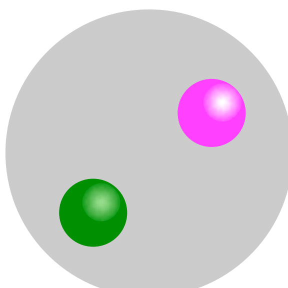
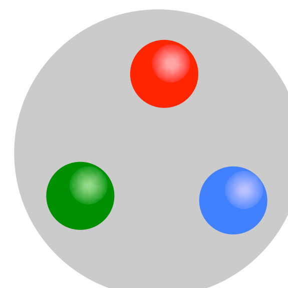
Perfect Abelian dominance of confinement in mesons and baryons in SU(3) lattice QCD

Naoyuki Sakumichi (RIKEN)

Hideo Suganuma (Kyoto Univ.)



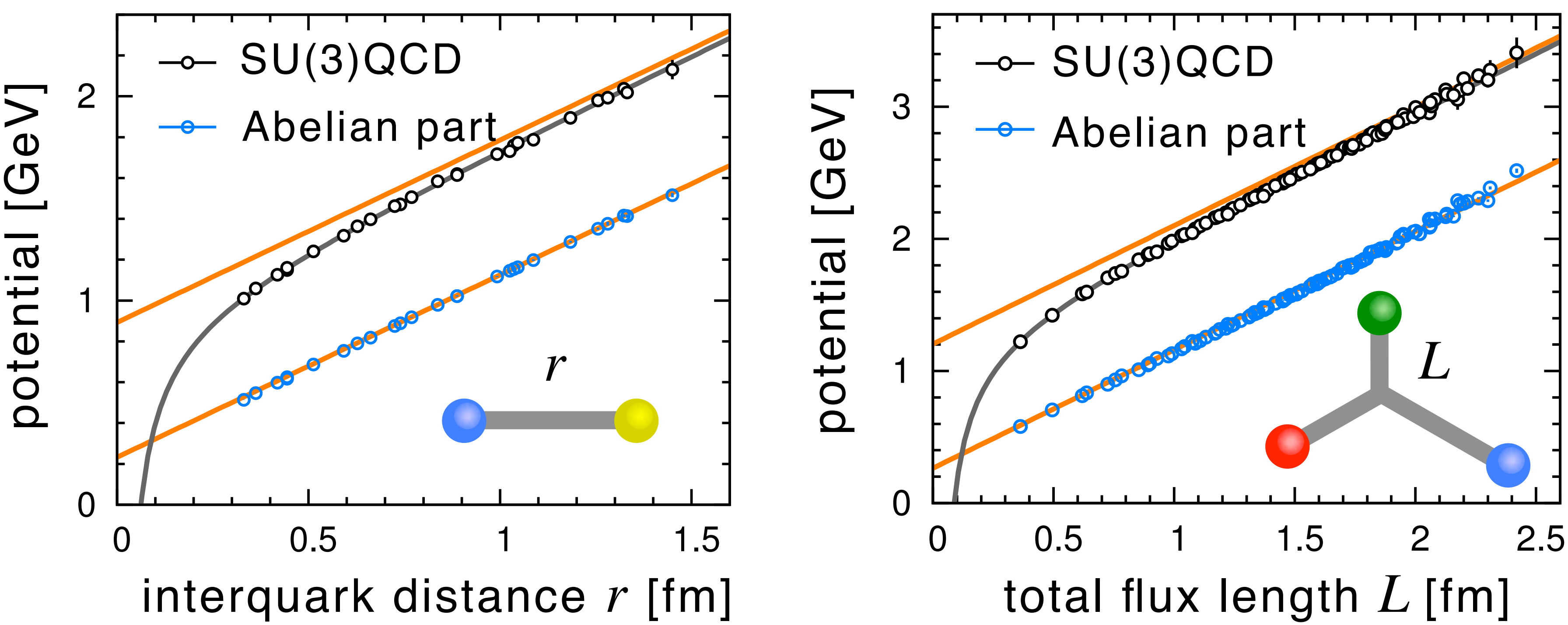


Mesons ($Q\bar{Q}$): Phys. Rev. D **90**, 111501(R) (29 Dec 2014).

Baryons (3Q): Phys. Rev. D **92**, 034511 (21 Aug 2015).

Published last week

Abstruct We study the static three-quark (3Q) potential for more than 300 different patters of 3Q systems with high statistics in SU(3) quenched lattice QCD. For all the distances, the 3Q potential is found to be well described by the Y-Ansatz, i.e., one-gluon-exchange Coulomb plus Y-type linear potential. We find quark-confining forces in both mesonic QQbar and baryonic 3Q systems can be described only with Abelian variables in the maximally Abelian gauge, which we call "perfect Abelian dominance" of quark confinement.



contents

Part 1: perfect Abelian dominance of confinement

in mesons [1]

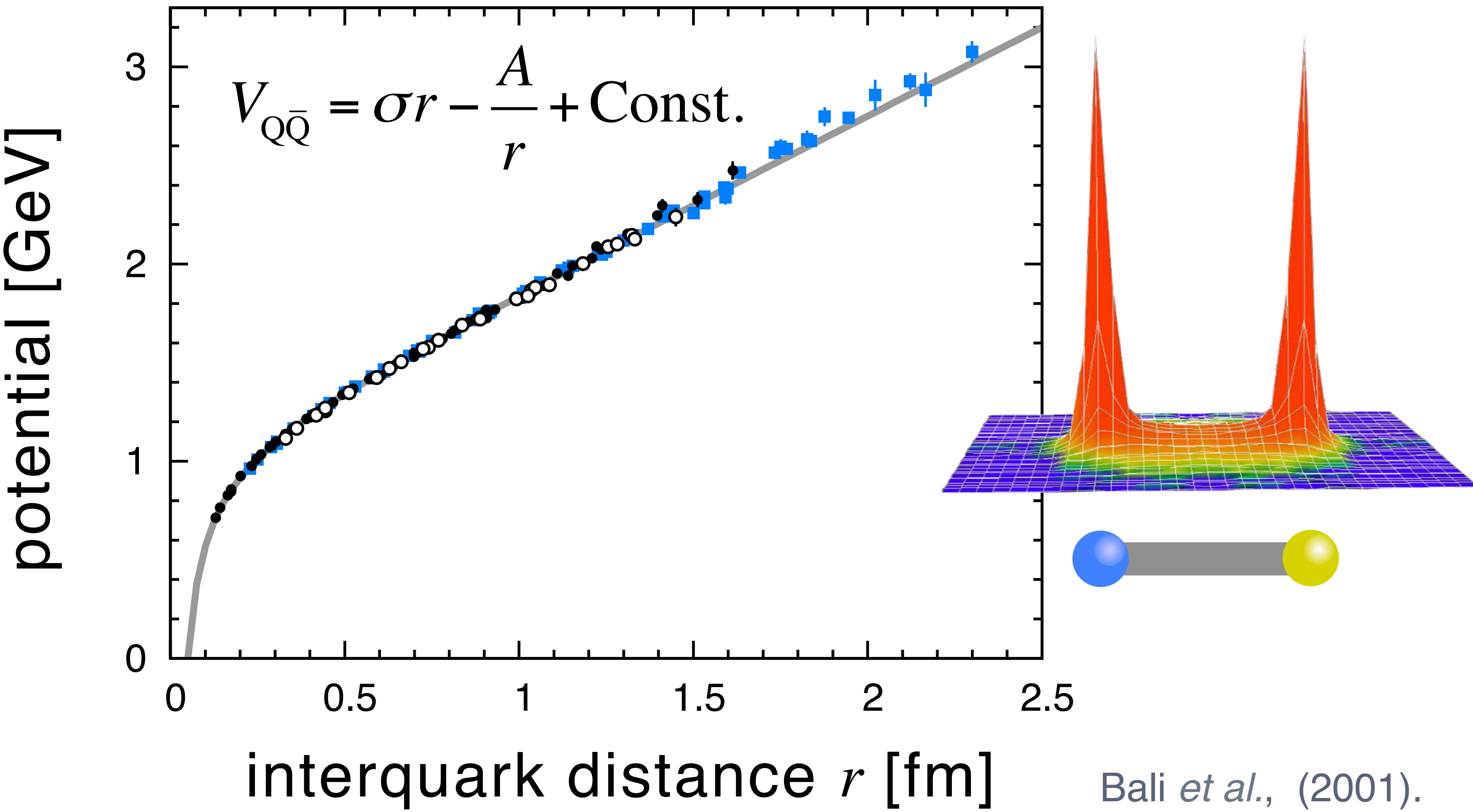
Part 2: perfect Abelian dominance of confinement

in baryons [2]

[1] NS and HS, Phys. Rev. D **90**, 111501(R) (29 Dec 2014).

