Chaotic vortical flows and their manifestations

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OUTLINE

Chaotic Arnold-Beltrami flows

Vorticity and hydrodynamical helicity modelling

Λ baryon polarization due to vorticity

Conclusions

Arnold-Beltrami flows

- Nonrelativistic incompressible fluids with vorticity parallel to velocity

$$\omega_i \equiv \epsilon_{ijk} \partial_j v_k = m v_i$$

- Compatible with Euler equation for steady flows

$$v_j \partial_j v_i = -\frac{1}{\rho} \partial_i p$$

- Bernoulli condition is valid in the whole volume of the fluid

$$\partial_i \phi = 0 \qquad \phi = \frac{p}{\rho} + \frac{v^2}{2}$$

Chaotic streamlines

$$\frac{d}{dt}x^i(t) = u^i(x(t),t)$$

Arnold's theorem:

For flows taking place on compact three manifolds, the only velocity fields able to produce chaotic streamlines are those satisfying Beltrami equation.

&

Topological conception of contact structures, each of which admits a representative contact vector field also satisfying Beltrami equation.

Chaotic thermalization?

- Arnold-Beltrami flows – Lagrangian turbulence
- Simple explanation: Bernoulli in the volume – the streamlines come close to each other
- Chaotic advection: laminar flows result in the chaotic motion of passive admixture
- Fast Dynamo problem, the spontaneous generation of a exponential growing magnetic field in a flow of conducting fluid with vorticity
- Possible role in the fast thermalization (complementary description)?!

Relation to Chern-Simons theory

- Euclidian (2+1)-D action

$$S = \frac{1}{2} \int d^3x \, (\omega_i - m \, v_i) \, \omega_i$$

- Applicable to CME

$$\epsilon_{ijk} \, \partial_j \, H_k = \kappa \, H_i$$

$$S = \frac{1}{2} \int d^3x \, (H_i - \kappa \, A_i) \, H_i \, .$$

Particular Solution: ABC-flow

- **Well known particular solution is Arnold-Beltrami-Childress (ABC) flows**

$$\mathbf{u}(x,y,z) = \mathbf{V}^{(ABC)}(x,y,z) \equiv \begin{pmatrix} C\cos(2\pi y) + A\sin(2\pi z) \\ A\cos(2\pi z) + B\sin(2\pi x) \\ B\cos(2\pi x) + C\sin(2\pi y) \end{pmatrix}$$

- **Generic solution with discrete symmetries?**

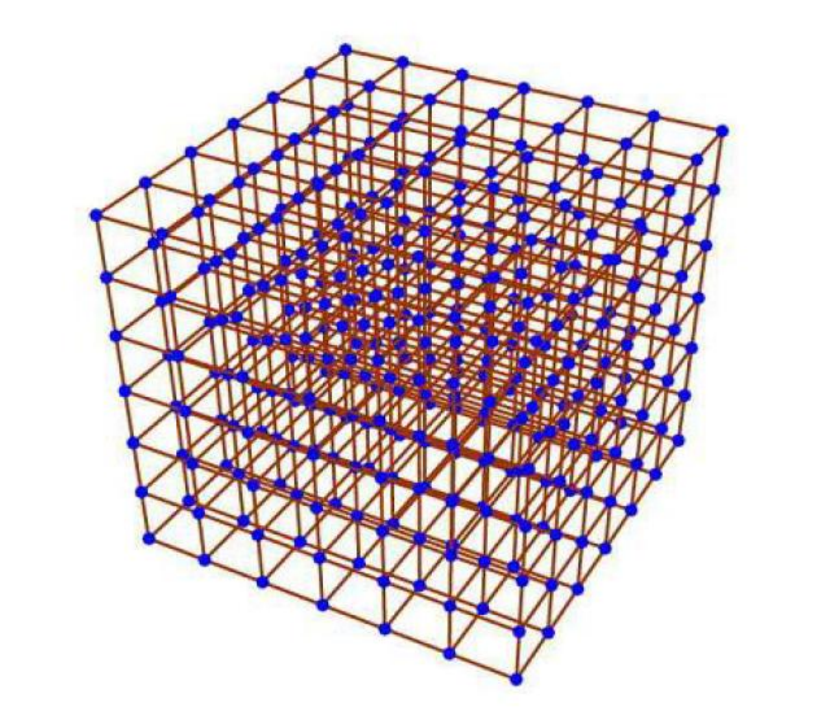
Solving Beltrami Equation

$$\omega_i \equiv \epsilon_{ijk}\,\partial_j\,v_k = m\,v_i$$

Solutions with discrete symmetries generalizing ABC flows

The Cubic Lattice and the Octahedral Point Group O_24

Harmonic analysis on the T^3 torus



Extension of the Point Group with the Translations:

constructing the Universal Classifying Group for the cubic lattice: G1536 and its irreducible representations

Constructing the spherical layers and the octahedral lattice orbits: 48 types of orbits

$$\mathbf{k}_{(n)} \in \mathfrak{S}_n \quad \Leftrightarrow \quad \langle \mathbf{k}_{(n)} \, , \, \mathbf{k}_{(n)} \rangle = r_n^2$$

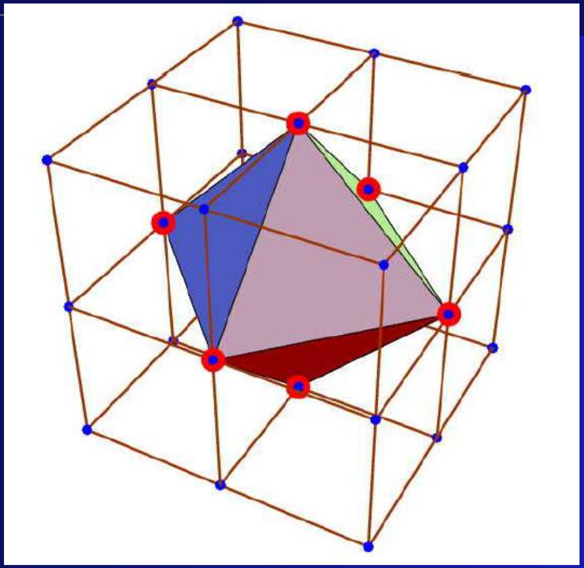
Explicit examples of generalized Arnold-Beltrami flows with discrete symmetries

$$Y\left(\mathbf{k} \,|\, \mathbf{x}\right) = a\left(\mathbf{k}\right) \cos\left(2\,\pi\,\mathbf{k}\cdot\mathbf{x}\right) + b\left(\mathbf{k}\right) \sin\left(2\,\pi\,\mathbf{k}\cdot\mathbf{x}\right) \quad , \quad \mathbf{k} \in \Lambda^\star$$

$$V_i\left(\mathbf{x}\right) = \sum_{\mathbf{x} \in \mathfrak{S}_n} Y_i\left(\mathbf{k} \,|\, \mathbf{x}\right)$$

