This research has been co-financed by the European Union (European Social Fund, ESF) and Greek national funds through the Operational Program "Education and Lifelong Learning" of the National Strategic Reference Framework (NSRF), under the grants schemes "Funding of proposals that have received a positive evaluation in the 3rd and 4th Call of ERC Grant Schemes" and the program "Thales"

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27 August 2015

Thermalization in an confining gauge theory at Strong Coupling

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Bibliography

Work with

Takaaki Ishii (U. of Crete), Christopher Rosen, (U. of Crete)

Published:

T. Ishii (Crete), E. Kiritsis (APC+Crete), C. Rosen (Crete)
arXiv: 1503.07766[hep-th]

and previous work with the Brussels group

B. Craps (Vrije U., Brussels), E. Kiritsis (APC+Crete), C. Rosen (Crete),
A. Taliotis, J. Vanhoof, H. Zhang (Vrije U., Brussels)
arXiv: 1311.7560[hep-th]

Introduction

- The process of thermalization in QFT is poorly understood even today.

- It has been brought forward recently with the heavy ion collisions at RHIC and CERN.

- The data indicate rapid thermalization of the initial energy density and the formation of a quark gluon plasma.

- The thermalization time is an order of magnitude smaller than what was expected at RHIC and seems even smaller at LHC.

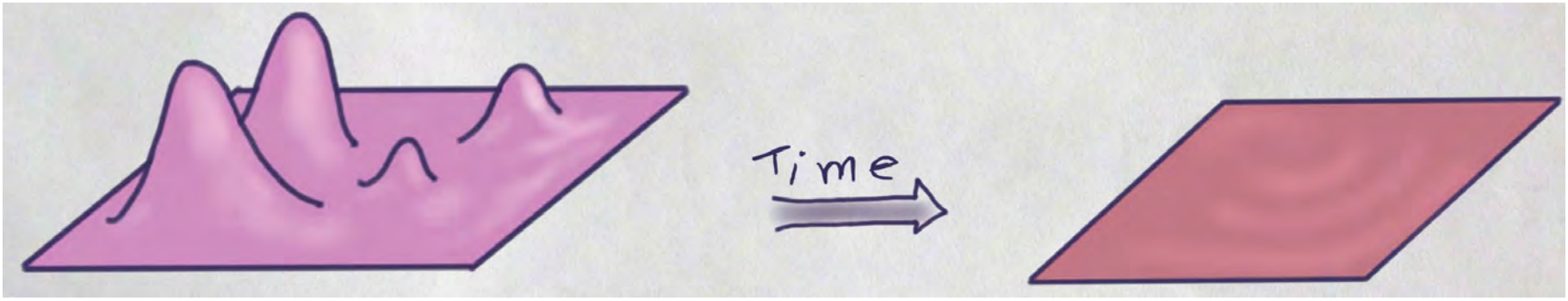
- The theory (QCD) is in a strongly coupled regime for most of the energy range of the experiments.

- The challenge is to understand thermalization in this context and more generally.

The setup for thermalization

• We consider the theory in its vacuum state and then perturb it by a time dependent coupling constant (this is a simplification)

$$L_{QFT} + f_0(t) \int d^4x \; O(x) \qquad \rightarrow \qquad \nabla^t \langle T_{tt} \rangle = \dot{f}_0 \; \langle O \rangle$$



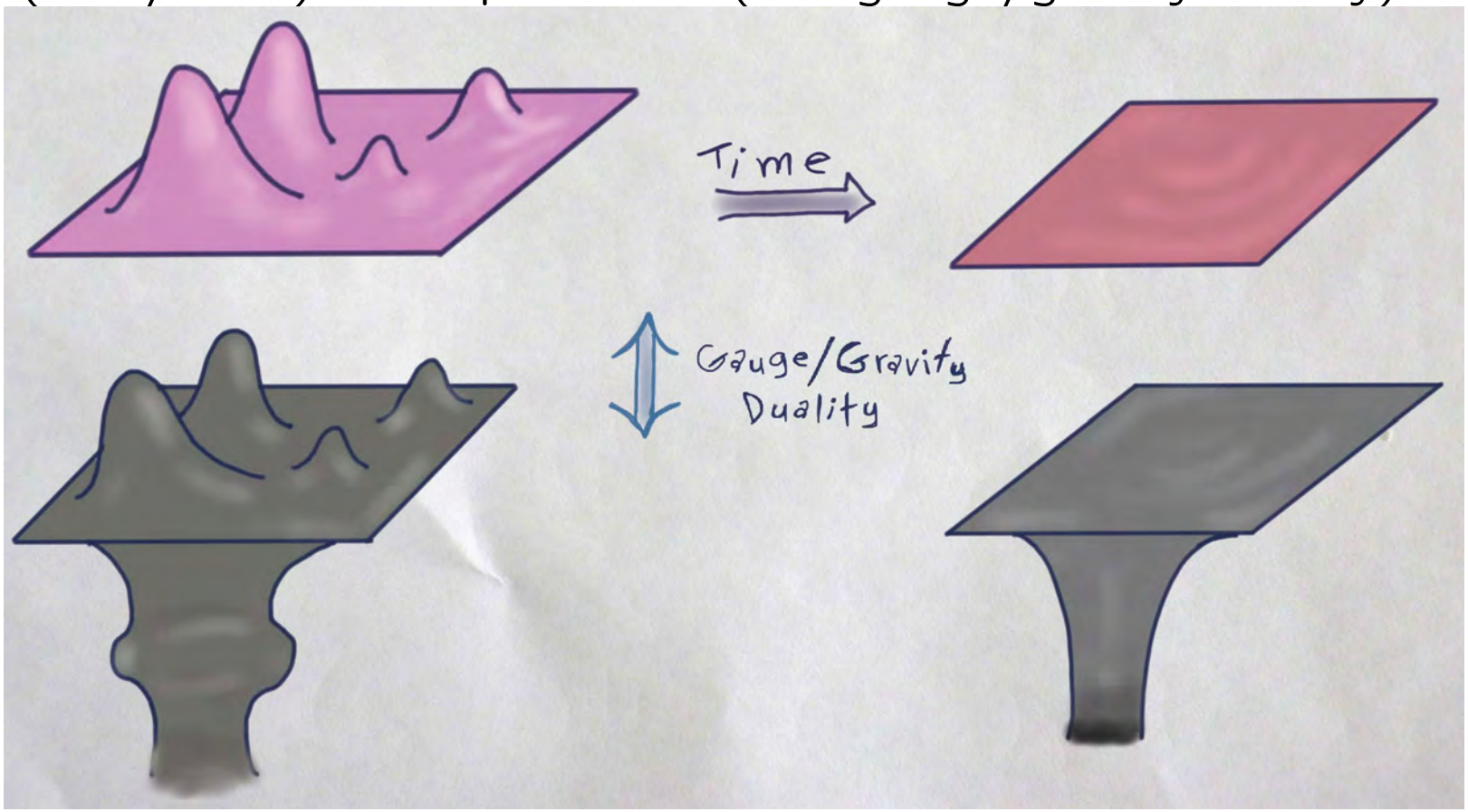
• The approach to equilibration is controlled by the expectation values $\langle T_{tt} \rangle (t)$, $\langle O \rangle (t)$.

