ITEP Lattice

Polarizability of pseudoscalar mesons from the lattice calculations

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2 REGIMES

Perturbative or weak coupling regime: quantum electrodynamics (QED), coupling constant is sufficiently small

$$\alpha = \frac{1}{137}.$$

Nonperturbative or strong coupling regime: quantum chromodynamics (QCD),

$$\alpha_s \approx 1,$$

analitical methods do not work, there is no small parameter in the theory for the expansions.

We use numerical methods.

Computer simulations of strongly interacting systems

Quantum chromodynamics:
1. spectrum of mesons ($\bar{q}_i q_j$) and baryons ($q_i q_j q_k$) (masses, decay constants, excited states);
2. confinement problem (computers can prove confinement ”numerically”);
3. the structure of QCD phase diagram;
QCD effects which occurs at very big magnetic fields in:
1. noncentral heavy ion collisions, $eB \sim \Lambda^2_{QCD}$;
2. early Universe. It is assumed that magnetic fields $\sim 2\ GeV$ existed in the Universe during the electroweak phase transition;
Graphene as quantum field theory

Quantum chromodymamics (QCD)

Quark field: $\psi_q^i(x)$, where q is the flavour index, $i = 1, 2, 3$ is the color index, $x = (x_1, x_2, x_3, x_0)$ is the coordinate.
Gluon field: $A_\mu^a(x)$, where $\nu, \mu = 1, ...4$ is the Lorentz index, $a, b, c = 1, ..., N^2 - 1$, $N = 3$ for $SU(3)_c$.

$$S_{QCD} = \int d^4x \mathcal{L} =$$

$$= \int d^4x \left(-\frac{1}{4} F_{\mu\nu}^a(x) F^{a\mu\nu}(x) + \sum_q \bar{\psi}_q^i(x)(i\gamma_\mu D_\mu(x) - m_q)_{ij} \psi_q^j(x) \right),$$

$$F_{\mu\nu}^a(x) = \partial_\mu A_\nu^a(x) - \partial_\nu A_\mu^a(x) + g_s f_{abc} A_\mu^b(x) A_\nu^c(x),$$

$$(D_\mu(x))_{ij} = \delta_{ij}\partial_\mu - ig_s \sum_a t_{ij}^a A_\mu^a(x), \quad [t^a, t^b] = i \sum_c f^{abc} t^c, \quad t_{ij}^a = \lambda_{ij}^a/2.$$

Lattice QCD

K.G.Wilson, Phys.Rev., vol. 010, p. 2445, 1974.
$x \Rightarrow an,\ n_i = 0, 1, ..., N_s - 1,\ \ i = 1, 2, 3,\ \ n_0 = 1, ...N_t, n = (n_i, n_0),$
a is the lattice spacing, $n \in Z$, $N_s^3 \times N_t$ is the lattice volume.

1. Make a transition to Euclidean space $x_0 \to it$.
2. Exchange the Lagrangian of the theory by it's discretized version.

$$\psi(x) \Rightarrow \psi(n), \quad \partial_\mu \psi(x) \Rightarrow \frac{\psi(n+\hat{\mu}) - \psi(n-\hat{\mu})}{2a} + O(a^2)$$

$$S_{QCD} \to iS^E_{QCD} \Rightarrow \exp\{iS_{QCD}\} \to \exp\{-S^E_{QCD}\}.$$

3. We generate numerically gauge field ensembles of gluonic configurations with weight $e^{-S^E_{QCD}}$, (corresponding to Boltzmann distribution) using Monte-Carlo methods.

Introduction

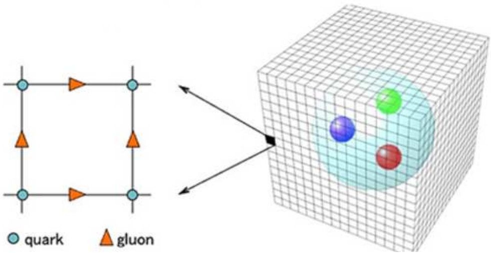
We work in $SU(3)$ lattice gauge theory without dynamical quarks in a constant external magnetic field directed along the third axis z and explore