P. Fre, A.S., Phys.Part.Nucl. 46 (2015) 4; arXiv:1501.04604

The lowest lying octahedral orbit of length 6 in the cubic lattice



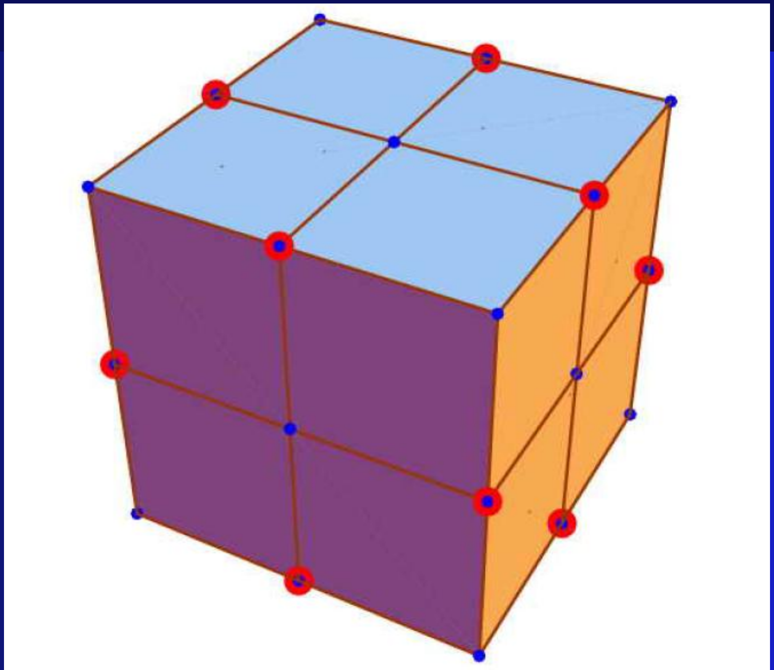
gives the ABC-flows

$$\mathbf{u}(x,y,z) = \mathbf{V}^{(ABC)}(x,y,z) \equiv \begin{pmatrix} C\cos(2\pi y) + A\sin(2\pi z) \\ A\cos(2\pi z) + B\sin(2\pi x) \\ B\cos(2\pi x) + C\sin(2\pi y) \end{pmatrix}$$

P. Fre, A.S., Phys.Part.Nucl. 46 (2015) 4; arXiv:1501.04604

The lowest lying octahedral orbit of length 12 in the cubic lattice

P. Fre, A.S.,
Phys.Part.Nucl. 46 (2015) 4;
arXiv:1501.04604



gives Beltrami vector field invariant under discrete group GP24

$$
\begin{aligned}
V_x &= \cos(2\pi(y-z)) + 2\cos(2\pi(y+z)) + \sqrt{2}\sin(2\pi(x-y)) \\
&\quad -\sqrt{2}\sin(2\pi(x+y)) - \sqrt{2}\sin(2\pi(x-z)) + \sqrt{2}\sin(2\pi(x+z)) \\
V_y &= 2\cos(2\pi(x-z)) + 2\cos(2\pi(x+z)) + \sqrt{2}\sin(2\pi(x-y)) \\
&\quad +\sqrt{2}\sin(2\pi(x+y)) + \sqrt{2}\sin(2\pi(y-z)) - \sqrt{2}\sin(2\pi(y+z)) \\
V_z &= 2\cos(2\pi(x-y)) + 2\cos(2\pi(x+y)) - \sqrt{2}\sin(2\pi(x-z)) \\
&\quad +\sqrt{2}\sin(2\pi(y-z)) - \sqrt{2}\sin(2\pi(x+z)) + \sqrt{2}\sin(2\pi(y+z))
\end{aligned}
$$

Microworld:
where is the fastest possible rotation?

- Non-central heavy ion collisions
- **Magnetic field – highest possible ever = CME**
- **Rotation – another pseudovector – angular velocity** ~ c/Compton wavelength – “small Bang”
- Differential rotation – vorticity
- P-odd
- **BUT can be smeared?**
- **Can it be quantified?**

Simulations in quark - gluon string model
Baznat, Gudima, A.S.,Teryaev

arXiv:1301.7003, PRC’13 & arXiv:1507.04652

Rotation in HIC and related quantities

- Non-central collisions – orbital angular momentum
- $L = \Sigma\, r \times p$
- Differential pseudovector characteristics – vorticity
- ϖ = curl v
- Pseudoscalar – helicity
- H ~ <(v curl v)>
- Maximal helicity – Beltrami chaotic flows v || curl v
- Investigation in QGSM

QGSM Simulation

$$50 \times 50 \times 100 \text{ cells}$$

$$dx = dy = 0.6 \text{ fm}, dz = 0.6/\gamma \text{ fm}$$

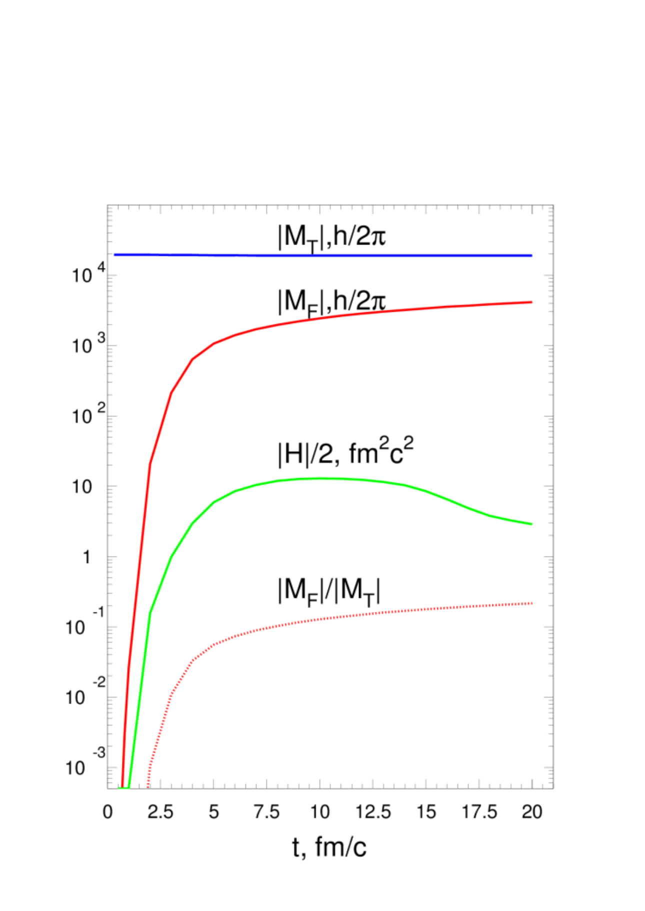
- Velocity

$$\vec{v}(x, y, z, t) = \frac{\sum_i \sum_j \vec{P}_{ij}}{\sum_i \sum_j E_{ij}}$$

