

Transport phenomena in a plasma of confining gluons

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- RHIC and LHC HIC data suggest that QGP is a strongly-interacting dissipative fluid
 ⇒ relativistic dissipative fluid dynamics (equation of state?, transport coefficients?)
- equation of state (at high T)
 - lattice QCD
 (M. Cheng et al., PRD 77, 014511 (2008); S. Borsanyi et al., JHEP 11, 077 (2010); S. Borsanyi et al., JHEP 07, 056 (2012))
 - re-summed perturbation theory
 (J.-P. Blaizot, E. Iancu, Phys. Rept. 359, 355 (2002); J. O. Andersen, M. Strickland, Annals Phys. 317, 281 (2005); G. D. Moore, O. Saremi, JHEP 09, 015 (2008); J. Hong, D. Teaney, PRC 82, 044908 (2010); N. Su, Commun. Theor. Phys. 57, 409 (2012);)
- transport coefficients
 (W. Israel, J. M. Stewart, Ann. Phys. (NY) 118, 341 (1979); A. Muronga, PRC 69, 034903 (2004); A. El et al., PRC 81, 041901(R) (2010); G. S. Denicol et al., PRL 105, 162501 (2010); A. Jaiswal, PRC 87, 051901(R) (2013))

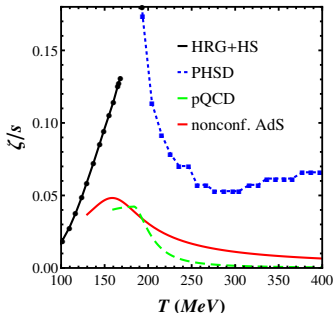
$$\begin{aligned} \dot{\Pi} + \frac{\Pi}{\tau_{\Pi}} &= -\frac{\zeta}{\tau_{\Pi}}\theta - \delta_{\Pi\Pi}\Pi\theta + \lambda_{\Pi\pi}\pi^{\mu\nu}\sigma_{\mu\nu} \\ \dot{\pi}^{\langle\mu\nu\rangle} + \frac{\pi^{\mu\nu}}{\tau_{\pi}} &= 2\frac{\eta}{\tau_{\pi}}\sigma^{\mu\nu} + 2\pi_{\gamma}^{\langle\mu}\omega^{\nu\rangle\gamma} - \delta_{\pi\pi}\pi^{\mu\nu}\theta - \tau_{\pi\pi}\pi_{\gamma}^{\langle\mu}\sigma^{\nu\rangle\gamma} + \lambda_{\pi\Pi}\Pi\sigma^{\mu\nu} \end{aligned}$$

$\zeta \rightarrow$ bulk viscosity $\eta \rightarrow$ shear viscosity

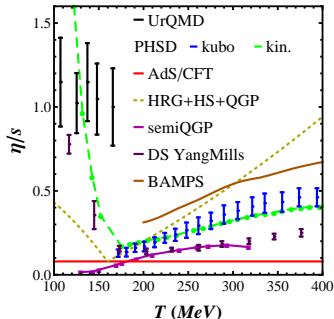
- by force of the Kubo formulas, they are sensitive to the long-distance dynamics of the underlying microscopic theory

- transport coefficients (cont'd)

- **pQCD** (P. B. Arnold et al., JHEP 11, 001 (2000); P. B. Arnold et al., JHEP 05, 051 (2003); G. D. Moore, O. Saremi, JHEP 09, 015 (2008); P. B. Arnold et al., PRD 74, 085021 (2006);)
- **functional renormalization techniques** (M. Haas et al., PRD90, 091501 (2014); N. Christiansen et al., (2014), arXiv:1411.7986 (hep-ph))
- **low energy theorems** (D. Kharzeev, K. Tuchin, JHEP 09, 093 (2008); F. Karsch et al., PLB 663, 217 (2008);)
- **I/QCD** (H. B. Meyer, PRL 100, 162001 (2008); H. B. Meyer, PRD 76, 101701 (2007);)
- **$\mathcal{N} = 4$ supersymmetric YM plasma with broken conformal symmetry** (G. Policastro et al., PRL 87, 081601 (2001); P. Benincasa et al., NPB 733, 160 (2006); A. Buchel, PRD 72, 106002 (2005); S. I. Finazzo et al., JHEP 02, 051 (2015))
- **HRG+HS** (G.P. Kadam, H. Mishra, NPA 934 (2014) 133-147))
- **parton-hadron-string dynamics transport approach** (V. Ozvenchuk, O. Linnyk, M. I. Gorenstein, E. L. Bratkovskaya and W. Cassing, PRC 87, no. 6, 064903 (2013))
- ...



