Hagedorn's Limiting Temperature and the Onset of Deconfinement

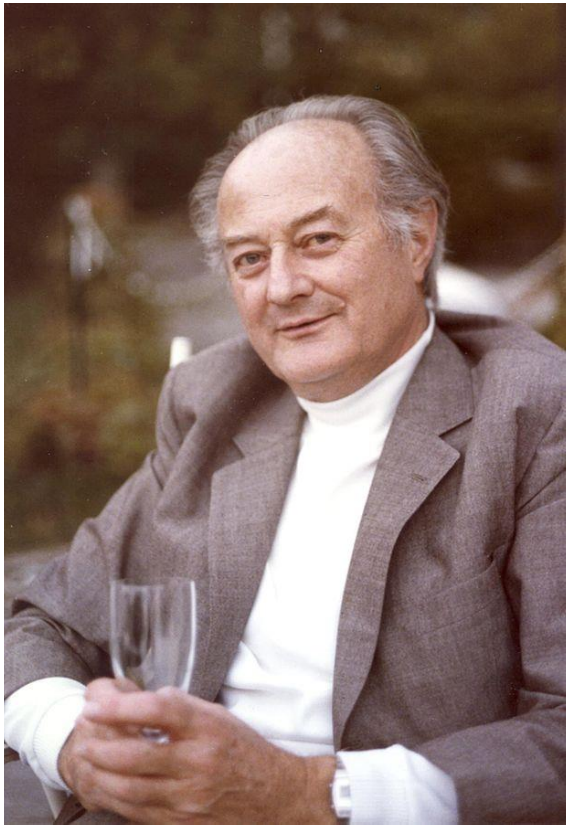
50 Years $T_0 = 160$ MeV

Rolf Hagedorn

"Statistical Thermodynamics of Strong Interactions at High Energies"

Nuovo Cim. Suppl. (1965),

1232 citations.



Hagedorn's Limiting Temperature and the Onset of Deconfinement

Mark I. Gorenstein

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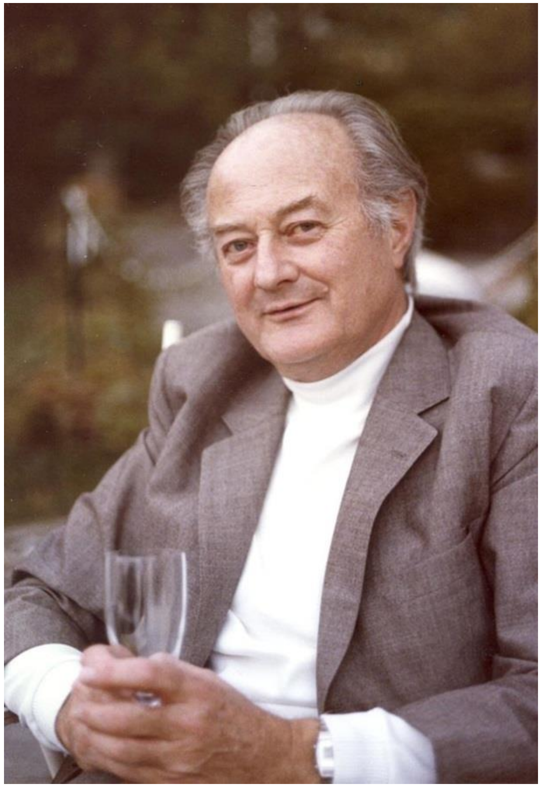
Kiev, Ukraine

1. Limiting Temperature of Fireballs
   1965- …
   Limiting Temperature of a Pion Gas
   …-2015
2. Phase Transitions in the Gas of Bags
   (1980-2000)
3. Onset of Deconfinement
   (2000-2015)
4. Summary



1. Limiting Temperature

Units: $\hbar = c = k = 1$ $1\text{fm}=10^{-13}\text{cm}$, $1\text{fm/c}=10^{-23}\text{sec}$

R. Hagedorn, "Statistical Thermodynamics of Strong Interactions at High Energies" Nuovo Cim. Suppl. (1965)

"We describe the thermodynamics of fireballs which consists of fireballs, which consists of fireballs, which…"

$$Z(T,V_0;m) = \sum_{N=0}^{\infty} \left(\frac{V_0}{2\pi^2} \int_0^{\infty} k^2 dk \; \exp\left(-\sqrt{k^2+m^2}\right)/T \right)^N \frac{1}{N!}$$

Relativistic, Ideal, Boltzmann, Multi-Component, Gas

$$= \exp\left(\frac{V_0 \; m^2 T \; K_2(m/T)}{2\pi^2} \right) = \exp\left(\bar{N}(T,m)\right) ,$$

$$Z(T,V_0;m_1,\ldots,m_L) = \prod_{j=1}^{L} Z(T,V_0;m_j) = \exp\left[\bar{N}(T,m_1)\right]\times\ldots$$

$$\times\exp\left[\bar{N}(T,m_L)\right] = \exp\left[\sum_{j=1}^{L} \bar{N}(T,m_j)\right] = \exp\left[\int_0^{\infty} dm\rho(m)\;\bar{N}(T,m)\right]$$

Interacting hadrons = (non-interacting) Hadrons + Resonances

E. Beth and G.E. Uhlenbeck, Physica (1937), calculated the level density for interacting particles. They derived an expression for this density which describes the interaction by the scattering phase shifts.

S.Z. Belenkij, Nucl. Phys. (1956), proposed to treat hadronic resonances exactly like stable particles in phase space calculations.

R. Dashen, S. Ma, and H.J. Bernstein, Phys. Rev. (1969), S-matrix formulation of statistical mechanics.

Asymptotic Bootsrap ("working hypothesis"):

$$Z(T,V_0) = \exp\left[\int_0^\infty dm\, \rho(m)\, \frac{V_0}{2\pi^2}\, m^2 T\, K_2\left(\frac{m}{T}\right)\right]$$

$$Z(T,V_0) = \int_0^\infty dE\, \sigma(E)\, \exp\left[-\frac{E}{T}\right]$$

$$\frac{\ln \sigma(x)}{\ln \rho(x)} \to 1 \qquad \text{for} \qquad x \to \infty$$

G. Veneziano, Nuovo Cim. (1968), dual resonance model (Veneziano model), Exponentially increasing mass spectrum, Strings (1020 citations)

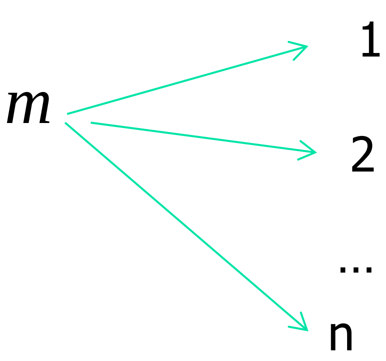
Fireballs=Strings

Asymptotic Bootstrap results in:

$$\rho(m) = C \; m^{-5/2} \exp\left(\frac{m}{T_0}\right) \qquad \text{for} \qquad m \to \infty$$

$$\sigma(E) = B \; E^{-\alpha} \exp\left(\frac{E}{T_0}\right) \qquad \text{for} \qquad E \to \infty$$

T_0 – limiting temperature,



decay of fireball with large mass:

$$\bar{n} \sim \ln(m)$$

Transverse momentum spectra

T_0 must govern the transverse momentum spectra of outgoing final particles in high energy collisions:

$$\frac{dN}{dp_T} \propto \exp\left(- \frac{\sqrt{p_T^2 + m^2}}{T_0} \right)$$

$$T_0 = 158 \pm 3 \text{ MeV}$$

$$T_0 = 160 \pm 5 \text{ MeV} \approx 10^{12} \ {}^0K$$

this estimate was used in further publications

T_0=158 MeV R. Hagedorn (1965)

p_T spectra

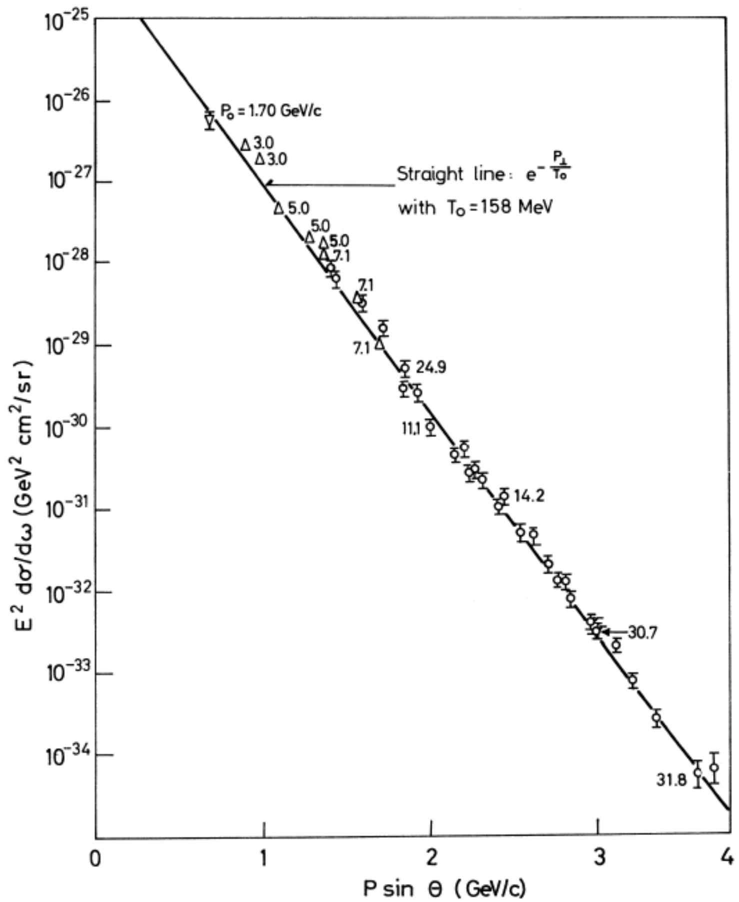


FIG. 1 $[E^2 d\sigma_{el}/d\omega]_{pp}$ as a function of the transverse momentum [taken from ref. 14)]

$\rho(m)$ versus m

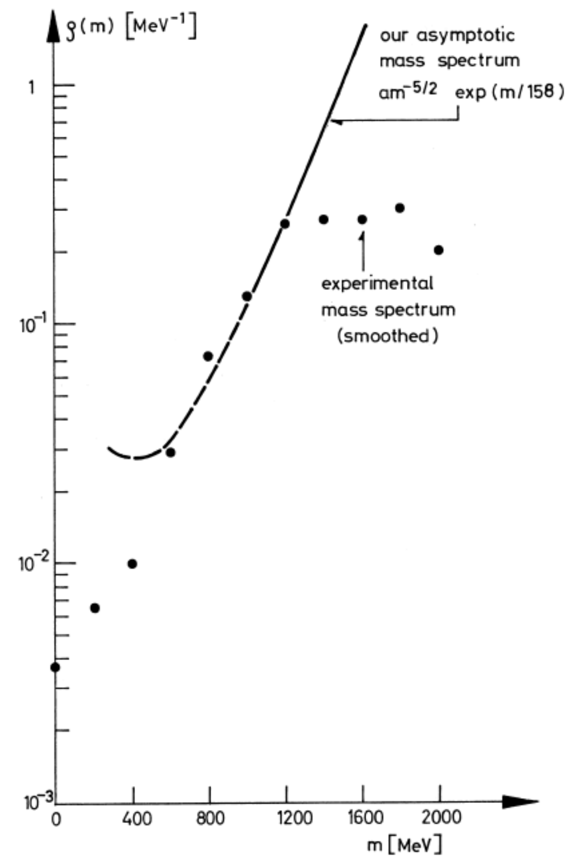


FIG. 2 The experimental mass spectrum 10) (smoothed) compared to our asymptotic ϱ(m); one-parameter fit with a≈6.5 x 10^3 [MeV$^{3/2}$]

R. Hagedorn and J. Ranft, Suppl. Nuovo Cim. (1968);
R. Hagedorn, Suppl. Nuovo Cim. (1968), Nuovo Cim. (1967, 1968).

R. Hagedorn, "Remarks on the Thermodynamical Model of Strong Interactions", Nucl. Phys. B (1970).

New terminology: Boiling Point

A firebal is
a statistical equilibrium of undetermined numbers of fireballs, each of which, in turn, is considered to be …

$$\rho(m) = \frac{C}{(m^2 + m_0^2)^{5/4}} \exp\left(\frac{m}{T_0}\right) \quad \text{for } m > 1000 \text{ MeV}$$

$$C = 2.63 \times 10^4 \text{ MeV}^{2/3}, \quad T_0 = 160 \text{ MeV}, \quad m_0 \cong 500 \text{ MeV}$$