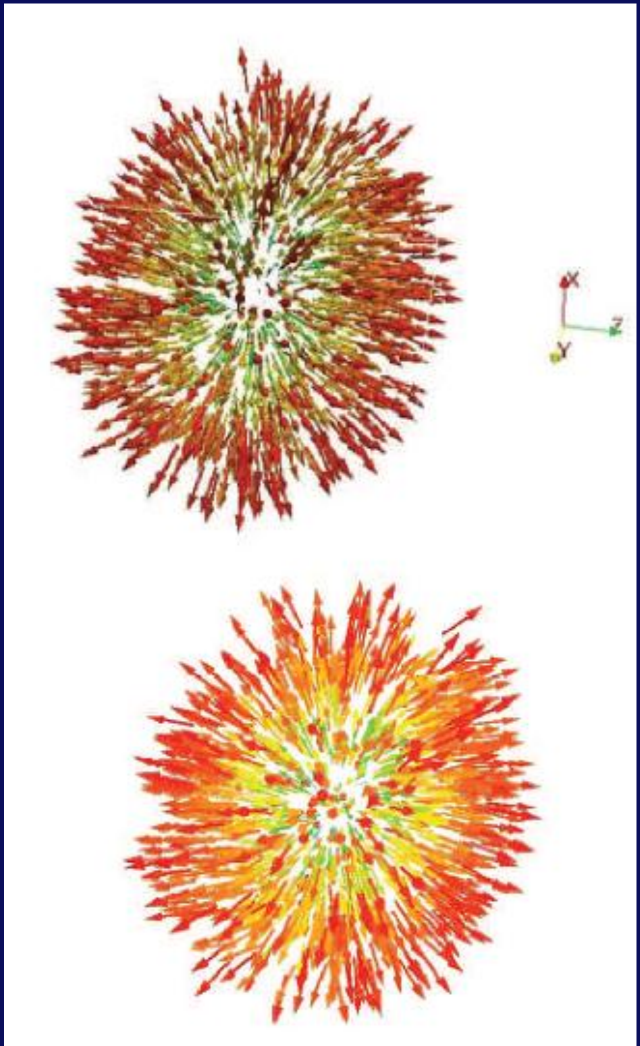
- Vorticity – from discrete partial derivatives

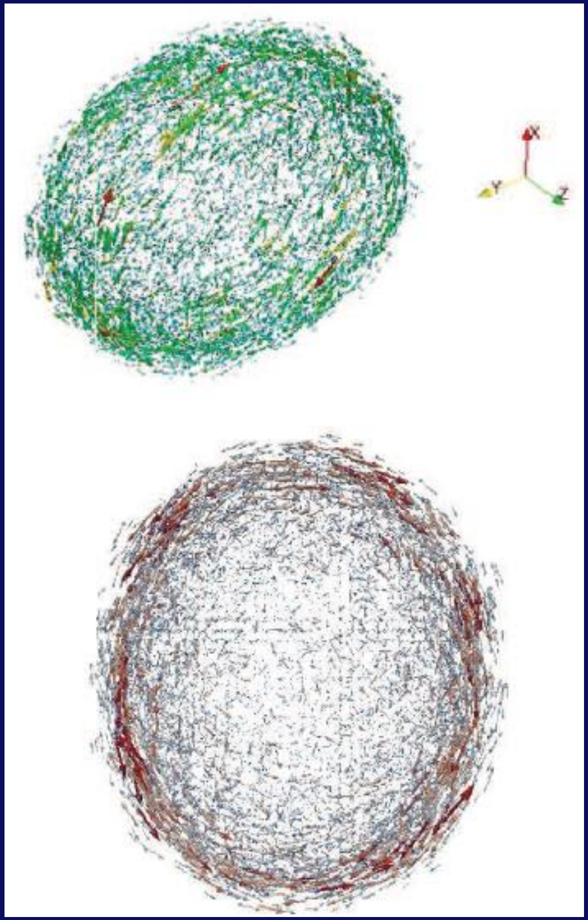
Angular momentum conservation and helicity

- Helicity vs orbital angular momentum (OAM) of fireball (~10% of total)
- Conservation of OAM with a good accuracy!



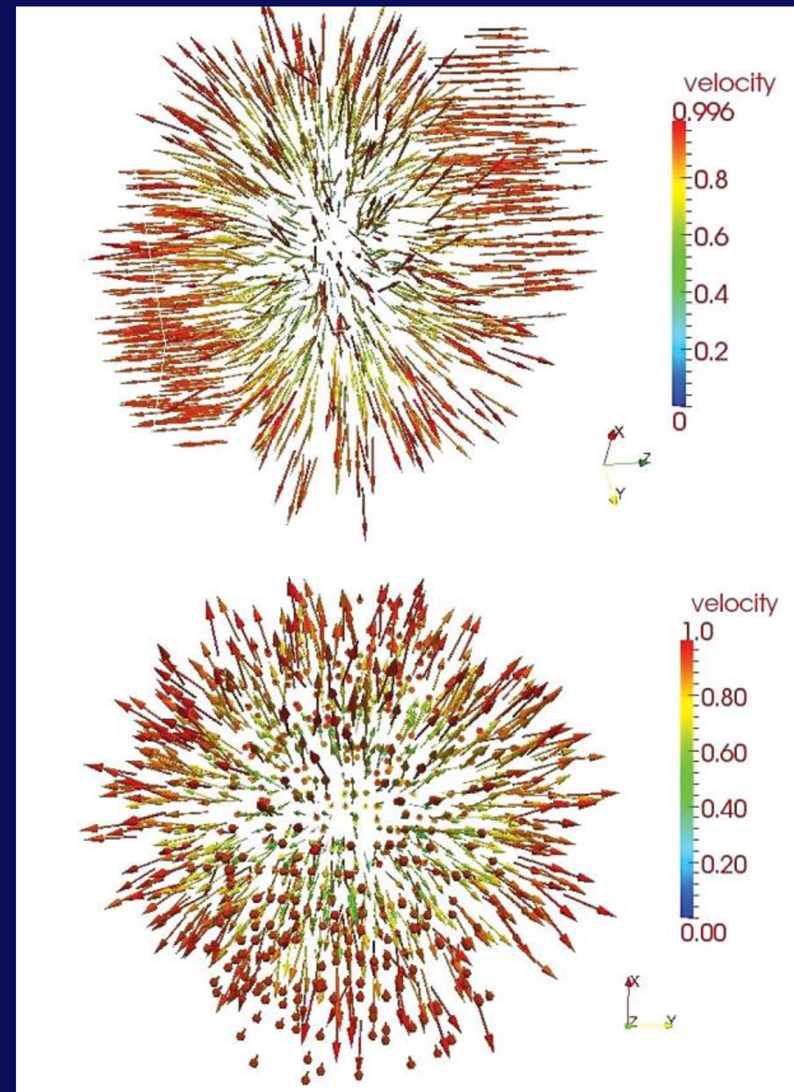
Structure of velocity and vorticity fields at 5 GeV/c c.m.s Gold-Gold collisions