around T_C ζ and η are comparable



- Gribov quantization of YM theory - fixing the infrared (IR) residual gauge transformations remaining after Faddeev-Popov procedure



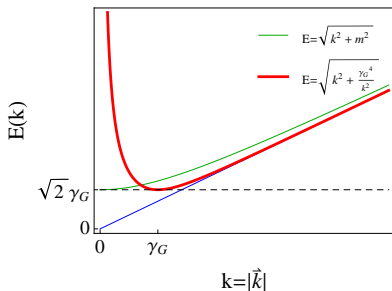
a new scale γ_G that leads to an IR-improved dispersion relation for gluons (Coulomb gauge)

(V. Gribov, NPB 139, 1 (1978);

D. Zwanziger, NPB 323, 513 (1989);

$$E(\mathbf{k}) = k \quad \rightarrow \quad E(\mathbf{k}) = \sqrt{k^2 + \frac{\gamma_G^4}{k^2}}$$

- attracted a lot of attention recently (G. Burgio, M. Quandt, H. Reinhardt, PRL 102, 032002 (2009); N. Su, K. Tywoniuk, PRL 114, 161601 (2015); D. E. Kharzeev, E. M. Levin, PRL 114, 242001 (2015); ...)



- reduction of the physical phase space due to the large energy cost of the excitation of soft gluons



essential feature of the confinement

(V. Gribov, NPB 139, 1 (1978);

R.P. Feynman, NPB 188, 479 (1981);

D. Zwanziger, NPB 485, 185 (1997);)

local rest frame

$$E(\mathbf{k}) = \sqrt{\mathbf{k}^2 + \frac{\gamma_G^4}{k^2}}$$

→

comoving frame

$$E(k \cdot u) = \sqrt{(k \cdot u)^2 + \frac{\gamma_G^4}{(k \cdot u)^2}}$$

- thermodynamics of the system of confining gluons

(D. Zwanziger, PRL 94, 182301 (2005);)

(W. Florkowski, R.R., N. Su, K. Tywoniuk, arXiv:1504.03176;)

$$\varepsilon = g_0 \int \frac{d^3k}{(2\pi)^3} E(\mathbf{k}) f(\mathbf{k}) \quad \rightarrow$$

$$\varepsilon = \int dK E(k \cdot u) f(x, k)$$

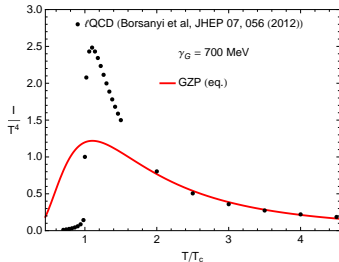
$$P = \frac{g_0}{3} \int \frac{d^3k}{(2\pi)^3} |\mathbf{k}| \frac{\partial E(\mathbf{k})}{\partial |\mathbf{k}|} f(\mathbf{k}) \quad \rightarrow$$

$$P = \frac{1}{3} \int dK \frac{(k \cdot u)^2}{E(k \cdot u)} \left(1 - \frac{\gamma_G^4}{(k \cdot u)^4} \right) f(x, k)$$

$$\int dK(\dots) = g_0 \int \frac{d^3k}{(2\pi)^3 k^0} (k \cdot u)(\dots)$$

$$g_0 = 2(N_c^2 - 1) \quad (SU(N_c))$$

(W. Florkowski, R.R., N. Su, K. Tywoniuk, arXiv:1504.03176;
S. Borsanyi et al, JHEP 07 (2012) 056;)



- in vacuum $\gamma_G^2 = \mu^2 \exp\left(\frac{5}{6} - \frac{64\pi^2}{3N_C g^2}\right)$

(D. Zwanziger, PRL 94, 182301 (2005);)

- in high-T limit $\gamma_G \rightarrow \frac{d}{d+1} \frac{N_C}{4\sqrt{2}\pi} g^2 T$

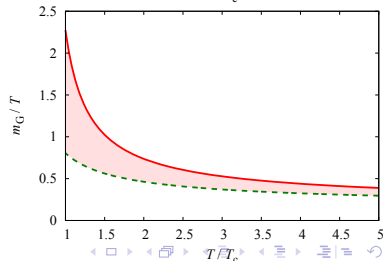
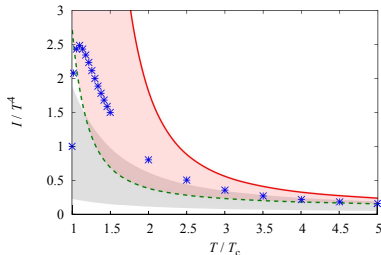
(D. Zwanziger, PRD 76, 125014 (2007);)

