New quantum effect: Emission of Cosmic $X-$ or $\gamma-$rays by Moving Unstable Particles at Late Times

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1. Introduction

Late time properties of unstable states arouse interest since the theoretical predictions of deviations from the exponential form of the decay law at suitably long times. The problem was that the predicted effect is almost negligible smal and therefore very difficult to its confirmations in laboratory experiments. On the other hand, numbers of created unstable particles during some astrophysical processes are so large that some of them can survive up to times t at which the survival probability depending on t transforms from the exponential form into the inverse power–like form. It appears that at this time region a new quantum effect is observed: A very rapid fluctuations of the instantaneous energy of unstable particles take place. These fluctuations of the instantaneous energy should manifest itself as fluctuations of the velocity of the particle. We show that this effect may cause unstable particles to emit electromagnetic radiation of a very wide spectrum: from radio frequencies through X–rays to γ–rays of very high energies and that the astrophysical processes are the place where this effect should be observed.

2. Preliminaries

Searching for the properties of unstable states $|\phi\rangle \in \mathcal{H}$ (where $\mathcal{H}$ is the Hilbert space of states of the considered system) one analyzes their decay law. The decay law, $\mathcal{P}_\phi(t)$ of an unstable state $|\phi\rangle$ decaying in vacuum is defined as follows

$$\mathcal{P}_\phi(t) = |a(t)|^2, \tag{1}$$

where $a(t)$ is the probability amplitude of finding the system at the time t in the initial state $|\phi\rangle$ prepared at time $t_0 = 0$,

$$a(t) = \langle\phi|\phi(t)\rangle. \tag{2}$$

and $|\phi(t)\rangle$ is the solution of the Schrödinger equation for the initial condition $|\phi(0)\rangle = |\phi\rangle$:

$$i\hbar \frac{\partial}{\partial t}|\phi(t)\rangle = H|\phi(t)\rangle, \tag{3}$$

where H denotes the total selfadjoint Hamiltonian for the system considered.

From basic principles of quantum theory it is known that the amplitude $a(t)$, and thus the decay law $\mathcal{P}_{\phi}(t)$ of the unstable state $|\phi\rangle$, are completely determined by the density of the energy distribution $\omega(\mathcal{E})$ for the system in this state (S. Krylov, V. A. Fock, Zh. Eksp. Teor. Fiz. **17**, (1947), 93.),

$$a(t) = \int_{Spec.(H)} \omega(\mathcal{E})\, e^{-\frac{i}{\hbar}\,\mathcal{E}\,t}\, d\mathcal{E}. \qquad (4)$$

where $\omega(\mathcal{E}) \geq 0$ and $a(0) = 1$.

In L. A. Khalfin paper, Zh. Eksp. Teor. Fiz. **33**, (1957), 1371; [Sov. Phys. – JETP **6**, (1958), 1053], assuming that the spect–rum of H must be bounded from below, ($Spec.(H) = [E_{min}, \infty)$ and $E_{min} > -\infty$), and using the Paley–Wiener Theorem it was proved that in the case of unstable states there must be

$$|a(t)| \;\geq\; A\, e^{-b\, t^{q}}, \qquad (5)$$

for $|t| \to \infty$. Here $A > 0$, $b > 0$ and $0 < q < 1$.

This means that the decay law $\mathcal{P}_\phi(t)$ of unstable states decaying in the vacuum, (1), can not be described by an exponential function of time t if time t is suitably long, $t \to \infty$, and that for these lengths of time $\mathcal{P}_\phi(t)$ tends to zero as $t \to \infty$ more slowly than any exponential function of t.

This effect was confirmed in experiment described in the Rothe paper (C. Rothe, S. I. Hintschich and A. P. Monkman, Phys. Rev. Lett. **96**,(2006), 163601.), where the experimental evidence of deviations from the exponential decay law at long times was reported.

If (and how) deviations from the exponential decay law at long times affect the energy of the unstable state and its decay rate at this time region.

