Anisotropic flow fluctuations in Pb+Pb collisions at LHC

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in collaboration with

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Diagram by Sasha Milov
Dynamic Regimes

Parton distribution, Nuclear geometry, Nuclear shadowing

Parton production & regeneration (or, sQGP)

Chemical freeze-out (Quark recombination)

Jet fragmentation functions

Hadron rescattering

Thermal freeze-out

Hadron decays

Diagram by Peter Steinberg
Initial spatial anisotropy is converted to anisotropy in momentum space

\[ \frac{dN}{d\phi} = \frac{N}{2\pi} \left( 1 + \sum 2v_n \cos(n(\phi - \psi_R)) \right) \]

Elliptic flow is quantified by the second Fourier coefficient \( (v_2) \) of the observed particle distribution

\[ v_2 = \langle \cos(2(\phi - \psi_R)) \rangle \propto \varepsilon \]
ECCENTRICITY

STANDARD

\[ \varepsilon_{RP} = \frac{\langle y^2 \rangle - \langle x^2 \rangle}{\langle y^2 \rangle + \langle x^2 \rangle} \]

PARTICIPANT

\[ \varepsilon_{part} = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 + 4\sigma_{xy}^2}}{\sigma_y^2 + \sigma_x^2} \]
TRIANGULAR FLOW

B. Alver and G. Roland, PRC 81 (2010) 054905

The triangular initial shape leads to triangular hydrodynamic flow

\[
\frac{dN}{d\phi} = \frac{N}{2\pi} \left(1 + \sum 2\nu_n \cos(n(\phi - \psi_n))\right)
\]

\[
\nu_2 = \langle \cos(2(\phi - \psi_R)) \rangle
\]
\[\nu_3 = 0\]

\[
\frac{dN}{d\phi} = \frac{N}{2\pi} \left(1 + \sum 2\nu_n \cos(n(\phi - \psi_n))\right)
\]

\[
\nu_2 = \langle \cos(2(\phi - \psi_2)) \rangle
\]
\[\nu_3 = \langle \cos(3(\phi - \psi_3)) \rangle\]
CROSS-TALK BETWEEN FLOW HARMONICS

Only the first few flow harmonics of final-state hadrons survive after hydrodynamic evolution.
The basic response of \( v_2 \) and \( v_3 \) to eccentricities is approx. linear

Higher flow coefficients show poor correlation with the eccentricities of the same order
II. HYDJET++ = FASTMC + PYQUEN
HYDJET++ (soft): hydrodynamics with resonances

Soft (hydro) part of HYDJET++ is based on the adapted FAST MC model:


- fast HYDJET-inspired MC procedure for soft hadron generation
- multiplicities are determined assuming thermal equilibrium
- hadrons are produced on the hypersurface represented by a parameterization of relativistic hydrodynamics with given freeze-out conditions
- chemical and kinetic freeze-outs are separated
- decays of hadronic resonances are taken into account (360 particles from SHARE data table) with severely modified decayer
- written within ROOT framework (C++)
- contains 16 free parameters (but this number may be reduced to 9)

See talks by G. Eyyubova (plenary) and B.H. Brushein Johansson (parallel) (26.08)
Parton rescattering and jet quenching

Collisional loss
(high momentum transfer approximation)

Radiative loss
(BDMS model, coherent radiation)

Strength of e-loss in PYQUEN is determined mainly by initial maximal temperature $T_0$ of hot matter in central ($b=0$) PbPb collisions (depends also on formation time $\tau_0$ and # of quark flavors $N_f$)
PYQUEN: physics frames

General kinetic integral equation:

\[
\Delta E(L, E) = \int_0^L dx \, \frac{dP}{dx}(x) \lambda(x) \frac{dE}{dx}(x, E), \quad \frac{dP}{dx}(x) = \frac{1}{\lambda(x)} \exp \left( -x / \lambda(x) \right)
\]

1. Collisional loss and elastic scattering cross section:

\[
\frac{dE}{dx} = \frac{1}{4T \lambda_s \sigma} \int_{\mu_D^2}^{\mu^2} dt \, \frac{d \sigma}{dt} t, \quad \frac{d \sigma}{dt} \simeq C \frac{2 \pi \alpha_s^2(t)}{t^2}, \quad \alpha_s = \frac{12 \pi}{(33 - 2N_f) \ln \left( t / \Lambda_{QCD}^2 \right)}, \quad C = 9/4(\%gg), 1(gq), 4/9(qq)
\]

