Polarizability of pseudoscalar mesons from the lattice calculations

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2 REGIMES

**Perturbative or weak coupling regime**: quantum electrodynamics (QED), coupling constant is sufficiently small

\[ \alpha = \frac{1}{137}. \]

**Nonperturbative or strong coupling regime**: quantum chromodynamics (QCD),

\[ \alpha_s \approx 1, \]

analytical methods do not work, there is no small parameter in the theory for the expansions.

*We use numerical methods.*
Quantum chromodynamics:
1. spectrum of mesons ($\bar{q}_i q_j$) and baryons ($q_i q_j q_k$) (masses, decay constants, excited states);
2. confinement problem (computers can prove confinement "numerically");
3. the structure of QCD phase diagram;

QCD effects which occurs at very big magnetic fields in:
1. noncentral heavy ion collisions, $eB \sim \Lambda_{QCD}^2$;
2. early Universe. It is assumed that magnetic fields $\sim 2 \text{ GeV}$ existed in the Universe during the electroweak phase transition;

Graphene as quantum field theory
Quantum chromodynamics (QCD)

Quark field: $\psi^i_q(x)$, where $q$ is the flavour index, $i = 1, 2, 3$ is the color index, $x = (x_1, x_2, x_3, x_0)$ is the coordinate.
Gluon field: $A^a_\mu(x)$, where $\nu, \mu = 1, \ldots, 4$ is the Lorentz index, $a, b, c = 1, \ldots, N^2 - 1$, $N = 3$ for $SU(3)_c$.

$$S_{QCD} = \int d^4x \mathcal{L} =$$

$$= \int d^4x \left( -\frac{1}{4} F^a_{\mu\nu}(x) F^{a\mu\nu}(x) + \sum_q \bar{\psi}^i_q(x) (i \gamma_\mu D_\mu(x) - m_q)_{ij} \psi^j_q(x) \right),$$

$$F^a_{\mu\nu}(x) = \partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) + g_s f_{abc} A^b_\mu(x) A^c_\nu(x),$$

$$(D_\mu(x))_{ij} = \delta_{ij} \partial_\mu - ig_s \sum_a t^a_{ij} A^a_\mu(x), \quad [t^a, t^b] = i \sum_c f^{abc} t^c, \quad t^a_{ij} = \lambda^a_{ij}/2.$$
Polarizability of pseudoscalar mesons from the lattice calculations

Lattice QCD


\( x \Rightarrow an, \ n_i = 0, 1, \ldots, N_s - 1, \ i = 1, 2, 3, \ n_0 = 1, \ldots N_t, n = (n_i, n_0) \)

\( a \) is the lattice spacing, \( n \in \mathbb{Z}, N_s^3 \times N_t \) is the lattice volume.

1. Make a transition to Euclidean space \( x_0 \rightarrow it \).

2. Exchange the Lagrangian of the theory by its discretized version.

\[
\psi(x) \Rightarrow \psi(n), \quad \partial_\mu \psi(x) \Rightarrow \frac{\psi(n + \hat{\mu}) - \psi(n - \hat{\mu})}{2a} + O(a^2)
\]

\[
S_{QCD} \rightarrow iS_{QCD}^E \Rightarrow \exp\{iS_{QCD}\} \rightarrow \exp\{-S_{QCD}^E\}.
\]

3. We generate numerically gauge field ensembles of gluonic configurations with weight \( e^{-S_{QCD}^E} \), (corresponding to Boltzmann distribution) using Monte-Carlo methods.
Introduction

We work in $SU(3)$ lattice gauge theory without dynamical quarks in a constant external magnetic field directed along the third axis $z$ and explore

1. the dependence of the $\pi^0$ and $\pi^\pm$ energies versus the value of the magnetic field;

2. calculate the magnetic polarizabilities of pions;

3. study mixing between $\pi^0$ and $\rho^0$;

$\pi^0(J^P = 0^-), \rho^0(J^P = 1^-)$: $\psi_{\pi^0,\rho^0} = (\psi_u \psi\bar{u} - \psi_d \psi\bar{d})/\sqrt{2}$

$\pi^\pm(J^P = 0^-), \rho^\pm(J^P = 1^-)$: $\psi_{\pi^+,\rho^+} = \psi_u \psi\bar{d}$, $\psi_{\pi^-,\rho^-} = \psi_d \psi\bar{u}$

$K^\pm(J^P = 0^-, 1^-)$: $\psi_{K^-} = \psi_s \psi\bar{u}$, $\psi_{K^+} = \psi_u \psi\bar{s}$
Technical details