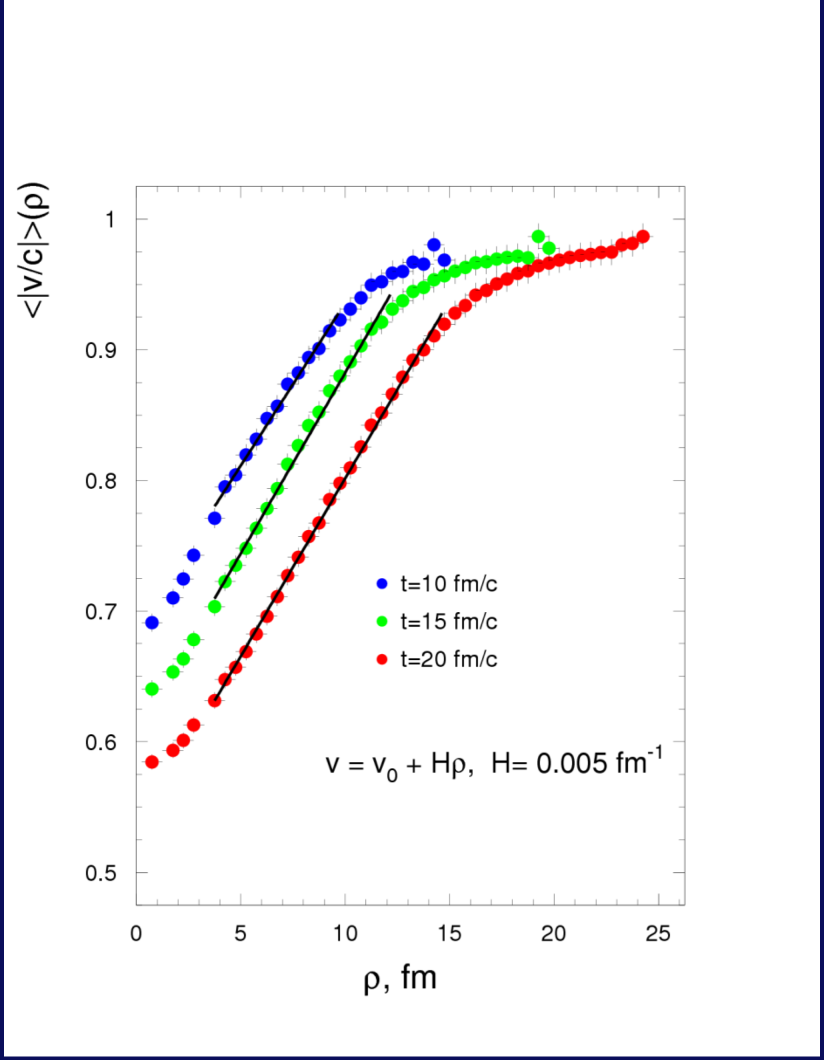


Distribution of velocity (“Small Bang”)