2. Radiative loss (BDMS):

\[
\frac{dE}{dx} (m_q = 0) = \frac{2 \alpha_s C_F}{\pi \tau_L} \int \frac{d \omega}{E_{\omega \mu} \lambda_s \mu_D^2} \ln |\cos(\omega_1 \tau_1)|, \quad \omega_1 = \sqrt{i \left( 1 - y + \frac{y^2}{2} \right) \ln \frac{16}{k}}, \quad k = \frac{\mu^2 \lambda_g}{\omega(1 - y)}, \quad \tau_1 = \frac{\tau_L}{2 \lambda_g}, \quad y = \frac{\omega}{E}, \quad C_F = \frac{4}{3}
\]

“dead cone” approximation for massive quarks:

\[
\frac{dE}{dx} (m_q \neq 0) = \frac{1}{\left[ 1 + (l \omega)^{3/2} \right]^2} \frac{dE}{dx} (m_q = 0), \quad l = \left( \frac{\lambda}{\mu_D^2} \right)^{1/3} \left( \frac{m_q}{E} \right)^{4/3}
\]
III. Triangular and elliptic flows in HYDJET++ : interplay of hydrodynamics and jets
Generation of elliptic and triangular flows

\[ V_2 \quad V_3 \]

\[ R(b, \phi) = R_s(b) \frac{\sqrt{1 - \epsilon^2(b)}}{\sqrt{1 + \epsilon(b) \cos(2\phi)}} \]

\[ R(b, \varphi) = R_{\text{ell}}(b, \varphi)[1 + \epsilon_3(b) \cos[3(\varphi - \Psi_3)]] \]
LHC data vs. HYDJET++ model
Elliptic flow

Pb+Pb @ 2.76 ATeV

Closed points: CMS data $v_2\{2\text{Part} & \text{LYZ}\}$;
Open points and histograms: HYDJET++ $v_2\{\text{EP} & \Psi_2\}$
LHC data vs. HYDJET++ model

Triangular flow

Pb+Pb @ 2.76 ATeV

Closed points: CMS data $v_3\{2\text{Part} & \text{LYZ}\}$; Open points and histograms: HYDJET++ $v_3\{\text{EP} & \Psi_3\}$

**Interplay of hydrodynamics and jets**

Triangular flow

**Pb+Pb @ 2.76 ATeV**

Hydrodynamics gives mass ordering of $v_3$

The model possesses crossing of baryon and meson branches

The reason for the mass ordering break at 2 GeV/c is traced to hard processes (jets)
Ridge as interplay of $v_2$ and $v_3$

The long-range correlations appear due to flow; $v_3$ is responsible for double-hump structure; the measured amplitude of the ridge at mid-central and semi-peripheral events is well described by a superposition of $v_2$ and $v_3$. 

G. Eyyubova et al., PRC 91, 064907 (2015)
IV. Anisotropic flow fluctuations
Intrinsic fluctuations in HYDJET++

Event-by-event flow vector $V_n$

$\text{Centrality} = 0\%, \text{ i.e. } \varepsilon = \delta = \varepsilon_3 = 0$

$p(V_n) = \frac{1}{2\pi\sigma_n^2} \exp[-V_n^2/(2\sigma_n^2)]$

$p(V_n) = \frac{V_n}{\sigma_n^2} \exp[-V_n^2/(2\sigma_n^2)]$

Intrinsic fluctuations in HYDJET++

Event-by-event flow vector $V_n$

Pb+Pb @ 2.76 ATeV

Centrality = 20-25%

Bayesian unfolding procedure

\[ \frac{dN}{d\varphi} \propto 1 + 2 \sum_{n=1}^{\infty} V_{n,\text{obs}} \cos [n(\varphi - \Psi_{n,\text{obs}})] = 1 + 2 \sum_{n=1}^{\infty} (V_{n,x}^\text{obs} \cos n\varphi + V_{n,y}^\text{obs} \sin n\varphi) \]

\[ V_n^\text{obs} = \sqrt{(V_{n,x}^\text{obs})^2 + (V_{n,y}^\text{obs})^2} \]

\[ V_{n,x}^\text{obs} = V_n^\text{obs} \cos n\Psi_n^\text{obs} = \langle \cos n\varphi \rangle \]

\[ V_{n,y}^\text{obs} = V_n^\text{obs} \sin n\Psi_n^\text{obs} = \langle \sin n\varphi \rangle \]

