Chaotic vortical flows and their manifestations

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Chaotic Arnold-Beltrami flows

Vorticity and hydrodynamical helicity modelling

A baryon polarization due to vorticity

Conclusions
Arnold-Beltrami flows

- Nonrelativistic incompressible fluids with vorticity parallel to velocity

\[ \omega_i \equiv \epsilon_{ijk} \partial_j v_k = m v_i \]

- Compatible with Euler equation for steady flows

\[ v_j \partial_j v_i = -\frac{1}{\rho} \partial_i p \]

- Bernoulli condition is valid in the whole volume of the fluid

\[ \partial_i \phi = 0 \]

\[ \phi = \frac{p}{\rho} + \frac{v^2}{2} \]
Arnold's theorem:
For flows taking place on compact three manifolds, the only velocity fields able to produce chaotic streamlines are those satisfying Beltrami equation.

\[ \frac{d}{dt} x^i(t) = u^i(x(t), t) \]

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Topological conception of contact structures, each of which admits a representative contact vector field also satisfying Beltrami equation.
Chaotic thermalization?

- Arnold-Beltrami flows – Lagrangian turbulence

- Simple explanation: Bernoulli in the volume – the streamlines come close to each other

- Chaotic advection: laminar flows result in the chaotic motion of passive admixture

- Fast Dynamo problem, the spontaneous generation of an exponential growing magnetic field in a flow of conducting fluid with vorticity

- Possible role in the fast thermalization (complementary description)?!
Relation to Chern-Simons theory

- Euclidian (2+1)-D action
  \[ S = \frac{1}{2} \int d^3x (\omega_i - m v_i) \omega_i \]

- Applicable to CME
  \[ \epsilon_{ijk} \partial_j H_k = \kappa H_i \]
  \[ S = \frac{1}{2} \int d^3x (H_i - \kappa A_i) H_i \]
Particular Solution: ABC-flow

- Well known particular solution is Arnold-Beltrami-Childress (ABC) flows

\[ u(x, y, z) = V^{(ABC)}(x, y, z) \equiv \begin{pmatrix} C \cos(2\pi y) + A \sin(2\pi z) \\ A \cos(2\pi z) + B \sin(2\pi x) \\ B \cos(2\pi x) + C \sin(2\pi y) \end{pmatrix} \]

- Generic solution with discrete symmetries?
Solving Beltrami Equation

\[ \omega_i \equiv \epsilon_{ijk} \partial_j v_k = m v_i \]

Solutions with discrete symmetries generalizing ABC flows

The Cubic Lattice and the Octahedral Point Group O\(_{24}\)

Harmonic analysis on the \(T^3\) torus
Extension of the Point Group with the Translations:

constructing the Universal Classifying Group for the cubic lattice: $G_{1536}$ and its irreducible representations

Constructing the spherical layers and the octahedral lattice orbits: 48 types of orbits

$$k_{(n)} \in \mathfrak{S}_n \iff \langle k_{(n)}, k_{(n)} \rangle = r_{n}^2$$

Explicit examples of generalized Arnold-Beltrami flows with discrete symmetries

$$Y(k|x) = a(k) \cos(2\pi k \cdot x) + b(k) \sin(2\pi k \cdot x) , \quad k \in \Lambda^*$$

$$V_i(x) = \sum_{x \in \mathfrak{S}_n} Y_i(k|x)$$

The lowest lying octahedral orbit of length 6 in the cubic lattice gives the ABC-flows

\[ u(x, y, z) = V^{(ABC)}(x, y, z) \equiv \begin{pmatrix} C \cos(2\pi y) + A \sin(2\pi z) \\ A \cos(2\pi z) + B \sin(2\pi x) \\ B \cos(2\pi x) + C \sin(2\pi y) \end{pmatrix} \]

The lowest lying octahedral orbit of length 12 in the cubic lattice gives Beltrami vector field invariant under discrete group GP24.

\[ V_x = \cos(2\pi(y - z)) + 2\cos(2\pi(y + z)) + \sqrt{2}\sin(2\pi(x - y)) - \sqrt{2}\sin(2\pi(x + y)) - \sqrt{2}\sin(2\pi(x - z)) + \sqrt{2}\sin(2\pi(x + z)) \]
\[ V_y = 2\cos(2\pi(x - z)) + 2\cos(2\pi(x + z)) + \sqrt{2}\sin(2\pi(x - y)) + \sqrt{2}\sin(2\pi(x + y)) + \sqrt{2}\sin(2\pi(y - z)) - \sqrt{2}\sin(2\pi(y + z)) \]
\[ V_z = 2\cos(2\pi(x - y)) + 2\cos(2\pi(x + y)) - \sqrt{2}\sin(2\pi(x - z)) + \sqrt{2}\sin(2\pi(y - z)) - \sqrt{2}\sin(2\pi(x + z)) + \sqrt{2}\sin(2\pi(y + z)) \]

Microworld: where is the fastest possible rotation?

