Massive Gravi-Electromagnetism in Terms of Octons

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Abstract

Although the origin of linear gravity is completely different from electromagnetism, it is possible that the description of linear gravity based on an analogy with electromagnetism. In this work, the octon algebra has been introduced to generalize the Proca-Maxwell type equations of electromagnetism and liner gravity in a compact and elegant form. Combining monopole terms with the massive field eq. of electromagnetism and linear gravity, the general form of wave equation has been derived in terms of octons. Similarly, the homogeneous Klein-Gordon eqation has been obtained. The proposed formulation in this work shows that the derived equations are in a similar form with their electromagnetic and linear gravity counterparts.

Octon

Algebraically associative but noncommutative octon $\breve{G}$ is defined as

$$\breve{G} = c_0 + \overleftrightarrow{c} + \tilde{d}_0 + \vec{d} = c_0\boldsymbol{e}_0 + c_1\boldsymbol{e}_1 + c_2\boldsymbol{e}_2 + c_3\boldsymbol{e}_3 + d_0\boldsymbol{a}_0 + d_1\boldsymbol{a}_1 + d_2\boldsymbol{a}_2 + d_3\boldsymbol{a}_3. \quad (1)$$

Here, $\boldsymbol{e}_0 = 1$ and $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$ are axial unit vectors, while $\boldsymbol{a}_0$ is the pseudoscalar unit, $\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3$ are polar unit vectors. Generally, c_n and d_n $(n = 0, 1, 2, 3)$ are complex numbers.

Table : The multiplication and commutation rules of octon's unit vectors

	$\boldsymbol{e}_1$	$\boldsymbol{e}_2$	$\boldsymbol{e}_3$	$\boldsymbol{a}_0$	$\boldsymbol{a}_1$	$\boldsymbol{a}_2$	$\boldsymbol{a}_3$
$\boldsymbol{e}_1$	1	$i\boldsymbol{e}_3$	$-i\boldsymbol{e}_2$	$\boldsymbol{a}_1$	$\boldsymbol{a}_0$	$i\boldsymbol{a}_3$	$-i\boldsymbol{a}_2$
$\boldsymbol{e}_2$	$-i\boldsymbol{e}_3$	1	$i\boldsymbol{e}_1$	$\boldsymbol{a}_2$	$-i\boldsymbol{a}_3$	$\boldsymbol{a}_0$	$i\boldsymbol{a}_1$
$\boldsymbol{e}_3$	$i\boldsymbol{e}_2$	$-i\boldsymbol{e}_1$	1	$\boldsymbol{a}_3$	$i\boldsymbol{a}_2$	$-i\boldsymbol{a}_1$	$\boldsymbol{a}_0$
$\boldsymbol{a}_0$	$\boldsymbol{a}_1$	$\boldsymbol{a}_2$	$\boldsymbol{a}_3$	1	$\boldsymbol{e}_1$	$\boldsymbol{e}_2$	$\boldsymbol{e}_3$
$\boldsymbol{a}_1$	$\boldsymbol{a}_0$	$i\boldsymbol{a}_3$	$-i\boldsymbol{a}_2$	$\boldsymbol{e}_1$	1	$i\boldsymbol{e}_3$	$-i\boldsymbol{e}_2$
$\boldsymbol{a}_2$	$-i\boldsymbol{a}_3$	$\boldsymbol{a}_0$	$i\boldsymbol{a}_1$	$\boldsymbol{e}_2$	$-i\boldsymbol{e}_3$	1	$i\boldsymbol{e}_1$
$\boldsymbol{a}_3$	$i\boldsymbol{a}_2$	$-i\boldsymbol{a}_1$	$\boldsymbol{a}_0$	$\boldsymbol{e}_3$	$i\boldsymbol{e}_2$	$-i\boldsymbol{e}_1$	1

