

**SPINORIAL SPACE-TIME**  
**AND THE ORIGIN**  
**OF QUANTUM**  
**MECHANICS**

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**TO THE MEMORY**  
**OF**  
**BERNARD**  
**D'ESPAGNAT**  
**AND**  
**YOICHIRO**  
**NAMBU**

# WHAT IS THE ORIGIN OF THE UNCERTAINTY PRINCIPLE ?

RELATION TO VACUUM PROPERTIES,  
SPACE-TIME, MATTER STRUCTURE ...<sup>3</sup> ?

# WHY DOES THE PATH INTEGRAL FORMULATION WORK?

**WHY RANDOM PATHS IN THE FUNDAMENTAL  
STRUCTURE OF MATTER AND SPACE-TIME ?<sub>4</sub>**

# Space-Time Approach to Non-Relativistic Quantum Mechanics

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Reprinted in “Quantum Electrodynamics”, edited by Julian Schwinger

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## Abstract

Non-relativistic quantum mechanics is formulated here in a different way. It is, however, mathematically equivalent to the familiar formulation. In quantum mechanics the probability of an event which can happen in several different ways is the absolute square of a sum of complex contributions, one from each alternative way. The probability that a particle will be found to have a path  $x(t)$  lying somewhere within a region of space time is the square of a sum of contributions, one from each path in the region. The contribution from a single path is postulated to be an exponential whose (imaginary) phase is the classical action (in units of  $\hbar$ ) for the path in question. The total contribution from all paths reaching  $x, t$  from the past is the wave function  $\psi(x, t)$ . This is shown to satisfy Schroedinger's equation. The relation to matrix and operator algebra is discussed. Applications are indicated, in particular to eliminate the coordinates of the field oscillators from the equations of quantum electrodynamics.

# **BUT WHAT IS THE ULTIMATE SPACE-TIME ORIGIN OF THIS SPACE-TIME APPROACH ?**

**AT THE PREVIOUS ICNFP,  
I SUGGESTED A LINK BETWEEN  
THE SPINORIAL SPACE-TIME (SST)  
AND THE GROUNDS  
OF QUANTUM MECHANICS**

**=> CAN CLASSICAL MECHANICS  
EXIST IN THE SST ?**

**WHAT IS THE REAL  
STRUCTURE OF VACUUM,**

**AND WHAT IS THE  
ACTUAL LINK WITH  
QUANTUM  
MECHANICS ?**

# POSSIBLE PROBLEMS FOR « CLASSICAL » MECHANICS IN THE SST

- STANDARD SPACE AND TIME ARE NO LONGER THE FUNDAMENTAL COORDINATES

- SIMILARLY FOR ENERGY AND MOMENTUM

- TIME-DEPENDENCE IS PECULIAR : IN THE SST, A SPIN-1/2 PARTICLE DESCRIBED BY AN EXTENDED OBJECT WOULD HAVE A NONZERO DENSITY FOR SEVERAL (ALL ?)

VALUES OF TIME SIMULTANEOUSLY => IS IT COMPATIBLE WITH « CLASSICAL » MOTION?



**HALF-INTEGER SPINS EXIST IN NATURE, BUT THEY CANNOT BE GENERATED THROUGH STANDARD ORBITAL ANGULAR MOMENTUM.**

**=> WHAT IS THE INTERNAL STRUCTURE OF CONVENTIONAL “ELEMENTARY” PARTICLES ?**

**=> A POSSIBLE WAY TO EXPLORE FERMION STRUCTURE : REPLACE THE STANDARD SPACE-TIME BY A SU(2) SPINORIAL ONE**

**THUS, THE SPIN-1/2 PARTICLES BECOME ACTUAL REPRESENTATIONS OF THE SU(2) GROUP OF SPACE TRANSFORMATIONS**

**=> ASSOCIATE TO EACH POINT OF SPACE-TIME A SU(2) COMPLEX SPINOR  $\xi$**

**=> USE A SPACE-TIME SPINOR  $\xi$  (TWO COMPLEX COMPONENTS INSTEAD OF THE FOUR REAL ONES) WITH A SU(2) GROUP THAT CONTAINS THE STANDARD ROTATION GROUP SO(3)**