- 3D/2D projection
- z-beams direction
- x-impact paramater

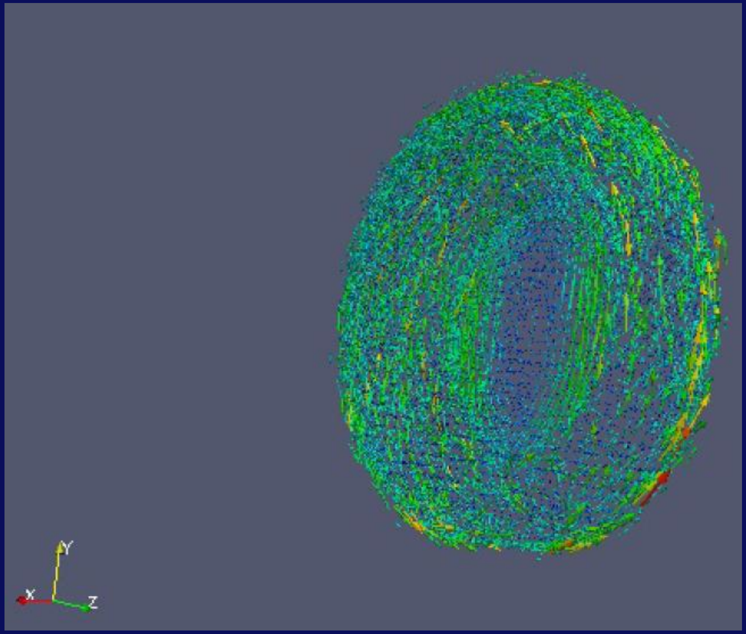


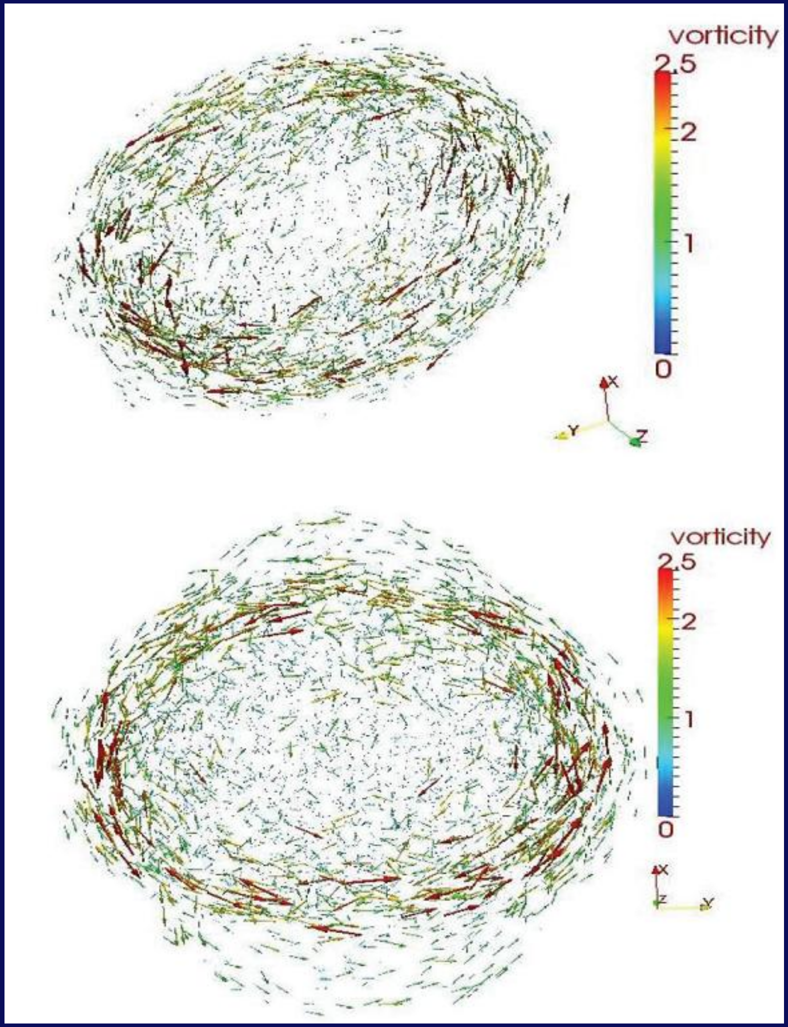
Little Hubble law



Distribution of vorticity ("small galaxies")