Studying properties of unstable states at late times it is useful to express $a(t)$ in the following form

$$a(t) = a_{exp}(t) + a_{lt}(t), \tag{6}$$

where $a_{exp}(t)$ is the exponential part of $a(t)$, that is

$$a_{exp}(t) = N \exp\left[-i\frac{t}{\hbar}(E_{\phi}^{0} - \frac{i}{2}\,\varGamma_{\phi}^{0})\right], \tag{7}$$

(E_{ϕ}^{0} is the energy of the system in the state $|\phi\rangle$ measured at the canonical decay times, i.e. when $\mathcal{P}_{\phi}(t)$ has the exponential form, $\varGamma_{\phi}^{0}$ is the decay width, N is the normalization constant), and $a_{lt}(t)$ is the late time non–exponential part of $a(t)$.
From the literature it is known that the characteristic feature of survival probabilities $\mathcal{P}_{\phi}(t)$ is the presence of sharp and frequent fluctuations at the transition times region, when contributions from $|a_{exp}(t)|^2$ and $|a_{lt}(t)|^2$ into $\mathcal{P}_{\phi}(t)$ are comparable and that the amplitude $a_{lt}(t)$ and thus the probability $\mathcal{P}_{\phi}(t)$ exhibits inverse power–law behavior at the late time region for times t much later than the crossover time T.

The crossover time T can be found by solving the following equation,

$$|a_{exp}(t)|^2 = |a_{lt}(t)|^2. \tag{8}$$

In general $T \gg \tau_\phi$, where $\tau_\phi = \hbar/\varGamma_\phi^0$ is the live–time of ϕ. Formulae for T depend on the model considered (i.e. on $\omega(E)$) in general). The standard form of the decay curve, that is the form of the probability $\mathcal{P}_\phi(t)$ as a function of time t is presented in Fig. (1). In this Figure the calculations were performed using the Breit–Wigner energy distribution function, $\omega(E) \equiv \omega_{BW}(E)$, where

$$\omega_{BW}(E) \stackrel{\text{def}}{=} \frac{N}{2\pi}\varTheta(E - E_{min}) \frac{\varGamma_\phi^0}{(E - E_\phi^0)^2 + (\frac{\varGamma_\phi^0}{2})^2}, \tag{9}$$

and $\varTheta(E)$ is the unit step function. In Fig. (1) calculations were performed for $s_0 \stackrel{\text{def}}{=} (E_\phi^0 - E_{min})/\varGamma_\phi^0 = 20$.

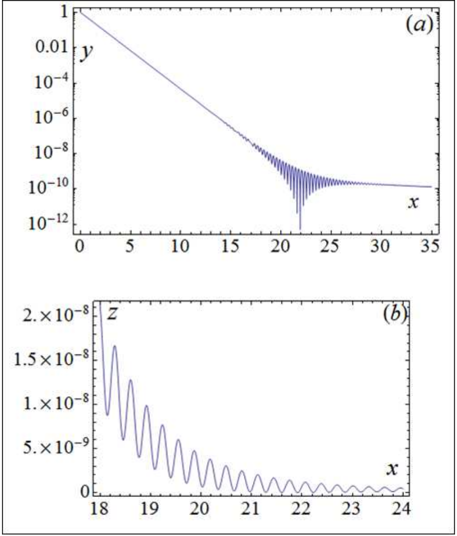


Figure: 1. Axes: $x = t/\tau_\phi, y = \mathcal{P}_\phi(t)$ — the logarithmic scale, $z = \mathcal{P}_\phi(t)$. (a) The general, typical form of the decay curve $\mathcal{P}_\phi(t)$. (b) An enlarged part of (a) showing a typical behavior of the survival probability $\mathcal{P}_\phi(t)$ at the transition times region when $t \sim T$.

