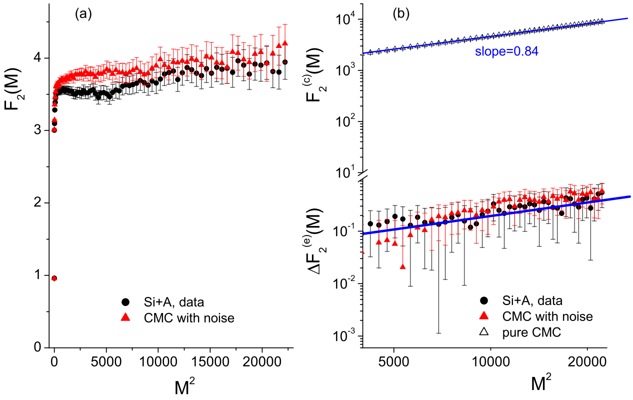
Analysis results - $\Delta F_2(M)$ for protons

- Fit with $\Delta F_2^{(e)}(M\,;\,\mathcal{C},\phi_2) = e^{\mathcal{C}} \cdot \left(M^2\right)^{\phi_2}$, for $M^2 \geq 6000$



Noisy CMC (baryons) – estimating the level of background

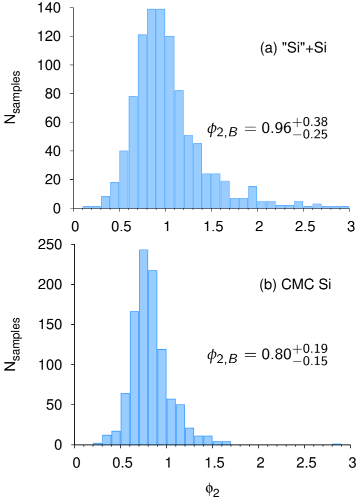
- $F_2(M)$ of noisy CMC approximates "Si"+Si for $\lambda \approx 0.99$
- Correlator $\Delta F_2^{(e)}(M)$ has slope $\phi_2 = 0.80^{+0.19}_{-0.15}$, very close to $\phi_2 = 0.84$ of pure $F_2^{(c)}(M)$



- The correlator reproduces the critical behaviour of pure CMC, even though their moments differ by orders of magnitude!
- Based on noisy CMC results, we conclude that omission of the cross-term is a reasonable approximation for dominant background.

Analysis results - φ_2 bootstrap distribution

- Distributions are highly asymmetric due to closeness of $F_2^{(d)}(M)$ to $F_2^{(m)}(M)$.



- CMC model with a dominant background can reproduce the spread of ϕ_2 values observed in the "Si"+Si dataset
- The spread is partly artificial due to pathological fits (negative $\Delta F_2(M)$ values in some bootstrap samples)
- Weighted fits, according to parameter error estimates, yield tighter confidence intervals – however, assigning weights is ambiguous.

Summary and outlook

Intermittency analysis in transverse momentum space of NA49 data for central "C"+C, "Si"+Si and Pb+Pb collisions has been performed.

- For protons at midrapidity we have found significant power-law fluctuations in "Si"+Si at 158A GeV. No significant intermittent behaviour is observed in "C"+C and low-intensity Pb+Pb (00B) data sets.
- The **intermittency index ϕ_2** for the **Si system overlaps with the critical QCD prediction**.
- Although the high-intensity Pb+Pb (01I) system shows some intermittency signal and possesses comparatively large statistics and event multiplicity, the q_{inv} analysis reveals an anomaly, possibly due to split tracks. Therefore, the intermittency analysis must be considered inconclusive for Pb+Pb.

Summary and outlook

- Study of self-similar (power-law) fluctuations of the net baryon provides us with a promising set of observables for detecting the location of the QCD critical point.
- **First experimental evidence** for the **approach to the vicinity of the critical point**.
- Analysis favors a critical baryochemical potential close to the freeze-out conditions of the "Si"+Si system ($T \sim 162$ MeV, $\mu_B \sim 260$ MeV)
  [T. Anticic *et al.* (NA49 Collaboration), arXiv:1208.5292v4]

Exploring peripheral Pb+Pb collision data of NA49 at 158 A GeV and performing a systematic **intermittency study in lighter systems** (Be+Be, Ar+Sc, Xe+La) as function of energy **in NA61** will hopefully lead to an **accurate determination of the critical point location**.

