SPINORIAL SPACE-TIME AND THE ORIGIN OF QUANTUM MECHANICS

Luis Gonzalez-Mestres
Cosmology Laboratory, John Naisbitt University, Belgrade and Paris

luis.gonzalez-mestres@megatrend.edu.rs
TO THE MEMORY OF

BERNARD D’ESPAGNAT

AND

YOICHIRO NAMBU
WHAT IS THE ORIGIN OF THE UNCERTAINTY PRINCIPLE?

RELATION TO VACUUM PROPERTIES, SPACE-TIME, MATTER STRUCTURE ...
WHY DOES THE PATH INTEGRAL FORMULATION WORK?

WHY RANDOM PATHS IN THE FUNDAMENTAL STRUCTURE OF MATTER AND SPACE-TIME?
**Space-Time Approach to Non-Relativistic Quantum Mechanics**

R.P. Feynman  
Cornell University, Ithaca, New York

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**Abstract**

Non-relativistic quantum mechanics is formulated here in a different way. It is, however, mathematically equivalent to the familiar formulation. In quantum mechanics the probability of an event which can happen in several different ways is the absolute square of a sum of complex contributions, one from each alternative way. The probability that a particle will be found to have a path \( x(t) \) lying somewhere within a region of space time is the square of a sum of contributions, one from each path in the region. The contribution from a single path is postulated to be an exponential whose (imaginary) phase is the classical action (in units of \( \hbar \)) for the path in question. The total contribution from all paths reaching \( x, t \) from the past is the wave function \( \psi(x, t) \). This is shown to satisfy Schroedinger’s equation. The relation to matrix and operator algebra is discussed. Applications are indicated, in particular to eliminate the coordinates of the field oscillators from the equations of quantum electrodynamics.
BUT WHAT IS THE ULTIMATE SPACE-TIME ORIGIN OF THIS SPACE-TIME APPROACH?

AT THE PREVIOUS ICNFP, I SUGGESTED A LINK BETWEEN THE SPINORIAL SPACE-TIME (SST) AND THE GROUNDS OF QUANTUM MECHANICS

=> CAN CLASSICAL MECHANICS EXIST IN THE SST?
WHAT IS THE REAL STRUCTURE OF VACUUM, AND WHAT IS THE ACTUAL LINK WITH QUANTUM MECHANICS?
POSSIBLE PROBLEMS FOR « CLASSICAL » MECHANICS IN THE SST

- STANDARD SPACE AND TIME ARE NO LONGER THE FUNDAMENTAL COORDINATES
- SIMILARLY FOR ENERGY AND MOMENTUM

-TIME-DEPENDENCE IS PECULIAR: IN THE SST, A SPIN-1/2 PARTICLE DESCRIBED BY AN EXTENDED OBJECT WOULD HAVE A NONZERO DENSITY FOR SEVERAL (ALL ?) VALUES OF TIME SIMULTANEOUSLY => IS IT COMPATIBLE WITH « CLASSICAL » MOTION?
HALF-INTEGER SPINS EXIST IN NATURE, BUT THEY CANNOT BE GENERATED THROUGH STANDARD ORBITAL ANGULAR MOMENTUM.

=> WHAT IS THE INTERNAL STRUCTURE OF CONVENTIONAL “ELEMENTARY” PARTICLES?

=> A POSSIBLE WAY TO EXPLORE FERMION STRUCTURE: REPLACE THE STANDARD SPACE-TIME BY A SU(2) SPINORIAL ONE

THUS, THE SPIN-1/2 PARTICLES BECOME ACTUAL REPRESENTATIONS OF THE SU(2) GROUP OF SPACE TRANSFORMATIONS

=> ASSOCIATE TO EACH POINT OF SPACE-TIME A SU(2) COMPLEX SPINOR ξ
USE A SPACE-TIME SPINOR $\xi$ (TWO COMPLEX COMPONENTS INSTEAD OF THE FOUR REAL ONES) WITH A SU(2) GROUP THAT CONTAINS THE STANDARD ROTATION GROUP SO(3)

EXTRACTING FROM A COSMIC SPINOR $\xi$ THE SCALAR $|\xi|^2 = \xi^\dagger \xi$ WHERE THE DAGGER STANDS FOR HERMITIC CONJUGATE, A POSITIVE COSMIC TIME $t = |\xi|$ IS DEFINED => NATURALLY EXPANDING UNIVERSE, ARROW OF TIME
THE CONVENTIONAL SPACE AT COSMIC TIME $t_0$ CORRESPONDS TO THE $|\xi| = t_0 \ S^3$ HYPERSPHERE WITH THE ADDITIONAL SPINORIAL STRUCTURE.