- $T \approx (2 - 4) T_c \quad \Rightarrow \quad \gamma_G \approx \text{const.}$

- $\gamma_G(T)$ derived from the gap equation with running coupling from IQCD

(K. Fukushima, N. Su, PRD 88, 076008 (2013);

J. O. Andersen et al., PRL 104, 122003 (2010);)



- consider the case of a **transversely homogeneous boost-invariant** system
- assume the **Bjorken flow** of matter in longitudinal direction (boost-invariance)
(J. D. Bjorken, PRD 27, 140 (1983);)

$$u^\mu = (t/\tau, 0, 0, z/\tau)$$

- introduce convenient boost-invariant variables
(A. Bialas et al., NPB 296, 611 (1988);)

$$v = k^0 t - k_{\parallel} z$$
$$w = k_{\parallel} t - k^0 z$$

- EOM follow from the conservation of $T^{\mu\nu}$

$$T^{\mu\nu} = \int dK k^\mu k^\nu f(x, k)$$

- $f = f(\tau, w, k_{\perp})$

- within the assumed symmetries the $T^{\mu\nu}$ has the spherically anisotropic form
(W. Florkowski, R. R., PRC 85, 044902 (2012);
M. Martinez et al., PRC 85, 064913 (2012);)

$$T^{\mu\nu} = (\varepsilon + P_{\perp})u^\mu u^\nu - P_{\perp}g^{\mu\nu} + (P_{\parallel} - P_{\perp})z^\mu z^\nu$$
$$z^\mu = (z/\tau, 0, 0, t/\tau)$$

$$\partial_\mu T^{\mu\nu} = 0 \quad \Rightarrow \quad \frac{d\varepsilon}{d\tau} + \frac{\varepsilon + P_{\parallel}}{\tau} = 0$$

Non-equilibrium fluid dynamics of GZ plasma

(A. Muronga, PRC 69, 034903 (2004);
R. Baier et al., PRC 73, 064903 (2006);)

$$\frac{d\varepsilon}{d\tau} + \frac{1}{\tau} (\varepsilon + P_{\text{GZ}} + \Pi - \pi) = 0 \quad \Leftrightarrow \quad \frac{d\varepsilon}{d\tau} + \frac{\varepsilon + P_{\parallel}}{\tau} = 0$$

- dissipative fluxes

$$\pi = \frac{4}{3} \frac{\eta}{\tau}$$

$$\pi = \frac{2}{3} (P_{\parallel} - P_{\perp})$$

$$\Pi = -\frac{\zeta}{\tau}$$

$$\Pi = P - P_{\text{GZ}} = \frac{1}{3} (P_{\parallel} + 2P_{\perp}) - P_{\text{GZ}}$$

(Navier-Stokes dissipative fluid dynamics)

- what is the form of ζ and η for GZ plasma?, close to equilibrium one can use the linear response approximation $f \approx f_{\text{GZ}} + \delta f + \dots$

$$\zeta(T, \gamma_{\text{G}}) = \frac{g_0 \gamma_{\text{G}}^5}{3\pi^2} \frac{\tau_{\text{rel}}}{T} \int_0^{\infty} dy \left[c_s^2 - \frac{1}{3} \frac{y^4 - 1}{y^4 + 1} \right] f_{\text{GZ}} (1 + f_{\text{GZ}})$$

$$\eta(T, \gamma_{\text{G}}) = \frac{1}{10} \frac{g_0 \gamma_{\text{G}}^5}{3\pi^2} \frac{\tau_{\text{rel}}}{T} \int_0^{\infty} dy \frac{(y^4 - 1)^2}{y^4 + 1} f_{\text{GZ}} (1 + f_{\text{GZ}})$$

$$\frac{d\varepsilon}{d\tau} + \frac{\varepsilon + P_{\parallel}}{\tau} = \int dK E(\tau, w, k_{\perp}) \frac{\partial f(\tau, w, k_{\perp})}{\partial \tau}$$

- (0+1)D kinetic equation in RTA

P.L. Bhatnagar et al., Phys. Rev. 94, 511 (1954);

G. Baym, PLB 138, 18 (1984);

G. Baym, NPA 418, 525C (1984);

$$\frac{\partial f(\tau, w, k_{\perp})}{\partial \tau} = \frac{f_{GZ}(\tau, w, k_{\perp}) - f(\tau, w, k_{\perp})}{\tau_{rel}(\tau)}$$

- satisfied as long as Landau matching condition is satisfied $\varepsilon_{GZ} = \varepsilon$

- formal solution

W. Florkowski et al., NPA 916, 249 (2013);

W. Florkowski et al., PRC 88, 024903 (2013);

W. Florkowski, et al., PRC 89, 054908 (2014);

$$f(\tau, w, k_{\perp}) = f_0(w, k_{\perp}) D(\tau, \tau_0) + \int_{\tau_0}^{\tau} \frac{d\tau'}{\tau_{rel}(\tau')} D(\tau, \tau') f_{GZ}(\tau', w, k_{\perp})$$