Response function

\[ p(V_n^\text{obs} | V_n) \propto V_n^\text{obs} \exp \left[ -\frac{(V_{n,\text{obs}})^2 + V_n^2}{2\delta^2} \right] I_0 \left( \frac{V_{n,\text{obs}} V_n}{\delta^2} \right) \]

Unfolding matrix

\[ M_{ij}^{\text{iter}} = \frac{A_{ji} c_i^{\text{iter}}}{\sum_{m,k} A_{mi} A_{jk} c_k^{\text{iter}}} \]

\[ \hat{c}_{\text{iter}+1} = \hat{M}^{\text{iter}} \hat{c}, A_{ji} = p(e_j | c_i) \]

The method excludes the non-flow effects and extracts the genuine flow fluctuations.

G.Aad et al. (ATLAS), JHEP 1311 (2013) 183
Fluctuations of elliptic flow in the model

Unsmereared parameter $\varepsilon$

Pb+Pb @ 2.76 ATeV

Note: the smearing width for $\varepsilon$ is optimized at one arbitrary centrality and then fixed for other centrality bins.
Fluctuations of triangular flow in the model

Unsmeared parameter $\varepsilon_3$

Pb+Pb @ 2.76 ATeV

Normally smeared $\varepsilon_3$

Note: the smearing width for $\varepsilon_3$ is optimized at one arbitrary centrality and then fixed for other centrality bins
The HYDJET++ model allows us to investigate flow of hydro and jet parts separately, to look at reconstruction of pure hydro flow and its modification due to jet part. It also enables to study cross-talk of $v_2$ and $v_3$, while other harmonics are absent.

1. Event-by-event distributions of anisotropic flow harmonics are studied in Pb+Pb collisions at LHC within the hydro-inspired freeze-out approach
2. To compare model results with the data, unfolding procedure was employed. It removes also the non-flow fluctuations
3. Dynamical origin of the flow fluctuations in the model: the fluctuations arise due to correlations between the $(p_i, x_i)$ of the hadrons and the velocities of the fluid cells
4. Gaussian smearing of two principal parameters permits to reproduce both elliptic and triangular flow fluctuations at LHC
Back-up Slides
Relativistic heavy ion event
generator HYDJET++

HYDJET++ (HYDrodynamics + JETs) - event generator to simulate heavy ion event by merging of two independent components (soft hydro-type part + hard multi-partonic state, the latter is based on PYQUEN - PYthia QUENched routine).

http://cern.ch/lokhtin/hydjet++

(latest version 2.1)


HYDJET++ (soft): main physics assumptions

A hydrodynamic expansion of the fireball is supposed to end by a sudden system breakup at given T and chemical potentials. Momentum distribution of produced hadrons keeps the thermal character of the equilibrium distribution.

Cooper-Frye formula:

$$p^0 \frac{d^3 N_i}{d^3 p} = \int d^3 \sigma(x) p^\mu f^{eq}_i (p^\nu u_\mu(x); T, \mu_i)$$

- HYDJET++ avoids straightforward 6-dimensional integration by using the special simulation procedure (like HYDJET): momentum generation in the rest frame of fluid element, then Lorentz transformation in the global frame → uniform weights → effective von-Neumann rejection-acceptance procedure.