**EXTRACTING FROM A COSMIC SPINOR  $\xi$  THE SCALAR  $|\xi|^2 = \xi^\dagger \xi$  WHERE THE DAGGER STANDS FOR HERMITIC CONJUGATE, A POSITIVE COSMIC TIME  $t = |\xi|$  IS DEFINED => NATURALLY EXPANDING UNIVERSE, ARROW OF TIME**

THE CONVENTIONAL SPACE AT COSMIC TIME  $t_0$  CORRESPONDS TO THE  $|\xi| = t_0 S^3$  HYPERSPHERE WITH THE ADDITIONAL SPINORIAL STRUCTURE.

THE  $S^3$  HYPERSPHERE IS THUS BUILT FROM THE FOUR REAL NUMBERS CONTAINED IN THE TWO SPINOR COMPONENTS  $\Rightarrow$  ON  $S^3$ , THE  $SU(2)$  TRANSFORMATIONS PROVIDE THE SPACE TRANSLATIONS.

SPACE ROTATIONS AROUND A POINT ARE PROVIDED BY  $SU(2)$  TRANSFORMATIONS ACTING ON THE TRANSLATIONS

# DESCRIPTION OF SPIN-1/2 PARTICLES

A FUNCTION  $\psi$  OF THE SPINORIAL POSITION  $\xi - \xi_0$ , WHERE  $\xi_0$  IS A SPACE ORIGIN, CAN REPRODUCE THE INTERNAL (**extended**) STRUCTURE OF A PARTICLE WITH SPIN 1/2 .

THE SPINORIAL « STRAIGHT LINE » BETWEEN  $\xi_0$  AND  $\xi$  CROSSES PREVIOUS VALUES OF TIME (**a straight line between two points of a circle**) AND VIOLATES CAUSALITY. **BUT SUCH A VIOLATION OF CAUSALITY CAN BE ALLOWED AT VERY SMALL DISTANCES IN SPACE AND TIME.**

# RELATIVITY IS NOT REQUIRED AS AN ABSOLUTE PROPERTY OF SPACE-TIME

THE SST GEOMETRY DOES NOT REQUIRE EXPLICIT MATTER TO AUTOMATICALLY REPRODUCE THE LUNDMARK – LEMAITRE – HUBBLE LAW ON THE  $v/d$  RATIO WITH  $v/d =$  INVERSE OF THE AGE OF THE UNIVERSE

INSTEAD, STANDARD RELATIVITY CAN BE A PROPERTY OF STANDARD MATTER AT LOW ENERGY, AND OTHER FORMS OF MATTER CAN EXIST (e.g. SUPERBRADYONS, POSSIBLE SUPERLUMINAL ULTIMATE CONSTITUENTS OF MATTER)

**IN PARTICULAR, THE SST VACUUM  
CAN BE A SUPERBRADYONIC VACUUM**

**=> QUANTUM MECHANICS MAY BE  
GENERATED IN A SUPERBRADYONIC  
VACUUM WITH SST GEOMETRY**

**THE COMPLEX SPACE-TIME COORDINATES  
OF SST CAN BE A NATURAL CONTEXT FOR  
COMPLEX WAVE FUNCTIONS OF MATTER.**

**BUT WHAT CAN MAKE NECESSARY  
THE TRANSITION FROM « CLASSICAL »  
TO QUANTUM MOTION ?**

**ASSUME, FOR SIMPLICITY, THAT THE INTERNAL STRUCTURE OF A SPIN-1/2 STANDARD PARTICLE BASED AT  $\xi = \xi_0$  IS DESCRIBED BY A SPINORIAL FUNCTION**