Model: quenched $SU(3)$ lattice gauge theory.
Generation of $A_\mu$: the tadpole-improved Wilson-Symanzik action, $16^4$, $18^4$ and $20^4$ lattice volumes, $a = 0.084 \, fm$, $0.095 \, fm$, $0.105 \, fm$, $0.115 \, fm$, $0.125 \, fm$ lattice spacings.
Fermion spectrum: chirally-invariant overlap operator, 50 lowest modes for the inversion, different quark masses $am_{lat}^q = 0.007, 0.010, 0.015, 0.02, 0.03, 0.04, 0.05$. 
We calculate the interpolators $\bar{\psi} O_\mu \psi$ (where $O_\mu = \gamma_\mu, \gamma_\mu \gamma_5, \gamma_5$) in the external $U(1)$ magnetic field and in the presence of vacuum $SU(3)$ nonabelian gluon fields.
Technical details

Solve the Dirac equation numerically:

\[ D \psi_k = i \lambda_k \psi_k, \quad D = \gamma^\mu (\partial_\mu - i A_\mu). \]

Calculate the propagators:

\[ D^{-1}(x, y) = \sum_{k < M} \frac{\psi_k(x)\psi_k^\dagger(y)}{i\lambda_k + m}. \]

The correlators with the quantum numbers corresponding to the particle considered:

\[ \langle \bar{\psi} O_\mu \psi \bar{\psi} O_\nu \psi \rangle_A = - tr[O_\mu D^{-1}(x, y) O_\nu D^{-1}(y, x)] + \]

\[ + tr[O_\mu D^{-1}(x, x)] tr[O_\nu D^{-1}(y, y)], \quad x = (na, n_t a), \quad y = (n'_a, n'_t a) \]

\[ n, n' \in \Lambda_3 = \{(n_1, n_2, n_3)|n_i = 0, 1, ..., N - 1\} \]
Correlators

\[ G(\vec{p}, \tau) = \frac{1}{N^{3/2}} \sum_{n \in \Lambda_3} \langle j_\mu (n, n_t) j^\dagger_\nu (0, 0) \rangle e^{-ianp} \]

\[ p_i = 2\pi k_i / (aN), \quad k_i = -N/2 + 1, \ldots, N/2. \]

We obtain the masses from the correlator of currents

\[ \langle \psi^\dagger (\vec{0}, n_t) O_\mu \psi (\vec{0}, n_t) \psi^\dagger (\vec{0}, 0) O_\nu \psi (\vec{0}, 0) \rangle_A = \sum_k \langle 0 | O_\mu | k \rangle \langle k | O_\nu^\dagger | 0 \rangle e^{-n t E_k}. \]

The main contributions comes from \( \langle 0 | O_\mu | k \rangle \langle k | O_\nu^\dagger | 0 \rangle e^{-n t E_0} \), We set \( \langle \mathbf{p} \rangle = 0 \). So \( E_0 = m_0 \) because \( E^2 - \mathbf{p}^2 = m^2 \).
The correlator for the neutral pion

We fit the correlator by the function

\[ \tilde{C}_{\text{fit}}(n_t) = A_0 e^{-n_t a E_0} + A_0 e^{-(N_T - n_t) a E_0} = 2A_0 e^{-N_T a E_0/2} \cosh((N_T - n_t) a E_0) \]

at \( 5 \leq n_t \leq N_T - 5 \).
Energy of $\pi^0$ meson vs. the magnetic field

Fit on $(eB)^2 \in [0 : 0.3 \text{ GeV}^4]$: $E = E(B=0) - 2\pi\beta(eB)^2$
Fit on \((eB)^2 \in [0 : 2.4 \text{ GeV}^2]\):
\[
E = E(B = 0) - 2\pi\beta(eB)^2 + k(eB)^4
\]
Higher degrees on \((eB)^2\) give a strong contribution to the pion energy at large magnetic fields \((> 1.5 \text{ GeV}^2)\).
The magnetic polarizability of $\pi^0$ meson

<table>
<thead>
<tr>
<th>$V_{latt}$</th>
<th>$m_q$ (MeV)</th>
<th>$a$ (fm)</th>
<th>$\beta$ (GeV$^{-3}$)</th>
<th>Error (GeV$^{-3}$)</th>
<th>$\chi^2$/d.o.f.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$18^4$</td>
<td>34.26</td>
<td>0.095</td>
<td>0.036</td>
<td>0.004</td>
<td>0.092</td>
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<td>$18^4$</td>
<td>34.26</td>
<td>0.105</td>
<td>0.037</td>
<td>0.003</td>
<td>0.050</td>
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<tr>
<td>$18^4$</td>
<td>34.26</td>
<td>0.115</td>
<td>0.042</td>
<td>0.006</td>
<td>1.034</td>
</tr>
<tr>
<td>$18^4$</td>
<td>34.26</td>
<td>0.125</td>
<td>0.049</td>
<td>0.002</td>
<td>0.0131</td>
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<tr>
<td>$18^4$</td>
<td>25.70</td>
<td>0.115</td>
<td>0.036</td>
<td>0.005</td>
<td>1.105</td>
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<tr>
<td>$18^4$</td>
<td>17.13</td>
<td>0.115</td>
<td>0.028</td>
<td>0.016</td>
<td>1.552</td>
</tr>
<tr>
<td>$18^4$</td>
<td>11.99</td>
<td>0.115</td>
<td>0.012</td>
<td>0.015</td>
<td>1.119</td>
</tr>
<tr>
<td>$20^4$</td>
<td>34.26</td>
<td>0.115</td>
<td>0.042</td>
<td>0.006</td>
<td>2.96</td>
</tr>
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$\beta = (2.14 \pm 1.22) \times 10^{-4} \text{ fm}^3$ at $m_q = 17.13 \text{ MeV}$, $m_\pi = 396 \text{ MeV}$.