3. Energy of unstable states at late times

The energy and the decay rate of the system in the state $|\phi\rangle$ under considerations, (to be more precise the instantaneous energy and the instantaneous decay rate), can be found using the following relations (for details see:

K. Urbanowski, Cent. Eur. J. Phys. **7**, 696, (2009),

$$\mathcal{E}_\phi \equiv \mathcal{E}_\phi(t) = \Re\,(h_\phi(t)), \tag{10}$$

$$\gamma_\phi \equiv \gamma_\phi(t) = -\,2\,\Im\,(h_\phi(t)), \tag{11}$$

where $\Re\,(z)$ and $\Im\,(z)$ denote the real and imaginary parts of z respectively and $h_\phi(t)$ denotes the "effective Hamiltonian" for the one–dimensional subspace of states $\mathcal{H}_{||}$ spanned by the normalized vector $|\phi\rangle$,

$$h_\phi(t) \stackrel{\rm def}{=} i\hbar\,\frac{\partial a(t)}{\partial t}\,\frac{1}{a(t)}. \tag{12}$$

It is easy to show that equivalently

$$h_{\phi}(t) \equiv \frac{\langle\phi|H|\phi(t)\rangle}{\langle\phi|\phi(t)\rangle}. \tag{13}$$

There is $\mathcal{E}_{\phi}(t) = E_{\phi}^{0}$ and $\varGamma_{\phi}(t) = \varGamma_{\phi}^{0}$ at the canonical decay times and at asymptotically late times

$$\mathcal{E}_{\phi}(t) \simeq E_{min} + \frac{c_2}{t^2} + \frac{c_4}{t^4} \ldots, \quad (\text{for } \ t \gg T), \tag{14}$$

$$\varGamma_{\phi}(t) \simeq \frac{c_1}{t} + \frac{c_3}{t^3} + \ldots \quad (\text{for } \ t \gg T), \tag{15}$$

where $c_i = c_i^*, \ i = 1, 2, \ldots,$ ($c_1 > 0$ and the sign of c_i for $i \geq 2$ depends on the model considered), so $\lim_{t\to\infty} \mathcal{E}_{\phi}(t) = E_{min}$ and $\lim_{t\to\infty} \varGamma_{\phi}(t) = 0$. Results (14) and (15) are rigorous.

The basic physical factor forcing the amplitude $a(t)$ to exhibit inverse power law behavior at $t \gg T$ is the boundedness from below of $\sigma(H)$. This means that if this condition is satisfied and $\int_{-\infty}^{+\infty} \omega(E)\, dE \;<\; \infty$, then all properties of $a(t)$, including the form of the time–dependence at $t \gg T$, are the mathematical consequence of them both. The same applies by (12) to the properties of $h_{\phi}(t)$ and concerns the asymptotic form of $h_{\phi}(t)$ and thus of $\mathcal{E}_{\phi}(t)$ and $\varGamma_{\phi}(t)$ at $t \gg T$.

Using relations (2), (4), (12) and assuming the form of $\omega(E)$ and performing all necessary calculations numerically one can find $\mathcal{E}_{\phi}(t)$ for all times t including times $t \gg T$. A typical behavior of the instantaneous energy $\mathcal{E}_{\phi}(t)$ at the transition time region is presented in Figs (2) and (3). In these figures the calculations were performed for the Breit–Wigner energy distribution function (9).

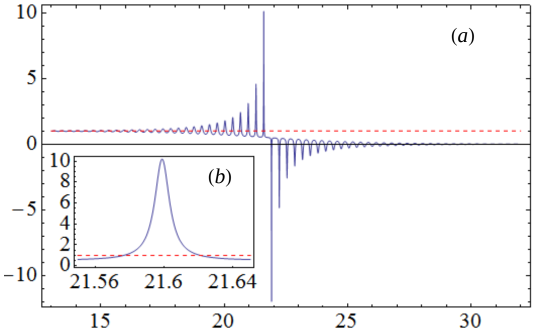


Figure: 2. Axes: $y = \kappa(t)$, $x = t/\tau_\phi$. The dashed line denotes the straight line $y = 1$.
(a) The instantaneous energy $\mathcal{E}_\phi(t)$ in the transitions time region: The case $s_0 \stackrel{\mathrm{def}}{=} (E_\phi^0 - E_{min})/\Gamma_\phi^0 = 20$.
(b) Enlarged part of (a): The highest maximum of $\kappa(t)$ in the transition times region.

where: $\kappa(t) = (\mathcal{E}_\phi(t) - E_{min})/(E_\phi^0 - E_{min})$.