• We expect that if the system thermalizes then

$$\langle O \rangle (t \rightarrow \infty) \qquad \rightarrow \qquad Tr[\rho_{\text{thermal}} \; O]$$

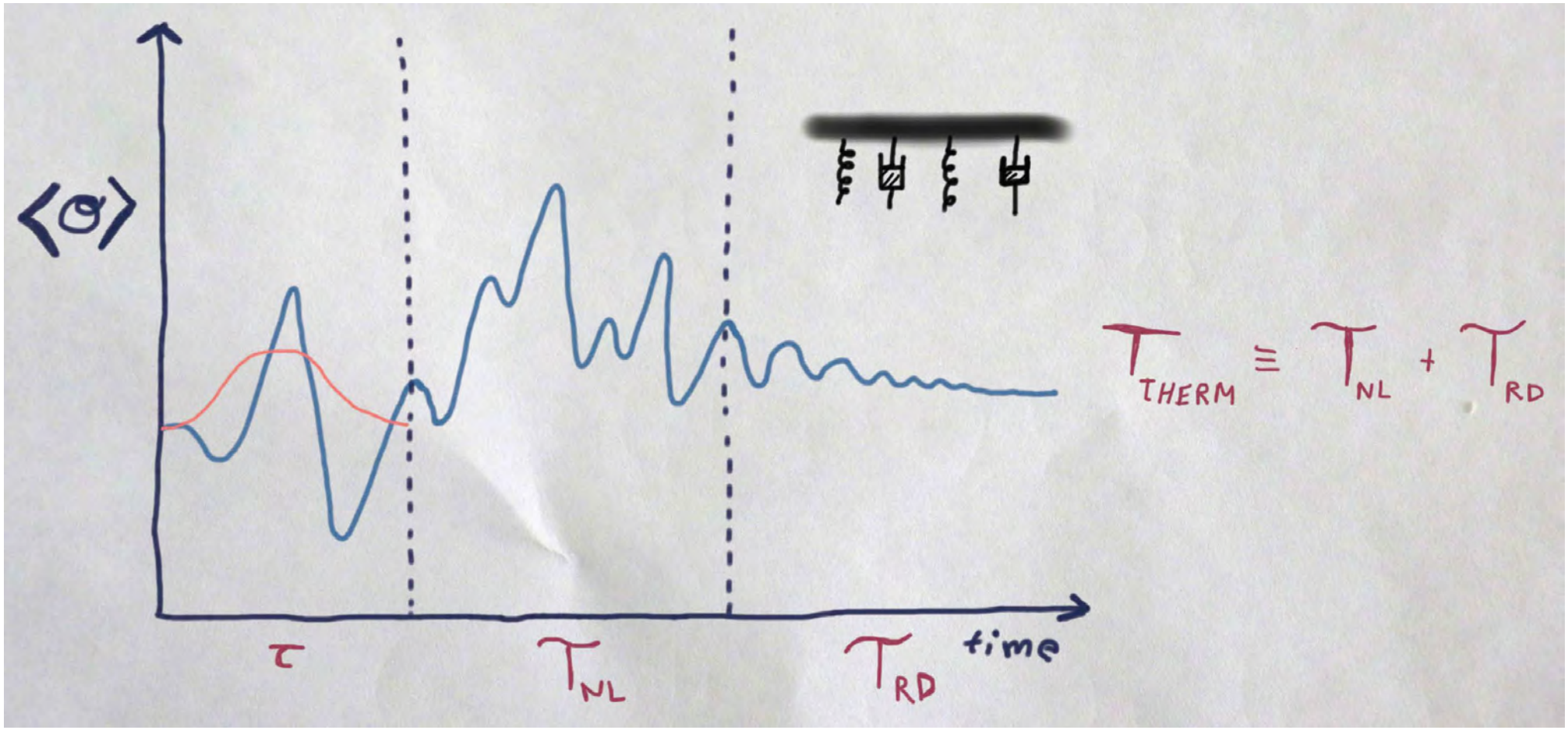
Thermalization at strong coupling

• To calculate the observables at strong coupling we will assume the holographic (AdS/CFT) correspondence (aka gauge/gravity duality).



• Thermalization corresponds to black hole formation in the bulk spacetime.

Gravitational expectations



- There are three possible characteristic times involved.

Thermalization calculations

• There have been studies of this setup in CFTs (AdS space). There is no consensus yet but in most cases there is thermalization.

Chessler+Yaffe, Heller+Janik+Witaszczyk
, Bizon+Rostorowski, Buchel+Liebling+Lehner

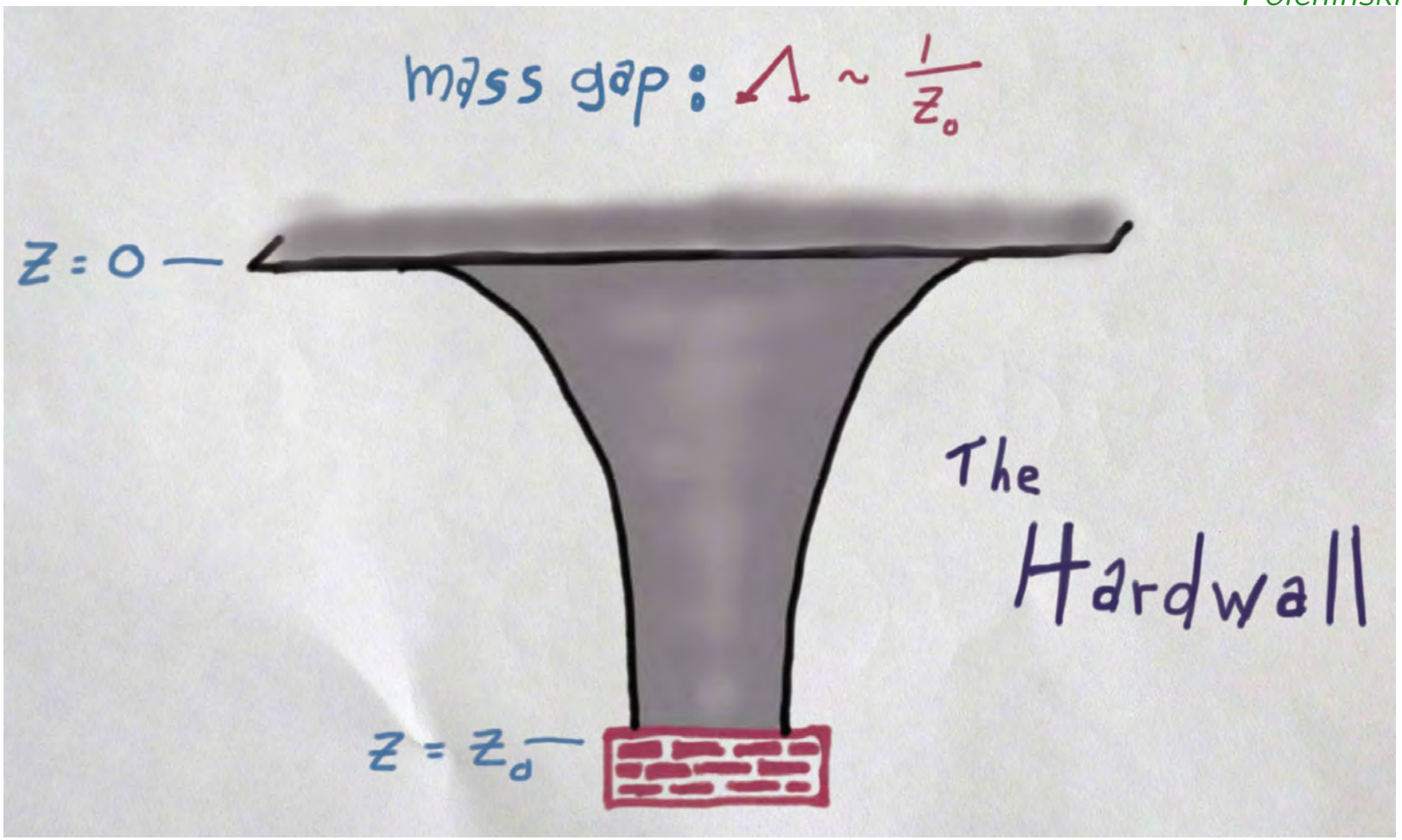
• There are similarities between a conformal (scale-invariant) gauge theory and a confining gauge theory (like QCD) that has a non-trivial scale, Λ_{QCD} but there are also important differences.