THE $S^3$ HYPERSPHERE IS THUS BUILT FROM THE FOUR REAL NUMBERS CONTAINED IN THE TWO SPINOR COMPONENTS => ON $S^3$, THE SU(2) TRANSFORMATIONS PROVIDE THE SPACE TRANSLATIONS.

SPACE ROTATIONS AROUND A POINT ARE PROVIDED BY SU(2) TRANSFORMATIONS ACTING ON THE TRANSLATIONS
DESCRIPTION OF SPIN-1/2 PARTICLES

A function $\psi$ of the spinorial position $\xi - \xi_0$, where $\xi_0$ is a space origin, can reproduce the internal (extended) structure of a particle with spin 1/2.

The spinorial «straight line» between $\xi_0$ and $\xi$ crosses previous values of time (a straight line between two points of a circle) and violates causality. But such a violation of causality can be allowed at very small distances in space and time.
RELATIVITY IS NOT REQUIRED AS AN ABSOLUTE PROPERTY OF SPACE-TIME

THE SST GEOMETRY DOES NOT REQUIRE EXPLICIT MATTER TO AUTOMATICALLY REPRODUCE THE LUNDMARK – LEMAITRE – HUBBLE LAW ON THE $v/d$ RATIO WITH $v/d =$ INVERSE OF THE AGE OF THE UNIVERSE

INSTEAD, STANDARD RELATIVITY CAN BE A PROPERTY OF STANDARD MATTER AT LOW ENERGY, AND OTHER FORMS OF MATTER CAN EXIST (e.g. SUPERBRADYONS, POSSIBLE SUPERLUMINAL ULTIMATE CONSTITUENTS OF MATTER)
IN PARTICULAR, THE SST VACUUM CAN BE A SUPERBRADYONIC VACUUM

=> QUANTUM MECHANICS MAY BE GENERATED IN A SUPERBRADYONIC VACUUM WITH SST GEOMETRY

THE COMPLEX SPACE-TIME COORDINATES OF SST CAN BE A NATURAL CONTEXT FOR COMPLEX WAVE FUNCTIONS OF MATTER.

BUT WHAT CAN MAKE NECESSARY THE TRANSITION FROM « CLASSICAL » TO QUANTUM MOTION?
ASSUME, FOR SIMPLICITY, THAT THE INTERNAL STRUCTURE OF A SPIN-1/2 STANDARD PARTICLE BASED AT $\xi = \xi_0$ IS DESCRIBED BY A SPINORIAL FUNCTION

$\psi (\xi - \xi_0)$ WITH THE RELATION:

$\psi (\xi - \xi_0) = (\xi - \xi_0) F (|\xi - \xi_0|)$

where the function $F (|\xi - \xi_0|)$ has an exponential decrease with the modulus $|\xi - \xi_0|$.
THEN, IF ONE ATTEMPTS TO DESCRIBE A « CLASSICAL » CONTINUOUS MOTION OF THE « CENTER » \( \xi_0 \) WRITING:

\[
\psi (\xi - \xi_C) = (\xi - \xi_C) F (|\xi - \xi_C|)
\]