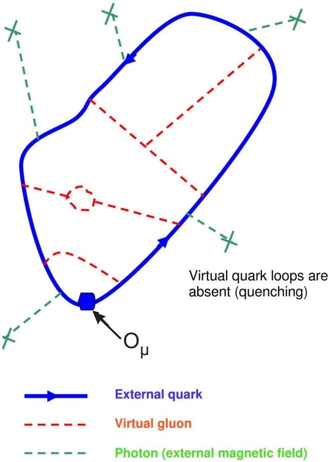
1. the dependence of the π^0 and $\pi^\pm$ energies versus the value of the magnetic field;
2. calculate the magnetic polarizabilities of pions;
3. study mixing between π^0 and ρ^0;

$\pi^0(J^P=0^-)$, $\rho^0(J^P=1^-)$: $\psi_{\pi^0,\rho^0}=(\psi_u\psi_{\bar{u}}-\psi_d\psi_{\bar{d}})/\sqrt{2}$
$\pi^\pm(J^P=0^-)$, $\rho^\pm(J^P=1^-)$: $\psi_{\pi^+,\rho^+}=\psi_u\psi_{\bar{d}}$, $\quad\psi_{\pi^-,\rho^-}=\psi_d\psi_{\bar{u}}$
$K^\pm(J^P=0^-,1^-)$: $\psi_{K^-}=\psi_s\psi_{\bar{u}}$, $\quad\psi_{K^+}=\psi_u\psi_{\bar{s}}$

Technical details

Model: quenched $SU(3)$ lattice gauge theory.
Generation of A_μ: the tadpole-improved Wilson-Symanzik action, 16^4, 18^4 and 20^4 lattice volumes,
$a = 0.084\ fm, 0.095\ fm, 0.105\ fm, 0.115\ fm,\ 0.125\ fm$ lattice spacings.
Fermion spectrum: chirally-invariant overlap operator, 50 lowest modes for the inversion, different quark masses
$am_q^{lat} = 0.007, 0.010, 0.015, 0.02, 0.03, 0.04, 0.05$.





We calculate the interpolators $\bar{\psi} O_\mu \psi$ (where $O_\mu = \gamma_\mu, \; \gamma_\mu\gamma_5, \; \gamma_5$) in the external $U(1)$ magnetic field and in the presence of vacuum $SU(3)$ nonabelian gluon fields.

Technical details

Solve the Dirac equation numerically:

$$D\psi_k = i\lambda_k\psi_k, \;\; D = \gamma^\mu(\partial_\mu - iA_\mu).$$

Calculate the propagators:

$$D^{-1}(x,y) = \sum_{k<M} \frac{\psi_k(x)\psi_k^\dagger(y)}{i\lambda_k + m}.$$

The correlators with the quantum numbers corresponding to the particle considered:

$$\langle\bar\psi O_\mu\psi\bar\psi O_\nu\psi\rangle_A = -tr[O_\mu D^{-1}(x,y)O_\nu D^{-1}(y,x)]+$$

$$+tr[O_\mu D^{-1}(x,x)]tr[O_\nu D^{-1}(y,y)], \;\; x = (\mathbf{n}a, n_t a), \;\; y = (\mathbf{n}'a, n_t' a)$$

$$\mathbf{n}, \mathbf{n}' \in \Lambda_3 = \{(n_1, n_2, n_3) | n_i = 0, 1, ..., N-1\}$$

Correlators

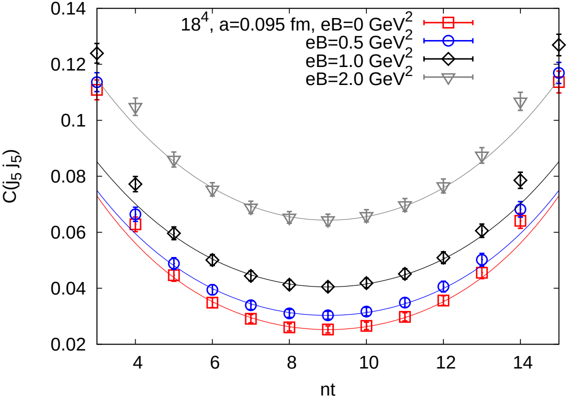
$$G(\vec{p},\tau) = \frac{1}{N^{3/2}} \sum_{\mathbf{n}\in\Lambda_3} \langle j_\mu(\mathbf{n}, n_t) j_\nu^\dagger(\mathbf{0},0)\rangle e^{-ia\mathbf{np}}$$