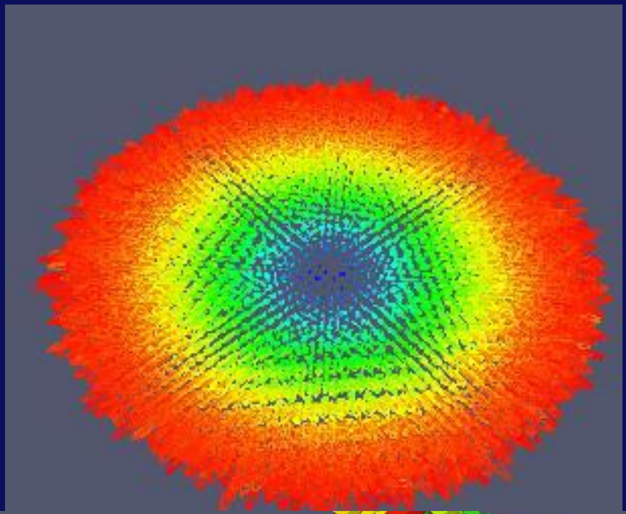
- Layer patterns



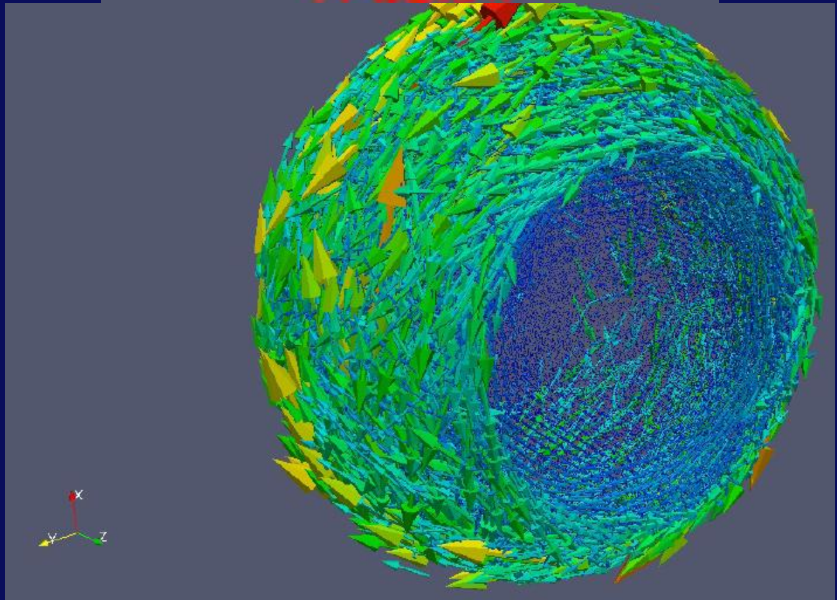


Velocity and vorticity patterns

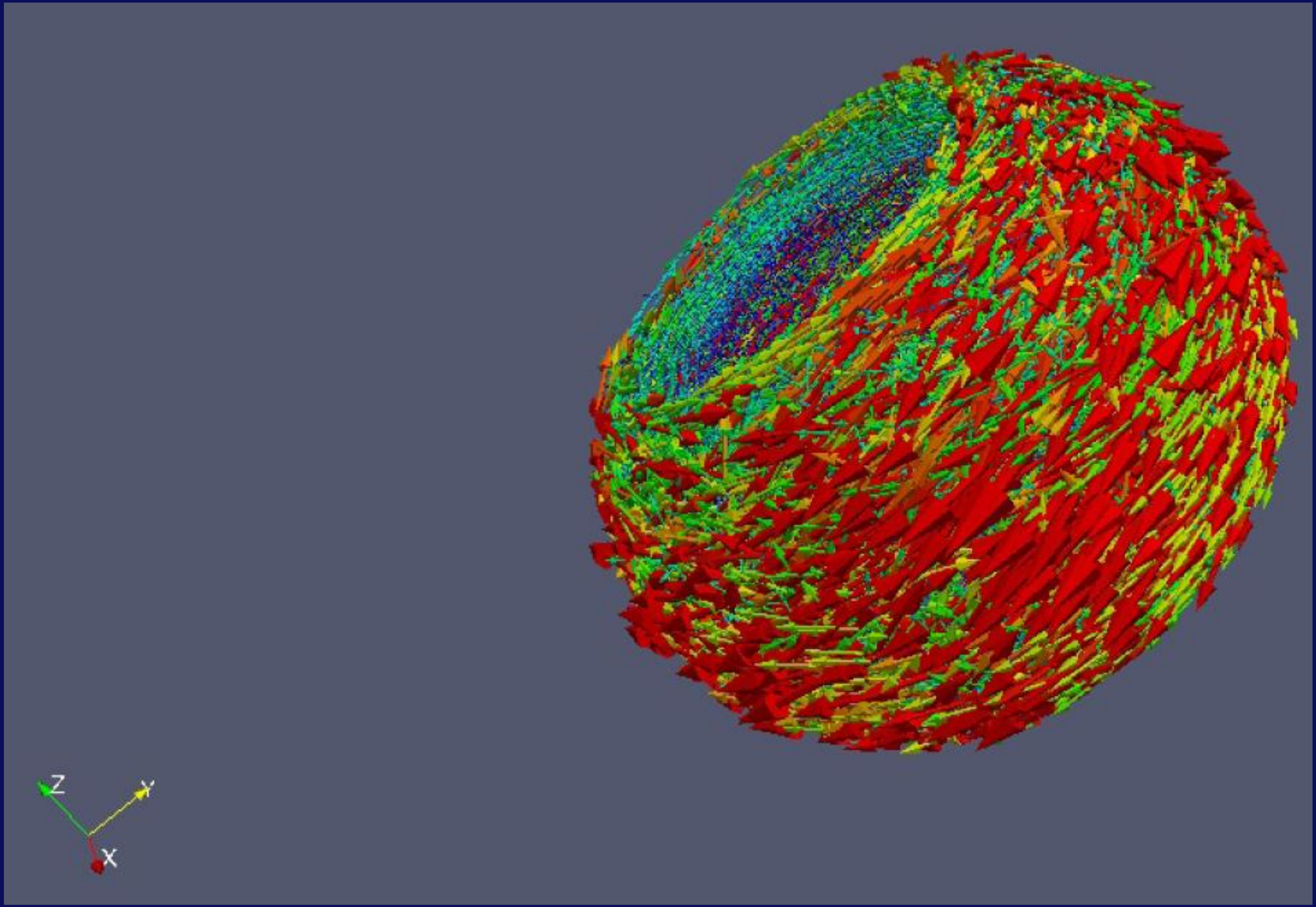
- Velocity



- Vorticity pattern - due to L BUT cylinder symmetry!