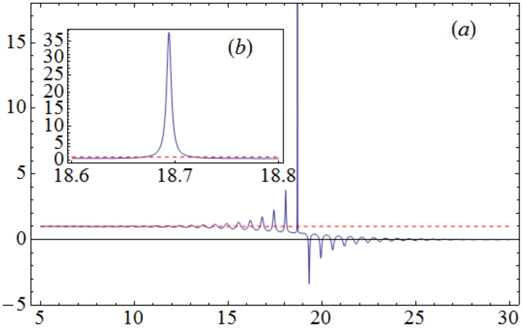


Figure: 3. The same as in Fig (2) for $s_0 = 10$.

As can be seen in Fig. (1) the characteristic feature decay curves is the presence of sharp and frequent oscillations at the transition times region.

This means that derivatives of the amplitude $a(t)$ may reach extremely large values for some times from the transition time region and the modulus of these derivatives is much larger than the modulus of $a(t)$, which is very small for these times. This explains why in this time region the real and imaginary parts of $h_\phi(t) \equiv \mathcal{E}_\phi(t) - \frac{i}{2}\gamma_\phi(t)$, which can be expressed by the relation (12), ie. by **a large derivative of $a(t)$ divided by a very small $a(t)$,** reach values much larger than the energy $\mathcal{E}_\phi^0$ of the the unstable state measured at times for which the decay curve has the exponential form (see Figs (2), (3)).

4. Possible observable effects

Remark:
Note that from the point of view of a frame of reference in which the time evolution of the unstable system was calculated the Rothe experiment as well as the picture presented in Figs (1), (2) and (3) refer to the rest coordinate system of the unstable system considered.

Properties of the ratio $\kappa(t)$ taking place for some time intervals mean that the instantaneous energy $\mathcal{E}_{\phi}(t)$ at these time intervals differs from the energy E_{ϕ}^{0} measured at times from the exponential decay time region.

The relation (13), that is

$$h_{\phi}(t) \equiv \frac{\langle \phi | H | \phi(t) \rangle}{\langle \phi | \phi(t) \rangle}.$$

explains why such an effect can occur.

Astrophysical sources of unstable particles emit them with relativistic or ultra–relativistic velocities in relation to an external observer so many of these particles move in space with ultra high energies. The question is what effects can be observed by an external observer when the unstable particle, say ϕ, which survived up to the transition times region, $t \sim T$, or longer is moving with a relativistic velocity in relation to this observer. The distance d from the source reached by this particle is of order

$$d \sim d_T,$$

where

$$d_T = v^{\phi} \cdot T',$$

and $T' = \gamma\, T$, $\gamma \equiv \gamma(v^{\phi}) = (\sqrt{1-\beta^2})^{-1}$, $\beta = v^{\phi}/c$, v^{ϕ} is the velocity of the particle ϕ. (For simplicity we assume that there is a frame of reference common for the source and observer both and that they do not move with respect to this frame of reference).

In the case of moving particles created in astrophysical processes one should consider the effect shown in Figs (2), (3) together with the fact that the particle gains extremely huge energy, W^{ϕ}, which have to be conserved.

If to assume that the unstable particle under considerations has a constant momentum $\vec{p}$, $|\vec{p}| = p > 0$, and that for simplicity $\vec{v} = (v_1, 0, 0) \equiv (v, 0, 0)$ then there is $\vec{p} = (p, 0, 0)$.