• Confinement is tracked by the Wilson loop that has area behavior in the confining phase.

The Hard Wall Model

- There is a very simple holographic model for a confining gauge theory: The hard wall model.

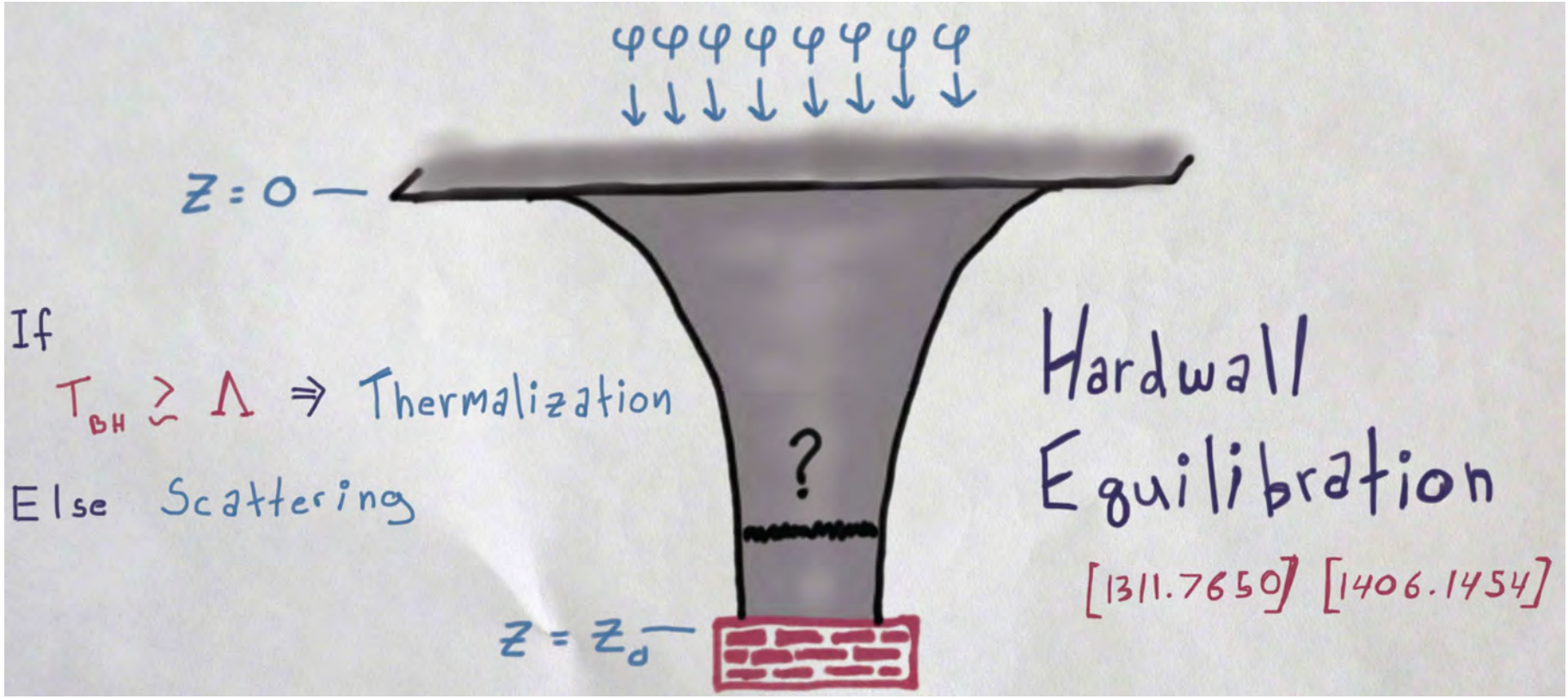
Polchinski+Strassler



- The hard wall induces an IR scale and confinement.

• In this simple model we can obtain analytic solutions for sufficiently small perturbations.

Craps+Kiritsis+Rosen+Taliotis+Vanhoof+Zhang



• Numerical solutions confirm these expectations.

Craps+Lindgren+Taliotis+Vanhoof+Zhang

• But the hard-wall model is too simple for many purposes.

Improved Holographic QCD

- Holographic models were developed that describe with rather good accuracy the strong coupling physics of YM theory.

Gursoy+Kiritsis+Nitti, Gubser+Nellore

- They contain the fields dual to the most important YM operators

$$\phi \qquad \Leftrightarrow \qquad Tr[F^2]$$

$$g_{\mu\nu} \qquad \Leftrightarrow \qquad T_{\mu\nu}$$

- The gravitational action is the Einstein Dilaton action with a potential.

$$S_{IHQCD} = M^3 \int d^5x \sqrt{g} \left[R - \frac{4}{3}(\partial\phi)^2 - V(\phi) \right]$$

- The potential is in one-to-one correspondence with the YM β-function.

- At large values of ϕ, $V \sim \sqrt{\phi}\, e^{\frac{4}{3}\phi}$.