New philosophy: If quarks exist as real free particles then the hadron mass contains states with non-integer charge and baryon number; if they do not exist as free particles, then the hadron spectrum does not contains such states. In both cases quarks need not be considered more elementary than other hadrons, they are just members of the family

S. Frautschi, "Statistical Bootstrap Model of Hadrons", Phys. Rev. D (1971)

$$\rho_{\text{out}}(m) = \sum_{n=2}^{\infty} \left(\frac{V}{(2\pi)^3} \right)^{n-1} \frac{1}{n!} \prod_{i=1}^{n} \int dm_i \rho_{\text{in}}(m_i) \int d^3k_i$$

$$\times\ \delta\left(\sum_{i=1}^{n} \sqrt{k_i^2 + m_i^2} - m \right) \delta^3\left(\sum_{i=1}^{n} \vec{k}_i \right)$$

$$\frac{\rho_{\text{out}}(m)}{\rho_{\text{in}}(m)} \to 1 \quad \text{for} \quad m \to \infty, \qquad \rho_{\text{in}}(m) = Cm^{-a} \exp(bm), \qquad a > 5/2$$

$$\bar{n} = 2.4, \qquad \text{P(n)} = \frac{(\ln 2)^{n-1}}{(n-1)!} \qquad \to \qquad \text{P(2)=0.69}, \quad \text{P(3)=0.24}$$

Selected papers on the Statistical Bootstrap Model:

W. Nahm, "Analytical Solution of the Statistical Bootstrap Model", Nucl. Phys. B (1972) a=3

S. Frautschi and C.J. Hamer, Phys. Rev. D (1971), Nuovo Cim. (1973), Inhomogenious Bootstrap Equation, a=3

C.J. Hamer, Nuovo Cim. (1972)

J. Yellin, Nucl. Phys. B (1973), "An Explicit Solution of the Statistical Bootstrap"

R. Hagedorn and I. Montway, "A Model Study in Hadron Statistical Bootstrap", Nucl. Phys. B (1973)

R. Hagedorn and U. Wambach, "Large Transverse Momenta from Statistical Bootstrap with Spin", Nucl. Phys. B (1977)

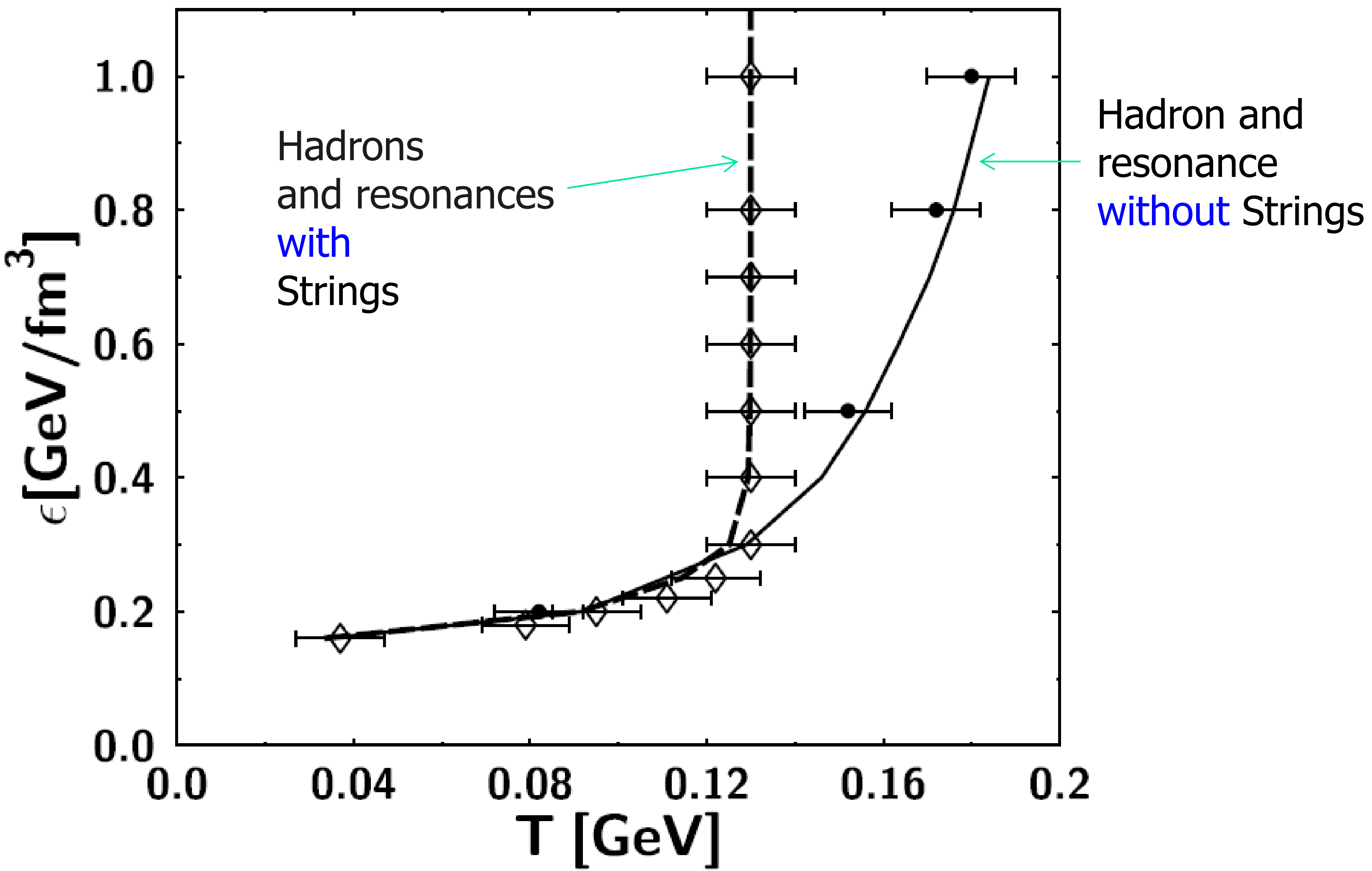
R. Hagedorn and J. Rafelski, Commun. Math. Phys. (1982) $z = 2Z - \exp(Z) + 1$

Belkasem,…, Bravina,…, M.I.G.,…, W. Greiner, Phys. Rev. C (1998)

UrQMD in the box with periodic boundary conditions

Strings = Fireballs

$$T_0 = 130 \pm 10 \text{ MeV}$$



Limiting Temperature (short summary)

$$\rho(m) = C \; m^{-a} \exp\left(\frac{m}{T_0}\right) \qquad \text{for} \qquad m \to \infty$$

$$p(T) = T \int_{M_0}^{\infty} dm \; \rho(m) \left(\frac{mT}{2\pi}\right)^{3/2} \exp\left(-\frac{m}{T}\right) \propto \left(T_0 - T\right)^{a-5/2}$$

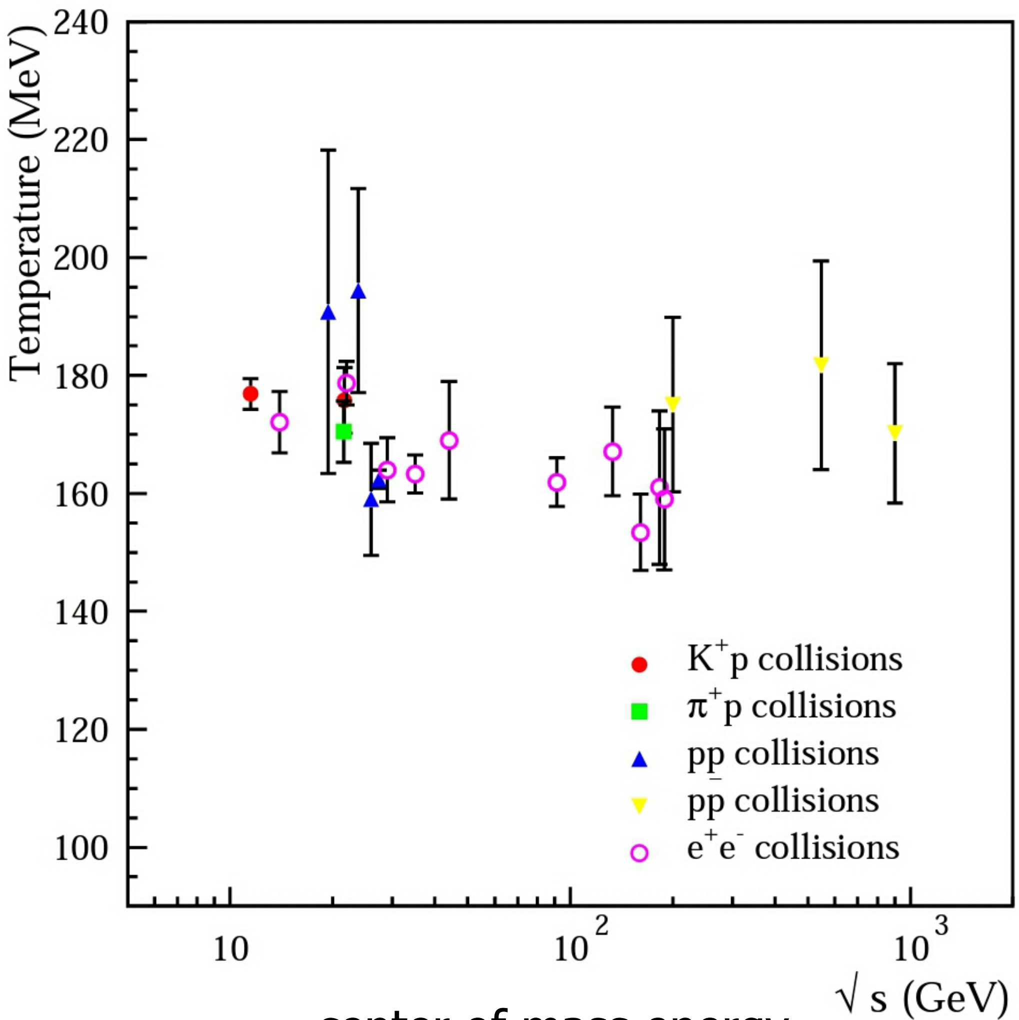
$$\varepsilon(T) = T \frac{dp}{dT} - p \;\propto\; \left(T_0 - T\right)^{a-7/2}$$

$$\text{At } T \to T_0: \quad p,\; \varepsilon \to \infty, \text{ for } a \le \frac{5}{2}$$

$$p \to const,\; \varepsilon \to \infty, \text{ for } \frac{5}{2} < a \le \frac{7}{2}$$

$$p \to const,\; \varepsilon \to const, \text{ for } a > \frac{7}{2}$$

T_0 is a limiting temperature in the Hadron World



center of mass energy

From fitting the data on hadron multiplicities within statistical model,
Becattini
arXiv:0901.3643 [hep-ph]

Limiting Temperature in pion gas with the van der Waals Equation of State

$$p = \frac{NT}{V - bN} - a\frac{N^2}{V^2} \equiv \frac{nT}{1 - bn} - an^2,$$

van der Waals, Ph.D, 1873

CE, Boltzmann Statistics

Nobel Prize in Physics 1910

GCE:

$$p(T,\mu) = p_{\rm id}(T,\mu^*) - an^2(T,\mu), \qquad n(T,\mu) = \frac{n_{\rm id}(T,\mu^*)}{1 + bn_{\rm id}(T,\mu)},$$

$$\mu^* = \mu - bp(T,\mu) - abn^2(T,\mu) + 2an(T,\mu)$$

Vovchenko, Anchishkin, and M.I.G. , J. Phys. A (2015)

Quantum Statistics in the VDW EoS:

$$p_{\rm id}(T,\mu^*) = \frac{d}{6\pi^2}\int_0^\infty k^2 dk \frac{k^2}{\sqrt{k^2+m^2}}\left[\exp\left(\frac{\sqrt{k^2+m^2}-\mu^*}{T}\right)+\eta\right]^{-1}$$

$$n_{\rm id}(T,\mu^*) = \frac{d}{2\pi^2}\int_0^\infty k^2 dk \left[\exp\left(\frac{\sqrt{k^2+m^2}-\mu^*}{T}\right)+\eta\right]^{-1}$$