Let Λ_p be the Lorentz transformation from the reference frame $\mathcal{O}$, where the momentum of unstable particle considered is zero, $\vec{p} = 0$, into the frame $\mathcal{O}'$ where the momentum of this particle is $\vec{p} \equiv (p, 0, 0) \neq 0$ or, equivalently, where its velocity $\vec{v}$ equals $\vec{v} = \vec{v}_p \equiv \frac{\vec{p}}{m\gamma}$, where $\gamma = \frac{1}{\sqrt{1+\beta^2}}$ and $\beta = \frac{v}{c}$ and let E be the energy of the particle in the reference frame $\mathcal{O}$ and E' be the energy of the moving particle and having the momentum $\vec{p} \neq 0$. In this case the corresponding 4–vectors are:

$$\wp = (E/c, 0, 0, 0) \in \mathcal{O} \quad \text{and} \quad \wp' = (E'/c, p, 0, 0) = \Lambda_p \wp \in \mathcal{O}'.$$

There is

$$\wp' \cdot \wp' \equiv (\Lambda_p \wp) \cdot (\Lambda_p \wp) = \wp \cdot \wp$$

in Minkowski space, which is an effect of the Lorentz invariance. (Here the dot "$\cdot$" denotes the scalar product in Minkowski space). Hence, in our case:

$$\wp' \cdot \wp' \equiv (E'/c)^2 - p^2 = (E/c)^2$$

because

$$\wp \cdot \wp \equiv (E/c)^2$$

and thus

$$(E')^2 = c^2\, p^2 + E^2. \tag{16}$$

One can show that in the case of a free particle $p \equiv \frac{E}{c}\,\gamma\,\beta$. Hence

$$c^2\, p^2 + E^2 \equiv E^2\,\gamma^2\,\beta^2 \;+\; E^2 = E^2(1 \,+\, \beta^2\,\gamma^2) \equiv E^2\,\gamma^2.$$

The last relation means that the energy, E', of moving particles equals

$$E' = \sqrt{E^2 + c^2 p^2} \equiv E\,\gamma \stackrel{\text{def}}{=} W^{\phi}.$$

So in the case of the instantaneous energy, $\mathcal{E}_{\phi}(t)$ considered earlier we can write that

$$W^{\phi} \equiv \mathcal{E}_{\phi}(t_k)\,\gamma_k = W^{\phi}(t_k),$$

at the instant t_k (where $\gamma_k = \gamma(t_k)$).

We have $\mathcal{E}_{\phi}(t) \equiv E_{\phi}^{0}$ at canonical decay times. So,

$$W^{\phi} \equiv E_{\phi}^{0}\,\gamma,$$

at these times (where $\gamma = \gamma(t \sim \tau_{\phi})$).

(At times $t \gg \tau_{\phi},\; t \sim T$ we have $\mathcal{E}_{\phi}(t) \neq E_{\phi}^{0}$).

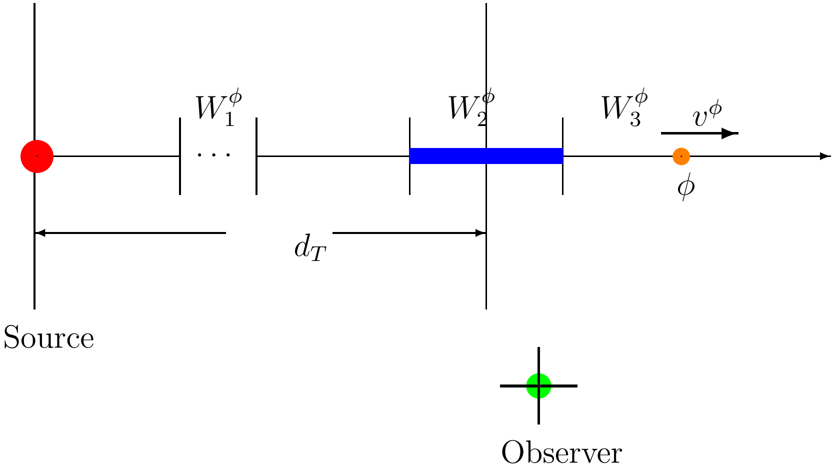


Figure: 4. Here $d_T = v^\phi \cdot T',\ T' = \gamma\ T,\ ; W_i^\phi = W^\phi(t_i),\ (i = 1, 2, 3)$ and $W_i^\phi(t_i)$ is the energy of moving relativistic particle ϕ, $t_1 \ll t_2 \ll t_3$, $W^\phi(t_i) = m_\phi^0(t_i)\, c^2\, \gamma(t_i),\ m_\phi^0(t_i) = \frac{\mathcal{E}_\phi(t_i)}{c^2}; \gamma = \gamma(t) = (\sqrt{1-\beta^2})^{-1}$, $\beta = v^\phi(t)/c$.