- Non-central heavy ion collisions
- Magnetic field – highest possible ever = CME
- Rotation – another pseudovector – angular velocity $\sim c/\text{Compton wavelength}$ – “small Bang”
- Differential rotation – vorticity
- P-odd
- BUT can be smeared?
- Can it be quantified?

Simulations in quark - gluon string model
Baznat, Gudima, A.S., Teryaev

Rotation in HIC and related quantities

- Non-central collisions – orbital angular momentum
- $L = \sum r \times p$
- Differential pseudovector characteristics – vorticity
- $\omega = \text{curl } v$
- Pseudoscalar – helicity
- $H \sim \langle (v \text{ curl } v) \rangle$
- Maximal helicity – Beltrami chaotic flows $v \parallel \text{curl } v$
- Investigation in QGSM
QGSM Simulation

50 × 50 × 100 cells

dx = dy = 0.6 fm, dz = 0.6/γ fm

• Velocity

\[ \vec{v}(x, y, z, t) = \frac{\sum_i \sum_j \vec{P}_{ij}}{\sum_i \sum_j E_{ij}} \]

• Vorticity – from discrete partial derivatives
Angular momentum conservation and helicity

- Helicity vs orbital angular momentum (OAM) of fireball (~10% of total)
- Conservation of OAM with a good accuracy!
Structure of velocity and vorticity fields at 5 GeV/c c.m.s. Gold-Gold collisions
Distribution of velocity ("Small Bang")

- 3D/2D projection
- z-beams direction
- x-impact parameter
Little Hubble law

\[ v = v_0 + \rho H, \quad H = 0.005 \text{ fm}^{-1} \]
Distribution of vorticity ("small galaxies")

- Layer patterns
Velocity and vorticity patterns

- Velocity

- Vorticity pattern - due to L BUT cylinder symmetry!
Vorticity pattern
Sections of vorticity patterns

- Front and side views
Helicity separation

- Total helicity integrates to zero BUT
- Mirror helicities below and above the reaction plane
What is the relative orientation of velocity and vorticity?

- Measure – Cauchy-Schwarz inequality
- Small but non-negligible correlation
- Maximal correlation - Beltrami flows
Polarization of hyperons, particularly $\Lambda$ – convenient observable

Relatively easy to measure via angular asymmetry of weak decay products (self-analyzing!)

Hadronic collisions – Single Spin Asymmetry: $P$-invariance – polarization normal to scattering plane
Polarization in Heavy-Ion collisions

- HIC: randomization – no scattering plane
- Disappearance of transverse polarization (Jacob, Rafelski (1987))
- Global rotation – polarization normal to reaction plane (Liang et al)
- Searched at RHIC (S. Voloshin et al.) – oriented plane (slow neutrons) - no signal observed
How to observe this effect?

- Bilinear effect of vorticity – quark axial current
  
  (Son, Surowka)

- Coupling of HD helicity to quark axial current via axial VVA anomaly

\[ e_j A_\alpha J^\alpha \Rightarrow \mu_j V_\alpha J^\alpha \]

- AH -> H(elicity) density

- VVA: QUARK polarization~helicity

- Strange quarks: May lead to POLARIZATION of hyperons (Rogachevsky, A.S., Teryaev (2010) )

  (cf other mechanisms)

\[ J^\mu_A \sim \mu^2 \left( 1 - \frac{2 \mu n}{3 (\epsilon + P)} \right) e^{\mu \nu \lambda \rho} V_\nu \partial_\lambda V_\rho \]

- Large chemical potential: appropriate for NICA/FAIR energies
Chemical potential

- TD and chemical equilibrium
- Conservation laws
- 2d section: y=0
Strange chemical potential
(polarization of Lambda is carried by strange quark!)

• Emergent effect
CONCLUSIONS

• Vortical structures
• Helicity separation effect (confirmed in HSD – Teryaev, Usubov)
• Lambda polarization of % order
• Result is surprisingly similar to thermal vorticity calculation (Becattini, Csernai, Wang)
• Rotation in heavy-ion collisions – essentially non-inertial frame
• Related P-odd effects are not numerically large (smearing) but may be observable
Thank you for attention!