Vovchenko, Anchishkin, and M.I.G. , Phys. Ev. C (2015)

Pion Gas: $m = m_\pi$, $d = 3$, $\mu=0$, $\eta = -1$

$b = 0.45 \text{ fm}^3$, i.e., $r = 0.3$ fm

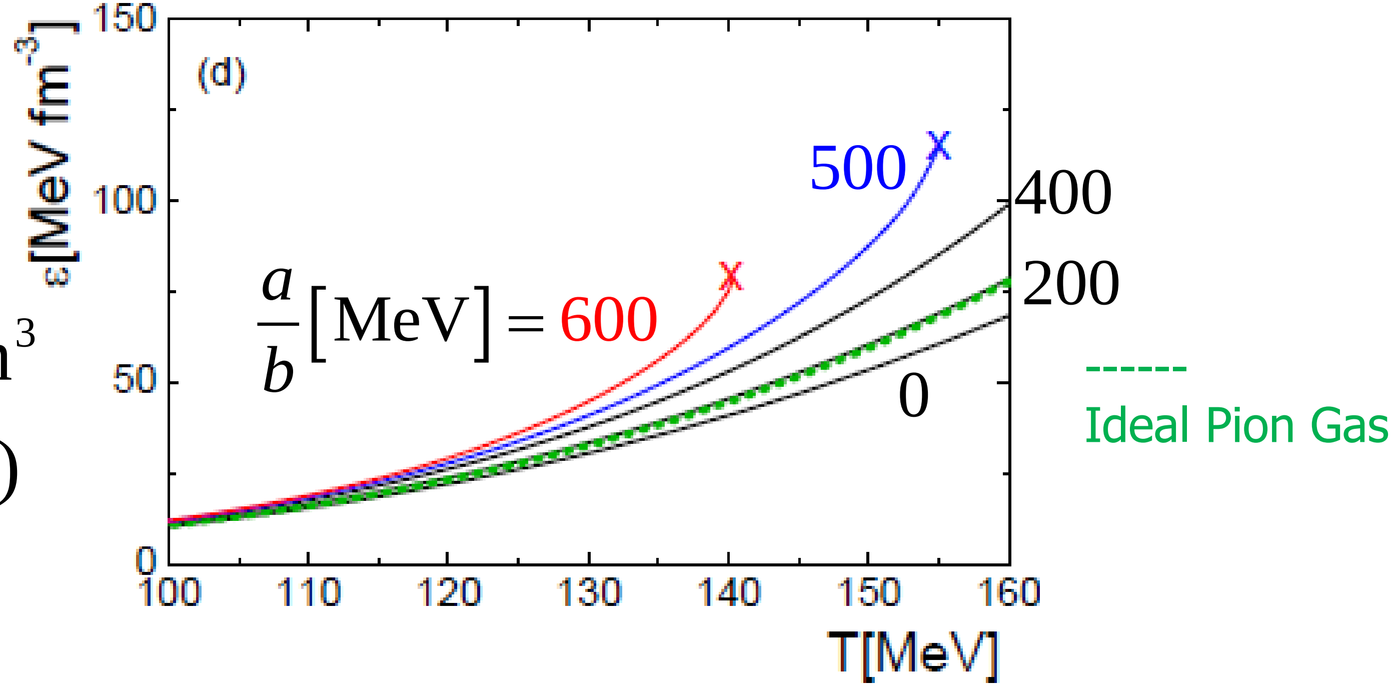
a is considered as a free parameter

Pion gas with the VDW EoS

×× limiting temperature: $T_0 = T_0(a,b)$

$b = 0.45\ \text{fm}^3$

$(r = 0.3\ \text{fm})$



Energy density of the pion gas with the VDW EoS

Vovchenko, Poberezhnyuk, Anchishkin, and M.I.G., arXiv (2015)

Limiting temperature in the pion gas with the VDW EoS

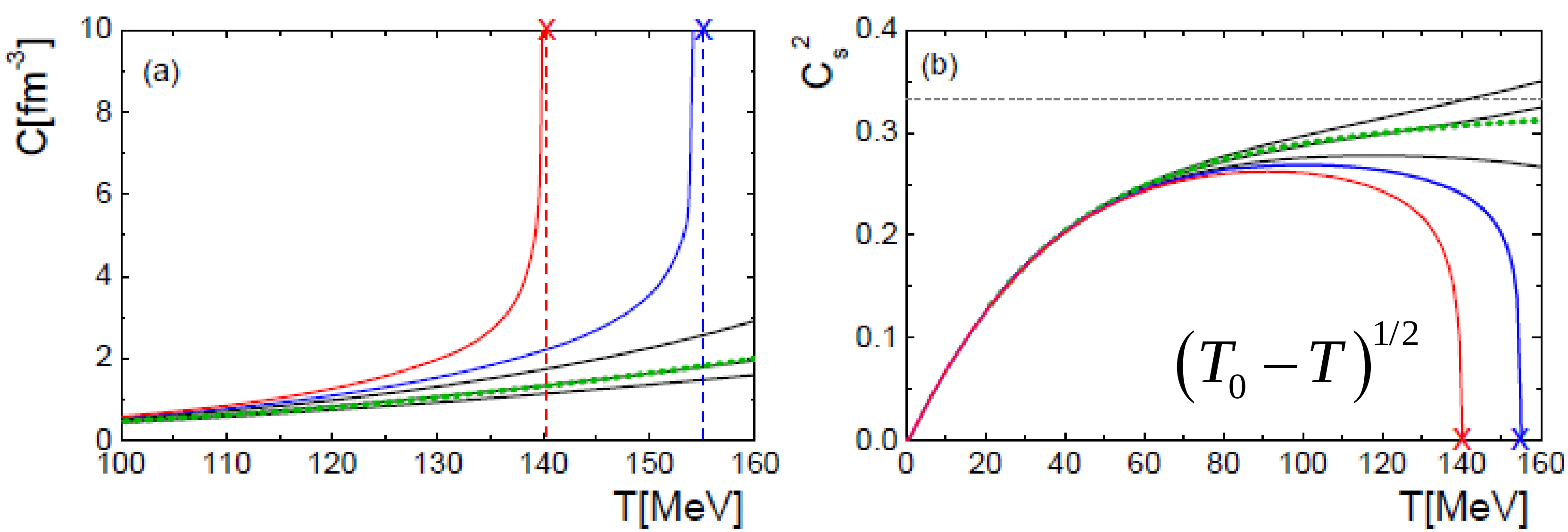
Heat Capacity

$$C = \frac{d\varepsilon}{dT}$$

$$\left(T_0 - T\right)^{-1/2}$$

Speed of Sound

$$c_s^2 = \frac{dp}{d\varepsilon}$$



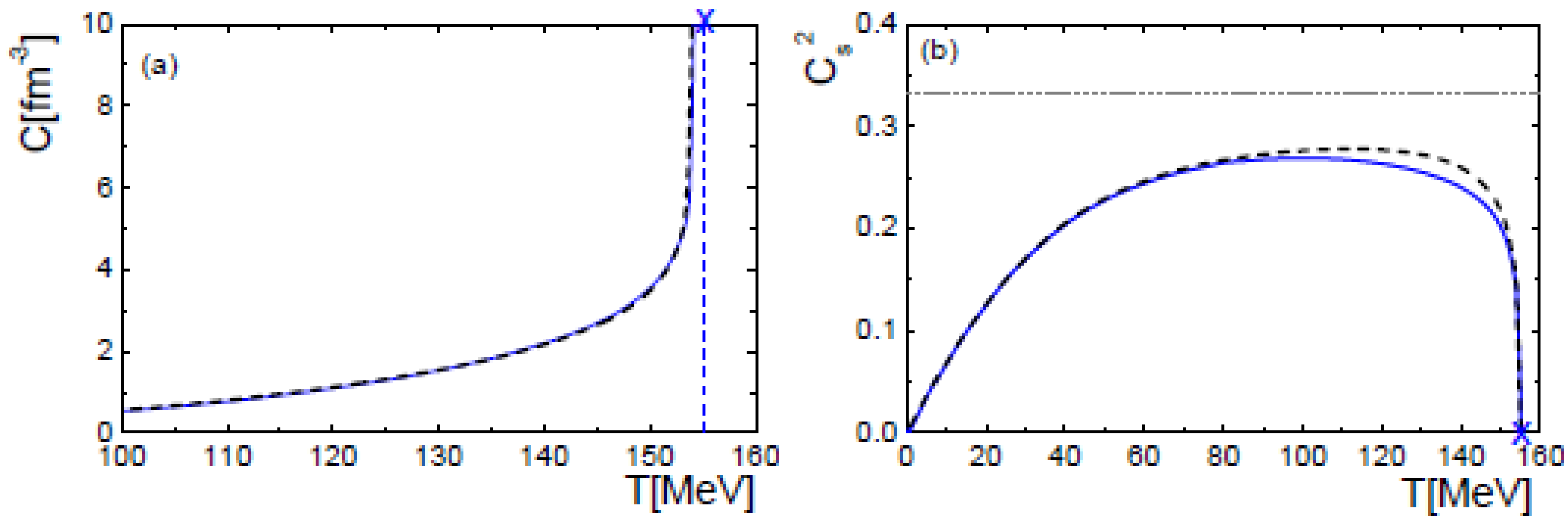
Hagedorn versus VDW

Hagedorn model: $\rho(m) = cm^{-\alpha}\exp(m/T_0),$

$\alpha = 4,\ T_0 = 155\ \text{MeV}$

VDW EoS for pions: $b = 0.45\ \text{fm}^3, \quad a/b = 500\ \text{MeV}$

solid blue line VDW dashed black line Hagedorn



2. Phase Transitions in the Gas of Bags

$$V(m) = \frac{m}{4B}, \qquad B - \text{bag constant}$$

R. Hagedorn and J. Rafelski, Phys. Lett. B (1980)
J. Kapusta, Phys. Rev. D (1981).

$$V_{av} = V - \frac{< E >}{4B},$$

$$\varepsilon(T,\mu) = \frac{\varepsilon_{pt}(T,\mu)}{1+\varepsilon_{pt}(T,\mu)/4B}, \qquad p(T,\mu) = \frac{p_{pt}(T,\mu)}{1+\varepsilon_{pt}(T,\mu)/4B}$$

Critical Line:

$$\varepsilon_{pt}(T,\mu) \to \infty, \qquad \varepsilon(T,\mu) \to 4B, \quad p(T,\mu) \to 0$$

$V_{av} \to 0$, We believe that beyond the critical line a transition to the quark gluon plasma occurs