Freeze-out surface parameterizations

1. The Bjorken model with hypersurface

   $$\tau = (t^2 - z^2)^{1/2} = const$$

2. Linear transverse flow rapidity profile

   $$\rho_u = \frac{r}{R} \rho_u^{max}$$

3. The total effective volume for particle production at

   $$V_{eff} = \int d^3 \sigma(x) u^\mu(x) = \tau \int_0^R \int_0^{2\pi} \int_{\eta_{min}}^{\eta_{max}} r dr d\phi d\eta = 2\pi \tau \Delta \eta \left( \frac{R}{\rho_u^{max}} \right)^2 (\rho_u^{max}) \sinh \rho_u^{max} - \cosh \rho_u^{max} + 1$$
HYDJET++ (soft): hadron multiplicities

1. The hadronic matter created in heavy-ion collisions is considered as a hydrodynamically expanding fireball with EOS of an ideal hadron gas.

\[ N_i = \rho_i(T, \mu_i)V_{\text{eff}} \]

2. "Concept of effective volume" \( T=\text{const} \) and \( \mu=\text{const} \): the total yield of particle species is

\[ T(\mu_B) = a - b\mu_B - c\mu_B^4; \mu_B(\sqrt{s_{NN}}) = \frac{d}{1 + e\sqrt{s_{NN}}} \]

3. Chemical freeze-out: \( T, \mu_i = \mu_B B_i + \mu_S S_i + \mu_c C_i + \mu_Q Q_i ; T, \mu_B -\text{can be fixed by particle ratios, or by phenomenological formulas} \)

\[ f_i^{eq}(p^0*; T, \mu_i) = \frac{1}{(2\pi)^3} \frac{g_i}{\exp([p^0* - \mu_i]/T) \pm 1} \]

4. Chemical freeze-out: all macroscopic characteristics of particle system are determined via a set of equilibrium distribution functions in the fluid element rest frame:

\[ \rho_i^{eq}(T, \mu_i) = \int_0^{\infty} d^2p^* f_i^{eq}(p^0*; T(x^*), \mu(x^*)) = 4\pi \int_0^{\infty} dp^* p^2 f_i^{eq}(p^0*; T, \mu_i) \]
HYDJET++ (soft): thermal and chemical freeze-outs

1. The particle densities at the chemical freeze-out stage are too high to consider particles as free streaming and to associate this stage with the thermal freeze-out

2. Within the concept of chemically frozen evolution, the conservation of the particle number ratios from the chemical to thermal freeze-out is assumed:

\[
\frac{\rho_i^{eq}(T^{ch}, \mu_{i}^{ch})}{\rho_{\pi}^{eq}(T^{ch}, \mu_{\pi}^{ch})} = \frac{\rho_i^{eq}(T^{th}, \mu_{i}^{th})}{\rho_{\pi}^{eq}(T^{th}, \mu_{\pi}^{th})}
\]

3. The absolute values of \( \rho_i^{eq}(T^{th}, \mu_{i}^{th}) \) are determined by the choice of the free parameter of the model: effective pion chemical potential \( \mu_{\pi}^{eff,th} \) at \( T^{th} \)

For hadrons heavier than pions the Boltzmann approximation is assumed:

\[
\mu_{i}^{th} = T^{th} \ln\left( \frac{\rho_i^{eq}(T^{ch}, \mu_{i}^{ch})}{\rho_i^{eq}(T^{th}, \mu_{i}^{th} = 0)} \cdot \frac{\rho_{\pi}^{eq}(T^{th}, \mu_{\pi}^{eff,th})}{\rho_{\pi}^{eq}(T^{ch}, \mu_{\pi}^{ch})} \right)
\]

Particle momentum spectra are generated on the thermal freeze-out hypersurface, the hadronic composition at this stage is defined by the parameters of the system at chemical freeze-out.
HYDJET++ (soft): input parameters

1-5. Thermodynamic parameters at chemical freeze-out: $T_{ch}$, $\{\mu_B, \mu_S, \mu_C, \mu_Q\}$ (option to calculate $T_{ch}$, $\mu_B$ and $\mu_S$ using phenomenological parameterization $\mu_B(\sqrt{s})$, $T_{ch}(\mu_B)$ is foreseen).

6-7. Strangeness suppression factor $\gamma_S \leq 1$ and charm enhancement factor $\gamma_c \geq 1$ (options to use phenomenological parameterization $\gamma_S (T_{ch}, \mu_B)$ and to calculate $\gamma_c$ are foreseen).

8-9. Thermodynamical parameters at thermal freeze-out: $T_{th}$, and $\mu_\pi$- effective chemical potential of positively charged pions.