**$\psi (\xi - \xi_0)$  WITH THE RELATION :**

$$\psi (\xi - \xi_0) = (\xi - \xi_0) F (|\xi - \xi_0|)$$

**where the function  $F (|\xi - \xi_0|)$  has an exponential decrease with the modulus  $|\xi - \xi_0|$**

THEN, IF ONE ATTEMPTS TO  
DESCRIBE A « CLASSICAL »  
CONTINUOUS MOTION OF THE  
« CENTER »  $\xi_0$  WRITING :

$$\psi (\xi - \xi_c) = (\xi - \xi_c) F (|\xi - \xi_c|)$$

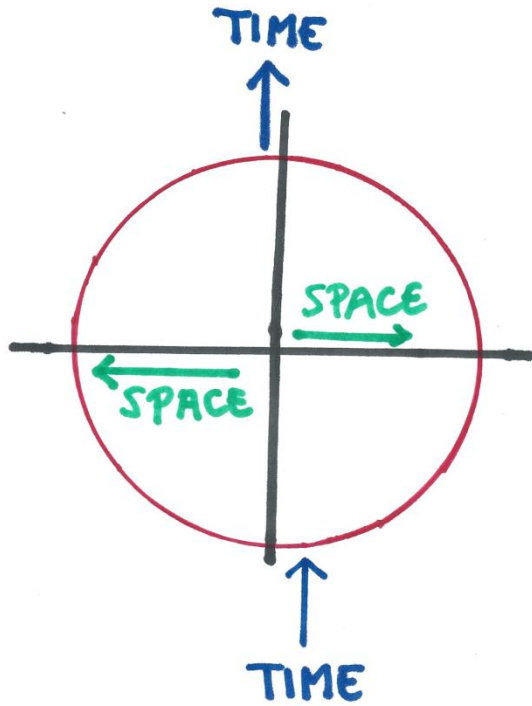
WHERE THE COSMIC TIME  $|\xi_c|$   
VARIES CONTINUOUSLY, TWO  
TIME VARIABLES ( $|\xi_c|$ ,  $|\xi|$ ) ARE  
INVOLVED FOR A GIVEN  $|\xi_c|$



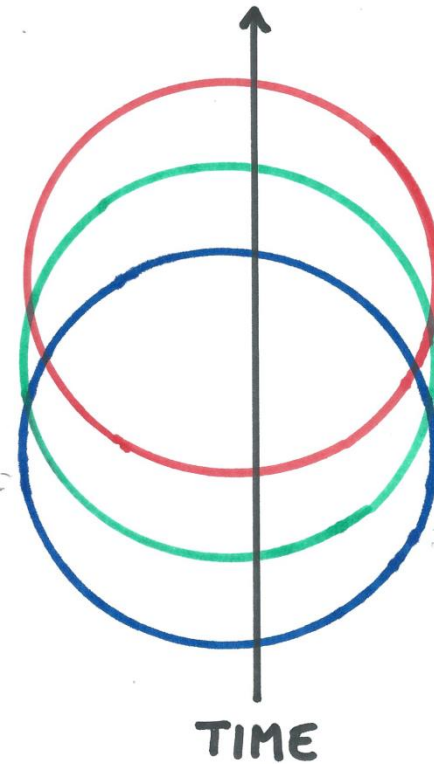
IF THE COSMIC TIME  $|\xi_c|$  VARIABLES  
CONTINUOUSLY, A GIVEN VALUE  
OF COSMIC TIME WILL RECEIVE  
CONTRIBUTIONS FROM THE  
SPINORIAL STRUCTURE  
FUNCTION  $\psi(\xi - \xi'_c)$  FOR ALL  
VALUES OF  $|\xi'_c| \Rightarrow$  THIS MAY  
BE REJECTED BY THE VACUUM  
DYNAMICS  $\Rightarrow$  **RANDOM PATHS?**

**INDEED, A CONTINUOUS  
PROPAGATION IN TIME OF A  
SPIN-1/2 PARTICLE STRUCTURE  
FUNCTION WOULD GENERATE  
A STRONG OVERLAP BETWEEN  
FUNCTIONS « CENTERED » AT  
DIFFERENT TIMES =>  
CAN BE INCOMPATIBLE WITH  
THE DYNAMICS GENERATING**

$$\psi (\xi - \xi_0)$$



AN "EXTENDED"  
 SPIN- $1/2$  PARTICLE  
 IN THE SST  $\Rightarrow$   
 A WHOLE CONTINUUM  
 OF VALUES OF TIME  
 IS SIMULTANEOUSLY  
 CONCERNED



CONTINUOUS MOTION  
 FORBIDDEN BY  
 VACUUM DYNAMICS  
 BECAUSE OF  
 OVERLAP IN TIME?