Phase Transition in the Gas Bags

$$V \to \left(V - \sum_{i=1}^{N} v_i \right)$$

M.I.G., Petrov, and Zinovjev, Phys. Lett. B (1981)

$$\rho(m) \to \rho\left(m, v\right) = \rho_0 + C v^{\gamma} \left(m - Bv\right)^{\delta} \exp\left[\frac{4}{3} \sigma^{1/4} v^{1/4} \left(m - Bv\right)^{3/4} \right]$$

$$\hat{Z}(T,s) = \int_0^{\infty} dV \exp(-sV)\; Z(T,V) = \frac{1}{s - f(T,s)}$$

$$f(T,s) \equiv \left(\frac{T}{2\pi} \right)^{3/2} \int dm dv\; m^{3/2} \rho(m,v) \exp\left(-\frac{m}{T} \right) \exp(-sv)$$

$$p(T) = Ts^{*}(T) = T \max\{ s_H(T), s_Q(T) \}$$

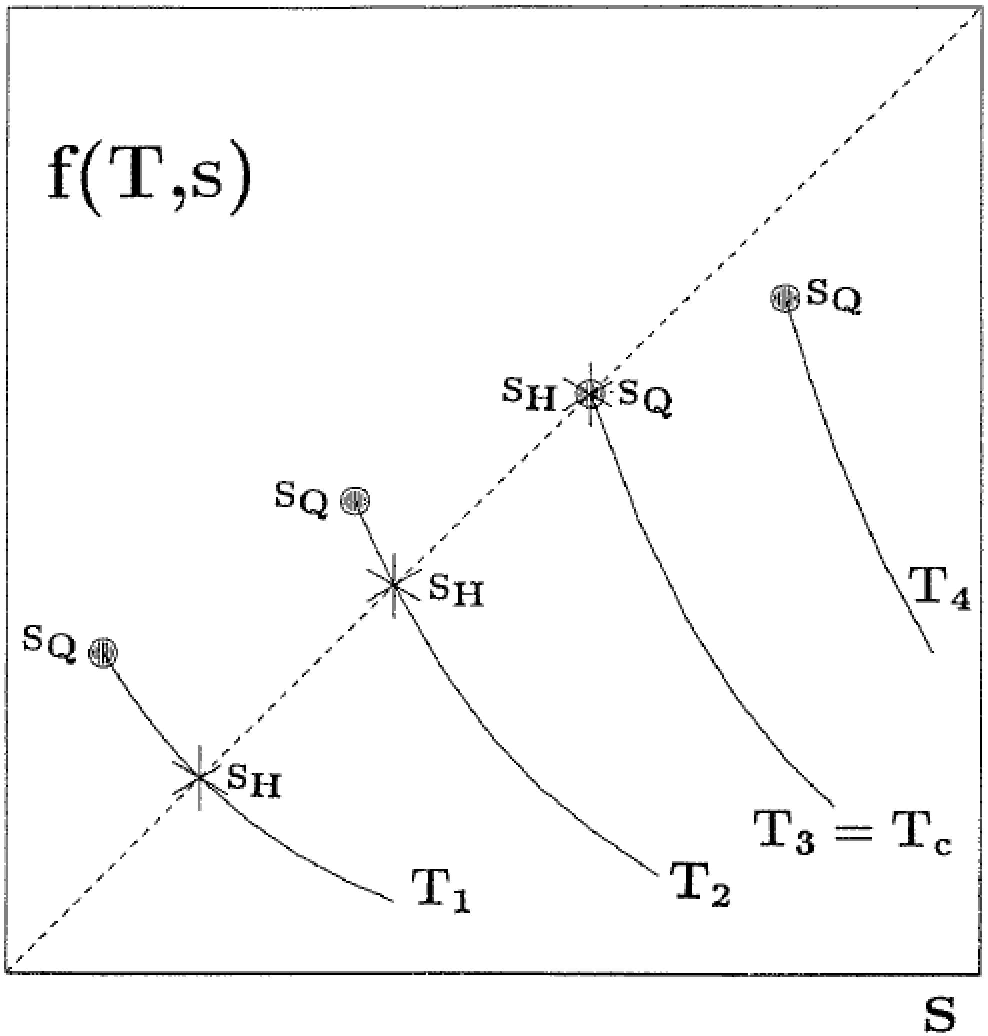
s^{*} is the farthest-right singularity of $\hat{Z}(T,s)$: $p(T) = Ts^{*}(T)$

$$s_H = f\left(T, s_H\right), \qquad f\left(T, s_Q\right)$$

$\gamma + \delta < -3, \quad \delta < -7/4$ conditions for the PTs

$$p(T) = Ts_Q(T) = \frac{\sigma}{3}T^4 - B$$

HG → QGP



M.I.G., Zinovjev, Petrov, and Shelest,
Teor. Mat. Fiz. (1982)
M.I.G. Yad. Fiz. (1984)

1st order PT

M.I.G., W. Greiner, and
Shin Nan Yang, J. Phys. G (1998)

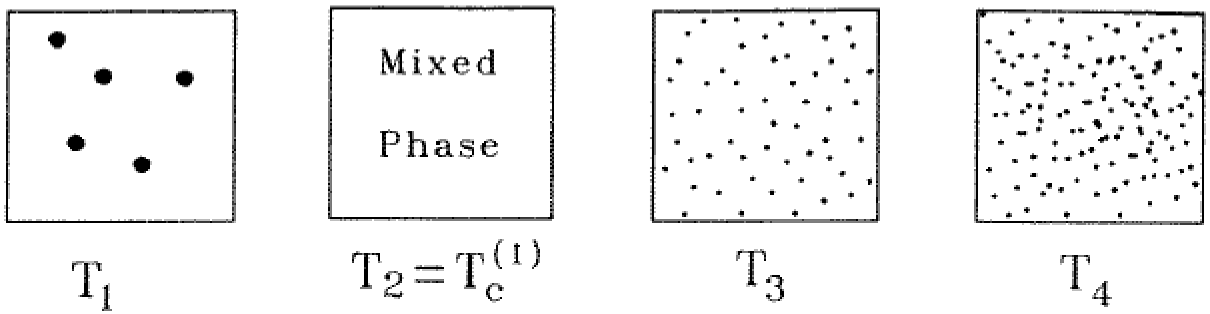
2nd order PT

M.I.G., Gazdzicki, and
W. Greiner, Phys. Rev. C (2005)

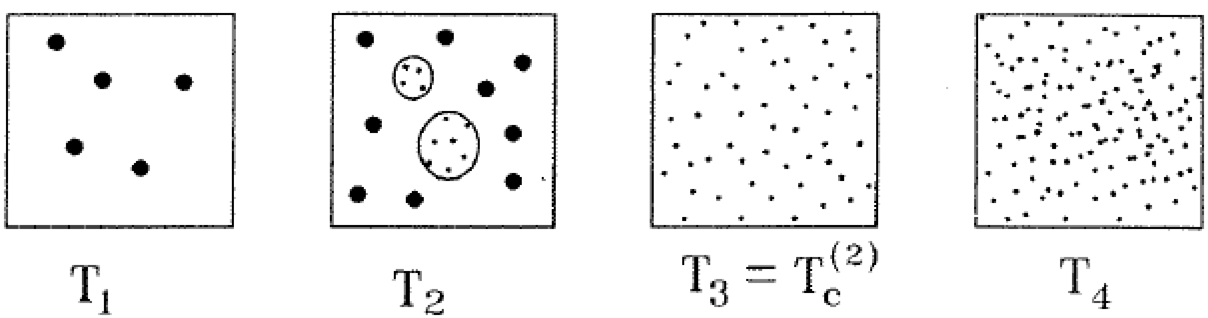
Higher order PTs

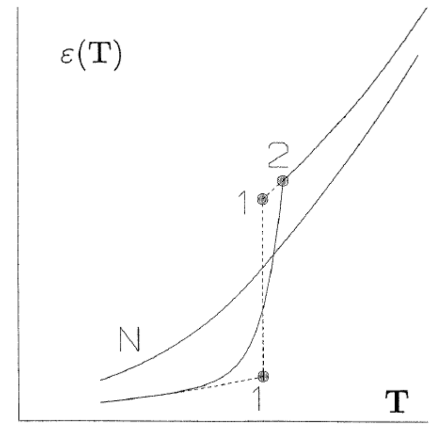
1^{st} order PT, 2^{nd} order PT, …, Cross-over