WHERE THE COSMIC TIME \( |\xi_C| \) VARIES CONTINUOUSLY, TWO TIME VARIABLES (\( |\xi_C|, |\xi| \)) ARE INVOLVED FOR A GIVEN \( |\xi_C| \)
IF THE COSMIC TIME $|\xi_c|$ VARIES CONTINUOUSLY, A GIVEN VALUE OF COSMIC TIME WILL RECEIVE CONTRIBUTIONS FROM THE SPINORIAL STRUCTURE FUNCTION $\psi(\xi - \xi'_c)$ FOR ALL VALUES OF $|\xi'_c| \Rightarrow$ THIS MAY BE REJECTED BY THE VACUUM DYNAMICS $\Rightarrow$ RANDOM PATHS?
INDEED, A CONTINUOUS PROPAGATION IN TIME OF A SPIN-1/2 PARTICLE STRUCTURE FUNCTION WOULD GENERATE A STRONG OVERLAP BETWEEN FUNCTIONS « CENTERED » AT DIFFERENT TIMES => CAN BE INCOMPATIBLE WITH THE DYNAMICS GENERATING \[ \psi (\xi - \xi_0) \]
AN "EXTENDED" SPIN-$\frac{1}{2}$ PARTICLE IN THE SST $\Rightarrow$

A WHOLE CONTINUUM OF VALUES OF TIME IS SIMULTANEOLUSLY CONCERNED

CONTINUOUS MOTION FORBIDDEN BY VACUUM DYNAMICS BECAUSE OF OVERLAP IN TIME?
THE SPACE-TIME STRUCTURE, A “CONDENSED MATTER” QUESTION?

THE STRUCTURE OF MATTER, A SPACE-TIME QUESTION?
IF A CONTINUOUS PROPAGATION IN TIME OF A SPIN-1/2 PARTICLE STRUCTURE FUNCTION IS NOT ALLOWED BY VACUUM DYNAMICS BECAUSE OF THE TIME OVERLAP, PROPAGATION WILL HAVE TO BE DISCRETE IN SPACE AND TIME => A FEYNMAN-LIKE « RANDOM » PATH => FEYNMAN PATH INTEGRAL ?
HOW « SENSIBLE » IS THAT THE VACUUM PREVENTS BY ITSELF A CONTINUOUS PROPAGATION IN TIME OF A SPIN-1/2 PARTICLE STRUCTURE FUNCTION?

IT CAN BE PERFECTLY NATURAL TO PREVENT TIME OVERLAP IF

$$\psi(\xi - \xi_0)$$

DESCRIBES A SPECIFIC EXCITATION OF VACUUM CORRESPONDING TO A DISCRETE, UNIQUE SOLUTION OF EQUATIONS
SPACE-TIME OVERLAP OF SPIN-1/2 STRUCTURE FUNCTIONS CAN NATURALLY BE FORBIDDEN IF THE SOLUTION TO THE EQUATIONS OF VACUUM DYNAMICS CANNOT BE CONTINUOUSLY DEFORMED AND IF TWO SOLUTIONS CANNOT BE SUPERIMPOSED IN THE SAME SPACE AND AT THE SAME TIME => CONTINUOUS MOTION « ALMOST » SUPERIMPOSES SPIN-1/2 ψ’s
SST GEOMETRY AND VACUUM $\Rightarrow$ QUANTUM MECHANICS

A « CONDENSED MATTER » AFFAIR ?
IF THE SPINORIAL SPACE-TIME, TOGETHER WITH VACUUM DYNAMICS, DO NOT ALLOW FOR A CONTINUOUS MOTION OF AN EXTENDED OBJECT WITH SPIN 1/2, WILL DISCRETE MOTION BE REALLY “RANDOM” IN AN UNDETERMINISTIC SENSE?
AT THIS STAGE, NO NEED FOR ANY ABSOLUTE INDETERMINISM

AS PRESENT UPPER BOUNDS ON THE ELECTRON SIZE REMAIN MUCH LARGER THAN THE PLANCK LENGTH, POSSIBLE ENERGY AND DISTANCE SCALES FOR THE FORMATION OF STANDARD MATTER IN THE SST ARE NOT KNOWN
HOW TO CHECK THIS POSSIBLE SCENARIO?

• LOOK FOR DEPARTURES FROM STANDARD PRINCIPLES IN ULTRA-HIGH ENERGY COSMIC-RAY DATA

• EXPLORE ALTERNATIVE COSMOLOGIES, INCLUDING POSSIBLE SST AND PRE-BIG BANG SIGNATURES

• TEST QUANTUM MECHANICS IN DETAIL