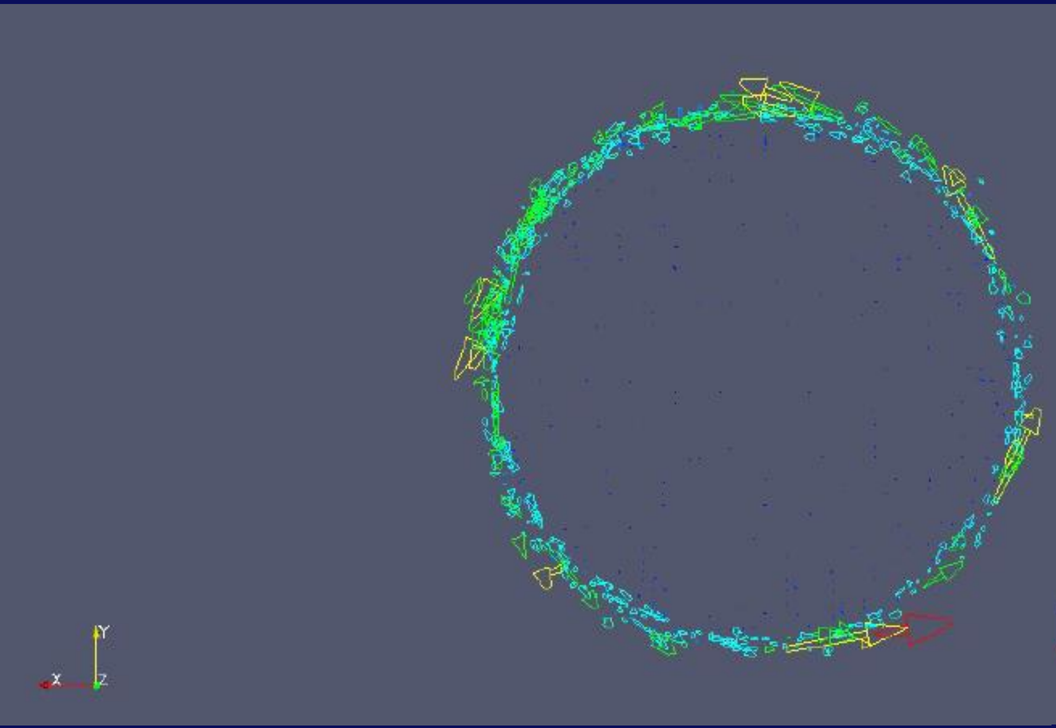


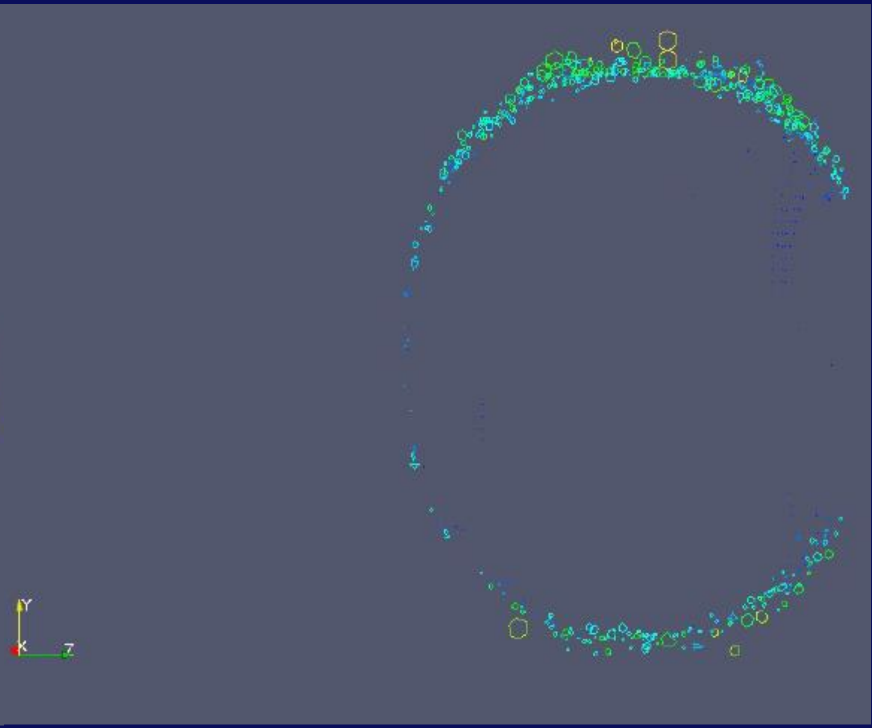
Vorticity pattern



Sections of vorticity patterns

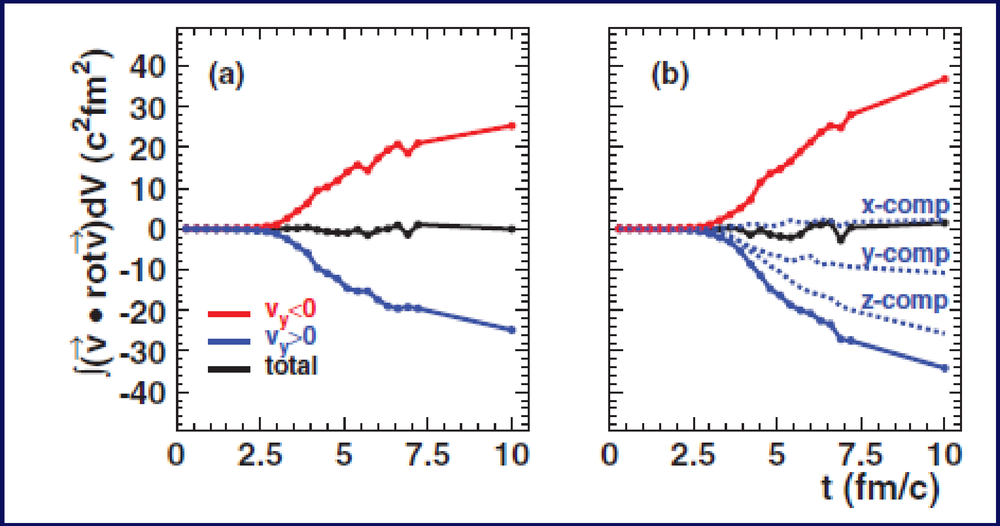
- Front and side views





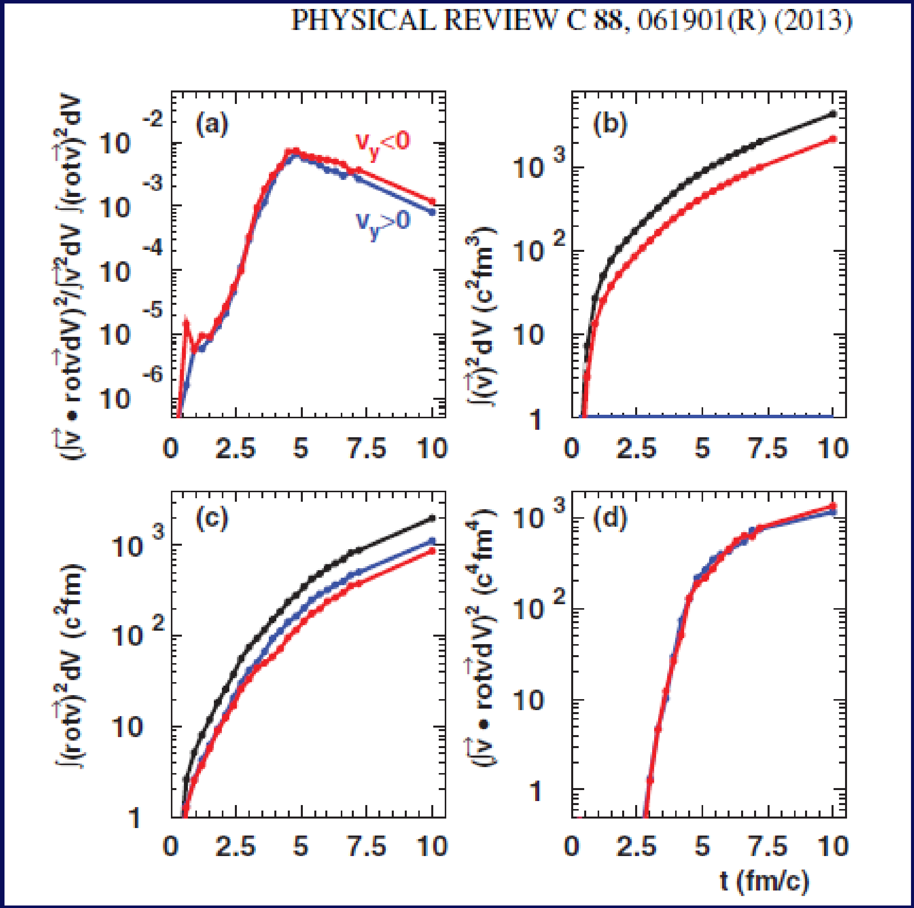
Helicity separation

- Total helicity integrates to zero BUT
- Mirror helicities below and above the reaction plane



What is the relative orientation of velocity and vorticity?

- Measure – Cauchy-Schwarz inequality
- Small but non-negligible correlation
- Maximal correlation - Beltrami flows



Vorticity and baryon polarization

- **Polarization of hyperons, particularly Λ – convenient observable**
- **Relatively easy to measure via angular asymmetry of weak decay products (self-analyzing!)**
- **Hadronic collisions – Single Spin Asymmetry: P-invariance – polarization normal to scattering plane**

Polarization in Heavy-Ion collisions

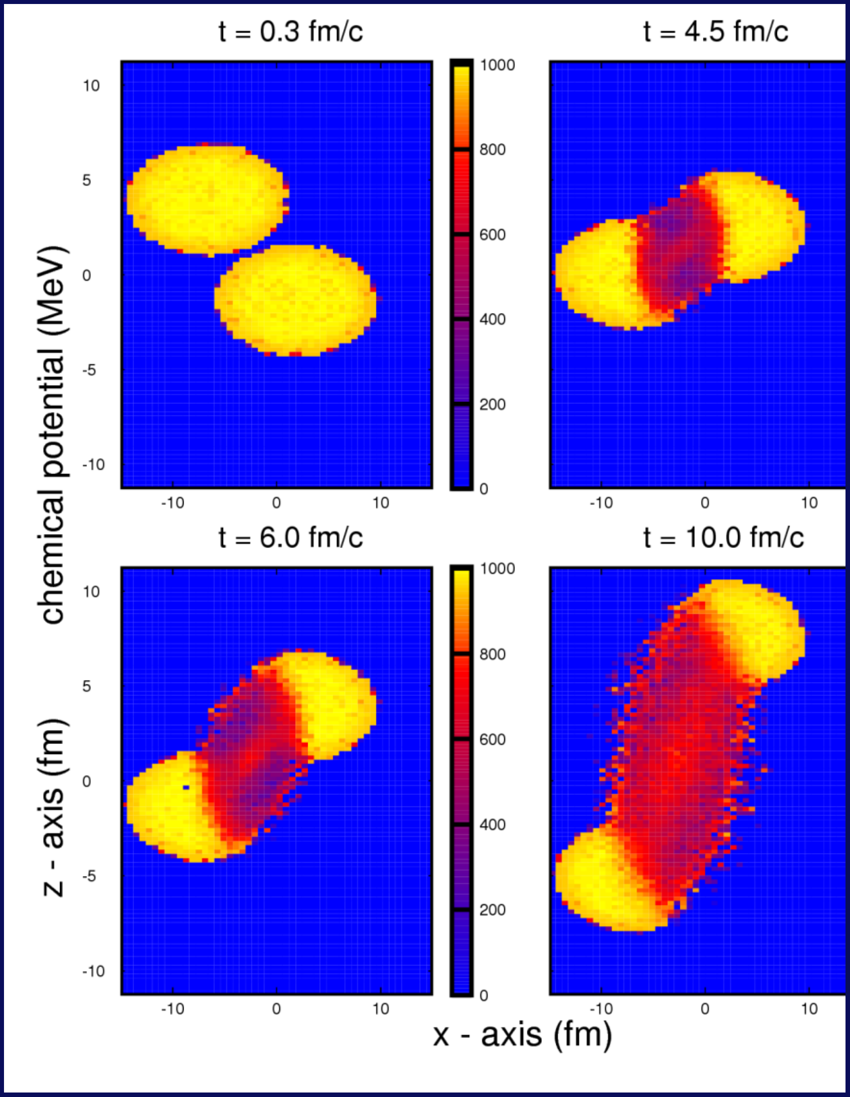
- HIC: randomization – no scattering plane
-
- Disappearance of transverse polarization (Jacob, Rafelski (1987))
- Global rotation – polarization normal to reaction plane (Liang et al)
- Searched at RHIC (S. Voloshin et al.) – oriented plane (slow neutrons) - no signal observed