hadrons=small bags 1^{st} order PT QGP=infinitely large bag



2^{nd} order PT

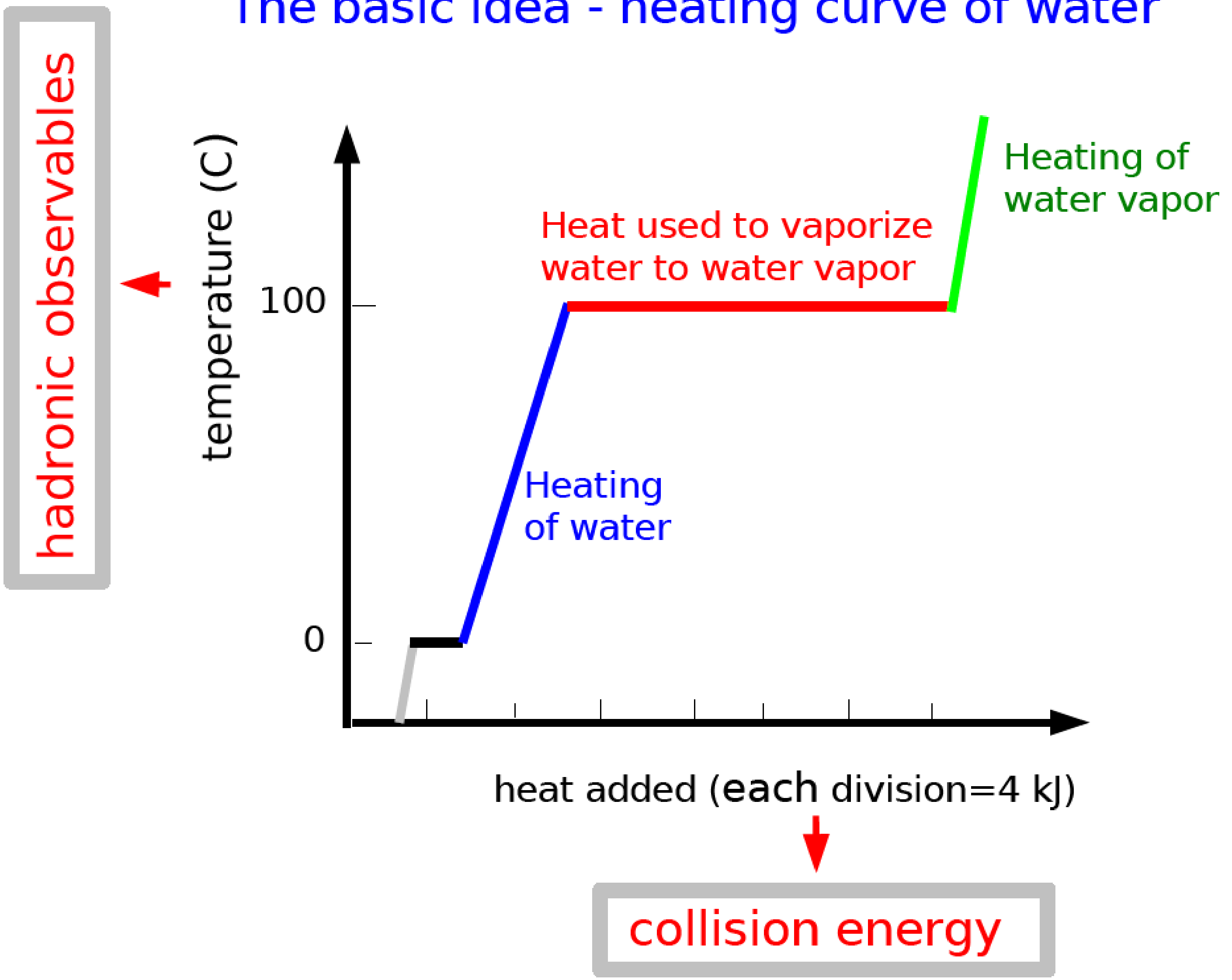




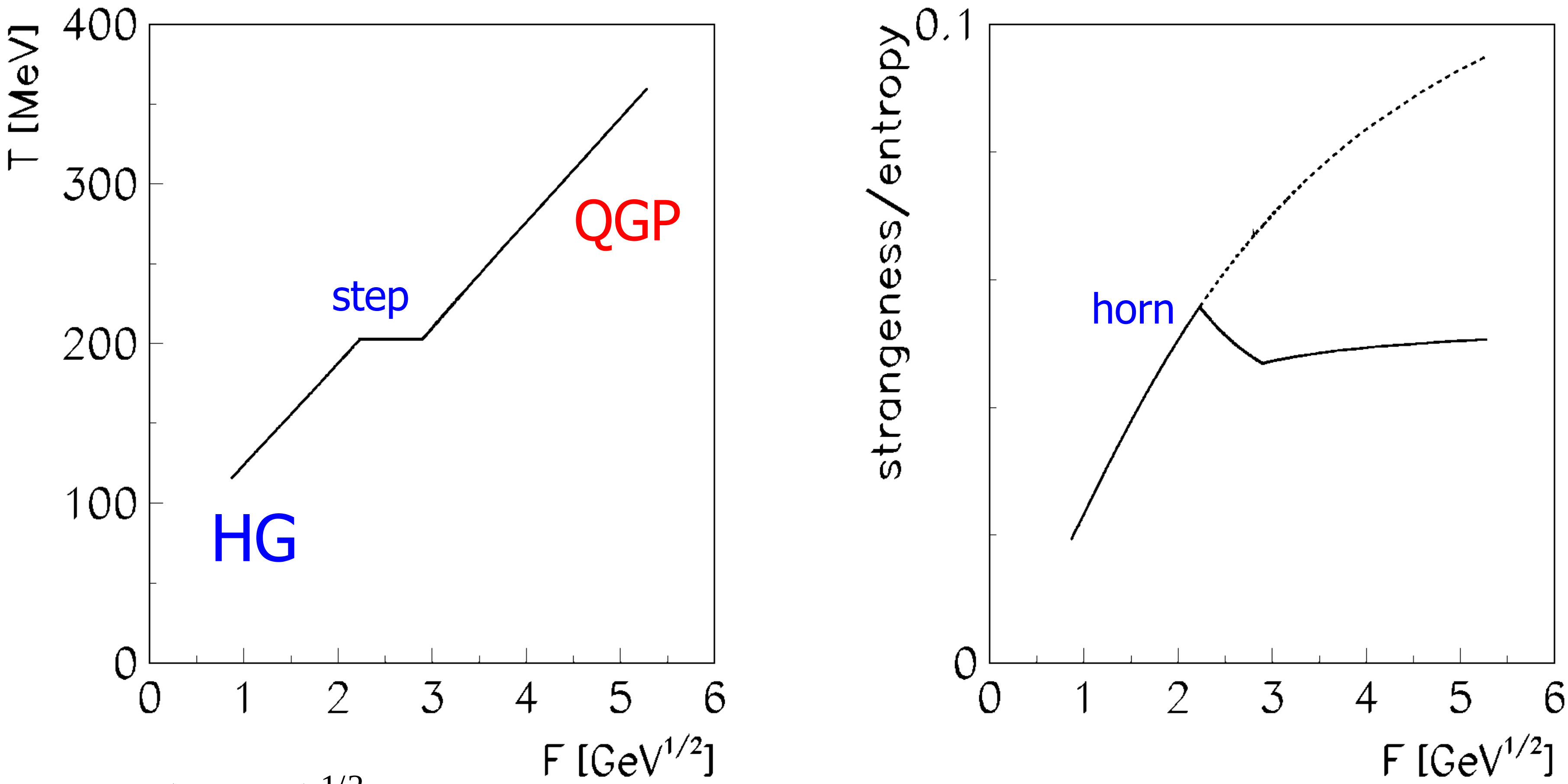
3. Onset of Deconfinement

Onset of deconfinement at the CERN SPS

The basic idea - heating curve of water



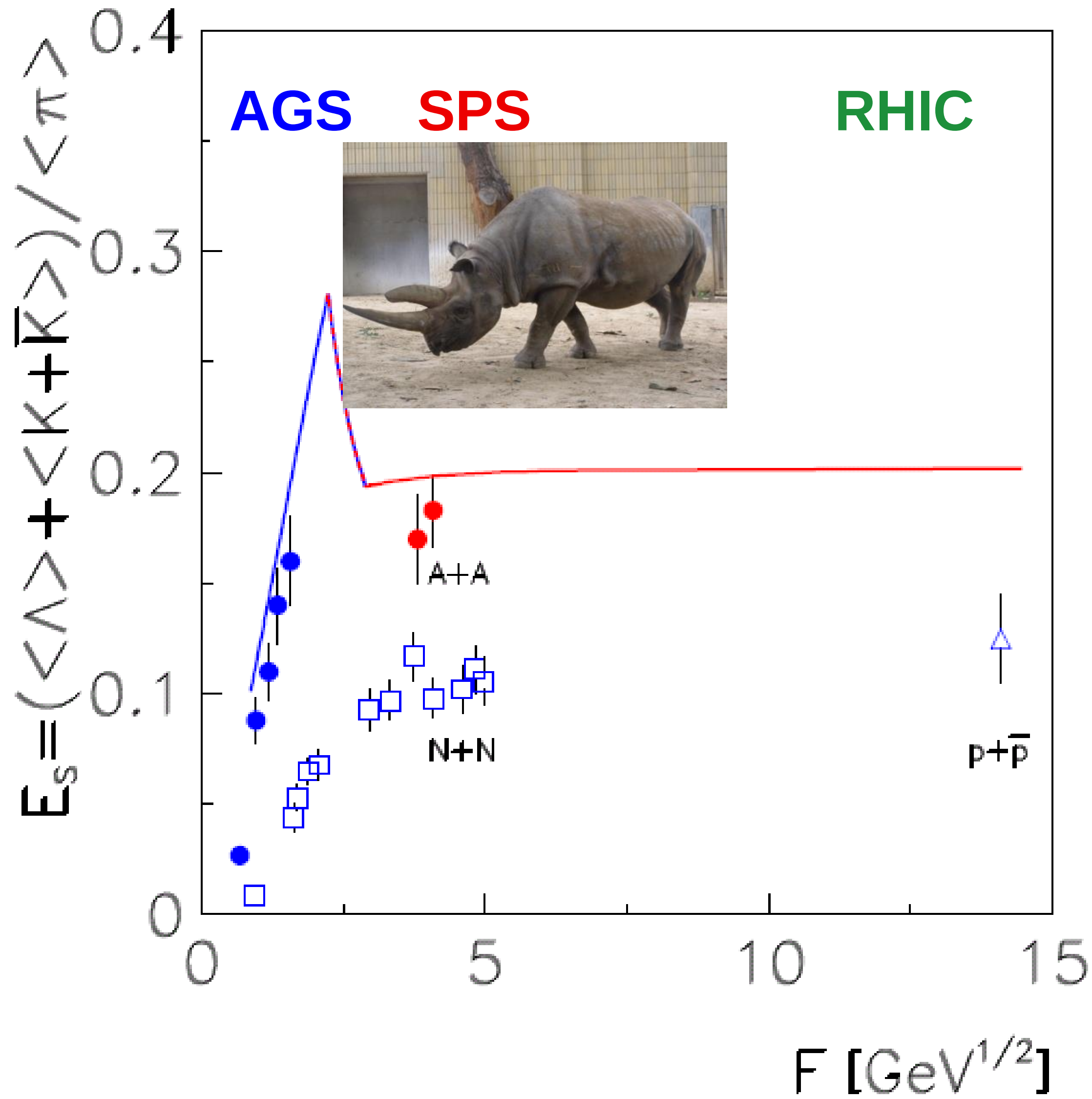
Statistical Model of Early Stage



$$F \cong \left(\sqrt{s_{NN}}\right)^{1/2}$$

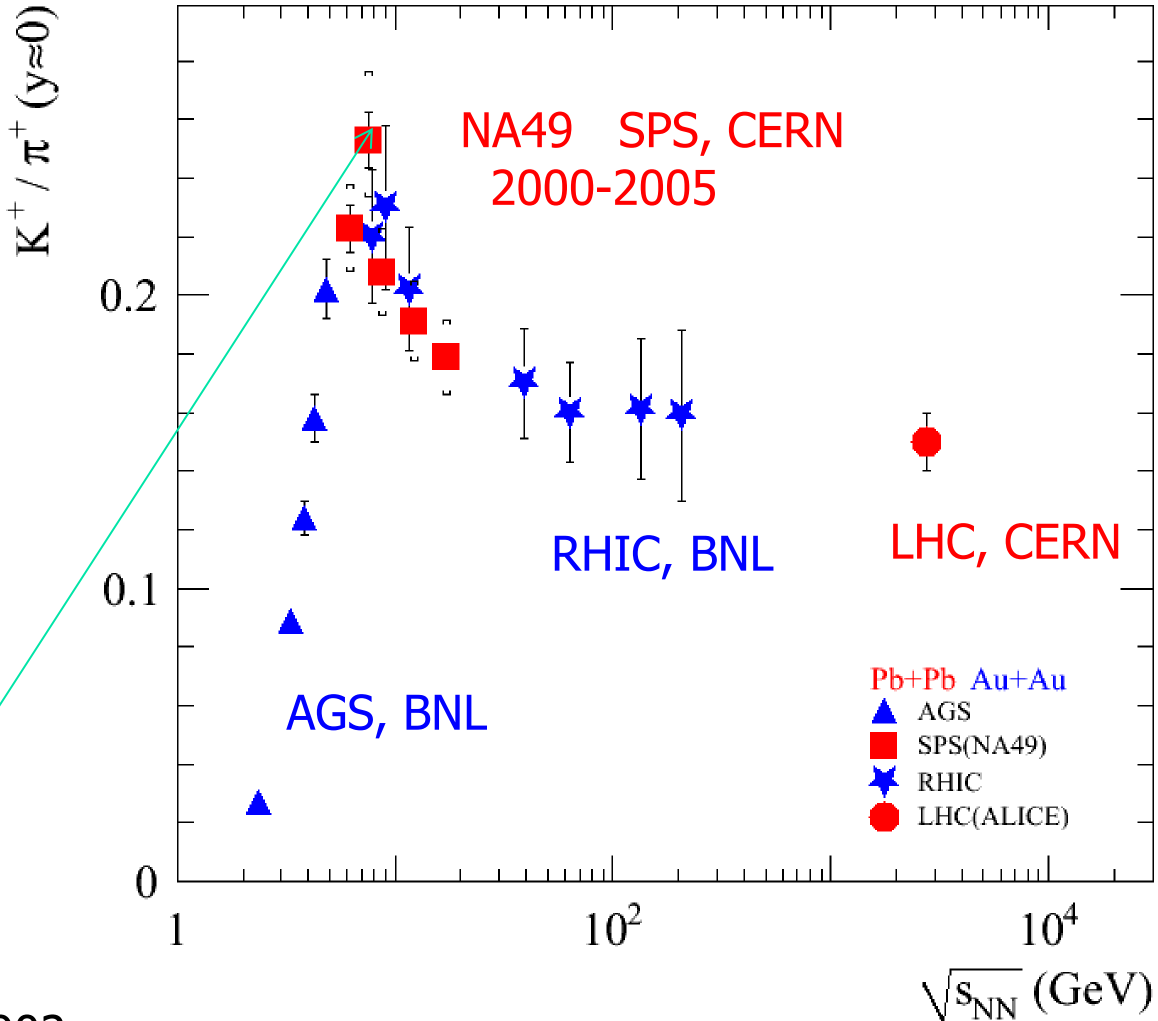
Gazdzicki and M.I.G., Acta Phys. Pol. (1998)

Experimental Data in 1998



$$\frac{\text{strangeness}}{\text{entropy}}$$

The Horn: Pb+Pb



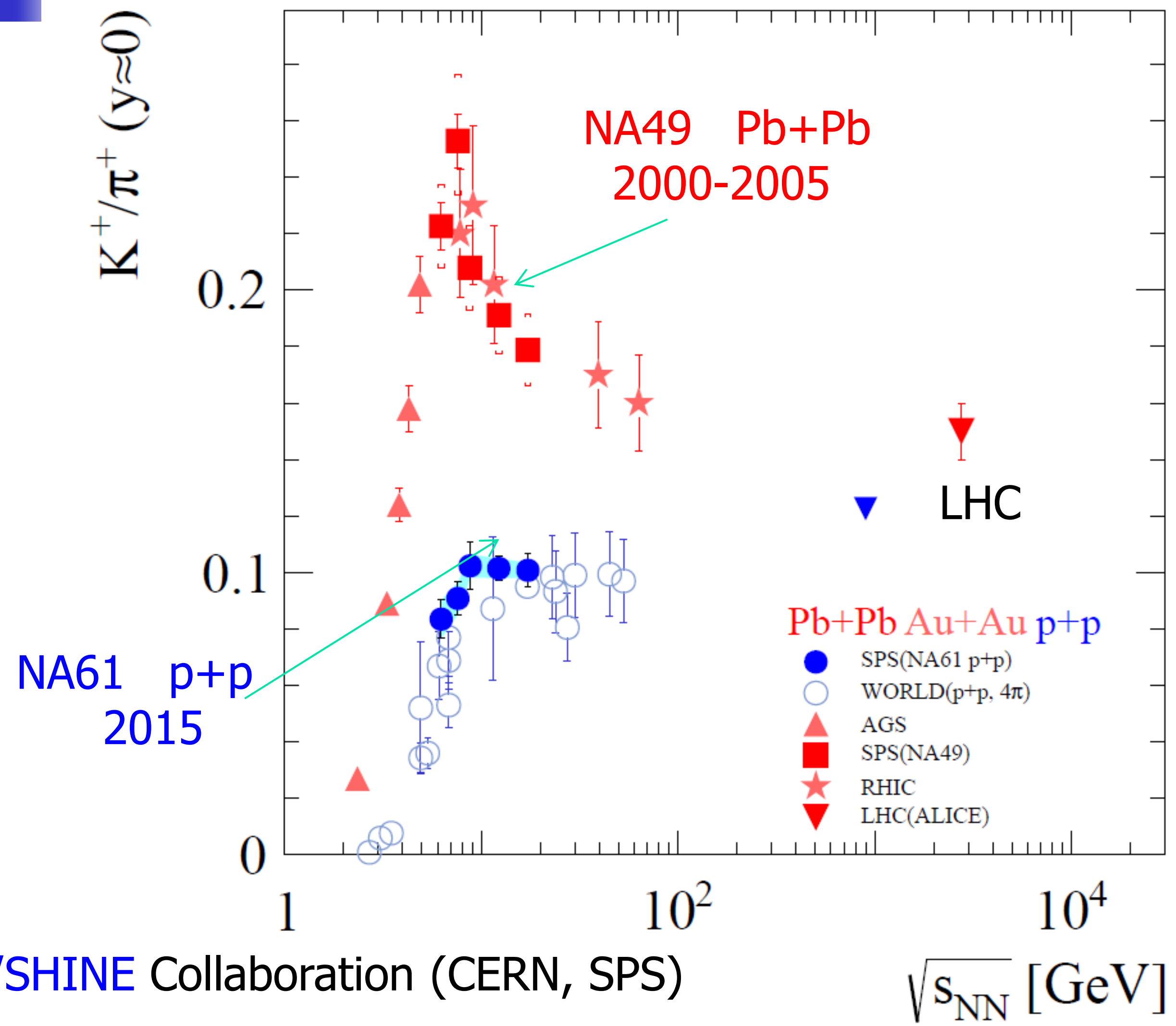
$$\frac{\text{strangeness}}{\text{entropy}}$$