[2] NS and HS, Phys. Rev. D **92**, 034511 (21 Aug 2015).

Confinement of $Q\bar{Q}$ systems:
one-dimensional squeezing of color flux leads to linear confinement potential.



Bali *et al.*, (2001).

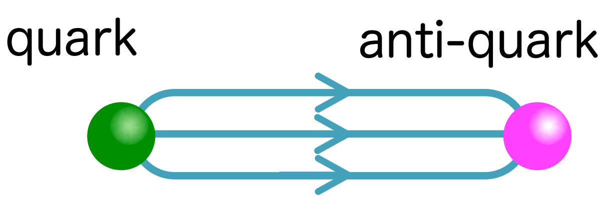
Dual-superconductor picture for confinement:

the analogy between QCD vacuum and superconductor

Nambu(1974); ’t Hooft(1975); Mandelstam(1976).

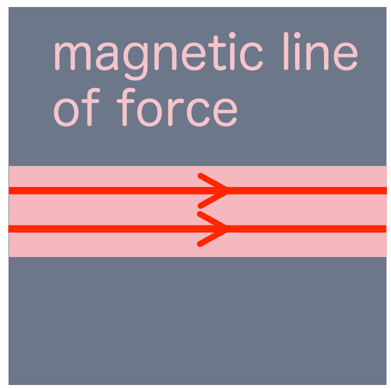
QCD

One-dimensional flux tube



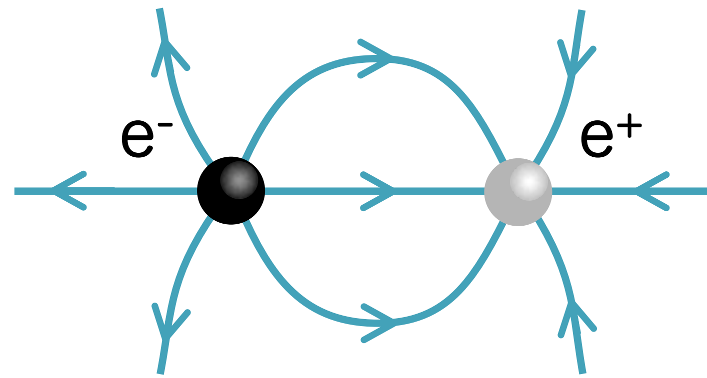
superconductor

Abrikosov vortex



QED

Three-dimensional electric line of force



<u>difference</u>

- electric or magnetic
- SU(3) non-Abelian gauge theory or U(1) Abelian gauge theory

In maximally Abelian (MA) gauge, QCD is reduced into an Abelian gauge theory with magnetic monopoles.

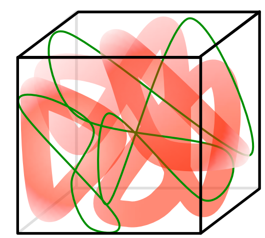
QCD

$U_\mu(s) \in SU(3)$ —MA gauge fixing→ $U_\mu^{MA}(s) \in SU(3)$

↓ MA projection

Abelian part (QED with monopole)

$u_\mu(s) \in U(1)^2$ with electric and magnetic-monopole current

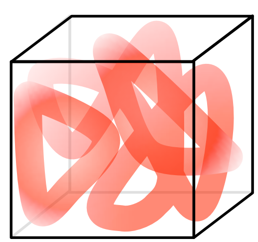


↓ Hodge decomposition

Photon part

$u_\mu^{Ph}(s) \in U(1)^2$

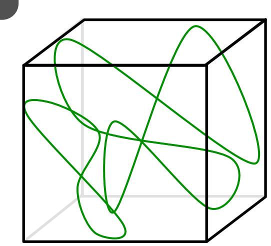
only electric current



Monopole part

$u_\mu^{Mo}(s) \in U(1)^2$

only magnetic-monopole current



Previous studies

◆ **Abelian projection**

t'Hooft (1981); Ezawa & Iwazaki (1983)

◆ **Abelian projection and monopole condensation**

Kronfeld, Scherholz, & Wiese (1987)

◆ **Abelian dominance of confinement**

Suzuki & Yotsuyanagi (1990)

◆ **monopole dominance of confinement**

Stack, Neiman, & Wensley (1994)

Previous lattice studies were performed mainly for simplified SU(2)QCD.

→ In this study, we investigate SU(3) quenched QCD.

How to extract U(1)² gauge theory from SU(3) QCD

$U(1)^2 \subset SU(3)$

$$U_\mu(s) = \exp(ia\sum_{\alpha=1}^{8} \tilde{A}_\mu^\alpha(s) T_\alpha) = \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ U_{21} & U_{22} & U_{23} \\ U_{31} & U_{32} & U_{33} \end{bmatrix}$$

maximally Abelian (MA) gauge fixing

maximaize

$$\sum_{\mu,s}\sum_{n=1}^{3}\left|\left[U_\mu(s)\right]_{nn}\right|^2$$

$$U_\mu^{\mathrm{MA}}(s) = \begin{bmatrix} U_{11}^{\mathrm{MA}} & U_{12}^{\mathrm{MA}} & U_{13}^{\mathrm{MA}} \\ U_{21}^{\mathrm{MA}} & U_{22}^{\mathrm{MA}} & U_{23}^{\mathrm{MA}} \\ U_{31}^{\mathrm{MA}} & U_{32}^{\mathrm{MA}} & U_{33}^{\mathrm{MA}} \end{bmatrix}$$

MA projection

maximaize

$$\mathrm{tr}\left[u_\mu^+(s) U_\mu^{\mathrm{MA}}(s)\right]$$

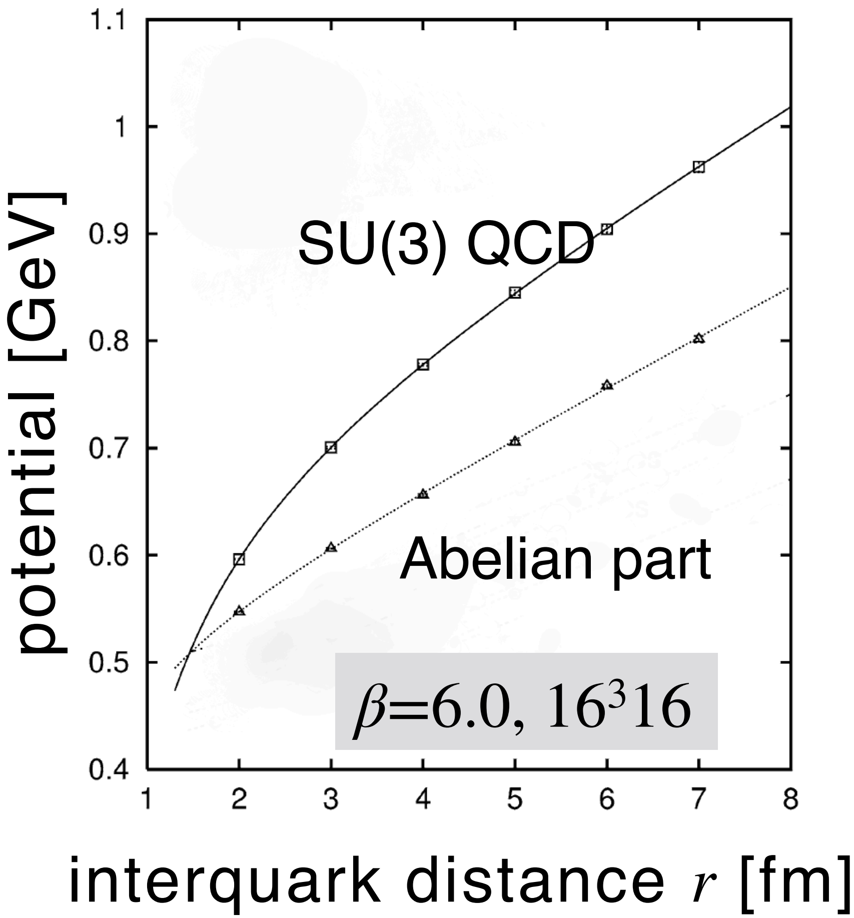
$$u_\mu(s) = \begin{bmatrix} \exp(iaA_\mu^1(s)) & 0 & 0 \\ 0 & \exp(iaA_\mu^2(s)) & 0 \\ 0 & 0 & \exp(iaA_\mu^3(s)) \end{bmatrix}$$

$$a\left(A_\mu^1(s) + A_\mu^2(s) + A_\mu^3(s)\right) \equiv 0 \quad (\mathrm{mod}\ 2\pi)$$

Pioneering work of Abelian dominance of confinement in SU(3) QCD

◆ Stack *et al.*, Nucl. Phys. B (2002).