The principle of conservation of energy $\Rightarrow$

$$W_1^\phi = W_2^\phi = W_3^\phi. \tag{17}$$

$$W_1^\phi \equiv E_\phi^0\,\gamma_1 \equiv \mathcal{E}_\phi(t_2)\,\gamma(t_2) = W_2^\phi, \tag{18}$$

where it is assumed that $t_2 \sim T'$ and $t_1 \ll T'$.
The consequences of (17) and (18):

$$E_\phi^0\,\gamma_1 = \mathcal{E}_\phi(t_2)\gamma(t_2), \tag{19}$$

or

$$\gamma_1 = \frac{\mathcal{E}_\phi(t_2)}{E_\phi^0}\gamma(t_2). \tag{20}$$

The ratio $\frac{\mathcal{E}_\phi(t_2)}{E_\phi^0}$ can be extracted from Figs (2), (3):

$$\frac{\mathcal{E}_\phi(t)}{E_\phi^0} = 1 + \big(\kappa(t) - 1\big)\,\frac{E_\phi^0 - E_{min}}{E_\phi^0}. \tag{21}$$

Conclusions:

$$\kappa \neq 1 \;\Rightarrow\; \gamma_2 \neq \gamma_1 \;\Rightarrow\; v_2^{\phi} \neq v_1^{\phi} \;\Rightarrow\; a_{average} = \frac{v_2^{\phi} - v_1^{\phi}}{t_2 - t_1} \neq 0.$$

So, the moving charged unstable particle ϕ has to emit electromagnetic radiation at the transition time region. The same concerns neutral unstable particles with non–zero magnetic moment.

An analogy: A conservation of the angular momentum and a pirouette like effect.

There is (the Larmor formula),

$$P = \frac{1}{6\pi\epsilon_0}\,\frac{q^2\,\dot{v^{\phi}}^{2}}{c^2}\,\gamma^6, \tag{22}$$

for the charged particle (P is total radiation power, q is the electric charge, ϵ_0 – permittivity for free space).

5. Some estimations

Fig. (2) $\Rightarrow$
$(x_{mx}, y_{mx}) = (21.60, 10.27) \Rightarrow \kappa(t_{mx}) = y_{mx} = 10.27.$
Coordinates of points of the intersection of this maximum with the straight line $y = 1$ are equal:
$(x_1, y_1) = (21.58, 1.0)$ and $(x_2, y_2) = (21.62, 1.0) \Rightarrow$
$\Delta x = x_2 - x_1 = 0.04.$ (Here $x = t/\tau_\phi$). $\Rightarrow$ $\Delta t = t_{mx} - t_1$.

Meson μ:
$\Rightarrow E^0_\mu - E_{min} = m_{\mu^\pm} - (m_e + m_{\bar{\nu}_e} + m_{\nu_\mu}) \simeq 105$ [MeV]
(and $\tau_\mu = 2,198 \times 10^{-6}[\mathrm{s}]$, $T^\mu \simeq 165\tau_\mu = 0,37 \times 10^{-3}[\mathrm{s}]$)
$\Rightarrow \gamma(t_1) \simeq 10.21\,\gamma(t_{mx}) \Rightarrow \Delta v^\mu$
$\Rightarrow P \sim 4.6$ [eV/s].

Astrophysical and cosmological processes in which extremely huge numbers of unstable particles are created seem to be a possibility for the above discussed effect to become manifest. The fact is that the probability $\mathcal{P}_{\phi}(t) = |a(t)|^2$ that an unstable particle ϕ survives up to time $t \sim T$ is extremely small.

According to estimations of the luminosity of some γ–rays sources the energy emitted by these sources can even reach a value of order 10^{52} [erg/s], and it is only a part of the total energy produced there.