**THE SPACE-TIME  
STRUCTURE,  
A “CONDENSED MATTER”  
QUESTION ?**

**THE STRUCTURE  
OF MATTER,  
A SPACE-TIME  
QUESTION ?**

**IF A CONTINUOUS PROPAGATION  
IN TIME OF A SPIN-1/2 PARTICLE  
STRUCTURE FUNCTION IS NOT  
ALLOWED BY VACUUM DYNAMICS  
BECAUSE OF THE TIME OVERLAP,  
PROPAGATION WILL HAVE TO BE  
DISCRETE IN SPACE AND TIME =>**

**A FEYNMAN-LIKE**

**« RANDOM » PATH =>**

**FEYNMAN PATH INTEGRAL ?<sup>1</sup>**

**HOW « SENSIBLE » IS THAT THE  
VACUUM PREVENTS BY ITSELF A  
CONTINUOUS PROPAGATION IN  
TIME OF A SPIN-1/2 PARTICLE  
STRUCTURE FONCTION ?**

**IT CAN BE PERFECTLY NATURAL  
TO PREVENT TIME OVERLAP IF**

**$\psi (\xi - \xi_0)$  DESCRIBES A SPECIFIC  
EXCITATION OF VACUUM**

**CORRESPONDING TO A DISCRETE,  
UNIQUE SOLUTION OF EQUATIONS**

**SPACE-TIME OVERLAP OF SPIN-1/2  
STRUCTURE FUNCTIONS CAN  
NATURALLY BE FORBIDDEN IF THE  
SOLUTION TO THE EQUATIONS OF  
VACUUM DYNAMICS CANNOT BE  
CONTINUOUSLY DEFORMED AND IF  
TWO SOLUTIONS CANNOT BE  
SUPERIMPOSED IN THE SAME  
SPACE AND AT THE SAME TIME =>  
CONTINUOUS MOTION « ALMOST »  
SUPERIMPOSES SPIN-1/2  $\psi$ 's**

**SST GEOMETRY  
AND VACUUM =>**

**QUANTUM  
MECHANICS**

**A « CONDENSED  
MATTER » AFFAIR ?**



**IF THE SPINORIAL SPACE-TIME,  
TOGETHER WITH  
VACUUM DYNAMICS,  
DO NOT ALLOW  
FOR A CONTINUOUS MOTION  
OF AN EXTENDED OBJECT  
WITH SPIN  $1/2$ ,**

**WILL DISCRETE MOTION BE  
REALLY “RANDOM” IN AN  
UNDETERMINISTIC SENSE ?**

**AT THIS STAGE, NO NEED  
FOR ANY ABSOLUTE  
INDETERMINISM**

**AS PRESENT UPPER BOUNDS ON THE  
ELECTRON SIZE REMAIN MUCH  
LARGER THAN THE PLANCK LENGTH,  
POSSIBLE ENERGY AND DISTANCE  
SCALES FOR THE FORMATION OF  
STANDARD MATTER IN THE SST  
ARE NOT KNOWN**

# HOW TO CHECK THIS POSSIBLE SCENARIO ?

- **LOOK FOR DEPARTURES FROM  
STANDARD PRINCIPLES IN  
ULTRA-HIGH ENERGY COSMIC-RAY  
DATA**

- **EXPLORE ALTERNATIVE  
COSMOLOGIES, INCLUDING POSSIBLE  
SST AND PRE-BIG BANG SIGNATURES**

- **TEST QUANTUM MECHANICS IN  
DETAIL**