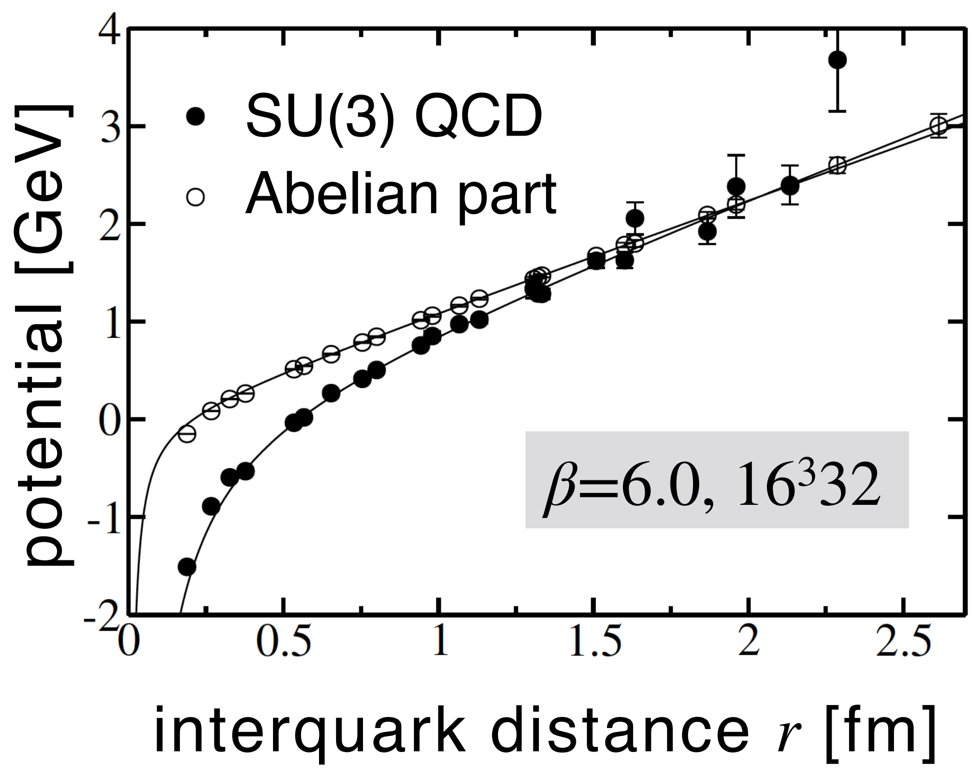
$$\sigma_{\text{Abel}} / \sigma = 0.90(4)$$



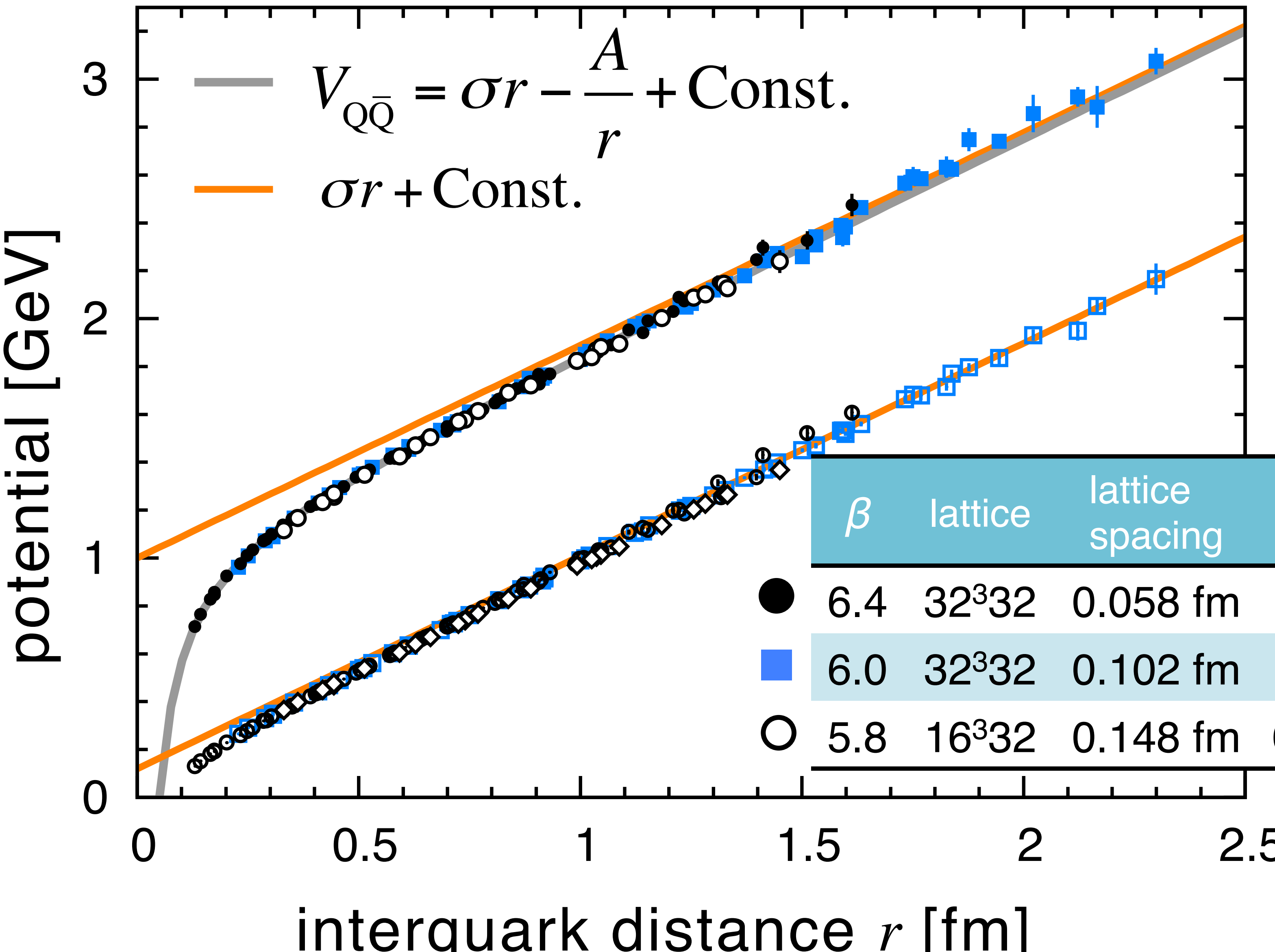
◆ Bornyakov *et al.*, PRD (2004).