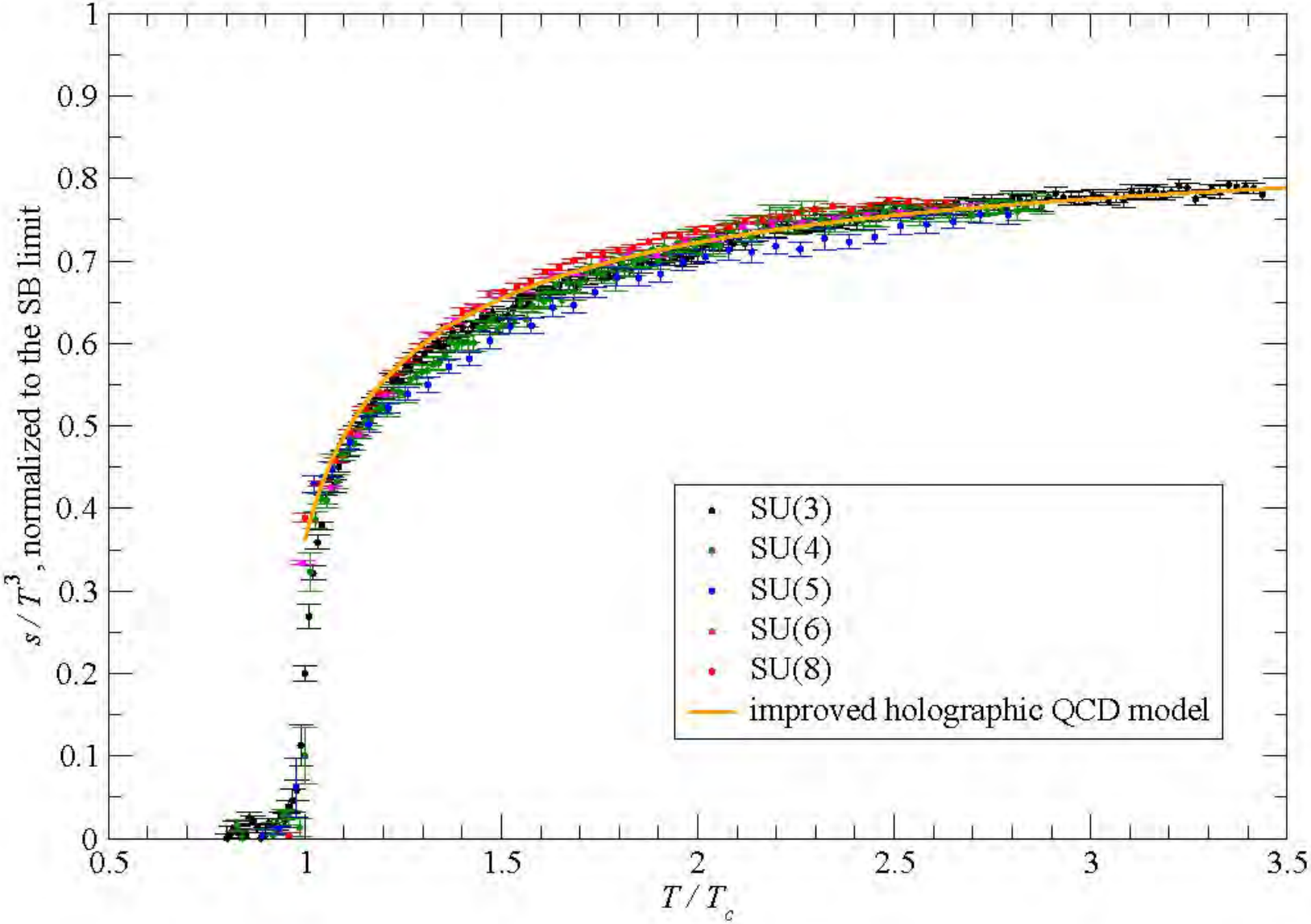


Figure 4: (Color online) Same as in fig. 1, but for the s/T^3 ratio, normalized to the SB limit.

Panero

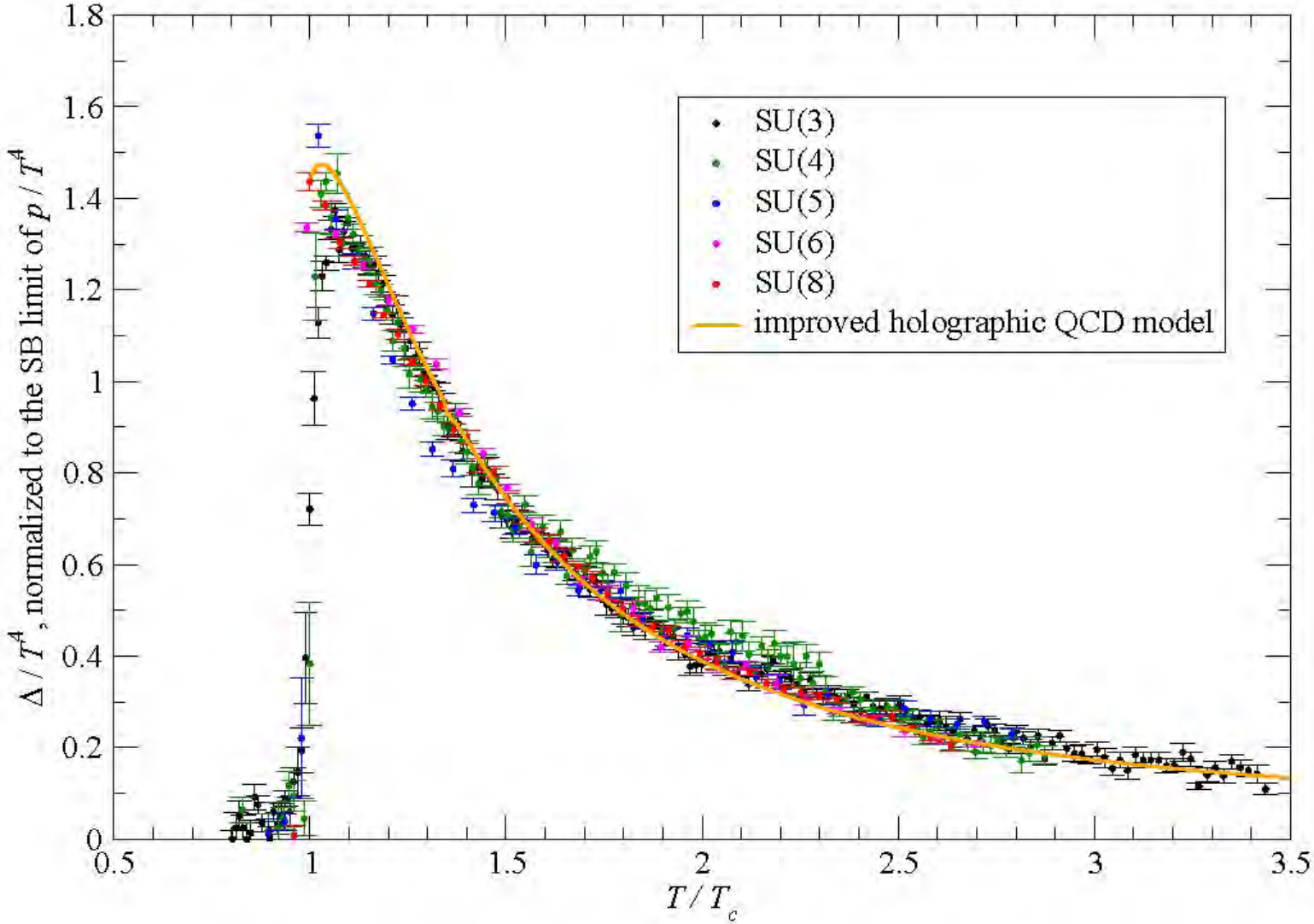
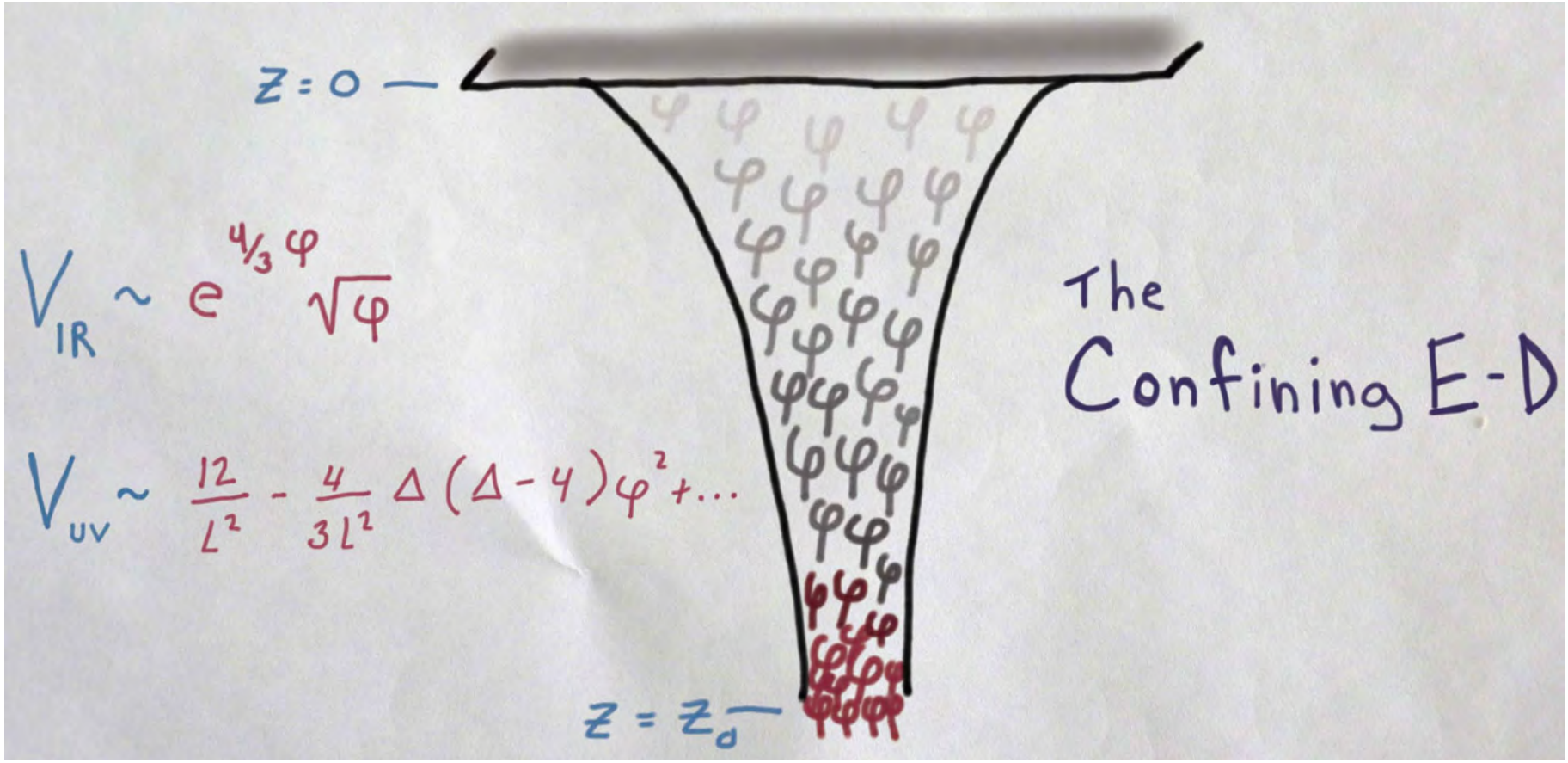


