Proton Structure and PDFs from HERA

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on behalf of the H1 and ZEUS Collaborations

- DIS / Proton Structure Functions / PDFs
- Combination of all inclusive NC&CC
data from H1&ZEUS at HERA
- $F_2$, $F_2^{\gamma Z}$, $xF_3^{\gamma Z}$, $F_L$, $\sigma_{CC}^{tot}$, ...
- HERAPDF 2.0 and its variants
- also with charm and jet HERA data; $\alpha_s$
Proton Structure Functions in Deep-Inelastic \( ep/\mu p/v p \) Scattering (DIS)

\( \rightarrow \) inclusive DIS cross section depends on three kinematical variables:

\[ Q^2 = -q^2 = -(k-k')^2 \quad \text{virtuality of } \gamma^*, Z^0, W \]
\[ x = Q^2/2(Pq) \quad \text{Bjorken } x \]
\[ y = (Pq)/(Pk) \quad \text{inelasticity} \]
\[ Q^2 = sxy \quad s=(k+P)^2 \]

\( \rightarrow \) inclusive cross section can be expressed via three proton structure functions, e.g.

\[
\tilde{\sigma}^\pm_{NC} = \frac{d^2\sigma^{e^+p}_{NC}}{dx dQ^2} \frac{xQ^4}{2\pi \alpha^2} \frac{1}{Y_+} \equiv \tilde{F}_2 - \frac{y^2}{Y_+} \tilde{F}_L + \frac{Y_+}{Y_+} x\tilde{F}_3
\]

\( Y_\pm = 1 \pm (1-y)^2 \)
QCD and Parton Distribution Functions (PDFs)

according to the QCD factorisation theorem for hard processes $\sigma = \hat{\sigma} \otimes \text{PDF}$ where universal PDFs containing long-distance structure of the proton can be measured at initial scale $Q_0^2$ and then be calculated at any other scale $Q^2$ using the QCD evolution equations, e.g. DGLAP.

In DIS inclusive cross sections are sums over partons in the proton:

\begin{align*}
F_2 &- \text{ over quarks and antiquarks,} \quad xF_3 - \text{ over valence quarks} \\
F_2(x,Q^2) &= \sum A_q(xq + \bar{xq}) \\
x F_3(x,Q^2) &= \sum B_q(xq - \bar{xq}) \\
F_L(x,Q^2) &\propto \alpha_s \cdot x g \quad \text{is a pure QCD effect (in QPM } F_L = 0) \\
\end{align*}

The universal parton distribution functions (of quarks of different flavor and gluon) in the proton measured in DIS can be applied to other hard processes which include proton, e.g. pp collisions at LHC.

→ PDFs are determined in QCD fits to measured cross sections
→ HERA inclusive DIS data are an indispensable input to any modern QCD PDF analysis

→ for HERAPDF the HERAFitter platform is used: [HERAFitter](http://www.herafitter.org)
The only *ep* collider HERA

15 years (1992-2007) of operation at DESY in Hamburg

electrons & positrons of \( E_e = 27.5 \text{ GeV} \)
collided with protons of
\( E_p = 920, 820, 575, \text{ and } 460 \text{ GeV} \)
corresponding to
\( \sqrt{s} = 318, 300, 251, \text{ and } 225 \text{ GeV} \)

Two multi-purpose collider experiments
H1 & ZEUS collected in total \( 1 \text{ fb}^{-1}. \)

In the second phase of HERA operation (2003-2007) the lepton beam was longitudinally polarised (~40%).
NC and CC inclusive data sets at HERA

41 NC and CC data sets from H1 and ZEUS corresponding to 1 fb\(^{-1}\)
0.045 \(\leq Q^2 \leq 50000\) GeV\(^2\), \(6 \times 10^{-7} \leq x \leq 0.65\)

21 data sets from HERA I
   NC & CC at \(E_p = 920\) and 820 GeV
and 20 data sets from HERA II (2003-2007)
   12 NC & CC sets at \(E_p = 920\) GeV
   4 NC sets at \(E_p = 575\) GeV
   4 NC sets at \(E_p = 460\) GeV

These data are collected over 15 years with changing beams and detectors conditions and different focus.
It is important to handle them properly, e.g. in view of possible correlations

\(\Rightarrow\) combine them into one coherent data set as it was done for HERA I before
(JHEP 1001:109, 2010 and HERAPDF 1.0)
Averaging Procedure