NA49 Collaboration
(CERN SPS)

Phys.Rev. C66 (2002) 054902

Phys.Rev. C77 (2008) 024903

The Horn: Pb+Pb vs p+p

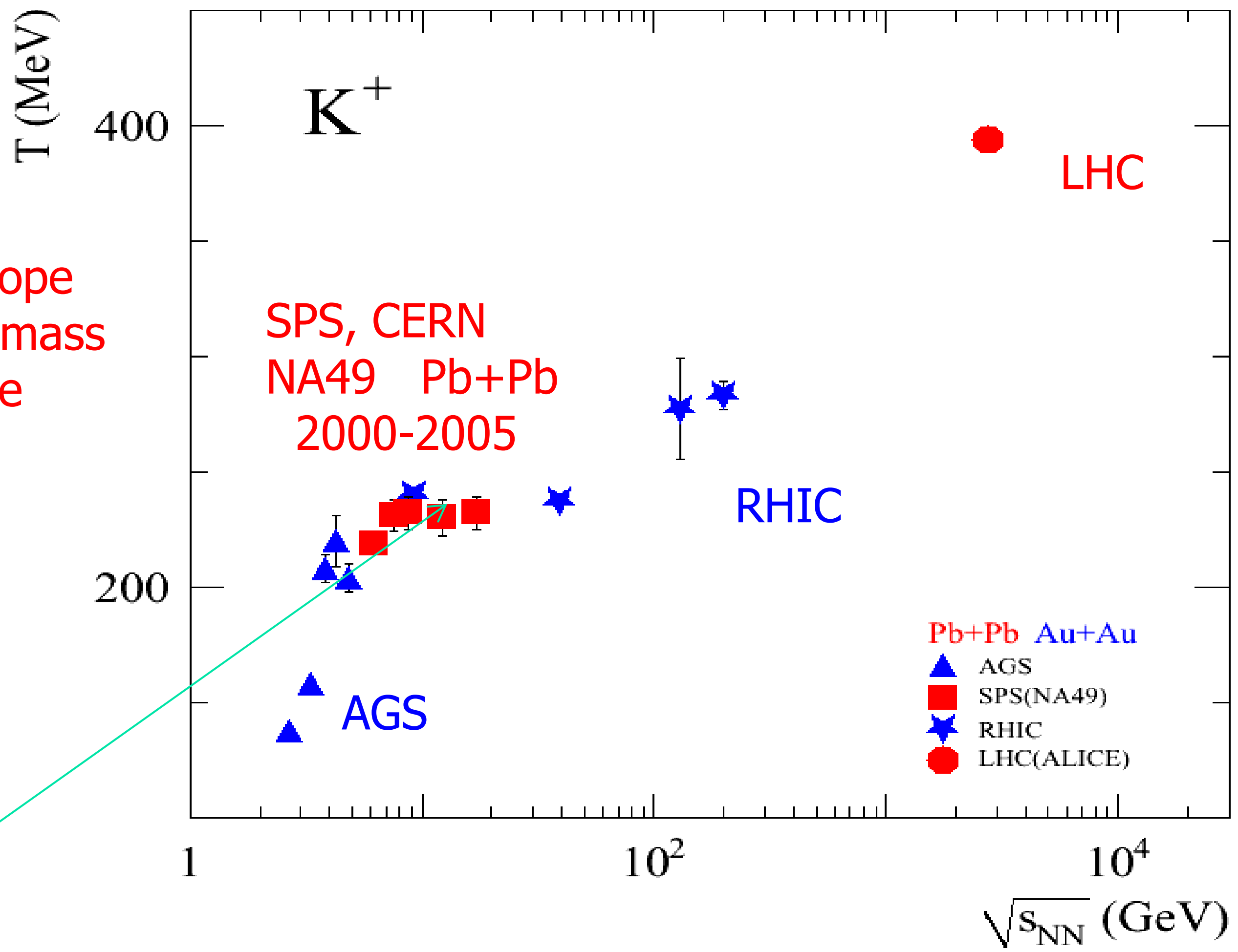


NA61/SHINE Collaboration (CERN, SPS)

arXiv:1502.07916 [nucl-ex]

The Step: Pb+Pb

T is the inverse slope of the transverse mass spectra = effective temperature.



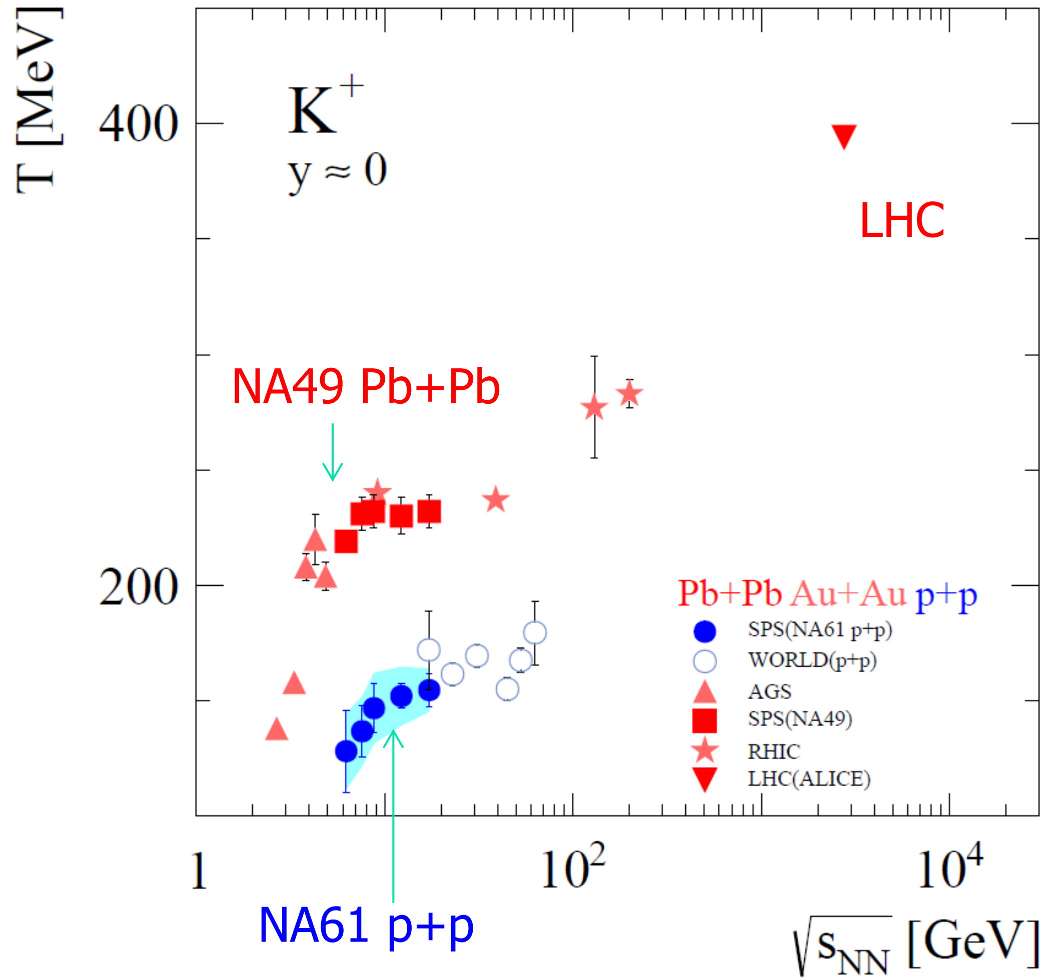
NA49

Phys.Rev. C66 (2002) 054902

Phys.Rev. C77 (2008) 024903

The Step: Pb+Pb vs p+p

K^+

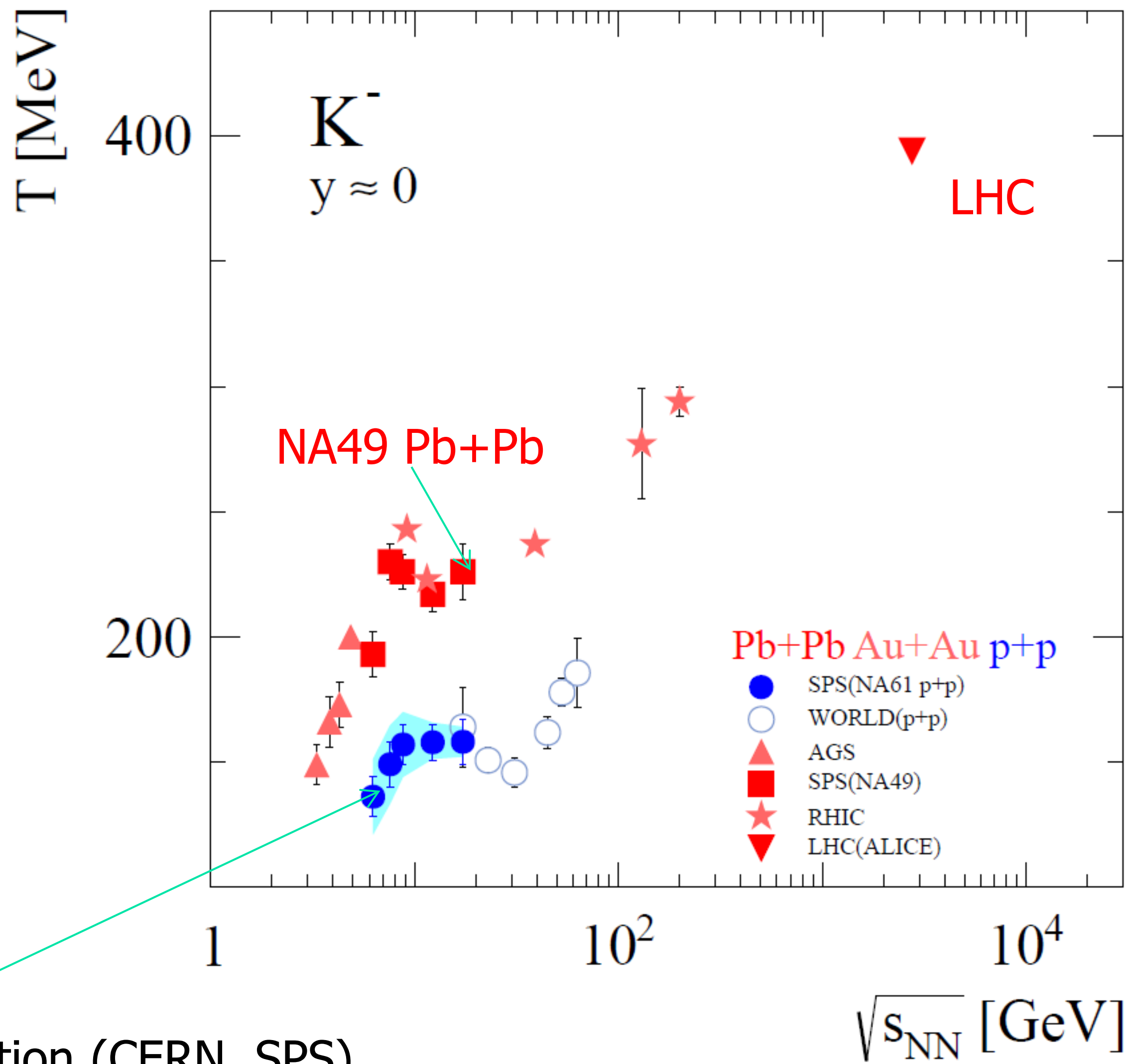


NA61/SHINE Collaboration (CERN, SPS)

arXiv:1502.07916 [nucl-ex]