Figure 2: (Color online) Same as in fig. 1, but for the Δ/T^4 ratio, normalized to the SB limit of p/T^4.

Panero

• Therefore the picture of the thermalization process is as follows:



• We will take for computational simplicity the operator dual to ϕ to have UV dimension 3. This is not expected to affect qualitatively the results.

Black holes in Improved Holographic QCD

Gursoy+Kiritsis+Mazzanti+Nitti

• Black holes can be characterized by their Hawking Temperature (two to one map), or the value of the scalar (dilaton) at the horizon $\lambda_H = e^{\phi_H}$ (one to one map).

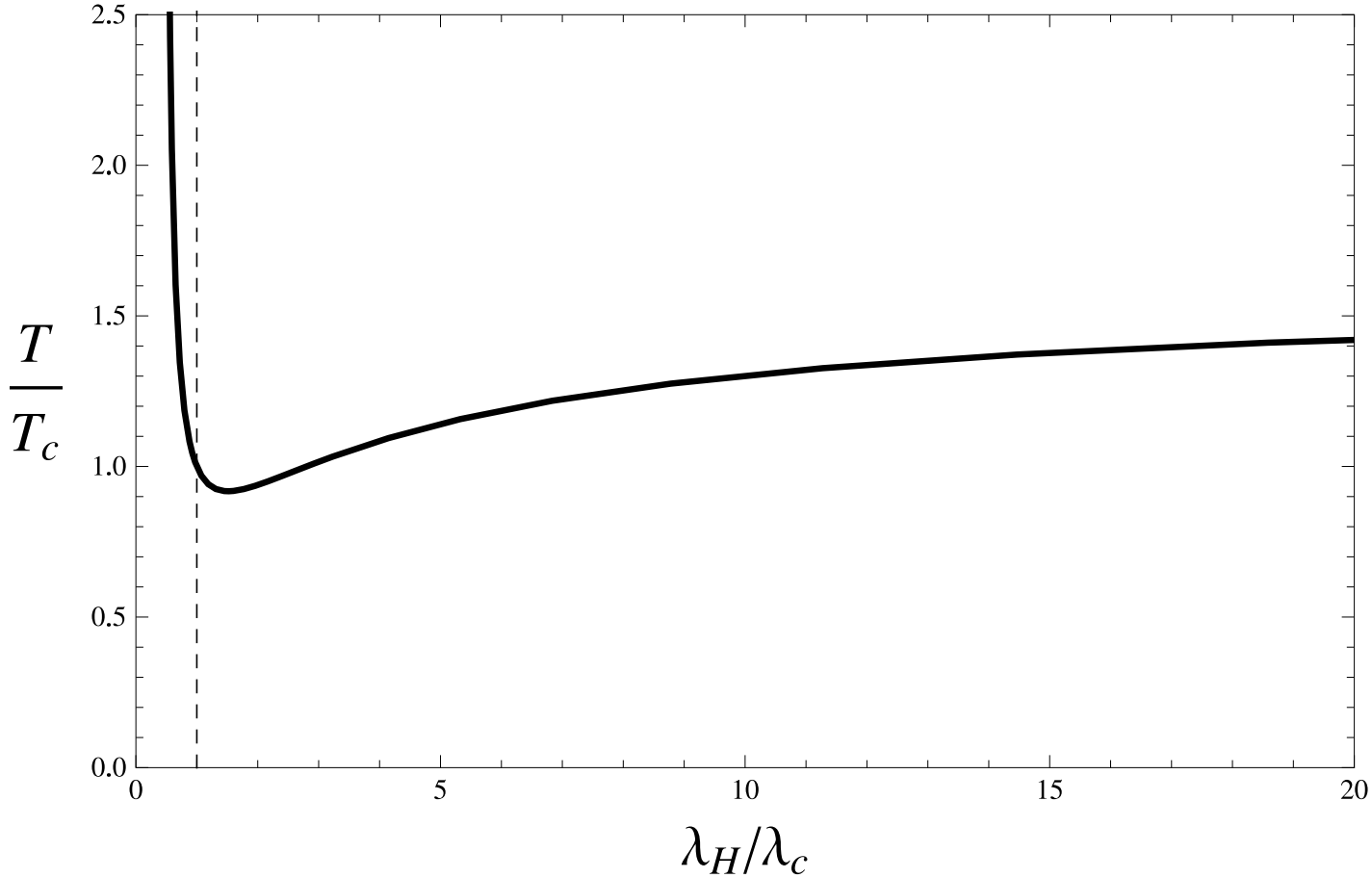
• There are two black hole branches

(a) The large black hole branch.