$$\sigma_{\text{Abel}} / \sigma = 0.83(3)$$

with simulated annealing algorithm

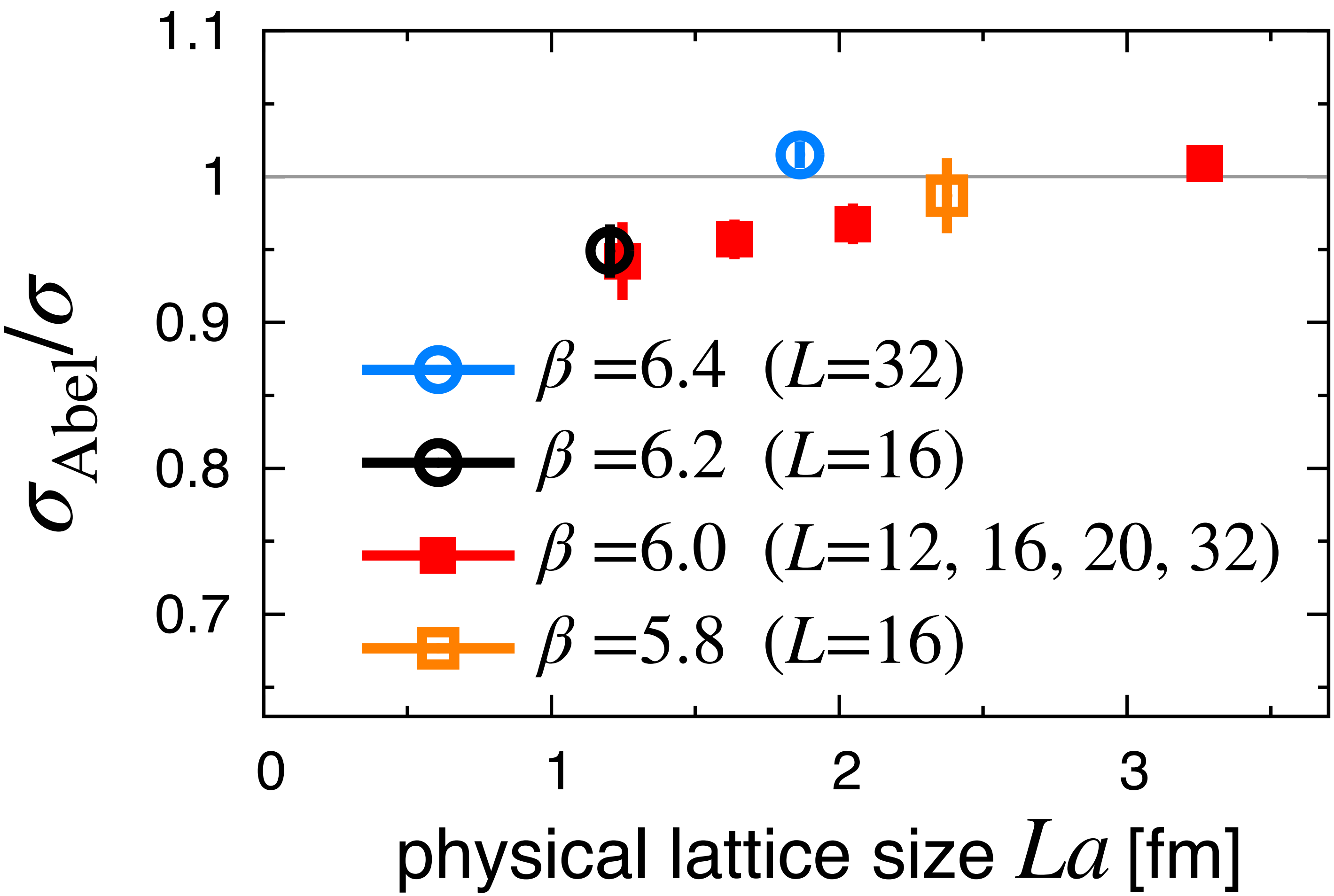


Main result: perfect Abelian dominance of confinement

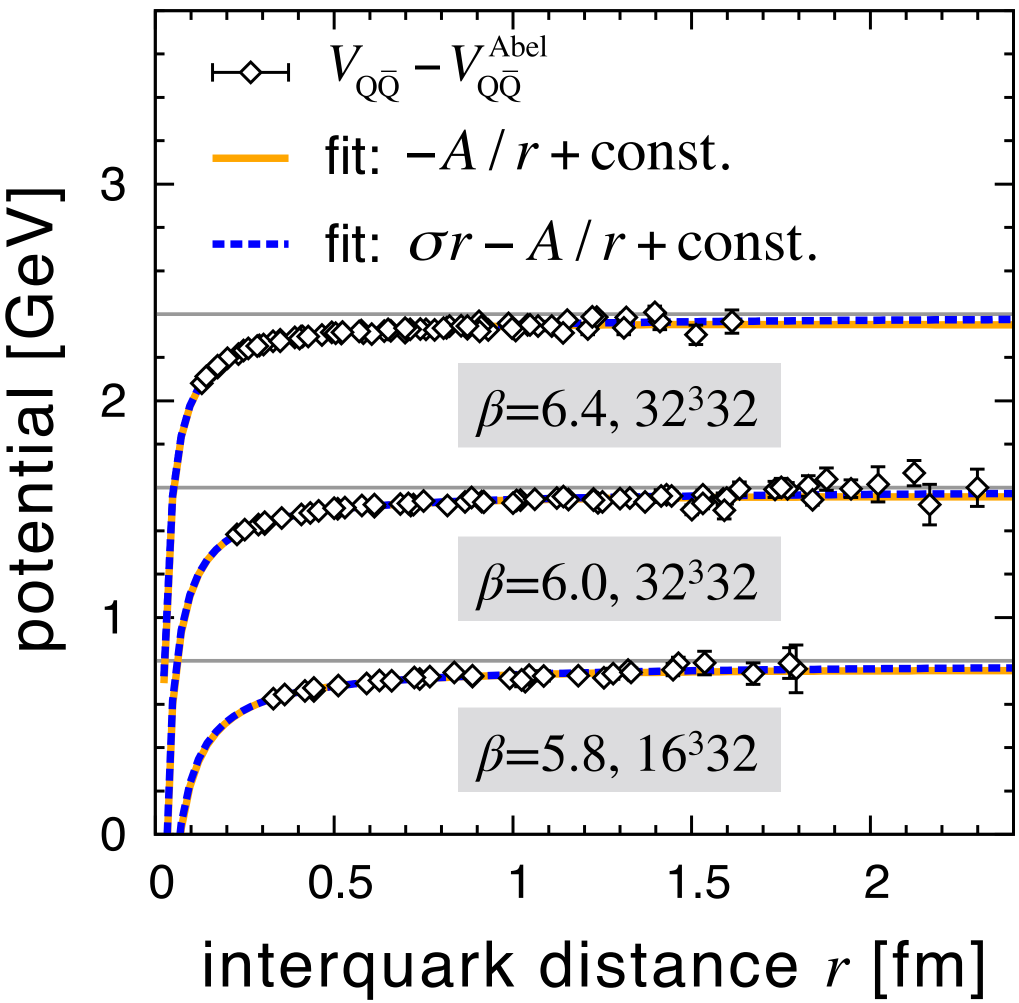


	β	lattice	lattice spacing	$\sigma_{\text{Abel}}/\sigma$
●	6.4	$32^3 32$	0.058 fm	1.015(09)
■	6.0	$32^3 32$	0.102 fm	1.009(10)
○	5.8	$16^3 32$	0.148 fm	0.99(3)

When physical spatial size *La* is larger than 2 fm,
Perfect Abelian dominance $\sigma_{\rm Abel}/\sigma \simeq 1$ is realized.
~ One of key quantities is the physical spatial volume.



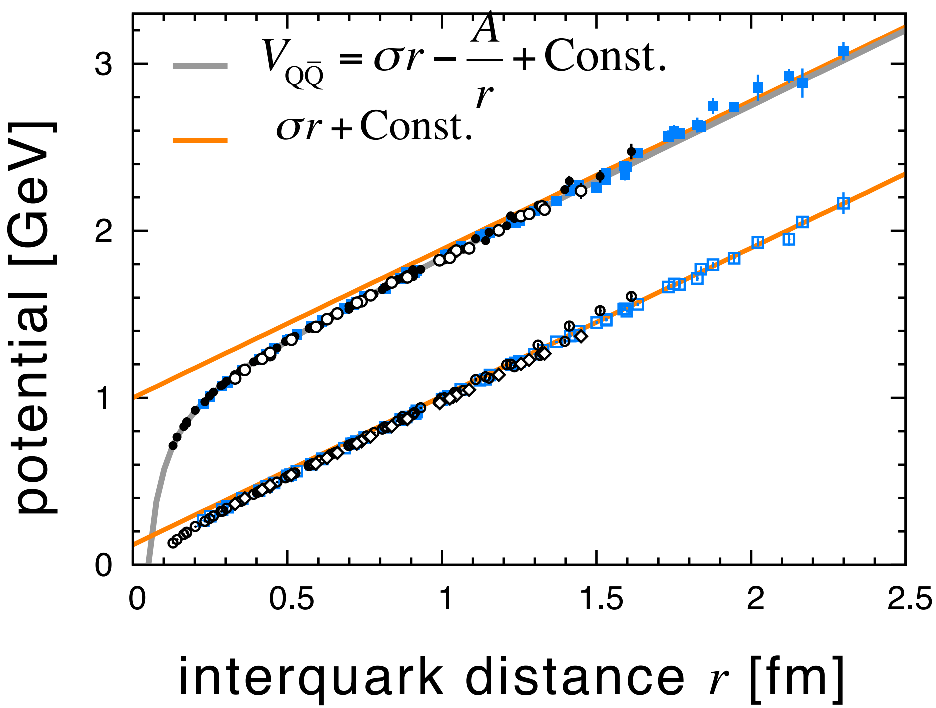
Difference between SU(3) potential V and Abelian part V^{Abel}



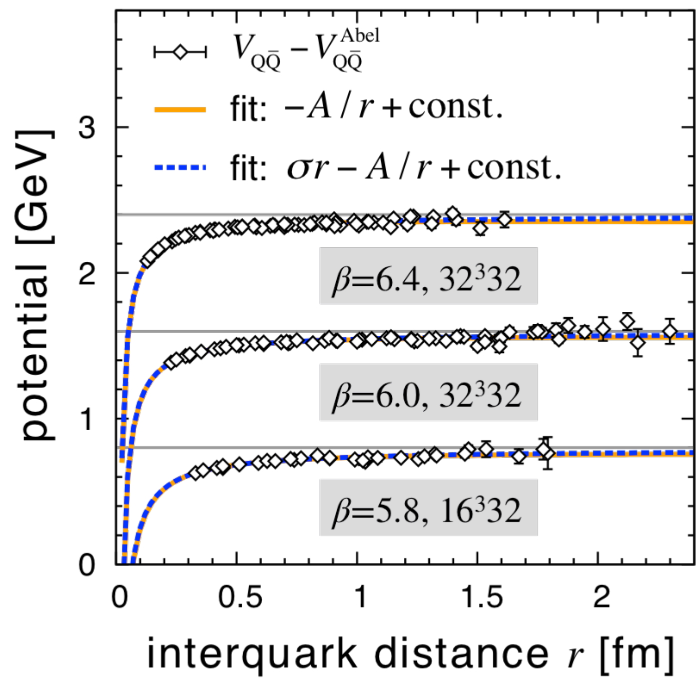
- No string tension in the difference $V(r) - V^{Abel}(r)$.
- The difference $V(r) - V^{Abel}(r)$ can be well fitted by pure Coulomb potential.