The Product of Octons

The product of two octons as $\breve{G}_1$ and $\breve{G}_2$ is defined as

$$\begin{aligned}\breve{G}_1\breve{G}_2 &= \{c_{10} + \overleftrightarrow{c_1} + \tilde{d}_{10} + \vec{d}_1\}\{c_{20} + \overleftrightarrow{c_2} + \tilde{d}_{20} + \vec{d}_2\} \\ &= c_{10}c_{20} + c_{10}\overleftrightarrow{c_2} + c_{10}\tilde{d}_{20} + c_{10}\vec{d}_2 + c_{20}\overleftrightarrow{c_1} + (\overleftrightarrow{c_1}\cdot\overleftrightarrow{c_2}) + [\overleftrightarrow{c_1}\times\overleftrightarrow{c_2}] \\ &+ \tilde{d}_{20}\overleftrightarrow{c_1} + (\overleftrightarrow{c_1}\cdot\vec{d}_2) + [\overleftrightarrow{c_1}\times\vec{d}_2] + \tilde{d}_{10}c_{20} + \tilde{d}_{10}\overleftrightarrow{c_2} + \tilde{d}_{10}\tilde{d}_{20} + \tilde{d}_{10}\vec{d}_2 \\ &+ c_{20}\vec{d}_1 + (\vec{d}_1\cdot\overleftrightarrow{c_2}) + [\vec{d}_1\times\overleftrightarrow{c_2}] + \tilde{d}_{20}\vec{d}_1 + (\vec{d}_1\cdot\vec{d}_2) + [\vec{d}_1\times\vec{d}_2].\end{aligned} \quad (2)$$

The octonic differential operator

The octonic differential operator can be introduced as

$$\square = \frac{\partial}{\partial t} + \vec{\nabla} = \frac{\partial}{\partial t}\boldsymbol{e}_0 + \frac{\partial}{\partial x}\boldsymbol{a}_1 + \frac{\partial}{\partial y}\boldsymbol{a}_2 + \frac{\partial}{\partial z}\boldsymbol{a}_3 \quad (3)$$

where its conjugate is

$$\overline{\square} = \frac{\partial}{\partial t} - \vec{\nabla} = \frac{\partial}{\partial t}\boldsymbol{e}_0 - \frac{\partial}{\partial x}\boldsymbol{a}_1 - \frac{\partial}{\partial y}\boldsymbol{a}_2 - \frac{\partial}{\partial z}\boldsymbol{a}_3. \quad (4)$$

As a consequence, the d'Alembertian operator can be obtained as

$$\boxdot = \square\overline{\square} = \overline{\square}\square = \frac{\partial^2}{\partial t^2} - \nabla^2 = \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} - \frac{\partial^2}{\partial z^2}. \quad (5)$$

The octonic generalized potential of gravi-electromagnetism

Assuming the existence of magnetic and gravitomagnetic monopoles, the following generalized octonic potential can be defined

$$\begin{aligned}\breve{\Psi} &= \varphi + \overleftrightarrow{\boldsymbol{A}} - \tilde{\phi} - \vec{\boldsymbol{A}} = (\varphi_e + i\varphi_g) + (\overleftrightarrow{\mathcal{A}_g} - i\overleftrightarrow{A_e}) - (\tilde{\phi}_g - i\tilde{\phi}_e) - (\vec{A}_e + i\vec{\mathcal{A}}_g) \\ &= (\varphi_e + i\varphi_g)\boldsymbol{e}_0 + (\mathcal{A}_x^m - iA_x^m)\boldsymbol{e}_1 + (\mathcal{A}_y^m - iA_y^m)\boldsymbol{e}_2 + (\mathcal{A}_z^m - iA_z^m)\boldsymbol{e}_3 \\ &-(\phi_g - i\phi_e)\boldsymbol{a}_0 - (A_x^e + i\mathcal{A}_x^e)\boldsymbol{a}_1 - (A_y^e + i\mathcal{A}_y^e)\boldsymbol{a}_2 - (A_z^e + i\mathcal{A}_z^e)\boldsymbol{a}_3.\end{aligned} \quad (6)$$

Here

$$\begin{aligned}\breve{A}_e = \varphi_e - i\overleftrightarrow{A_e} + i\tilde{\phi}_e - \vec{A}_e &= \varphi_e\boldsymbol{e}_0 - iA_x^m\boldsymbol{e}_1 - iA_y^m\boldsymbol{e}_2 - iA_z^m\boldsymbol{e}_3 \\ &+ i\phi_e\boldsymbol{a}_0 - A_x^e\boldsymbol{a}_1 - A_y^e\boldsymbol{a}_2 - A_z^e\boldsymbol{a}_3\end{aligned} \quad (7)$$