(b) The small black hole branch. They are thermodynamically unstable with negative specific heat. As the horizon size vanishes the small black hole turns into the vacuum state solution.

• There is a minimum temperature $T > T_0$, that separates the large from the small black hole branch.

• The first order (deconfining) phase transition to the deconfined (black hole phase) happens at $T_c > T_0$ inside the large black hole branch.



Plots of the temperature scaled by the critical temperature as a function of λ_H/λ_c (left) and the entropy density scaled by the third power of the temperature as a function of T/T_c (right). The rightmost plot becomes "dotted" as one passes through the phase transition by lowering the temperature from above. This is meant to indicate that in the field theory, this low temperature phase is governed by the thermal gas solutions, whose entropy is subleading in the number of colors N_c.

Quench dynamics

- The quench profile is:

$$f_0(v) = \tilde{f}_0 - \delta f_0 \, e^{-\frac{v^2}{2\tau^2}}$$

- For numerical simplicity we start with the theory in a thermal state that corresponds to the small black hole branch.

- The "smallest" the initial black hole, the closest we are to the initial (confining) ground state of the theory.

We find the following

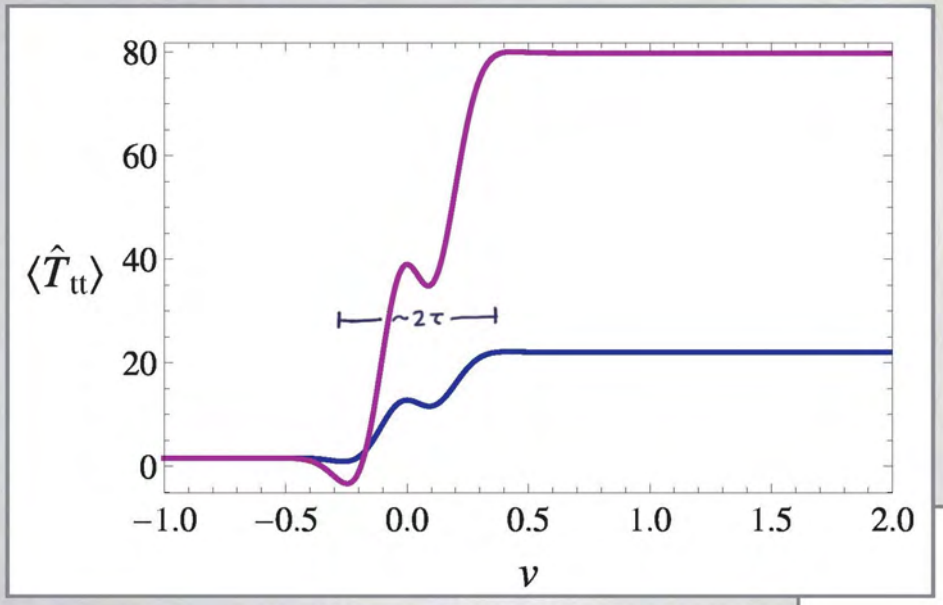
- The characteristic time associated with the intermediate non-linear regime is negligible compared to τ and T_{RD}.

- This seems to be a generic occurrence in holography/gravity and a clean explanation is still lacking.

- Therefore

$$T_{\text{thermalization}} \simeq \frac{1}{\Gamma}$$

- For adiabatic perturbations, $\tau \gg 1$ the system does NOT oscillate but goes continuously to the final-state black hole.