10-12. Volume parameters at thermal freeze-out: proper time $\tau_f$, its standard Deviation (emission duration) $\Delta\tau_f$, maximal transverse radius $R_f$.

13. Maximal transverse flow rapidity at thermal freeze-out $\rho_{umax}$.

14. Maximal longitudinal flow rapidity at thermal freeze-out $\eta_{max}$.

15. Flow anisotropy parameter: $\delta(b) \rightarrow u\mu = u\mu(\delta(b), \varphi)$

16. Coordinate anisotropy:

$$
\varepsilon(b) \rightarrow R_f(b) = R_f(0) \left[ \frac{V_{eff}(\varepsilon(0), \delta(0))}{V_{eff}(\varepsilon(b), \delta(b))} \right]^{1/2} \left[ \frac{N_{part}(b)}{N_{part}(0)} \right]^{1/3}
$$

For impact parameter range $b_{min}$-$b_{max}$:

$$
V_{eff}(b) = V_{eff}(0) \frac{N_{part}(b)}{N_{part}(0)}, \quad \tau_f(b) = \tau_f(0) \left[ \frac{N_{part}(b)}{N_{part}(0)} \right]^{1/3}
$$
Freeze-out hypersurface for non-central collisions

In non-central collisions the shape of the emission region is approximated by an ellipse

\[ R(b, \phi) = R_s(b) \frac{\sqrt{1 - \epsilon^2(b)}}{\sqrt{1 + \epsilon(b) \cos(2\phi)}} \]

the freezeout fireball radius in the reaction plane

\[ \epsilon(b) \] ellipse eccentricity in the z-x plane

\[ R_s(b) = \sqrt{(R_x^2 + R_y^2)/2} \]

We use a scaling approximation

\[ R_s(b) = R_s(b=0) \sqrt{1 - \epsilon_s(b)} \] where \( R_s(b=0) = R \) is the fireball radius in central collisions

\[ \epsilon_s(b) \] is used as an input parameter which depends on collision energy, radius of colliding nuclei and impact parameter.

Since we do not trace the hydro evolution, we have to introduce also a parameter for the momenta anisotropy developed during the hydro evolution.

The azimuthal angle of the fluid velocity vector is rotated so that is not identical with the spatial azimuthal angle.

\[ u^\mu(t, \vec{x}) = \{ y_\phi \cosh \rho_u \cosh \eta_u, \sqrt{1 + \delta(b)} \sinh \rho_u \cos \phi, \sqrt{1 - \delta(b)} \sinh \rho_u \sin \phi, y_\phi \cosh \rho_u \sinh \eta_u \} \]

where \[ y_\phi = \sqrt{1 + \delta(b) \tanh^2 \rho_u \cos(2 \phi)} \]

The fluid velocity azimuthal angle is related to the spatial azimuthal angle through:

\[ \tan \phi_u = \sqrt{\frac{1 - \delta(b)}{1 + \delta(b)}} \tan \phi \]
Monte-Carlo simulation of hard component (including nuclear shadowing) in HYDJET/HYDJET++

• Calculating the number of hard NN sub-collisions $N_{\text{jet}} (b, P_{\text{tmin}}, \sqrt{s})$ with $P_t > P_{\text{tmin}}$ around its mean value according to the binomial distribution.

• Selecting the type (for each of $N_{\text{jet}}$) of hard NN sub-collisions ($pp$, $np$ or $nn$) depending on number of protons ($Z$) and neutrons ($A-Z$) in nucleus $A$ according to the formula: $Z = A / (1.98 + 0.015A^{2/3})$.

• Generating the hard component by calling PYQUEN $n_{\text{jet}}$ times.