is octonic generalized potential related to electromagnetism and

$$\begin{aligned}\breve{A}_g = i\varphi_g + \overleftrightarrow{\mathcal{A}_g} - \tilde{\phi}_g - i\vec{\mathcal{A}}_g &= i\varphi_g\boldsymbol{e}_0 + \mathcal{A}_x^m\boldsymbol{e}_1 + \mathcal{A}_y^m\boldsymbol{e}_2 + \mathcal{A}_z^m\boldsymbol{e}_3 \\ &- \phi_g\boldsymbol{a}_0 - i\mathcal{A}_x^e\boldsymbol{a}_1 - i\mathcal{A}_y^e\boldsymbol{a}_2 - i\mathcal{A}_z^e\boldsymbol{a}_3\end{aligned} \quad (8)$$

is octonic generalized potential of linear gravity.

Octonic Formulations of Massive Field Equations

If conjugation of differential operator acts on the generalized potential,

$$\begin{aligned}\overline{\square}\breve{\Psi} &= \left\{\frac{\partial}{\partial t} - \vec{\nabla}\right\}\left\{(\varphi_e + i\varphi_g) + (\overleftrightarrow{\mathcal{A}_g} - i\overleftrightarrow{A_e}) - (\tilde{\phi}_g - i\tilde{\phi}_e) - (\vec{A}_e + i\vec{\mathcal{A}}_g)\right\} \\ &= \left\{\frac{\partial\varphi_e}{\partial t} + (\vec{\nabla}\cdot\vec{A}_e)\right\} - \vec{\nabla}\varphi_e - \frac{\partial\vec{A}_e}{\partial t} + i[\vec{\nabla}\times\overleftrightarrow{A_e}] \\ &+ i\left\{\frac{\partial\varphi_g}{\partial t} + (\vec{\nabla}\cdot\vec{\mathcal{A}}_g)\right\} - i\vec{\nabla}\varphi_g - i\frac{\partial\vec{\mathcal{A}}_g}{\partial t} - [\vec{\nabla}\times\overleftrightarrow{\mathcal{A}_g}] \\ &- \left\{\frac{\partial\tilde{\phi}_g}{\partial t} + (\vec{\nabla}\cdot\overleftrightarrow{\mathcal{A}_g})\right\} + \vec{\nabla}\tilde{\phi}_g + \frac{\partial\overleftrightarrow{\mathcal{A}_g}}{\partial t} + i[\vec{\nabla}\times\vec{\mathcal{A}}_g] \\ &+ i\left\{\frac{\partial\tilde{\phi}_e}{\partial t} + (\vec{\nabla}\cdot\overleftrightarrow{A_e})\right\} - i\vec{\nabla}\tilde{\phi}_e - i\frac{\partial\overleftrightarrow{A_e}}{\partial t} + [\vec{\nabla}\times\vec{A}_e].\end{aligned} \quad (9)$$

the expressions in the Eq.(9) introduce the new definitions of fields related to electromagnetism and linear gravity in terms of octons

$$\vec{E} = -\vec{\nabla}\varphi_e - \frac{\partial\vec{A}_e}{\partial t} + i[\vec{\nabla}\times\overleftrightarrow{A_e}], \quad (10)$$

$$\vec{\mathcal{E}} = -\vec{\nabla}\varphi_g - \frac{\partial\vec{\mathcal{A}}_g}{\partial t} + i[\vec{\nabla}\times\overleftrightarrow{\mathcal{A}_g}], \quad (11)$$

$$\overleftrightarrow{\mathcal{H}} = -\vec{\nabla}\tilde{\phi}_g - \frac{\partial\overleftrightarrow{\mathcal{A}_g}}{\partial t} - i[\vec{\nabla}\times\vec{\mathcal{A}}_g], \quad (12)$$

$$\overleftrightarrow{H} = -\vec{\nabla}\tilde{\phi}_e - \frac{\partial\overleftrightarrow{A_e}}{\partial t} - i[\vec{\nabla}\times\vec{A}_e]. \quad (13)$$

Thus, Eq.(9) can be rewritten by the following compact form,

$$\overline{\square}\breve{\boldsymbol{A}} = \breve{\boldsymbol{F}} \quad (14)$$

where $\breve{\boldsymbol{F}}$ is the octonic generalized field defined by

$$\breve{\boldsymbol{F}} = \vec{\boldsymbol{E}} - \overleftrightarrow{\boldsymbol{H}} = (\vec{E} + i\vec{\mathcal{E}}) - (\overleftrightarrow{\mathcal{H}} - i\overleftrightarrow{H}). \quad (15)$$