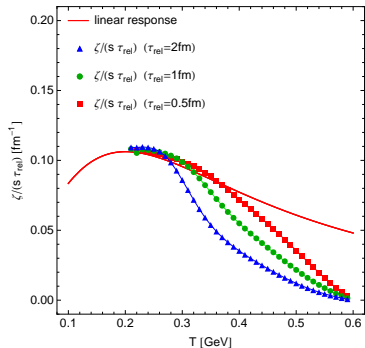
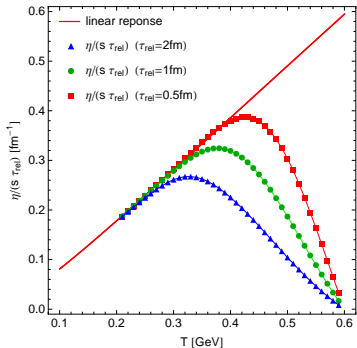
$$\varepsilon = \int dK E(\tau, w, k_{\perp}) f(\tau, w, k_{\perp})$$

$$P_{\parallel} = \int dK \frac{w^2}{\tau^2 E(\tau, w, k_{\perp})} \left[1 - \frac{\gamma_G^4}{(w^2/\tau^2 + k_{\perp}^2)^2} \right] f$$

$$P_{\perp} = \int dK \frac{k_{\perp}^2}{2E(\tau, w, k_{\perp})} \left[1 - \frac{\gamma_G^4}{(w^2/\tau^2 + k_{\perp}^2)^2} \right] f$$

$$D(\tau_2, \tau_1) = \exp \left[- \int_{\tau_1}^{\tau_2} \frac{d\tau}{\tau_{rel}(\tau)} \right]$$

Results: Bulk and shear viscosity



Results: ζ/η dependence

$$\frac{\zeta}{\eta} = 15 \left(\frac{1}{3} - c_s^2 \right)^2$$

- photon gas coupled to hot matter

(S. Weinberg, *Astrophys. J.* 168, 175 (1971);)

scalar theory

(A. Hosoya et al., *Ann. Phys. (N.Y.)* 154, 229 (1984);

R. Horsley, W. Schoenmaker, *NPB* 280, 716 (1987);)

weakly-coupled QCD (large- T limit)

(P. B. Arnold et al., *JHEP* 0011 (2000) 001;

P. B. Arnold et al., *JHEP* 0305 (2003) 051;

P. B. Arnold et al., *PRD* 74 (2006) 085021;)

$$\frac{\zeta}{\eta} = \kappa \left(\frac{1}{3} - c_s^2 \right)$$

- strongly-coupled nearly-conformal gauge theory plasma using gauge theory-gravity duality (large- T limit)

($\kappa = 2$, $\kappa = 4.558 - 4.935$)

(P. Benincasa et al., *NPB* 733, 160 (2006);

A. Buchel, *PRD* 72, 106002 (2005);)

- **Gribov's plasma of confining gluons**

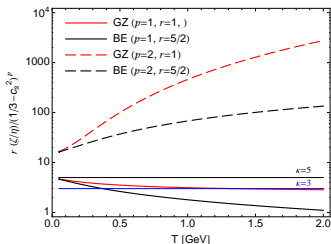
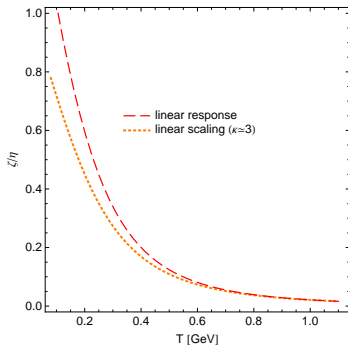
($\kappa = 3$)

(W. Florkowski, R.R., N. Su, K. Tywoniuk [arXiv:1504.03176](https://arxiv.org/abs/1504.03176);

W. Florkowski, R.R., N. Su, K. Tywoniuk, *forthcoming*.)

different from massive BE gas ! \rightarrow

(C. Sasaki, K. Redlich, *PRC* 79, 055207 (2009);)



Summary

- a **dynamic and non-equilibrium description** of a plasma consisting of confining gluons (obtained from the Gribov quantization of SU(3) YM theory) introduced for the first time
- the expressions for the **shear and bulk viscosities** of the Gribov-Zwanziger plasma were derived
- **ζ/η T-scaling** which is in line with the strong-coupling methods results was found

Outlook

- include the running $\gamma_G(T)$
- use our formula in the hydrodynamic simulations

Thank you for your attention!

ACKNOWLEDGMENTS

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