The combination of all H1 and ZEUS unpolarised NC and CC data is performed using HERAverager (wiki-zeuthen.desy.de/HERAverager)

- the data points are moved to common $x,Q^2$ grid (previous slide)
- in each grid point the same cross section is expected to be measured
- all 162 systematic sources of uncertainties are treated as multiplicative in one simultaneous minimization of $\chi^2$
- expert knowledge in the treatment of the correlations between individual data sets is taken into account

The following $\chi^2$ definition is used:

$$
\chi^2_{\text{exp,ds}}(m, b) = \sum_{i,ds} + \sum_{j,b} = \sum_i \frac{\left[ m^i - \sum_j \gamma_j^i m^j b_j - \mu_i^i \right]^2}{\delta_{i,\text{stat}}^2 \mu_i^i (m^i - \sum_j \gamma_j^i m^j b_j) + (\delta_{i,\text{uncor}} m^i)^2} + \sum_j b_j^2
$$

7 additional procedural errors correspond to:
- multiplicative vs. additive, correlation over all data sets of photoproduction bkg and hadronic energy scale uncertainties and in addition 4 procedural errors related to cross correlations between different syst. uncertainties
Averaging of all NC and CC data at HERA

2927 cross sections are combined to 1307 points with 169 correlated systematic errors and $\chi^2$/d.o.f. = 1685/1620

Coherent set of unpolarised $e^\pm p$ NC&CC at three $\sqrt{s} =$ 318, 300, 251, 225 GeV:

→ www.desy.de/h1zeus/herapdf20/
→ precise, complete and easy in use
→ with reduced stat. and syst. errors

$e^\pm p$ NC&CC ($E_p =$920 GeV)
$e^p$ NC ($E_p =$ 820, 575, 460 GeV)
$0.045 \leq Q^2 \leq 50000 \text{ GeV}^2$, $6 \times 10^{-7} \leq x_{\text{Bj}} \leq 0.65$
total unc. $< 1.5\%$ for $Q^2$ up to 500 GeV$^2$

→ up to 6 measurements are combined into one averaged point
→ correlated shifts are propagated to all points (even measured by single experim.)
Consistency of the input data sets

H1 and ZEUS

**NC $e^+p$, $Q^2 < 3.5$**

**3.5 – 100**

**> 100 GeV**

Very good overall (close to one)

$\chi^2$/d.o.f. = 1685/1620

Checks in different corners of the phase space are in agreement with expected one sigma gaussian distributions of pulls.

Pulls are defined as

$$ p^{i,k} = \frac{\mu^{i,k} - \mu^i (1 - \sum_j \gamma_{j,k} b_j)}{\sqrt{\Delta_{i,k}^2 - \Delta_i^2}} $$

ICNFP 2015
Crete, 25.08.2015
V. Chekelian, Proton Structure
and PDFs from HERA
Combined NC and CC $e^\pm p$ data

- single differential cross sections are obtained by integration over $x$ of the combined NC and CC $e^\pm p$ data at $\sqrt{s}=318$ GeV and $y<0.9$

- $e^+p$ NC and $e^-p$ NC are the same in the $\gamma$-exchange domain at low $Q^2$ and start to differ at high $Q^2$ due to $\gamma Z$ interference.

- CC is two orders of magnitude smaller than NC at $Q^2=200$ GeV$^2$ and about the same at $Q^2$ around $M_Z^2$, $M_W^2$, demonstrating electroweak unification.

- remaining differences in CC are related to $u$, $d$ content of the proton and to helicity factors.
Proton structure function $F_2$

$F_2$ scaling (independence of $Q^2$) at moderate $x$ and scaling violations at high $x_{Bj}$ and low $x_{Bj}$ due to gluon emission and gluon splitting

**H1 and ZEUS**

- HERA NC $e^{-}p$ 0.4 fb$^{-1}$
- HERA NC $e^{-}p$ 0.5 fb$^{-1}$
- $\sqrt{s} = 318$ GeV
- Fixed Target
- HERAPDF2.0 $e^{-}p$ NNLO
- HERAPDF2.0 $e^{-}p$ NNLO

$Q^2 = 6.5$ GeV$^2$

$Q^2 = 120$ GeV$^2$

- HERA NC $e^{+}p$ 0.5 fb$^{-1}$
- $\sqrt{s} = 318$ GeV
- HERAPDF2.0 NLO

$Q^2 = 12$ GeV$^2$

$Q^2 = 1200$ GeV$^2$

A steep rise of $F_2$ towards low $x_{Bj}$, becoming steeper as $Q^2$ increases

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**$e^\pm p$ CC probe u/d composition of proton**

$$\tilde{\sigma}_{CC} = \frac{2\pi x}{G_F^2} \left[ \frac{M_W^2 + Q^2}{M_W^2} \right]^2 \frac{d^2\sigma_{CC}}{dx dQ^2}$$