$p_i = 2\pi k_i/(aN), \; k_i = -N/2+1, ..., N/2.$
We obtain the masses from the correlator of currents

$$\langle \psi^\dagger(\vec{0}, n_t) O_\mu \psi(\vec{0}, n_t) \psi^\dagger(\vec{0},0) O_\nu \psi(\vec{0},0)\rangle_A = \sum_k \langle 0|O_\mu|k\rangle\langle k|O_\nu^\dagger|0\rangle e^{-n_t E_k}.$$

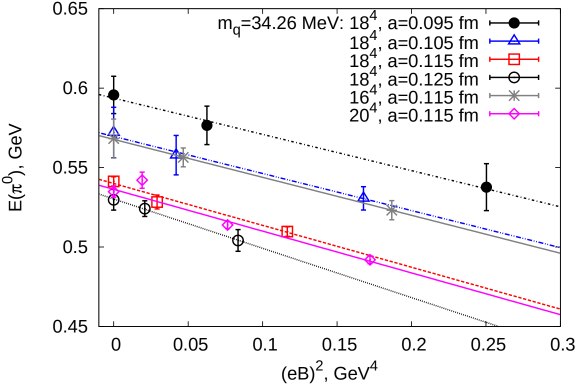
The main contributions comes from $\langle 0|O_\mu|k\rangle\langle k|O_\nu^\dagger|0\rangle e^{-n_t E_0}$, We set $\langle \mathbf{p}\rangle = 0$. So $E_0 = m_0$ because $E^2 - \mathbf{p}^2 = m^2$.

The correlator for the neutral pion

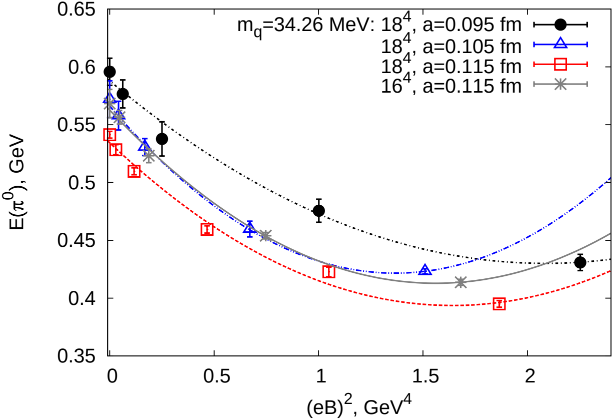


We fit the correlator by the function $\tilde{C}_{fit}(n_t) = A_0 e^{-n_t aE_0} + A_0 e^{-(N_T - n_t) aE_0} = 2A_0 e^{-N_T aE_0/2} \cosh((N_T - n_t)aE_0)$ at $5 \leq n_t \leq N_T - 5.$.

Energy of π^0 meson vs. the magnetic field

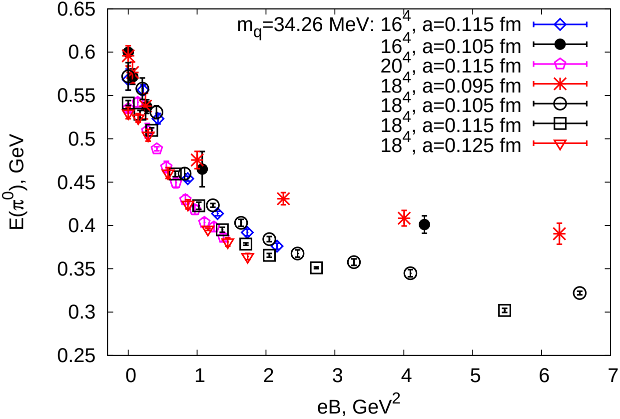


Fit on $(eB)^2 \in [0 : 0.3\ GeV^4]$: $E = E(B=0) - 2\pi\beta(eB)^2$



Fit on $(eB)^2 \in [0 : 2.4\ GeV^2]$:

$$E = E(B=0) - 2\pi\beta(eB)^2 + k(eB)^4$$



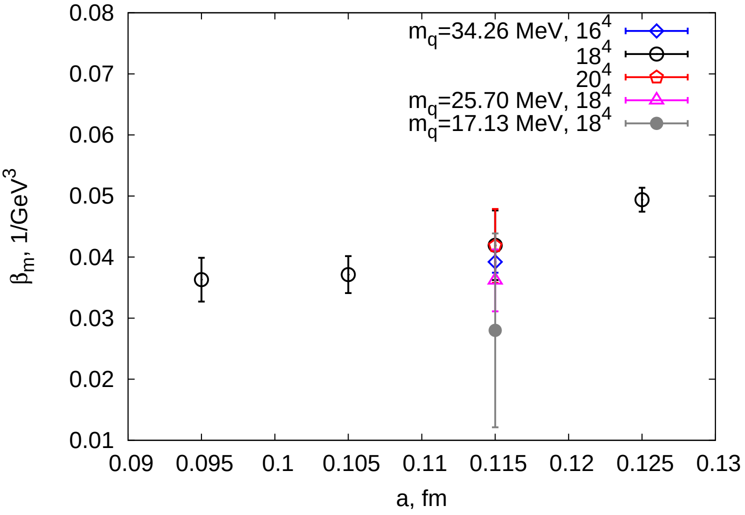
Higher degrees on $(eB)^2$ give a strong contribution to the pion energy at large magnetic fields ($> 1.5\ GeV^2$).

The magnetic polarizability of π^0 meson

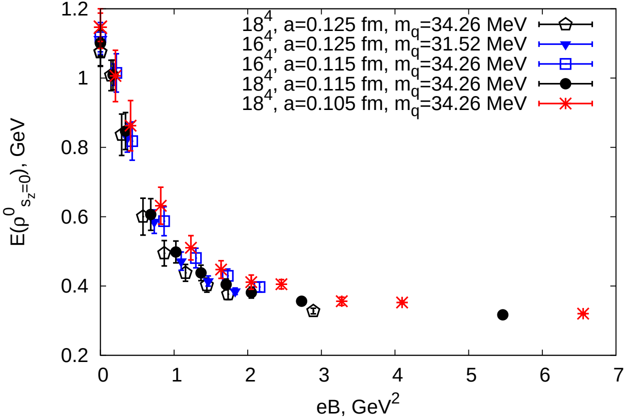
V_{latt}	m_q (MeV)	a (fm)	$\beta\,(GeV^{-3})$	Error (GeV^{-3})	χ^2/d.o.f.
18^4	34.26	0.095	0.036	0.004	0.092
18^4	34.26	0.105	0.037	0.003	0.050
18^4	34.26	0.115	0.042	0.006	1.034
18^4	34.26	0.125	0.049	0.002	0.0131
18^4	25.70	0.115	0.036	0.005	1.105
18^4	17.13	0.115	0.028	0.016	1.552
18^4	11.99	0.115	0.012	0.015	1.119
20^4	34.26	0.115	0.042	0.006	2.96

$\beta = (2.14 \pm 1.22)10^{-4}\ fm^3$ at
$m_q = 17.13\,MeV$, $m_\pi = 396\,MeV$.
Chiral Perturbation theory at 2 loops: $\beta_{\pi^0} = (1.5 \pm 0.3) \cdot 10^{-4}\ fm^3$ (0.5 at one loop); $\alpha_\pi + \beta_\pi = 0$ at the leading order of χPT, α_π is the electric polarizability.
COMPASS from JLab data: $\alpha_{\pi^0} = (-1.0 : -0.3) \cdot 10^{-4}\ fm^3$.