⇒ This also suggests perfect Abelian dominance for confinement.

Summary of 1st part: We study MA projection of Q-Qbar potential in SU(3) quenched lattice QCD with large physical-volume lattices, and find *almost perfect Abelian dominance* of quark confinement.



SU(3) potential and the Abelian part in MA gauge in lattice QCD at b=6.0~6.4 and 32^4. They have almost the same slope at large distance.



Difference between SU(3) potential and the Abelian part is almost pure Coulomb form, which suggests perfect Abelian dominance of confinement.

[Ref] NS and HS, Phys. Rev. D **90**, 111501(R) (2014).

contents

Part 1: perfect Abelian dominance of confinement

in mesons [1]

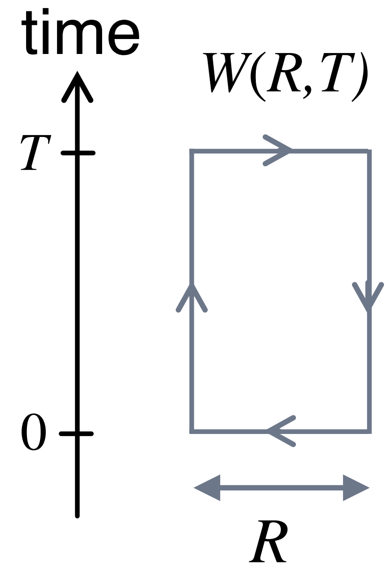
Part 2: perfect Abelian dominance of confinement

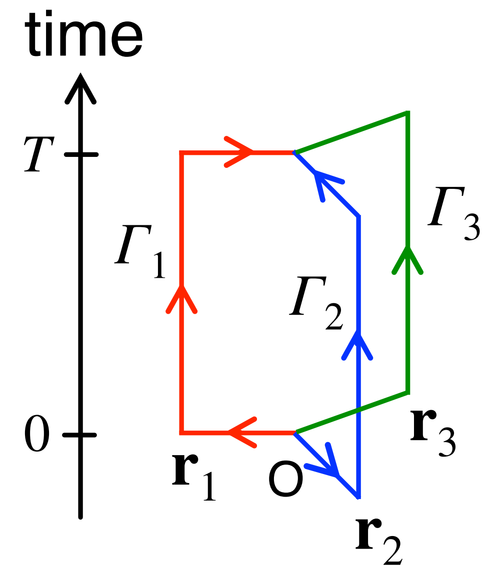
in baryons [2]

[1] NS and HS, Phys. Rev. D **90**, 111501(R) (29 Dec 2014).

[2] NS and HS, Phys. Rev. D **92**, 034511 (21 Aug 2015).

Similar to the case of QQbar potential, the color-singlet baryonic 3Q potential can be calculated from 3Q Wilson loop





$$W(R,T) \cong C \exp\left[-TV_{Q\bar{Q}}(R)\right] \quad (T \to \infty)$$

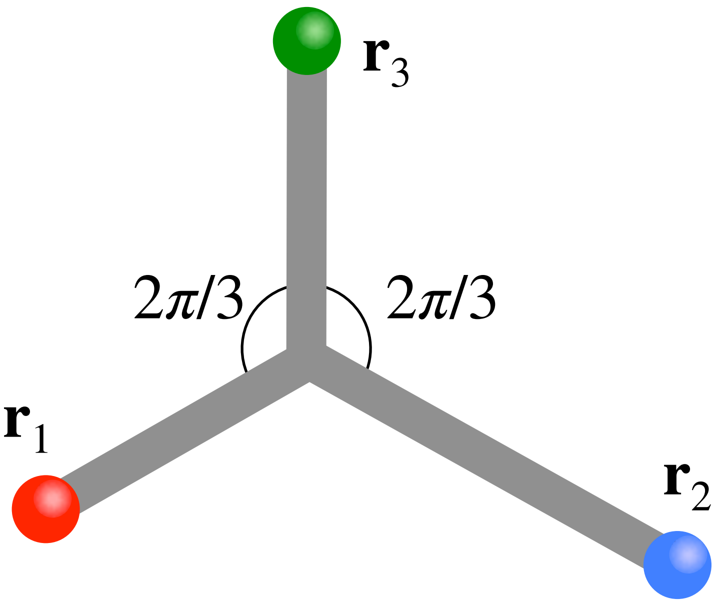
$$W_{3Q}(\vec{r}_1,\vec{r}_2,\vec{r}_3,T) \cong C_{3Q} \exp\left[-TV_{3Q}(\vec{r}_1,\vec{r}_2,\vec{r}_3)\right] \quad (T \to \infty)$$

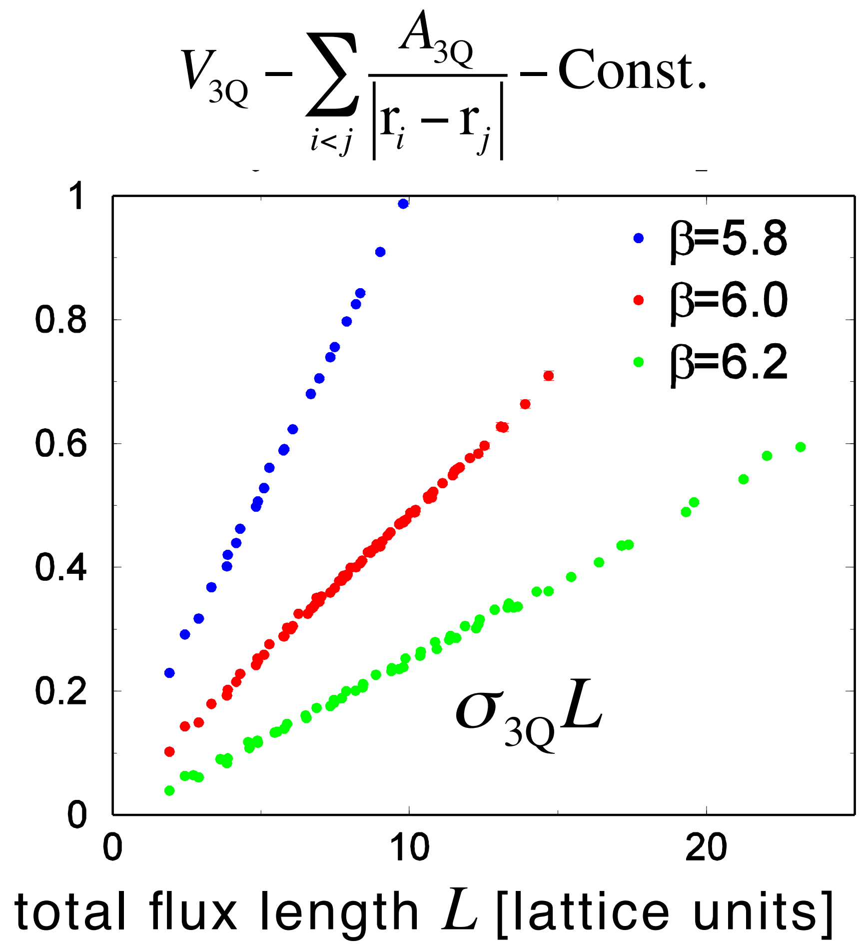
The 3Q Wilson loop physically means that gauge-invariant 3Q state is created at t=0 and is annihilated at t=T with three quarks spatially fixed for 0<t<T.