**H1 and ZEUS**

<table>
<thead>
<tr>
<th>$\sigma_{CC}$</th>
<th>HERA</th>
<th>HERA PDF2.0 NNLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sqrt{s} = 318$ GeV</td>
<td>$\sqrt{s} = 318$ GeV</td>
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<tr>
<td>$\bullet$ CC $e^p 0.5$ fb$^{-1}$</td>
<td>$\bullet$ CC $e^p 0.4$ fb$^{-1}$</td>
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<tr>
<td>$\bullet$ CC $e^p 0.4$ fb$^{-1}$</td>
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<tr>
<td>$\bullet$ CC $e^p 0.4$ fb$^{-1}$</td>
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<tr>
<td>$x_B = 0.008$ (x15000)</td>
<td>$x_B = 0.013$ (x3000)</td>
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<tr>
<td>$x_B = 0.032$ (x700)</td>
<td>$x_B = 0.08$ (x170)</td>
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<tr>
<td>$x_B = 0.032$ (x700)</td>
<td>$x_B = 0.13$ (x20)</td>
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<td>$x_B = 0.25$ (x2)</td>
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**$e^+p$ CC**

**$e^-p$ CC**

$Q^2/\text{GeV}^2$

$e^+p$ CC at high $x$ is related to $d$-quark ($Q^2$ dependence is due to helicity factor $(1-y)^2$)

$e^-p$ CC is dominated by $u$-quark and depends weakly on $Q^2$ at given $x$
$e^\pm p$ NC: lepton charge dependence and $x F_3$

$$x \tilde{F}_3 = \frac{Y^+}{2Y^-} \left( \tilde{\sigma}_{NC}^- - \tilde{\sigma}_{NC}^+ \right)$$

charge asymmetry of $e^\pm p$ NC cross sections is mostly due to $\gamma Z$ interference

$$x F_3^{\gamma Z} = -x \tilde{F}_3 \cdot (Q^2 + M_Z^2)/(a_\gamma k Q^2)$$

transform the $x F_3^{\gamma Z}(x,Q^2)$ measurements to $Q^2 = 1000 \text{ GeV}^2$ and average them to get $x F_3^{\gamma Z}(x)$ at $Q^2 = 1000 \text{ GeV}^2$

→ related to valence quark:

$$F_3^{\gamma Z} \approx (2u_v + d_v)/3$$

→ integration over the measured range $0.016 < x_{Bj} < 0.725$ gives $1.165 \pm 0.042 - 0.053$ for data and $1.314 \pm 0.057(\text{stat}) \pm 0.057(\text{syst})$ using HERAPDF2.0
NC: The parity violating structure function $F_2^{\gamma Z}$

\[
\frac{\sigma^\pm (P_L^\pm) - \sigma^\pm (P_R^\pm)}{P_L^\pm - P_R^\pm} = \frac{\kappa Q^2}{Q^2 + M_Z^2} \left[ \mp a_e F_2^{\gamma Z} + \frac{Y_-}{Y_+} v_e x F_3^{\gamma Z} - \frac{Y_-}{Y_+} Q^2 + M_Z^2 \left( v_e^2 + a_e^2 \right) x F_3 \right]
\]

taking the difference for $e^+p$ and $e^-p$, the terms with $xF_3^{\gamma Z}$ and $xF_3^{Z}$ cancel and $F_2^{\gamma Z}$ can be directly extracted from measured polarised cross sections.

\[
\kappa^{-1} = 4 \frac{M_W^2}{M_Z^2} \left( 1 - \frac{M_W^2}{M_Z^2} \right)
\]

**longitudinal polarisation** of e-beam:

$P_e = (N_r - N_l) / (N_r + N_l)$, where

$N_l(N_r)$ - number of left- (right-) handed leptons in e-beam

“left” and “right” data periods are with

$P_L = <P_e>$ below zero (<0)

$P_R = <P_e>$ above zero (>0)

transform the $F_2^{\gamma Z}(x,Q^2)$ measurements to $Q^2 = 1500 \text{ GeV}^2$ and average them to get $F_2^{\gamma Z}(x)$ at $Q^2 = 1500 \text{ GeV}^2$

\[
F_2^{\gamma Z} = \sum q e_q v_q (xq + x\bar{q})
\]
Probe (V-A) structure of CC