Chiral Perturbation theory at 2 loops: $\beta_{\pi^0} = (1.5 \pm 0.3) \cdot 10^{-4} \text{ fm}^3$ (0.5 at one loop); $\alpha_\pi + \beta_\pi = 0$ at the leading order of $\chi PT$, $\alpha_\pi$ is the electric polarizability.

COMPASS from JLab data: $\alpha_{\pi^0} = (-1.0 : -0.3) \cdot 10^{-4} \text{ fm}^3$. 
Lattice spacing and quark mass dependence

![Graph showing lattice spacing and quark mass dependence](image)
Neutral $\rho^0$ meson with zero spin projection to $B$

The $\rho^0(s_z = 0)$ meson mass versus the magnetic field.
Mixing between $\pi^0$ and $\rho^0(s_z = 0)$

$$\tilde{C}_{fit}(n_t) = 2A_0 e^{-N_T a E_0/2} \cosh((N_T - n_t) a E_0).$$

$$< j_{v3} a_0 > = < \psi^{\dagger} \gamma_3 \psi \psi^{\dagger} \gamma_5 \gamma_0 \psi >, \quad < j_{v0} a_3 > = < \psi^{\dagger} \gamma_0 \psi \psi^{\dagger} \gamma_5 \gamma_3 \psi >.$$

Results

Polarizability of pseudoscalar mesons from the lattice calculations
Polarizability of pseudoscalar mesons from the lattice calculations

Results

\[ \tilde{C}_{\text{fit}}(n_t) = 2A_0 e^{-N_T aE_0/2} \cosh((N_T - n_t)aE_0) . \]
The energy levels of free charged pointlike particles in a background magnetic field parallel to z axis

\[ E^2 = p_z^2 + (2n + 1)|qB| - gs_z qB + E^2(B = 0) \]

So doesn’t take into account polarizability of particle. In our case

\[ E^2 = E^2(B = 0) + |qB| - 2\pi\beta(eB)^2. \]
Polarizability of pseudoscalar mesons from the lattice calculations

Results

Energy of $\pi^\pm$ meson vs. the magnetic field

$$C^{PSPS} = \langle \bar{\psi}_u(\vec{0}, n_t) \gamma_5 \psi_u(\vec{0}, n_t) \bar{\psi}_d(\vec{0}, 0) \gamma_5 \psi_d(\vec{0}, 0) \rangle$$

$E^2(\pi), \text{GeV}^2$

eB, GeV$^2$

$18^4$, $m_q=34.26$ MeV, $\pi^-$, $\pi^+$, $a=0.095$ fm

$a=0.115$ fm

$a=0.125$ fm

$20^4$, $m_q=34.26$ MeV, $a=0.115$ fm
The magnetic polarizability of $\pi^\pm$ meson

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<td>0.015</td>
<td>0.001</td>
<td>0.19656</td>
</tr>
<tr>
<td>$18^4$</td>
<td>34.26</td>
<td>0.115</td>
<td>0.017</td>
<td>0.002</td>
<td>3.68394</td>
</tr>
<tr>
<td>$18^4$</td>
<td>34.26</td>
<td>0.125</td>
<td>0.012</td>
<td>0.005</td>
<td>0.41927</td>
</tr>
<tr>
<td>$20^4$</td>
<td>34.26</td>
<td>0.115</td>
<td>0.016</td>
<td>0.003</td>
<td>7.94917</td>
</tr>
</tbody>
</table>

$\beta = (0.015 \pm 0.001) \text{ GeV}^{-3} \rightarrow \beta = (1.15 \pm 0.08\pm) \cdot 10^{-4} \text{ fm}^3$

COMPASS: $\alpha_{\pi^\pm} \in (+2 : +6) \cdot 10^4 \text{ fm}^3$.

$\chi^2 PT: \alpha_{\pi^\pm} = 2.83 \cdot 10^{-4} \text{ fm}^3$, $\beta_{\pi^\pm} = -2.76 \cdot 10^{-4} \text{ fm}^3$ at $O(p^4)$ (A.Aleksejevs, S.Barkanova, arXiv: 1309.3313).
Conclusions

1. Explore the dependences of $\pi^0$ and $\pi^\pm$ energies on the magnetic field;

2. Calculate the magnetic polarizabilities at several quark masses, agreement with chiral perturbation theory for neutral pion;

3. Observe the mixing between $\pi^0$ and $\rho^0$ in strong magnetic field;