If to consider a source emitting energy 10^{50} [erg/s] then, eg., an emission of $\mathcal{N}_0 \simeq 6.25 \times 10^{47}$ [1/s] particles of energy 10^{18} [eV] is energetically allowed. The same source can emit $\mathcal{N}_0 \simeq 6.25 \times 10^{56}$ [1/s] particles of energy 10^{9} [eV] and so on.

Within the model defined by $\omega_{BW}(E)$ the cross–over time T is given by the following approximate relation (valid for $E_{\phi}^{0}/\Gamma_{\phi}^{0} \gg 1$),

$$\Gamma_{\phi}^{0}\, T \equiv \frac{T}{\tau_{\phi}} \sim 2 \ln \left[2\pi \left(\frac{E_{\phi}^{0} - E_{min}}{\Gamma_{\phi}^{0}} \right)^{2} \right], \qquad (23)$$

A typical value of the ratio $(E_{\phi}^{0} - E_{min})/\Gamma_{\phi}^{0}$ is $(E_{\phi}^{0} - E_{min})/\Gamma_{\phi}^{0} \geq O(10^{3} - 10^{6})$.

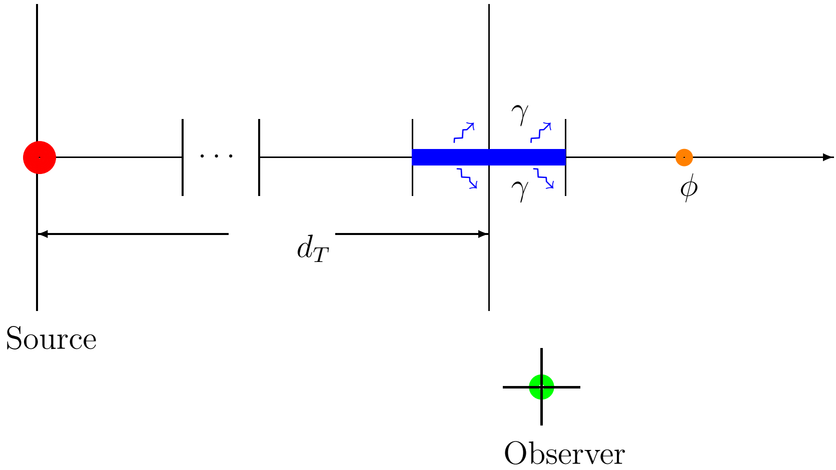
Taking $(E_{\phi}^{0} - E_{min})/\Gamma_{\phi}^{0} = 10^{6}$ one obtains from (23) that

$$\mathcal{N}_{\phi}(T) \equiv \mathcal{P}_{\phi}(T)\,\mathcal{N}_{0} \sim 2.53 \times 10^{-26}\,\mathcal{N}_{0}.$$

So there are $\mathcal{N}_{\phi}(T) \sim 14 \times 10^{21}$ particles per second of energy $W^{\phi} = 10^{18}$ [eV] or $\mathcal{N}_{\phi}(T) \sim 14 \times 10^{30}$ particles of energy $W^{\phi} = 10^{9}$ [eV] in the case of the considered example and T calculated using (23).

6. Conclusions

- Fluctuations of the instantaneous energy, $\mathcal{E}_\phi(t)$, of the moving unstable particles at transition times region cause fluctuations of the velocity of these particles at this time region, which forces the charged particles to emit electromagnetic radiation.
- Similarly to the charged particle, a moving neutral unstable particle with non–zero magnetic moment have to emit electromagnetic radiation in the transition time region.
- Ultra relativistic unstable charged particles can emit X– or γ–rays in the transition time region.
- Astrophysical sources are able to create such numbers $\mathcal{N}_0$ of unstable particles that sufficiently large number $\mathcal{N}_\phi(T) \gg 1$ of them has to survive up to times T and therefore to emit electromagnetic radiation at transition times. The expected spectrum of this radiation can be very wide:
  From radio frequencies up to γ–rays.



γ
γ
ϕ
d_T
Source
Observer

Thank you for your attention

The talk was based on the paper:

K. Urbanowski, K. Raczyńska, Phys. Letters, **B731**, 236 (2014); arXiv: 1303.6975.