The 3Q potential is well described by Y-Ansatz, i.e., sum of Y-type linear potential and one-gluon-exchange Coulomb.

$$V_{3Q} = \sigma_{3Q} L - \sum_{i<j} \frac{A_{3Q}}{|\mathbf{r}_i - \mathbf{r}_j|} + \text{Const.}$$

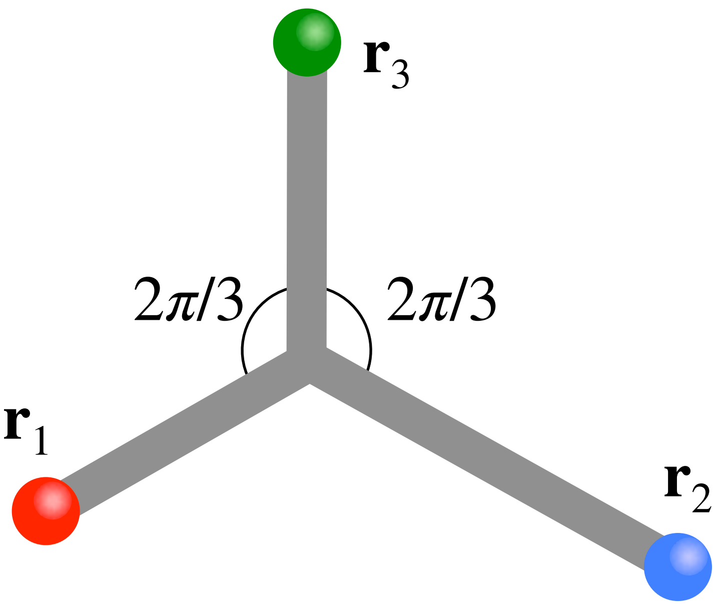
L : total length of string linking three valence quarks

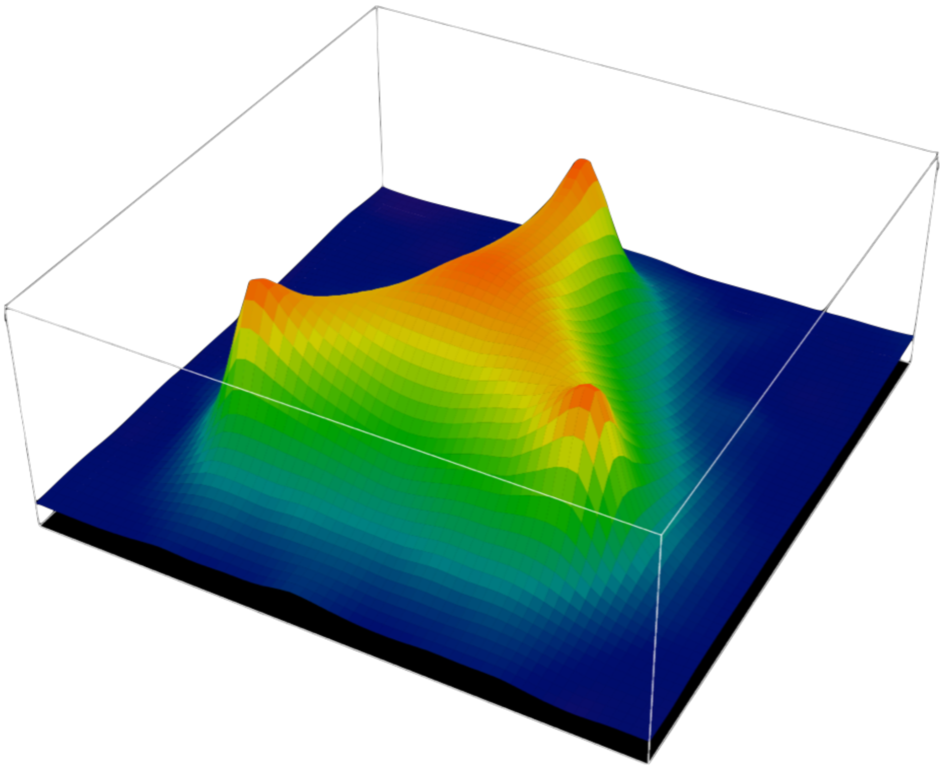




T.T.Takahashi, et al. PRL (2001); PRD (2002).
F.Okiharu, et al., PRD (2005).

For 3Q systems, Y-shaped flux-tube formation has been observed in lattice QCD, and such one-dimensional squeezing of color flux leads to Y-type linear confinement potential.





H. Ichie et al., Nucl. Phys. A721, 899 (2003).
V.G. Bornyakov et al., PRD70, 054506 (2004).
F. Bissey et al., Phys.Rev.D76,114512 (2007).

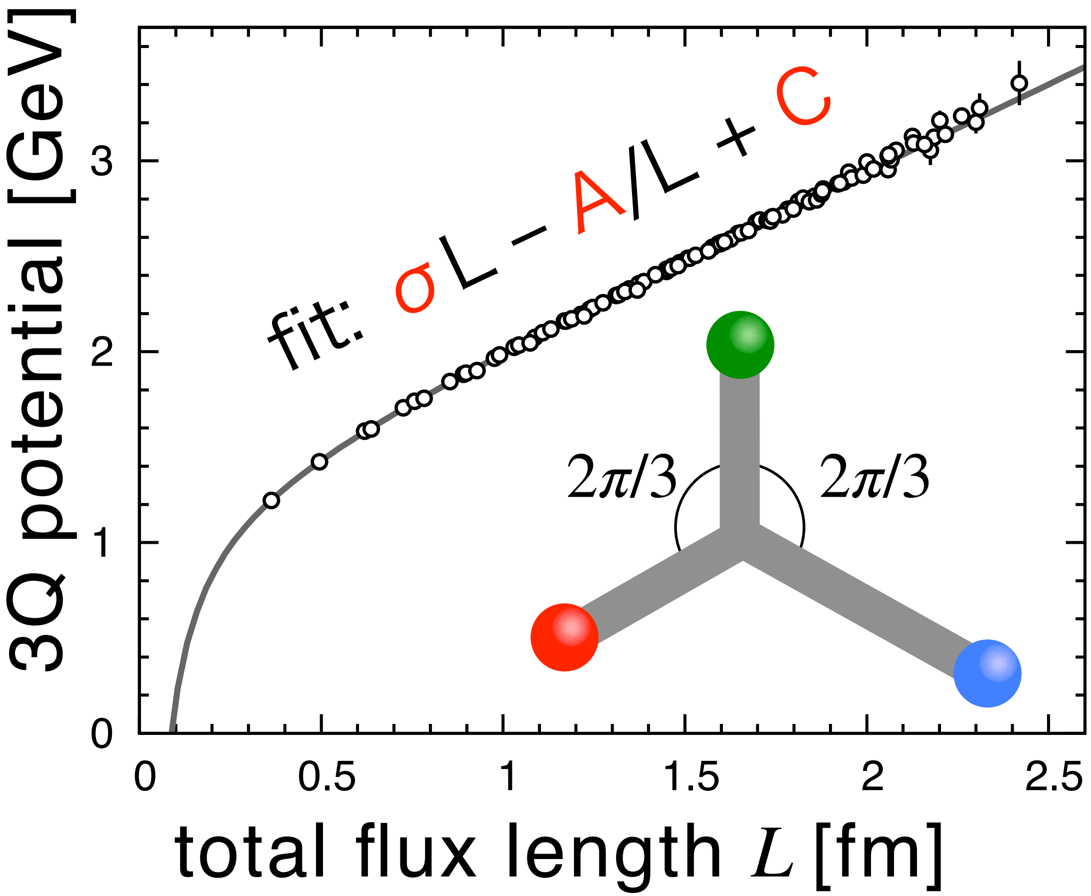
Our calculation: Numerical condition for 3Q potential

lattice parameter β	lattice spacing a [fm]	number of gauge configurations	lattice $L^3 L_t$	physical lattice size La [fm]
5.8	0.148(2)	2000	$16^3 32$	2.37(3)
6.0	0.102(1)	1000	$20^3 32$	2.05(1)

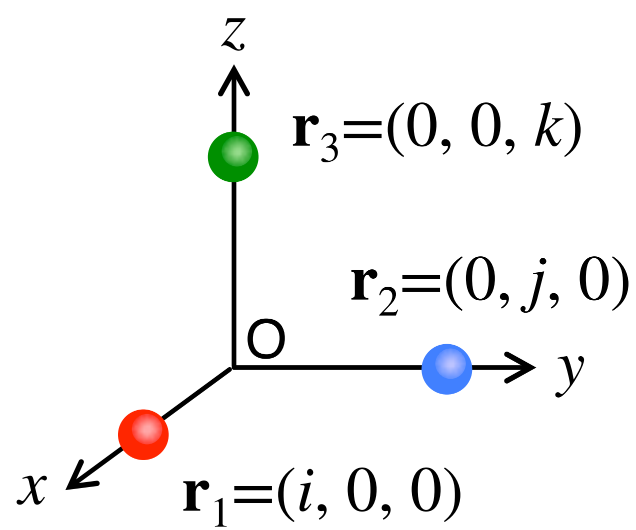
- SU(3) standard plaquette action at quenched level
- over-relaxation method for MA gauge fixing
- smearing method for accurate measurement

The 3Q potential is well described by Y-Ansatz, i.e., sum of Y-type linear potential and one-gluon-exchange Coulomb.