Thank you!

Back Up Slides

Event & track cuts for Si+A

Event cuts:

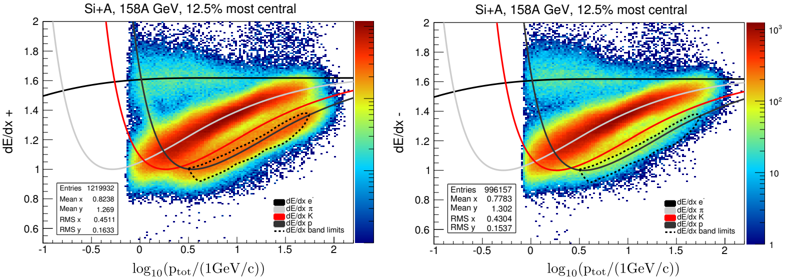
- Iflag $= 0$, $\mathrm{chi}^2 > 0$
- Beam charge cuts (Al,Si,P)
- Vertex cuts:
  - -0.4 cm $\leq V_x \leq 0.4$ cm
  - -0.5 cm $\leq V_y \leq 0.5$ cm
  - -580.3 cm $\leq V_z \leq -578.7$ cm

Track cuts:

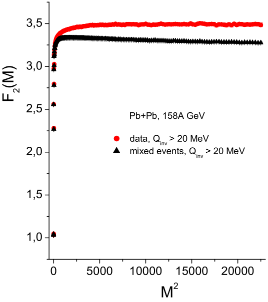
- Iflag $= 0$
- Npoints ≥ 30 (for the whole detector)
- Ratio $\frac{\mathrm{Npoints}}{\mathrm{NMaxPoints}} \geq 0.5$
- ZFirst ≤ 200
- Impact parameters: $|B_x| \leq 2$, $|B_y| \leq 1$
- dE/dx cuts for particle identification
- p_{tot} cuts (via dE/dx cut)
- rapidity cut

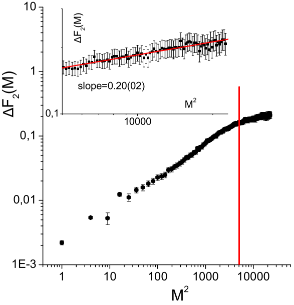
NA49 analysis – applied cuts and particle ID

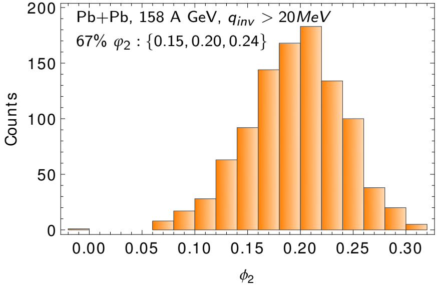
- Cuts based on the standard set of event & track cuts used in NA49 experiment [Anticic et al., PRC,83:054906 (2011)]
- Beam components merged for analysis in "Si"+Si, "C"+C
- Quality cuts to minimize split track effect
- Proton identification through cuts in particle energy loss dE/dx vs p_{TOT}:
  - Inclusive dE/dx distribution fitted in 10 bands of $\log[p_{TOT}/1\text{GeV/c}]$
  - Fit with 4 gaussian sum for $\alpha = \pi, K, p, e$
  - Probability for a track with energy loss x_i of being a proton:
    $P = f^{\text{p}}(x_i, p_i)/(f^{\pi}(x_i, p_i) + f^{\text{K}}(x_i, p_i) + f^{\text{p}}(x_i, p_i) + f^{\text{e}}(x_i, p_i))$



q_{inv} cut – Pb+Pb 158A GeV







- $\Delta F_2(M)$ shows intermittent behaviour for $M^2 > 5000$ with $\varphi_2 = 0.20(02)$
- Bootstrap φ_2 distribution is almost symmetric and centered around 0.20
- q_{inv} cut, being aggressive, distorts the signal, however the intermittency effect remains, with a reduced φ_2 value.