• Correcting the PDF in nucleus by the accepting/rejecting procedure for each of $N_{\text{jet}}$ hard NN sub-collisions: comparison of random number generated uniformly in the interval $[0,1]$ with shadowing factor $S(r_1,r_2,x_1,x_2,Q_2) \leq 1$ taken from the adapted impact parameter dependent parameterization based on Glauber-Gribov theory (K. Tywoniuk et al., Phys. Lett. B 657 (2007) 170).
Ratio  $\frac{v_3}{v_2}$ for different production modes

(a) all; (b) hydro + decays; (c) directly produced; (d) hydro w/o decays

B.H. Brusheim Johansson et al., to be submitted
The $p_T$ spectra of $\pi$, $K$, $p$, $\Lambda$ with HYDJET++ model (RHIC)

The slope for the hydro part depends strongly on mass:

- the heavier the particle -- the harder the spectrum

The hydro part dies out earlier for light particles than for heavy ones.
The secondary pion spectrum is much softer than proton spectrum.
IV. Influence of resonance decays
Influence of resonance decays on v3 of different particles at LHC

Pions, Kaons, Baryons: the maximum of the resulting flow is shifted to higher pT compared to the flow of directly produced particles.
Triangular flow of direct and secondary particles at LHC

(Similar to elliptic flow): At low transverse momenta pions from baryon resonances enhance the flow whereas pions from meson resonances reduce it.

J. Crkovska et al., to be submitted
LHC data vs. HYDJET++ model

Transverse momentum

Pb+Pb @ 2.76 ATeV

Rapidity

Correlation radii (femtoscopy)
V. Number-of-constituent-quark (NCQ) scaling
Number-of-constituent-quark scaling at RHIC

Direct particles: scaling is not good. All particles: KET/nq scaling

One of the explanations of KET/nq scaling is partonic origin of the elliptic flow. However, final state effects (such as resonance decays and jets) may also lead to appearance of the scaling.
NCQ scaling for $v2$ at LHC

LHC: NCQ scaling will be only approximate (prediction, 2009)
Experimental results (LHC)

ALICE Collaboration, M. Krzewicki et al., JGP 38 (2011) 124047

The NCQ scaling is indeed only approximate (2011)
NCQ-scaling of elliptic flow at beam energy scan (RHIC)

STAR Collab., PRL 110 (2013) 142301

- NCQ scaling holds for particles and anti-particles separately at lower energies

Our explanation: jets (more influential at higher energies) violate the NCQ-scaling, whereas Hydro+Resonances work towards its fulfilment
NCQ scaling for v3 at LHC

No scaling for direct particles

After resonance decays

Approximate scaling for all particles

NCQ scaling for triangular flow is similar to that for the elliptic flow
VI. Interplay of v2 and v3
Hexagonal flow: \[ V_6 \propto \alpha V_2^3 + \beta V_3^2 \]

Bravina et al., PRC 89, 024909 (2014)

It would be interesting to study \( V_6(\Psi_2) \) and \( V_6(\Psi_3) \) in experiment
Centrality dependence in HYDJET++ is correct
Methods for $v_2$ calculation

(1) Event plane method

\[ v_2^{obs} \{EP\} = \langle \cos 2(\varphi_i - \Psi_2) \rangle \]

$\Psi_2$ is the calculated reaction plane angle: \[
\tan n\psi_n = \frac{\sum \omega_i \sin n\varphi_i}{\sum \omega_i \cos n\varphi_i}, \quad n \geq 1, \quad 0 \leq \psi_n < 2\pi / n
\]

\[ v_2 \{EP\} = \frac{v_2^{obs} \{EP\}}{R} = \frac{v_2^{obs} \{EP\}}{\langle \cos 2(\Psi_2 - \Psi_R) \rangle} \]

(2) Two particle correlation method

\[ v_2 \{2\} = \sqrt{\langle \cos 2(\varphi_i - \varphi_j) \rangle} \]

(3) Lee-Yang zero method

\[ G(ir) = \langle e^{irQ} \rangle, Q = \sum \cos(2\varphi) \]

Integral $v_2$ is connected with the first minimum $r_0$ of the module of the $G(ir)$:

\[ v_2 = \frac{j_0}{N\rho_0} \]

Differential flow is calculated by the formula:

\[ \frac{v_2(\rho_T)}{Nv_2} = \text{Re} \left( \frac{\langle \cos(2\varphi)e^{inQ} \rangle}{\langle Qe^{inQ} \rangle} \right) \]
Comparison of Event Plane and Lee-Yang zeroes methods (c=30%)

Event Plane method overestimates $v_2$ at high $p_T$ due to non-flow correlation (mostly because of jets).