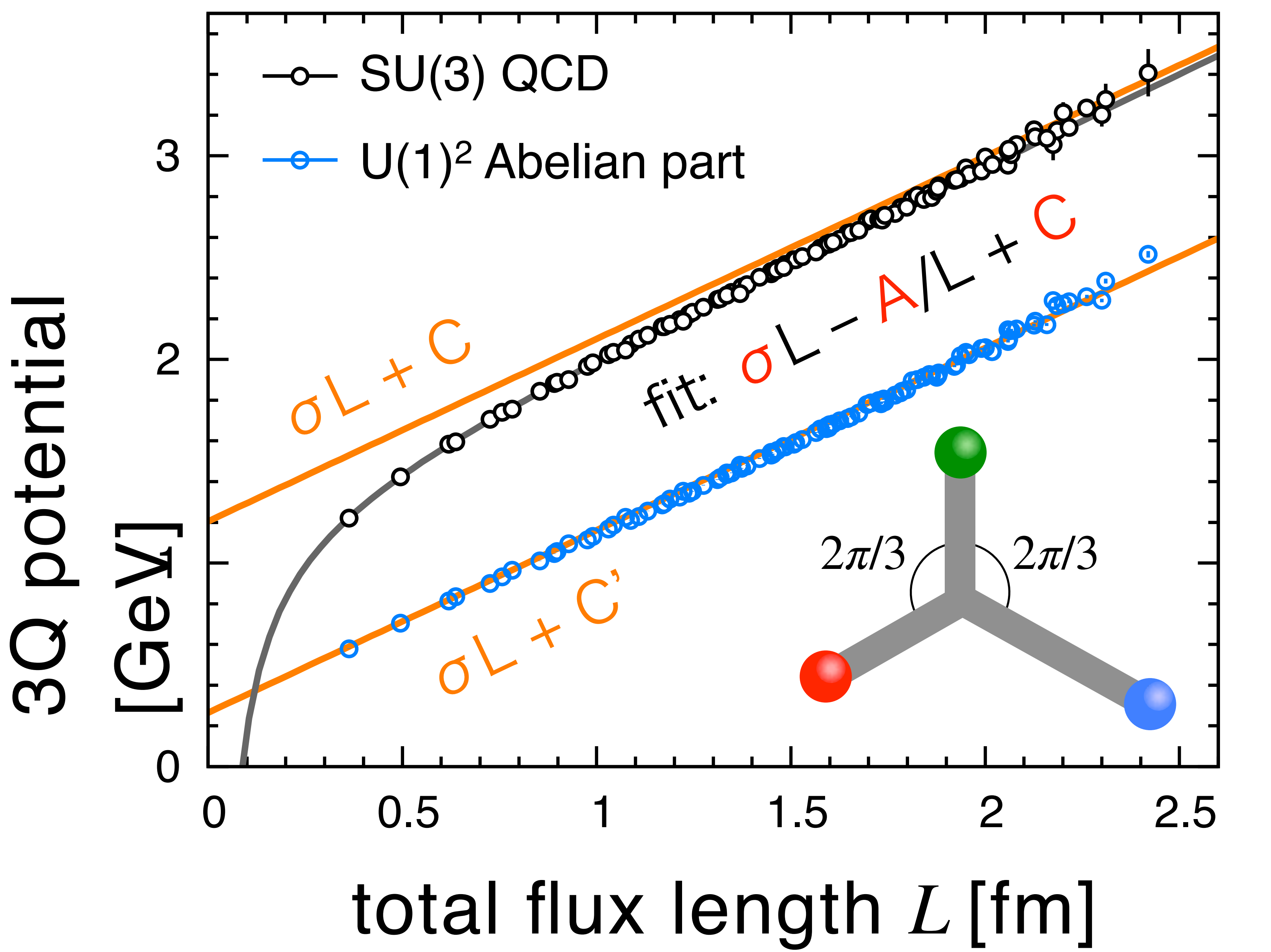
$$V_{3Q} = \sigma_{3Q} L - \sum_{i<j} \frac{A_{3Q}}{|r_i - r_j|} + \text{Const.}$$



We have calculated 101 patterns of 3Q system at β=5.8 and 211 patterns of 3Q system at β=6.0.

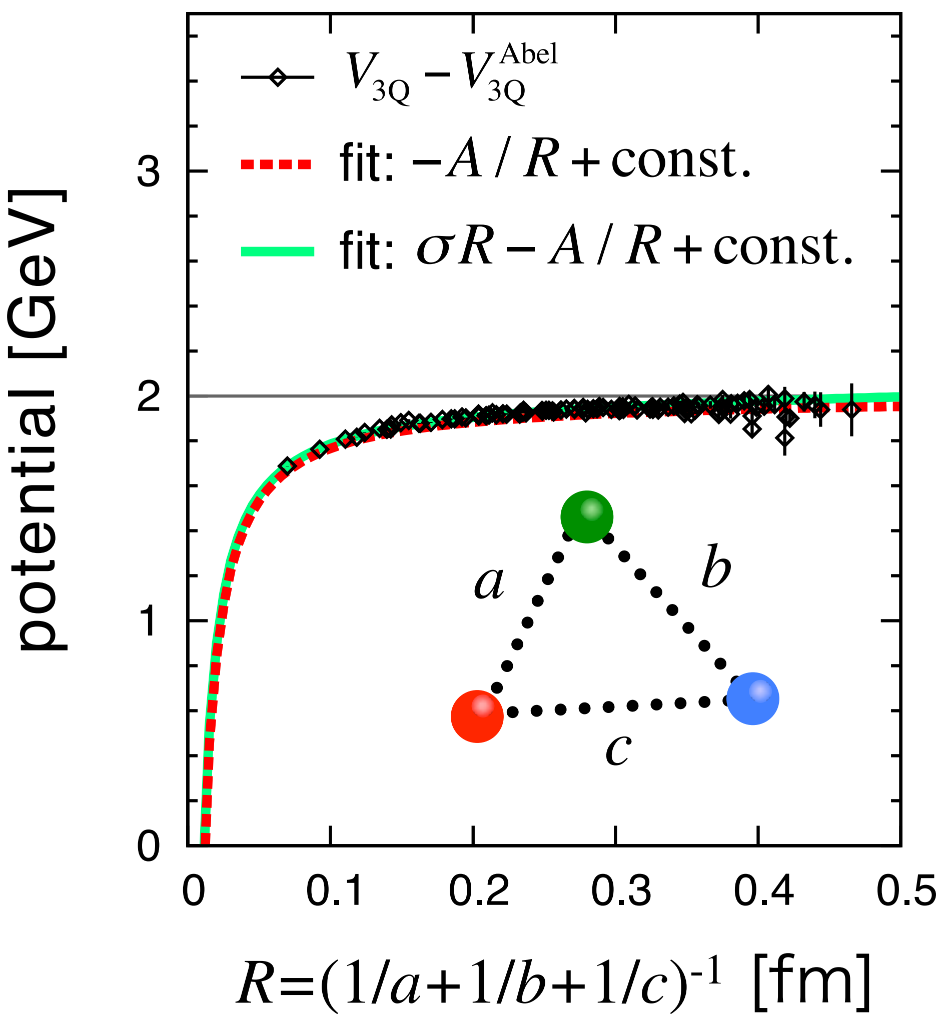


Perfect Abelian dominance of confinement in 3Q potential



Difference between SU(3) potential V_{3Q} and Abelian part V_{3Q}^{Abel}

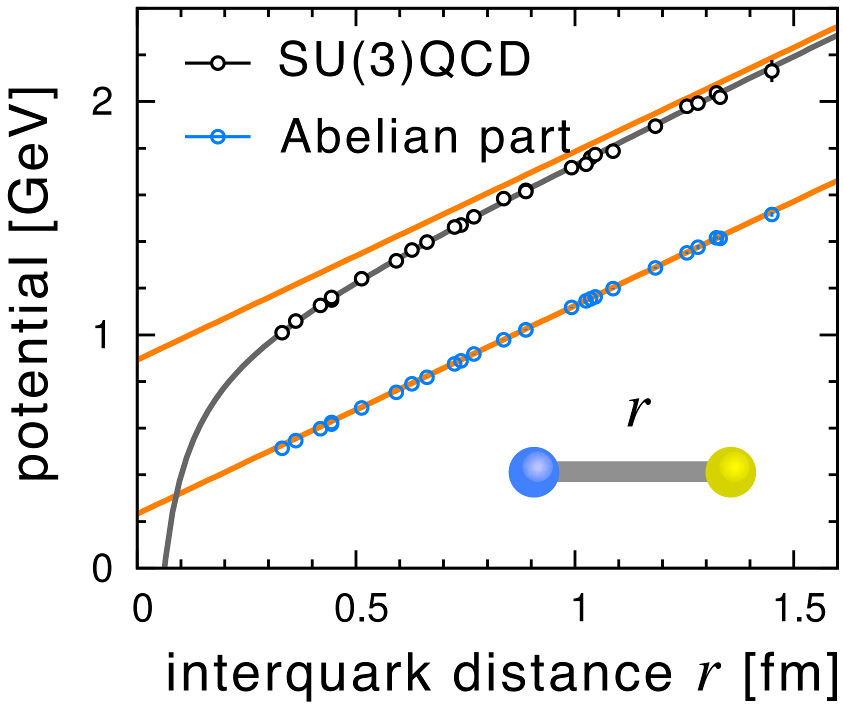
$$V_{3Q} = \sigma_{3Q} L - \sum_{i<j} \frac{A_{3Q}}{|r_i - r_j|} + \text{Const.}$$