The Step: Pb+Pb vs p+p

K^-



NA61/SHINE Collaboration (CERN, SPS)

arXiv:1502.07916 [nucl-ex]

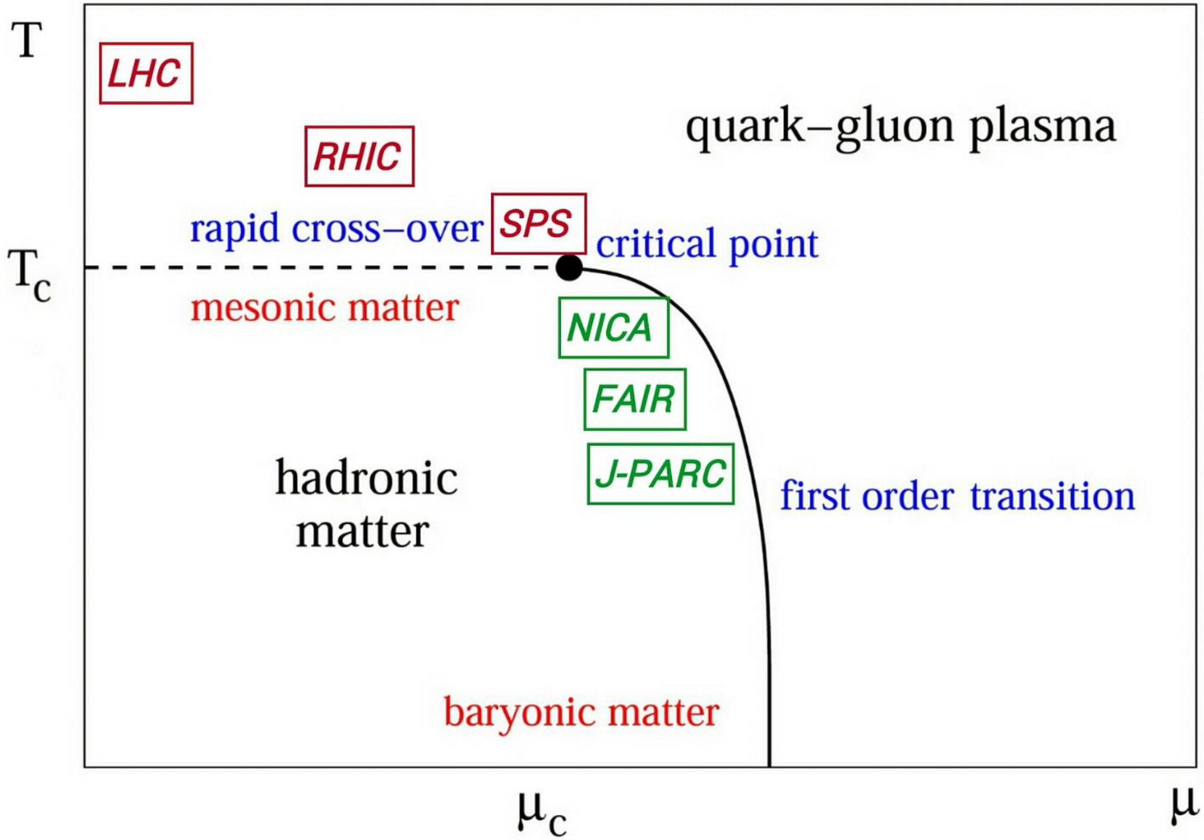
4. Summary

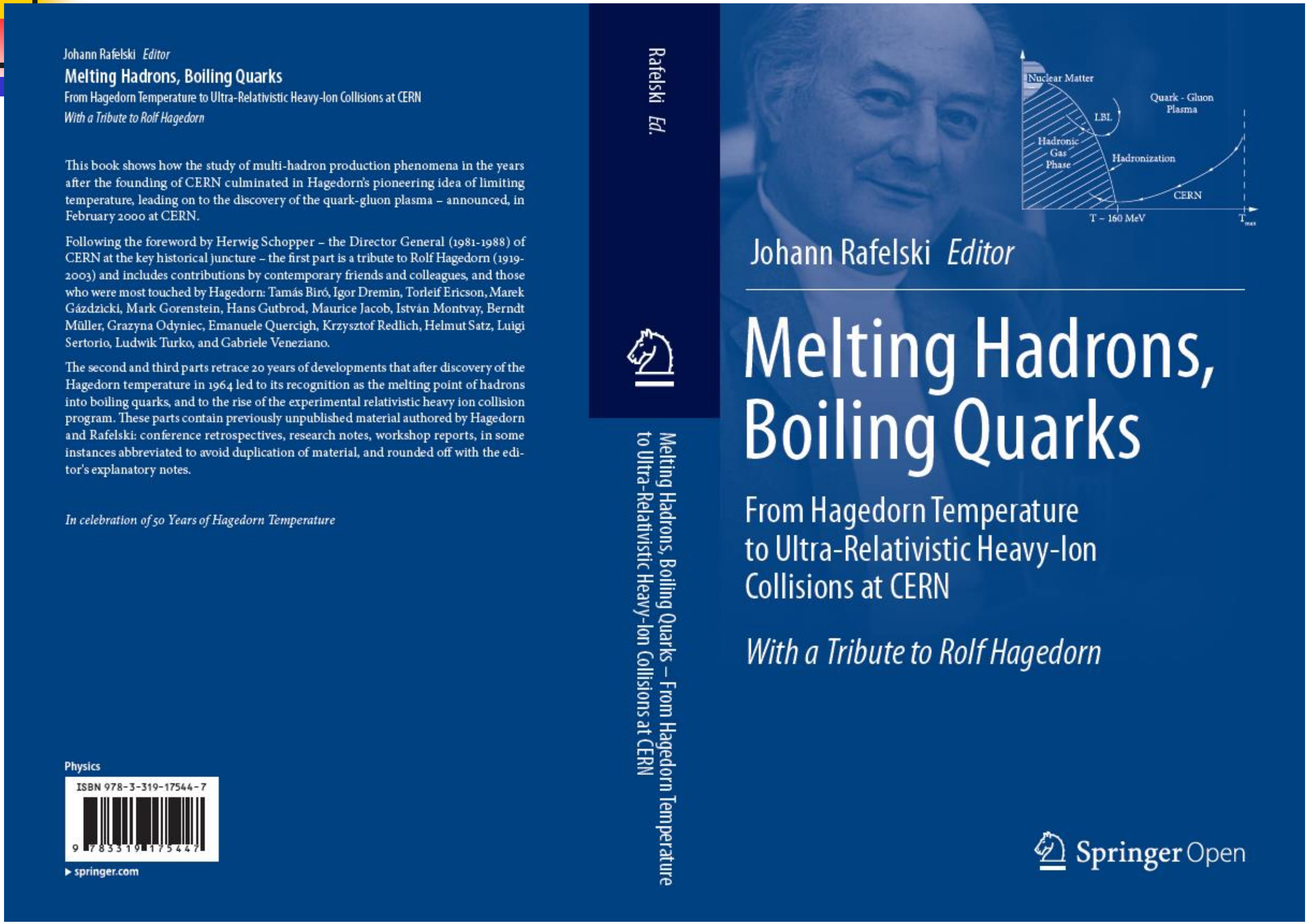
Hadron Gas $T_0 \rightarrow \begin{cases} 1^{st} \text{ order PT} & (\text{large } \mu) \\ \text{Cross-over} & (\text{small } \mu) \end{cases} \rightarrow$ Quark Gluon Plasma

$T_0 = T_c$

Hagedorn Temperature

160 MeV





Johann Rafelski *Editor*

Melting Hadrons, Boiling Quarks

From Hagedorn Temperature to Ultra-Relativistic Heavy-Ion Collisions at CERN

With a Tribute to Rolf Hagedorn

This book shows how the study of multi-hadron production phenomena in the years after the founding of CERN culminated in Hagedorn's pioneering idea of limiting temperature, leading on to the discovery of the quark-gluon plasma – announced, in February 2000 at CERN.

Following the foreword by Herwig Schopper – the Director General (1981-1988) of CERN at the key historical juncture – the first part is a tribute to Rolf Hagedorn (1919-2003) and includes contributions by contemporary friends and colleagues, and those who were most touched by Hagedorn: Tamás Biró, Igor Dremin, Torleif Ericson, Marek Gázdzicki, Mark Gorenstein, Hans Gutbrod, Maurice Jacob, István Montvay, Berndt Müller, Grazyna Odyniec, Emanuele Quercigh, Krzysztof Redlich, Helmut Satz, Luigi Sertorio, Ludwik Turko, and Gabriele Veneziano.

The second and third parts retrace 20 years of developments that after discovery of the Hagedorn temperature in 1964 led to its recognition as the melting point of hadrons into boiling quarks, and to the rise of the experimental relativistic heavy ion collision program. These parts contain previously unpublished material authored by Hagedorn and Rafelski: conference retrospectives, research notes, workshop reports, in some instances abbreviated to avoid duplication of material, and rounded off with the editor's explanatory notes.

In celebration of 50 Years of Hagedorn Temperature

Physics

ISBN 978-3-319-17544-7

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Rafelski *Ed.*

Melting Hadrons, Boiling Quarks – From Hagedorn Temperature to Ultra-Relativistic Heavy-Ion Collisions at CERN

Johann Rafelski *Editor*

Melting Hadrons, Boiling Quarks

From Hagedorn Temperature to Ultra-Relativistic Heavy-Ion Collisions at CERN

With a Tribute to Rolf Hagedorn

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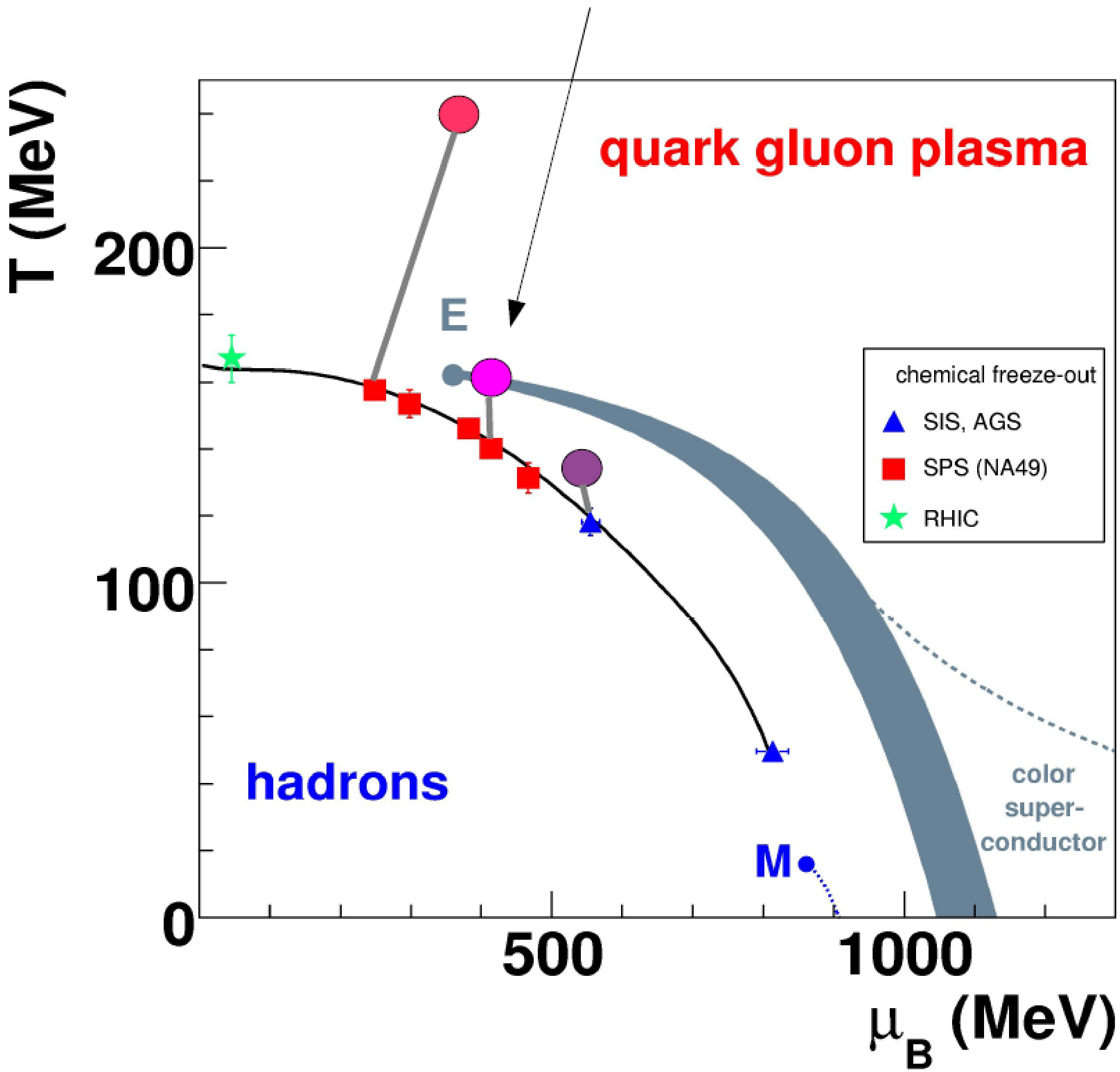
Thank You !