How to observe this effect?

- Bilinear effect of vorticity – quark axial current
- (Son, Surowka)
- Coupling of HD helicity to quark axial current via axial VVA anomaly
- V: $$e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha$$
- AH -> H(elicity) density
- VVA: QUARK polarization~helicity
- Strange quarks: May lead to POLARIZATION of hyperons (Rogachevsky, A.S., Teryaev (2010))
- (cf other mechanisms) $$j_A^\mu \sim \mu^2 \left(1 - \frac{2\,\mu\,n}{3\,(\epsilon + P)}\right) \epsilon^{\mu\nu\lambda\rho}\, V_\nu\, \partial_\lambda V_\rho$$
- Large chemical potential: appropriate for NICA/FAIR energies

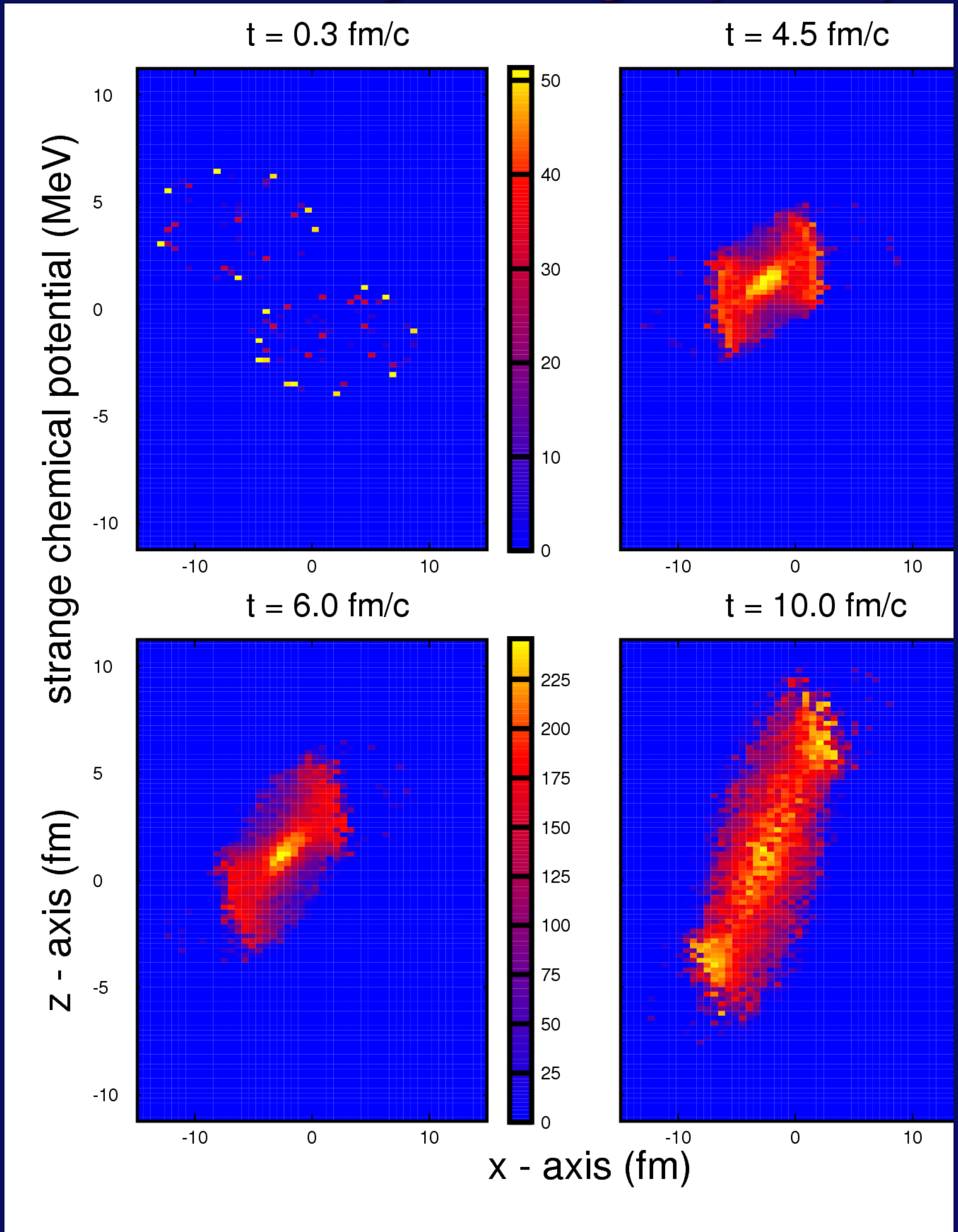
Chemical potential

- TD and chemical equilibrium
- Conservation laws
- 2d section: y=0



Strange chemical potential
(polarization of Lambda is carried by strange quark!)

- Emergent effect



CONCLUSIONS

- Vortical structures
- Helicity separation effect (confirmed in HSD – Teryaev, Usubov)
- Lambda polarization of % order
- Result is surprisingly similar to thermal vorticity calculation (Becattini, Csernai, Wang)
- Rotation in heavy-ion collisions – essentially non-inertial frame
- Related P-odd effects are not numerically large (smearing) but may be observable

Thank you for attention!