Lattice spacing and quark mass dependence

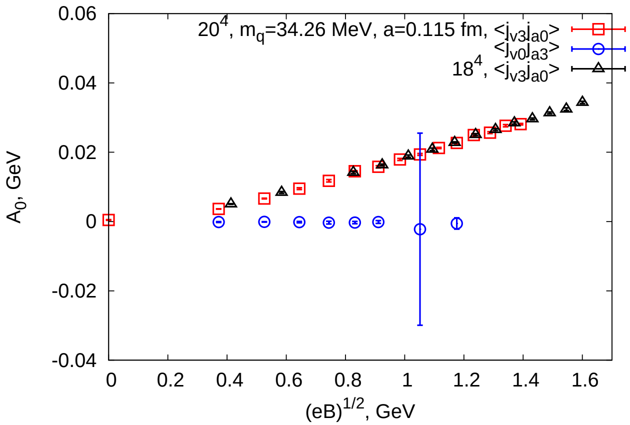


Neutral ρ^0 meson with zero spin projection to B



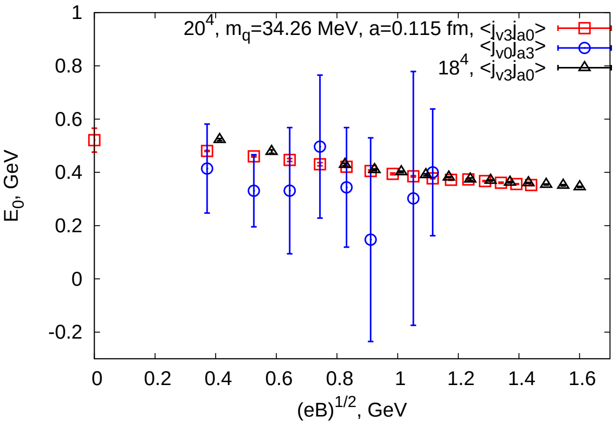
The $\rho^0(s_z = 0)$ meson mass versus the magnetic field.

Mixing between π^0 and $\rho^0(s_z = 0)$



$$\tilde{C}_{fit}(n_t) = 2A_0 e^{-N_T aE_0/2} \cosh((N_T - n_t)aE_0)\ .$$

$$< j_{v3} j_{a0} > = < \psi^\dagger \gamma_3 \psi \psi^\dagger \gamma_5 \gamma_0 \psi >, < j_{v0} j_{a3} > = < \psi^\dagger \gamma_0 \psi \psi^\dagger \gamma_5 \gamma_3 \psi >$$



$$\tilde{C}_{fit}(n_t) = 2A_0 e^{-N_T aE_0/2} \cosh((N_T - n_t)aE_0)\ .$$

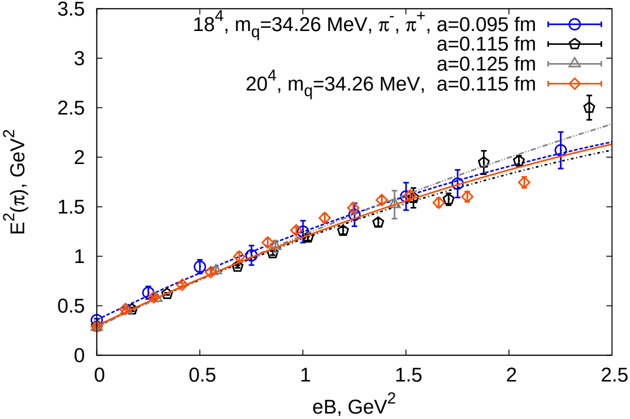
The energy levels of free charged pointlike particles in a background magnetic field parallel to z axis

$$E^2 = p_z^2 + (2n+1)|qB| - g s_z qB + E^2(B=0)$$

So doesn't take into account polarizability of particle. In our case $E^2 = E^2(B=0) + |qB| - 2\pi\beta(eB)^2$.

Energy of $\pi^{\pm}$ meson vs. the magnetic field

$$C^{PSPS} = \langle \bar{\psi}_u(\vec{0}, n_t)\gamma_5\psi_u(\vec{0}, n_t)\bar{\psi}_d(\vec{0}, 0)\gamma_5\psi_d(\vec{0}, 0)\rangle$$



The magnetic polarizability of $\pi^{\pm}$ meson

V_{latt}	m_q (MeV)	a (fm)	$\beta\,(GeV^{-3})$	Error (GeV^{-3})	χ^2/d.o.f.
18^4	34.26	0.095	0.015	0.001	0.19656
18^4	34.26	0.115	0.017	0.002	3.68394
18^4	34.26	0.125	0.012	0.005	0.41927
20^4	34.26	0.115	0.016	0.003	7.94917

$\beta = (0.015 \pm 0.001)\; GeV^{-3} \rightarrow \beta = (1.15 \pm 0.08\pm) \cdot 10^{-4}\; fm^3$
COMPASS: $\alpha_{\pi^{\pm}} \in (+2 : +6) \cdot 10^4\; fm^3$.
χPT : $\alpha_{\pi^{\pm}} = 2.83 \cdot 10^{-4}\; fm^3$, $\beta_{\pi^{\pm}} = -2.76 \cdot 10^{-4}\; fm^3$ at $O(p^4)$
(A.Aleksejevs, S.Barkanova, arXiv: 1309.3313).

Conclusions

1. Explore the dependences of π^0 and $\pi^\pm$ energies on the magnetic field;
2. calculate the magnetic polarizabilities at several quark masses, agreement with chiral perturbation theory for neutral pion;
3. observe the mixing between π^0 and ρ^0 in strong magnetic field;