Polarisation dependence of the total CC cross section \((Q^2>400 \text{ GeV}^2, y<0.9)\)

\[
\sigma_{CC}^{e^\pm p} = (1 \pm P_e)\sigma_{CC}^{e^\pm p} (P_e = 0)
\]

linear dependence with \(\sigma=0\) intercept at \(P_e=1\) for \(e^-p\) and \(P_e=-1\) for \(e^+p\)

\[
P_e = \frac{(N_r-N_l)}{(N_r+N_l)}
\]

\(\rightarrow\) absence of right-handed weak currents

V-A structure of CC
(pure left-handed)

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Longitudinal structure function $F_L$

$F_L$ is a pure QCD effect sensitive to gluon density $F_L(x,Q^2) = \frac{\alpha_s}{4\pi} x^2 \int_x^1 \frac{dz}{z^3} \left[ \frac{16}{3} F_2 + 8 \sum_q e_q^2 (1 - \frac{x}{z}) \cdot x g \right]$

H1 and ZEUS

Consistency of the H1 and ZEUS $F_L$ data was checked accounting for corr. errors: $\chi^2/d.o.f. = 11/8$ (p-value=20%).

$R = \sigma_L/\sigma_T = F_L/(F_2-F_L) = 0.23 \pm 0.04$ (H1, $1.5 \leq Q^2 \leq 800$ GeV$^2$)

$R = 0.105 + 0.055 - 0.037$ (ZEUS, $9 \leq Q^2 \leq 110$ GeV$^2$)
HERAPDF2.0 QCD Fit

The combined $e^\pm p$ NC/CC HERA data set is the only input

- no nuclear, heavy target or HT corrections; consistency of data, $\Delta \chi^2 = 1$ criterion
- parametrisation of PDFs at starting scale $Q_o^2=1.9$ GeV$^2$ with 14 free parameters

\[
xg(x) = A_g x^B_g (1 - x)^C_g - A'_g x^{B'}_g (1 - x)^{C'}_g,
\]
\[
xu(x) = A_u x^{B_u} (1 - x)^{C_u} \left(1 + E_u x^2\right), \quad x\bar{U}(x) = A_{\bar{U}} x^{B_{\bar{U}}} (1 - x)^{C_{\bar{U}}} (1 + D_{\bar{U}} x),
\]
\[
xd(x) = A_d x^{B_d} (1 - x)^{C_d}, \quad x\bar{D}(x) = A_{\bar{D}} x^{B_{\bar{D}}} (1 - x)^{C_{\bar{D}}}.
\]

- QCD evolution of PDFs using DGLAP equations at NLO and NNLO
- Thorne-Roberts general mass variable-flavor-number scheme RTOPT (as used in MMHT)
- default $Q^2_{\text{min}}=3.5$ GeV$^2$, $f_s=0.40$ ($xs = f_s x\bar{D}$ at $Q_o^2$)
- $M_c$ and $M_b$ values are optimized using HERA charm and beauty production data
- $\alpha_s(M^2_Z)=0.118$ is consistent with HERA jet data

→ at [www.desy.de/h1zeus/herapdf20/](http://www.desy.de/h1zeus/herapdf20/) and will be available on LHAPDF:

HERAPDF2.0 at NLO and NNLO
also with a scan of $\alpha_s(M^2_Z)$ from 0.110 to 0.130 in steps of 0.001

additional PDF sets:

HERAPDF2.0HiQ2 at NLO and NNLO - $Q^2_{\text{min}}=10$ GeV$^2$
HERAPDF2.0AG at LO, NLO and NNLO - alternative gluon parameterisation (strictly positive)
HERAPDF2.0FF3A and FF3B - fixed flavor number schemes at NLO
Uncertainties of HERAPDF2.0

Three types of PDF uncertainties are considered:

**Experimental uncertainty band**
- Hessian method with $\Delta \chi^2 = 1$
  verified by MC method - replicas of data

**Model uncertainty band**
- variation of model assumptions

<table>
<thead>
<tr>
<th>Variation</th>
<th>Standard Value</th>
<th>Lower Limit</th>
<th>Upper Limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{\text{min}}^2$ [GeV$^2$]</td>
<td>3.5</td>
<td>2.5</td>
<td>5.0</td>
</tr>
<tr>
<td>$Q_{\text{min}}^2$ [GeV$^2$] HiQ2</td>
<td>10.0</td>
<td>7.5</td>
<td>12.5</td>
</tr>
<tr>
<td>$M_r$(NLO) [GeV]</td>
<td>1.47</td>
<td>1.41</td>
<td>1.53</td>
</tr>
<tr>
<td>$M_c$(NNLO) [GeV]</td>
<td>1.43</td>
<td>1.37</td>
<td>1.49</td>
</tr>
<tr>
<td>$M_b$ [GeV]</td>
<td>4.5</td>
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<td>4.75</td>
</tr>
<tr>
<td>$f_s$</td>
<td>0.4</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>$\alpha_s(M_b^2)$</td>
<td>0.118</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\mu_b$ [GeV]</td>
<td>1.9</td>
<td>1.6</td>
<td>2.2</td>
</tr>
</tbody>
</table>

**Parameterisation uncertainty band**
- variation of the starting scale $Q_o^2$ and
- form of parameterisation (number of free parameters)
  → valid in the $x$-range covered by the QCD fit to HERA data

H1 and ZEUS

HERAPDF2.0 NNLO $\mu_r^2 = 10$ GeV$^2$

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ICNFP 2015
Crete, 25.08.2015
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HERAPDF2.0 dependence on $Q^2_{\text{min}}$

$\chi^2$/d.o.f. is improving from 1.20 to 1.15 with increasing $Q^2_{\text{min}}$ from 3.5 to 10 GeV$^2$ (similar behavior was for HERAPDF1.0 although at smaller values of $\chi^2$/d.o.f.)

HERAPDF2.0HiQ2 is very similar to HERAPDF2.0 apart from low $x$ measured at low $Q^2 < 10$ GeV$^2$

→ this difference plays no role at large scales, for example at LHC
Low $Q^2$ (and $x$) domain and $F_L$ description

low $Q^2$ / low $x$ domain (with increased $\chi^2$/d.o.f.) is very interesting for study of low $x$ phenomenology

it seems that in this domain the order of the $F_L$ calculation is more important then other QCD fit settings:

→ order of $O(\alpha_s)$ is preferred

arxiv:1506.06042
Comparison with modern PDFs from global fits

vs. PDFs using variable-flavor-number scheme: MMHT2014, CT10, NNPDF3.0

vs. PDFs using fixed-flavor-number scheme: ABM11 FF, NNPDF3.0FF(3N)

→ differences in valence quarks at high x: new HERA data

→ sea and gluon are consistent
Charm and bottom mass parameters in HERAPDF2.0

$M_c$ and $M_b$, charm and bottom mass parameters, are determined in $\chi^2$ scans of the HERA charm and bottom data together with combined inclusive data.

$M_c = 1.47 \ (1.43) \ \text{GeV at NLO (NNLO)}$

$M_b = 4.5 \ \text{GeV at NLO, NNLO}$

→ reduction of the $M_c$ and $M_b$ uncertainties in HERAPDF fits
HERAPDF2.0Jets
(inclusive + charm + jets)

**include also HERA combined charm production and selected jet production data:**

at NLO, with free $\alpha_s$ and additional error band related to hadronisation of jets

$\rightarrow \alpha_s$, determined in a simultaneous fit with PDFs:

$$\alpha_s(M_Z^2) = 0.1183 \pm 0.0009\text{(exp)} \pm 0.0005\text{(model/param)} \pm 0.0012\text{(hadronisation)} \pm^{37}_{-30}\text{(scale)}$$

PDFs and the error bands are very close to HERAPDF2.0 obtained using inclusive data and $M_c$ and $M_b$ already optimized using charm and bottom HERA data and

$\alpha_s=0.118$, consistent with the HERA multi-jet data.

(slight increase of err. band is due to hadronisation).
Conclusions

H1 and ZEUS completed the inclusive DIS program at HERA by combining all inclusive unpolarised measurements into one coherent data set of NC & CC e⁺p and e⁻p at √s = 319, 302, 251 and 225 GeV with 169 common correlated systematic errors.

All three proton structure functions $F_2$, $F_2^\gamma Z$, $xF_3^\gamma Z$ and $F_L$ are measured exploiting charge and polarity dependencies of the cross section measurements at HERA.

This combined inclusive HERA data set of the NC and CC cross sections is used as a sole input to the QCD analysis of the data resulting in the set of parton distribution functions HERAPDF2.0 which will be available on LHAPDF together with its variants.