Additional Slides

Onset of deconfinement: the early stage hits the transition line



Phase Transition in the Gas Bags

$p_{\rm id} V = NT$, Ideal Boltzmann Gas,

$p(V - bN) = NT$, van der Waals excluded volume

$$Z(T,V) = \sum_{N=0}^{\infty} \frac{\left[(V - bN)\varphi_m(T)\right]^N}{N!}\theta(V - bN), \quad \text{GCE, } \mu = 0$$

$$\varphi_m(T) = \frac{1}{2\pi^2}\int_0^\infty k^2 dk \exp\left(-\frac{\sqrt{k^2+m^2}}{T}\right) = \frac{m^2 T}{2\pi^2} K_2(m/T),$$

$$\hat{Z}(T,s) = \int_0^\infty dV \exp(-sV) Z(T,V) = \frac{1}{s}\sum_{N=0}^{\infty}\left[\frac{\exp(-bs)\varphi_m(T)}{s}\right]^N$$

$$= \frac{1}{s - \exp(-bs)\varphi_m(T)}$$

M.I.G., Petrov, and Zinovjev, Phys. Lett. B (1981)

s^* is the farthest-right singularity of $\hat{Z}(T,s)$: $p(T) = Ts^*(T)$

$$Z(T,V) = \exp\left[\frac{p(T)V}{T}\right], \qquad \hat{Z}(T,s) = \int_0^\infty dV \exp\left(-sV + \frac{p(T)V}{T}\right)$$

$$\hat{Z}(T,s) = \frac{1}{s - \exp(-bs)\varphi_m(T)} \quad \to \quad s^* = \exp(-bs^*)\varphi_m(T)$$

$$p(T) = T\exp\left[-\frac{bp(T)}{T}\right]\varphi_m(T) = \exp\left[-\frac{bp(T)}{T}\right]p_{\text{id}}(T)$$

$$p_{\text{id}}(T,\mu) = \exp\left(\frac{\mu}{T}\right)\varphi_m(T)$$

$$p(T,\mu) = p_{\text{id}}\left[T, \mu - bp(T,\mu)\right]$$

Rischke, M.I.G., Stocker, and W. Greiner, Z. Phys. C (1991)
Excluded volume hadron gas model

$\rho(m,v)$ mass-volume spectrum of the quark-gluon bags

$$Z(T,V) = \sum_{N=0}^{\infty} \frac{1}{N!} \prod_{i=1}^{N} \int dm_i dv_i \rho(m_i, v_i) \varphi(T, m_i) \left(V - \sum_{i=1}^{N} v_i \right)^N \theta\left(V - \sum_{i=1}^{N} v_i \right)$$

$$\hat{Z}(T,s) = \int_0^{\infty} dV \exp(-sV)\, Z(T,V) = \frac{1}{s - f(T,s)}$$

$$f(T,s) \equiv \int dm dv\ \rho(m,v)\ \varphi(T,m)\ \exp(-sv)$$

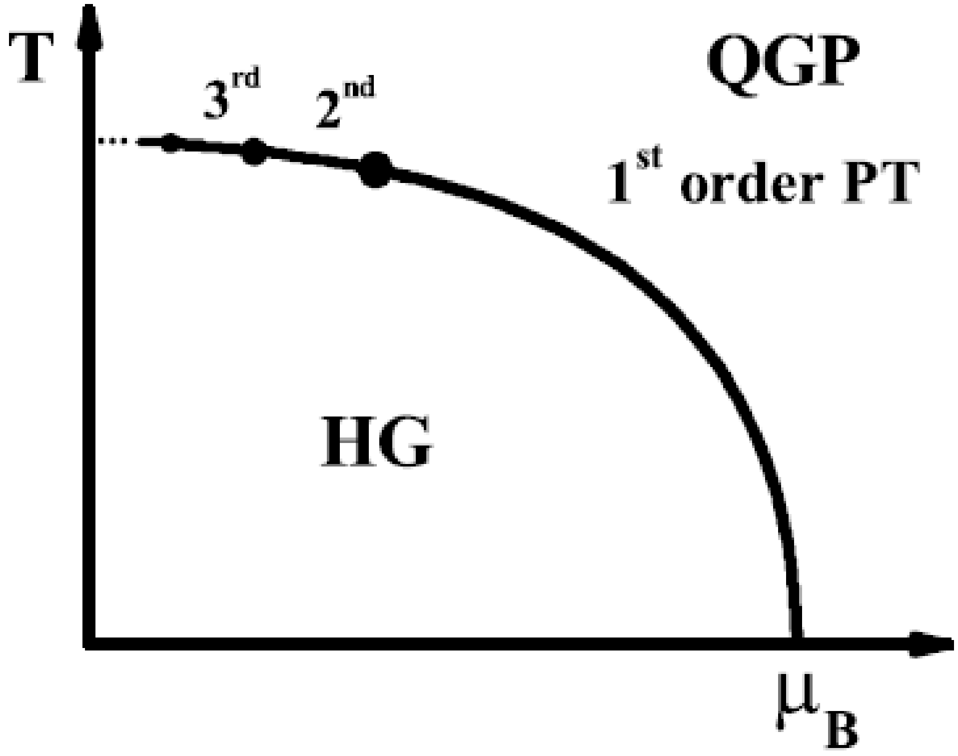
$$p(T) = Ts^*(T) = T \max\{s_H(T), s_Q(T)\}$$

$$s_H(T) = f\left[T, s_H(T)\right],$$ s_Q(T) is a singularity of the function $f(T,s)$

$$\rho(m,v) = \rho_0 + Cv^{\gamma}(m - Bv)^{\delta} \exp\left[\frac{4}{3}\sigma^{1/4} v^{1/4} (m - Bv)^{3/4}\right]$$

$$\rho_0 = \sum_i d_i \delta(v - v_i)\, \delta(m - m_i), \qquad \sigma = \frac{\pi^2}{30}\left(d_g + \frac{7}{8} d_{q\bar{q}}\right)$$

Different types of the Phase Transitions in the $\mu_B - T$ plane

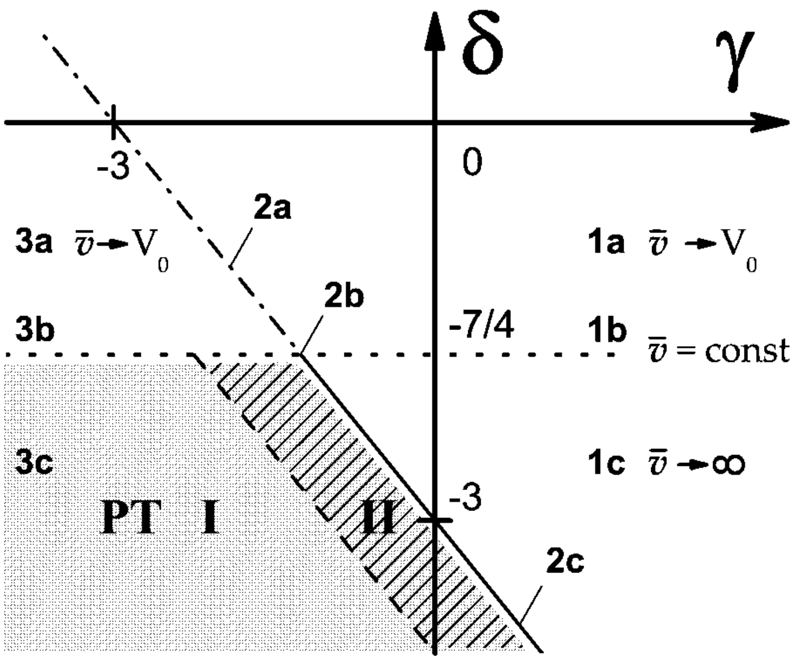


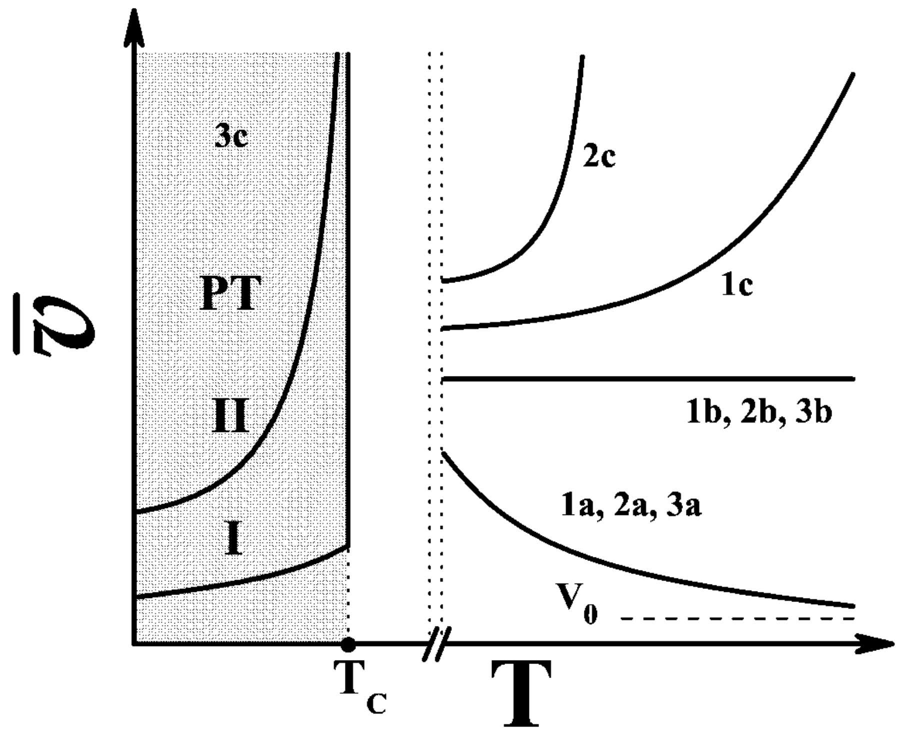
M.I.G., Gazdzicki,
and W. Greiner,
Phy. Rev. C (2005)

Begun, M.I.G., and W. Greiner, "Crossover to the cluster QGP in the gas of bags", J. Phys. G (2009)

$$1)\ \ \gamma+\delta>-3\ ,\qquad 2)\ \ \gamma+\delta=-3\ ,\qquad 3)\ \ \gamma+\delta<-3$$

$$a)\ \ \delta>-7/4\ ,\qquad b)\ \ \delta=-7/4\ ,\qquad c)\ \ \delta<-7/4$$





$$\varepsilon(T)\ \cong\ \sigma_Q\ T^4$$