Maxwell-Proca-Dirac type Equations of Gravi-Electromagnetim

Operating differential operator on the generalized field

$$\begin{aligned}\square\breve{\boldsymbol{F}} &= \left\{\tfrac{\partial}{\partial t} + \vec{\nabla}\right\}\left\{\vec{\boldsymbol{E}} - \overleftrightarrow{\boldsymbol{H}}\right\} = \left\{\tfrac{\partial}{\partial t} + \vec{\nabla}\right\}\left\{(\vec{E} + i\vec{\mathcal{E}}) - (\overleftrightarrow{\mathcal{H}} - i\overleftrightarrow{H})\right\} \\ &= \left\{(\vec{\nabla}\cdot\vec{E}) + i(\vec{\nabla}\cdot\vec{\mathcal{E}})\right\} + \left\{-\tfrac{\partial\overleftrightarrow{\mathcal{H}}}{\partial t} + i[\vec{\nabla}\times\vec{\mathcal{E}}]\right\} + i\left\{\tfrac{\partial\overleftrightarrow{H}}{\partial t} - i[\vec{\nabla}\times\vec{E}]\right\} \\ &+ \left\{-(\vec{\nabla}\cdot\overleftrightarrow{\mathcal{H}}) + i(\vec{\nabla}\cdot\overleftrightarrow{H})\right\} + \left\{\tfrac{\partial\vec{E}}{\partial t} + i[\vec{\nabla}\times\overleftrightarrow{H}]\right\} + i\left\{\tfrac{\partial\vec{\mathcal{E}}}{\partial t} + i[\vec{\nabla}\times\overleftrightarrow{\mathcal{H}}]\right\}.\end{aligned} \quad (16)$$

and defining the octonic generalized source density

$$\begin{aligned}\breve{\boldsymbol{J}} &= \rho - \overleftrightarrow{\boldsymbol{J}} + \tilde{\rho} - \vec{\boldsymbol{J}} = (\varrho_e - i\varrho_g) - (\overleftrightarrow{\mathcal{J}_g} + i\overleftrightarrow{J_e}) + (\tilde{\varrho}_g + i\tilde{\varrho}_e) - (\vec{J}_e - i\vec{\mathcal{J}}_g) \\ &= (\varrho_e - i\varrho_g)\boldsymbol{e}_0 - (\mathcal{J}_x^m + iJ_x^m)\boldsymbol{e}_1 - (\mathcal{J}_y^m + iJ_y^m)\boldsymbol{e}_2 - (\mathcal{J}_z^m + iJ_z^m)\boldsymbol{e}_3 \\ &+ (\rho_g + i\rho_e)\boldsymbol{a}_0 - (J_x^e - i\mathcal{J}_x^e)\boldsymbol{a}_1 + (J_y^e - i\mathcal{J}_y^e)\boldsymbol{a}_2 - (J_z^e - i\mathcal{J}_z^e)\boldsymbol{a}_3.\end{aligned} \quad (17)$$

the generalized octonic electric potential

$$\breve{\Psi}_e = \varphi - \vec{\boldsymbol{A}} = (\varphi_e + i\varphi_g) - (\vec{A}_e + i\vec{\mathcal{A}}_g)$$

then we can reach the following expression

$$\square\breve{\boldsymbol{F}} + \lambda_\gamma^2\breve{\Psi}_e = \breve{\boldsymbol{J}}. \quad (18)$$

Wave Equations of Massive Gravi-Electromagnetism

After the following operation

$$\square\overline{\square}\breve{\Psi} = \square\breve{\boldsymbol{F}} \quad (19)$$

and using Eq. (18), the following generalized Proca type wave equation in compact form can be written

$$\boxdot\,\breve{\Psi} + \lambda_\gamma^2\breve{\Psi}_e = \breve{\boldsymbol{J}}. \quad (20)$$

For the source free region, $\breve{\boldsymbol{J}} = 0$, this equation becomes

$$\boxdot\breve{\Psi} + \lambda_\gamma^2\breve{\Psi}_e = 0 \quad (21)$$

and it is called the generalized Klein Gordon equation in octon form.

References

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