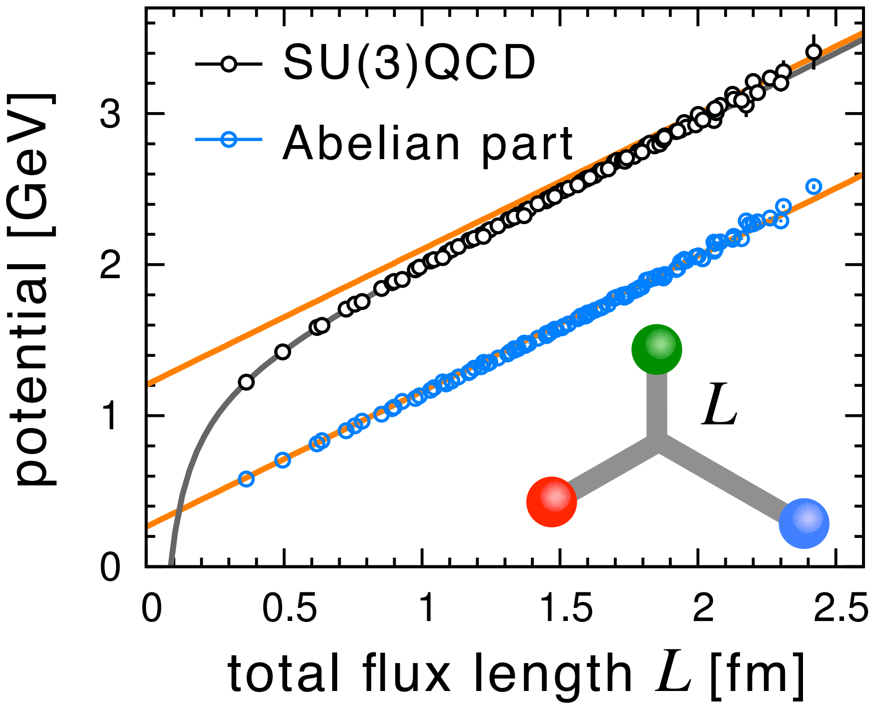


- No string tension in the difference $V_{3Q}(R) - V_{3Q}^{Abel}(R)$.
- The difference $V_{3Q}(R) - V_{3Q}^{Abel}(R)$ can be well fitted by 2-body pure Coulomb potential.

⇒ This also suggests perfect Abelian dominance for 3Q confinement.

[Sammary] perfect Abelian dominance of confinement is found in mesons and baryons





Mesons ($Q\bar{Q}$): Phys. Rev. D **90**, 111501(R) (29 Dec 2014).

Baryons (3Q): Phys. Rev. D **92**, 034511 (21 Aug 2015).

Published last week

Our study

Numerical condition for QQbar potential calculation

- SU(3) standard plaquette action at quenched level
- various lattice parameter: β = 5.8~6.4,
  corresponding to lattice spacing: a = 0.058~0.148fm
- various lattice size: La = 2~3fm for main calculations
- large number (200~600) of gauge configurations
- over-relaxation method for MA gauge fixing
- smearing method for accurate measurement

β	L^3L_t	$N_{\rm con}$	a [fm]	La [fm]	$\sigma_{\rm Abel}/\sigma$
6.4	32^4	200	0.0582(2)	1.86(1)	1.015(09)
6.0	32^4	200	0.1022(5)	3.27(1)	1.009(10)
5.8	$16^3 32$	600	0.148(1)	2.37(2)	1.00(2)
6.0	$16^3 32$	600	0.102(1)	1.64(1)	0.94(1)
6.0	$12^3 32$	400	0.104(1)	1.25(4)	0.94(3)
6.2	$16^3 32$	400	0.075(1)	1.20(1)	0.95(2)

MA gauge fixing and Gribov copy effect

In MA gauge, we maximize

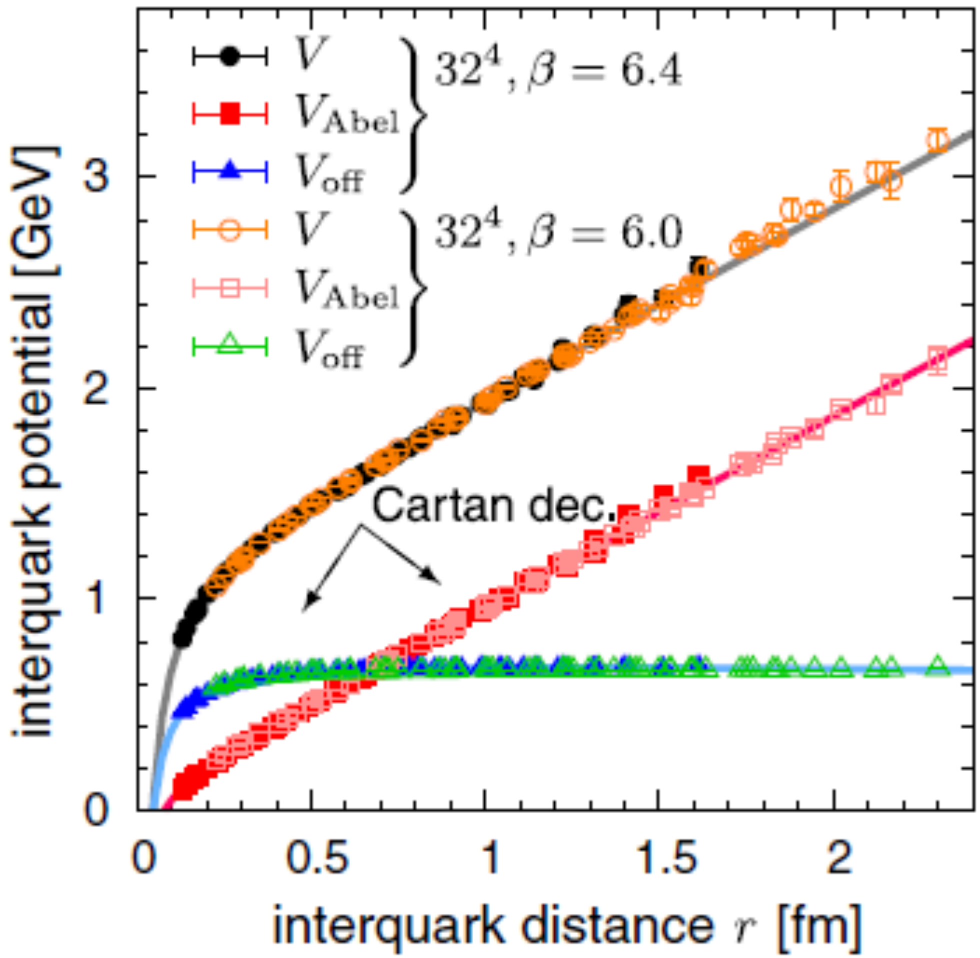
$$R\left[U_\mu(s)\right] \equiv \mathrm{Re}\sum_{s,\mu}\mathrm{Tr}\left(U_\mu^\dagger(s)\,\vec{H}\,U_\mu(s)\,\vec{H}\,\right)$$

In our over-relaxation method, the maximized value of *R* is almost the same over 200~600 gauge configurations.

Actually, the converged values of *R* are 0.7072(6) with $16^3 32$ at β=5.8; 0.7321(11), 0.7322(7), and 0.7318(3) with $12^3 32$, $16^3 32$, and 32^4 at β=6.0; 0.7510(7) with $16^3 32$ at β=6.2; and 0.7656(3) with 32^4 at β=6.4.
Here, the values in parentheses denote the standard deviation.

In fact, our procedure seems to escape bad local minima, where *R* is relatively small. Then, we expect that the Gribov copy effect is not so significant in our calculation.

Abelian Dominance for Confinement in QQbar Potential



SU(3) potential V(r)

Abelian part $V_{Abel}(r)$

SU(3) potential and Abelian-projected potential in MA gauge in lattice QCD with β=6.0~6.4 and 32^4.

summation formula