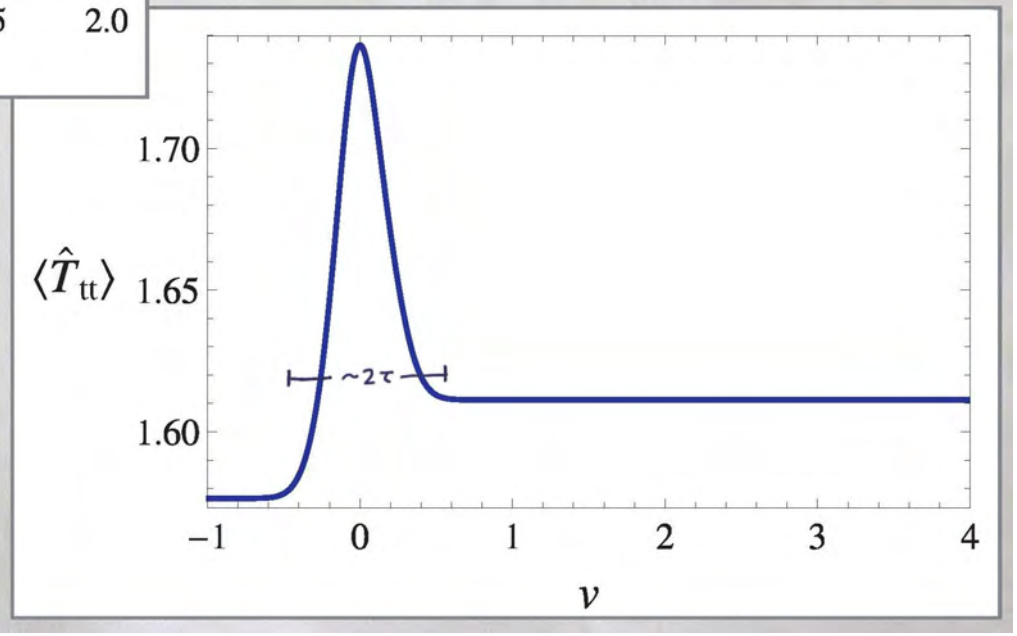


Large Amplitude

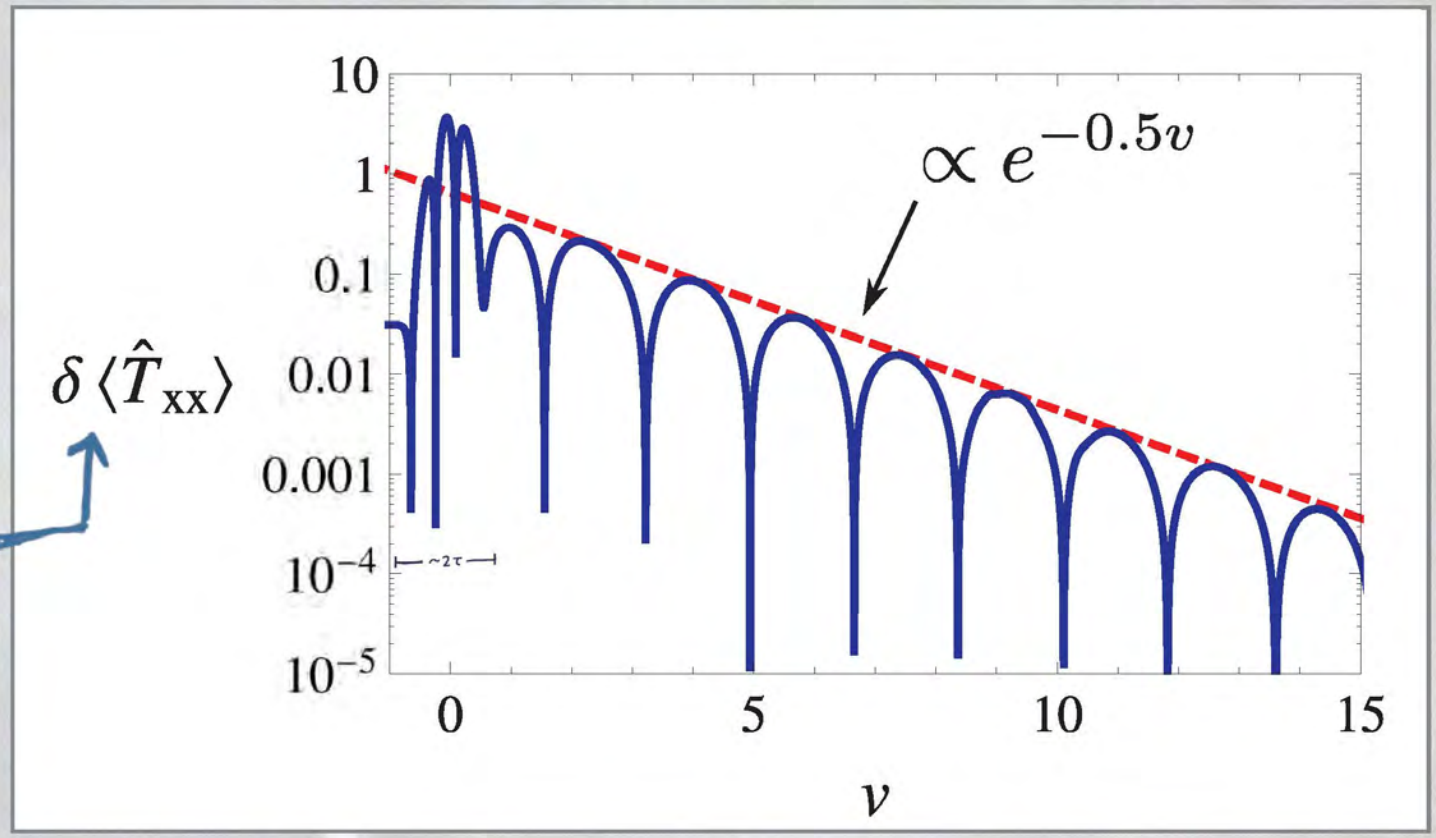
- Small BH → Big BH
- τ_{THERM} small

Small Amplitude

- Small BH → Small BH
- τ_{THERM} large

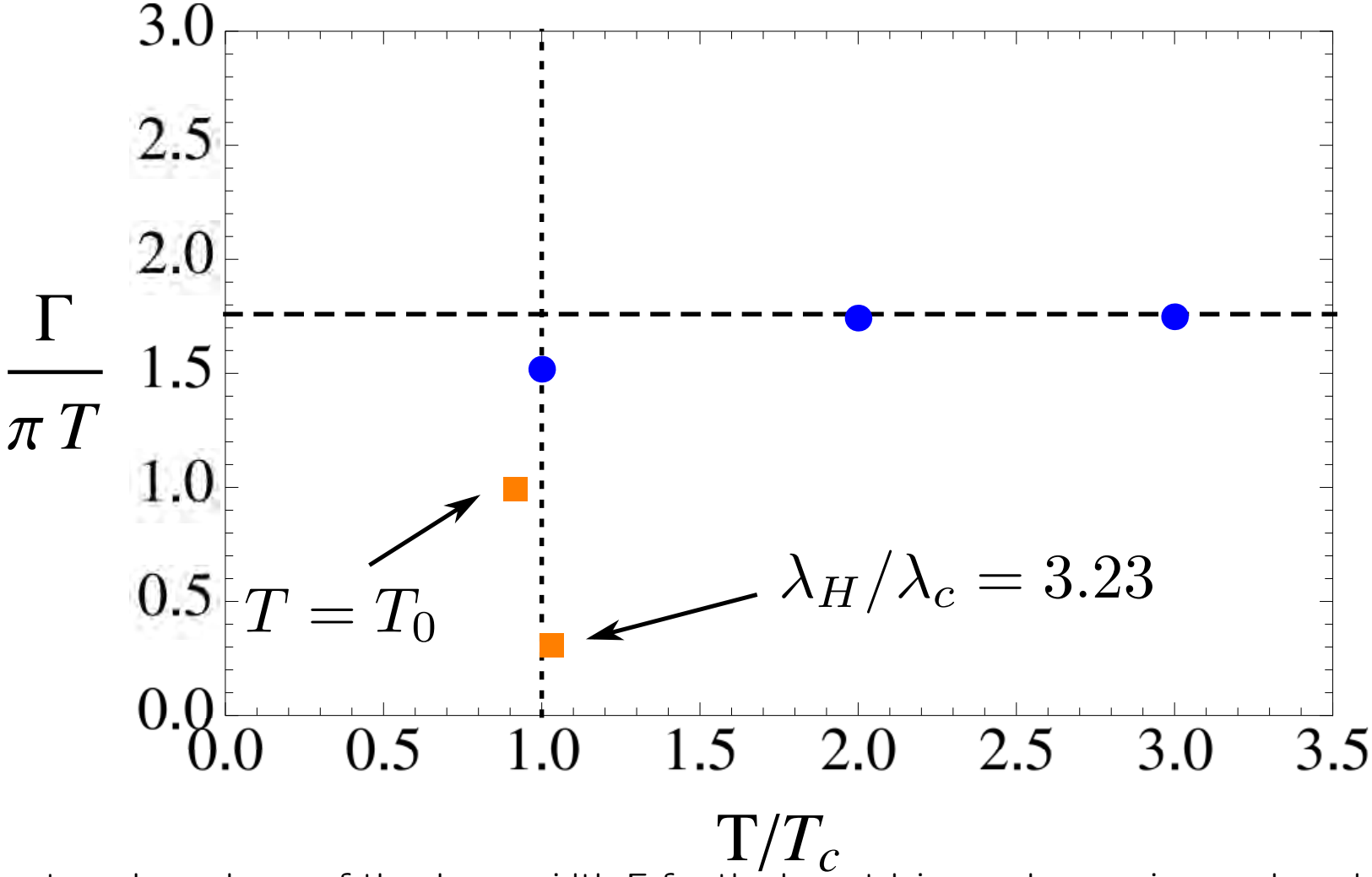


The ring-down phase



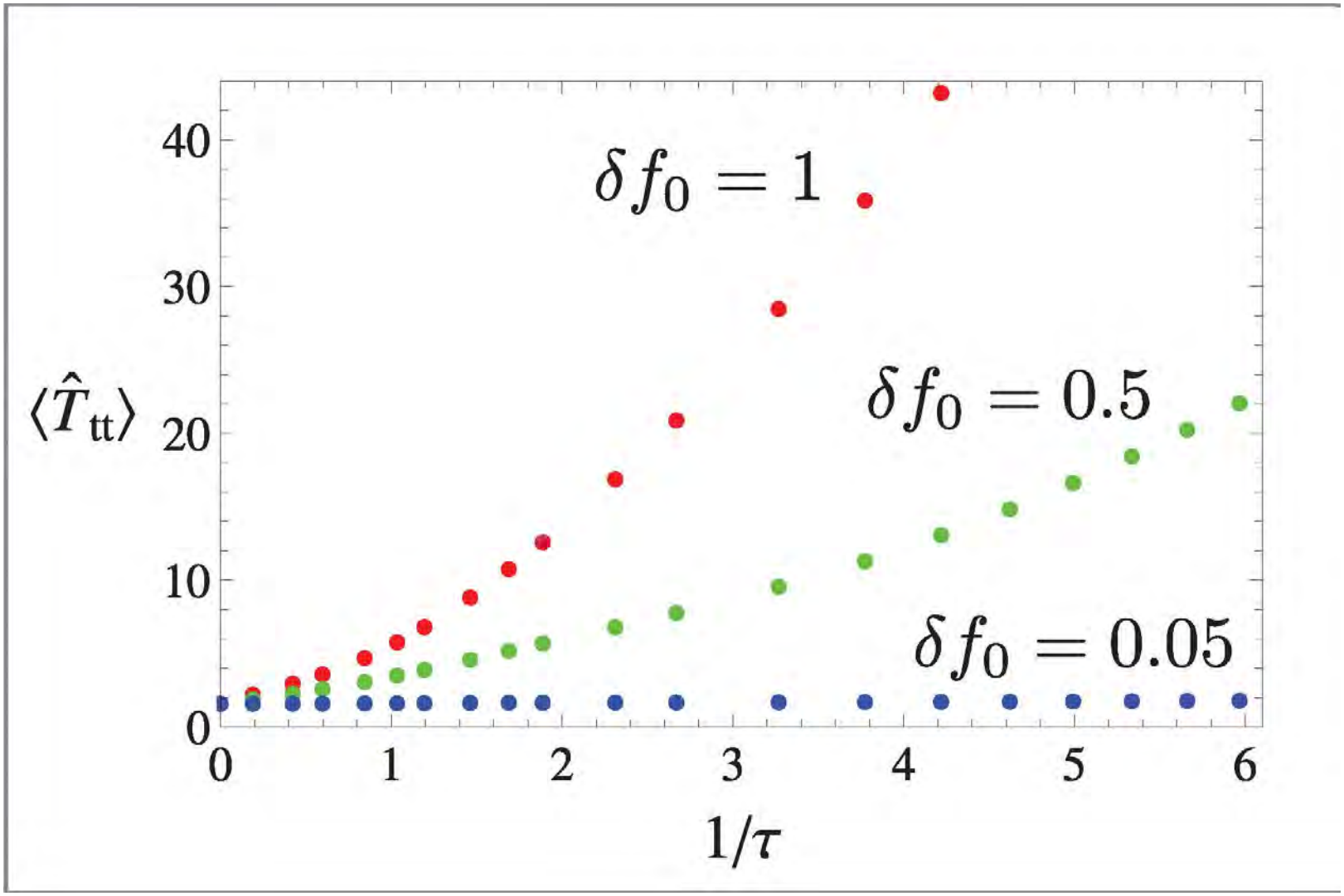
$$\langle \hat{T}_{xx}(v)\rangle - \langle \hat{T}_{xx}(\infty)\rangle$$

$$\boxed{\tau_{THERM} \sim \frac{1}{\Gamma} \sim 2}$$

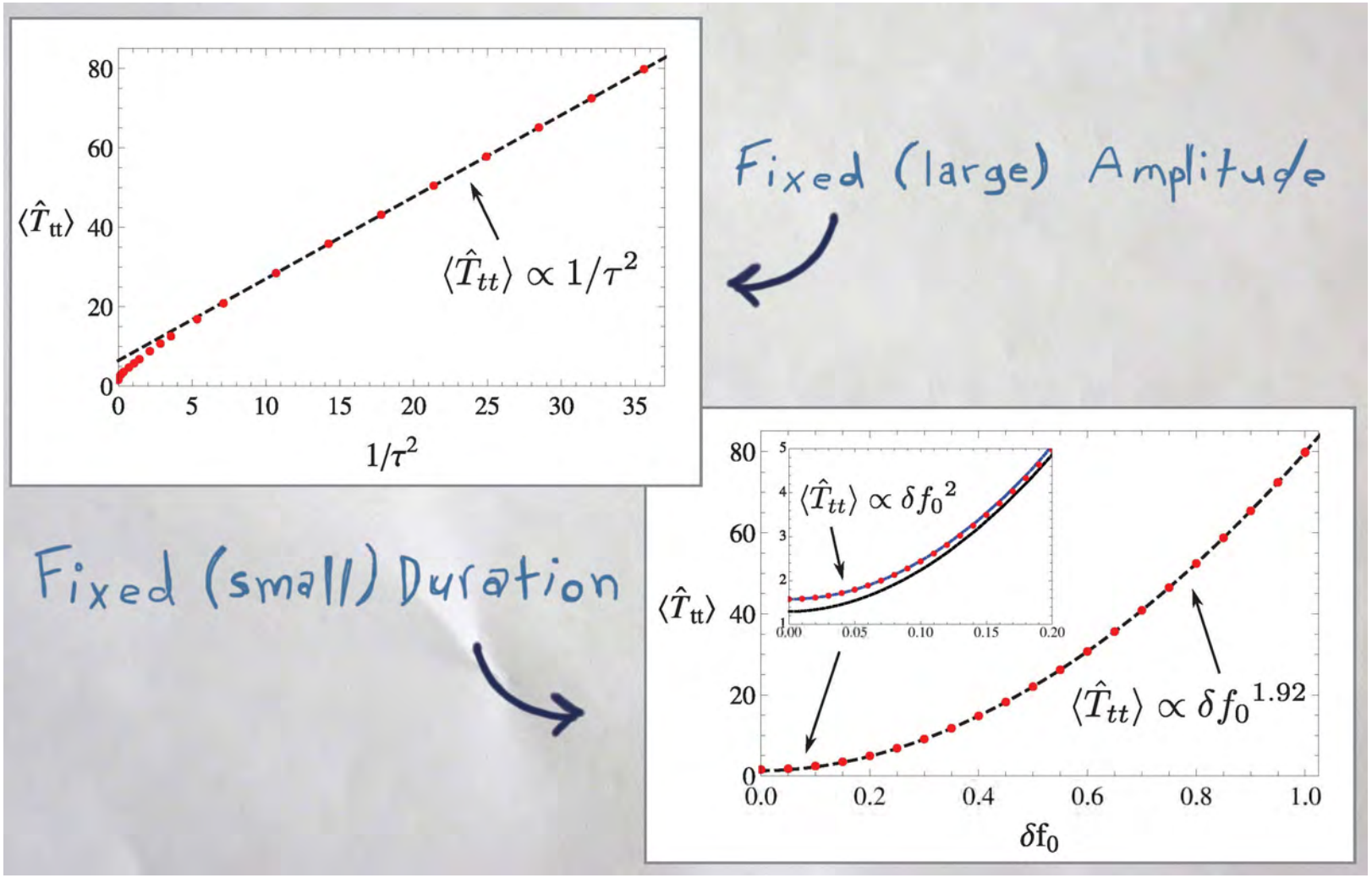


The temperature dependence of the decay width Γ for the lowest lying scalar quasi-normal mode in several states of our theory. The blue circles are large black branes whose temperature is an integer multiple of T_c. The orange squares correspond to the minimum temperature black brane (top) and the smallest black hole we perturb in our study (bottom). The ratio $\Gamma/\pi T$ approaches 1.75953 (the dashed line) at high temperatures, which coincides with the expected value for perturbations of AdS_5 Schwarzschild by a dimension 3 scalar operator

The numerical data



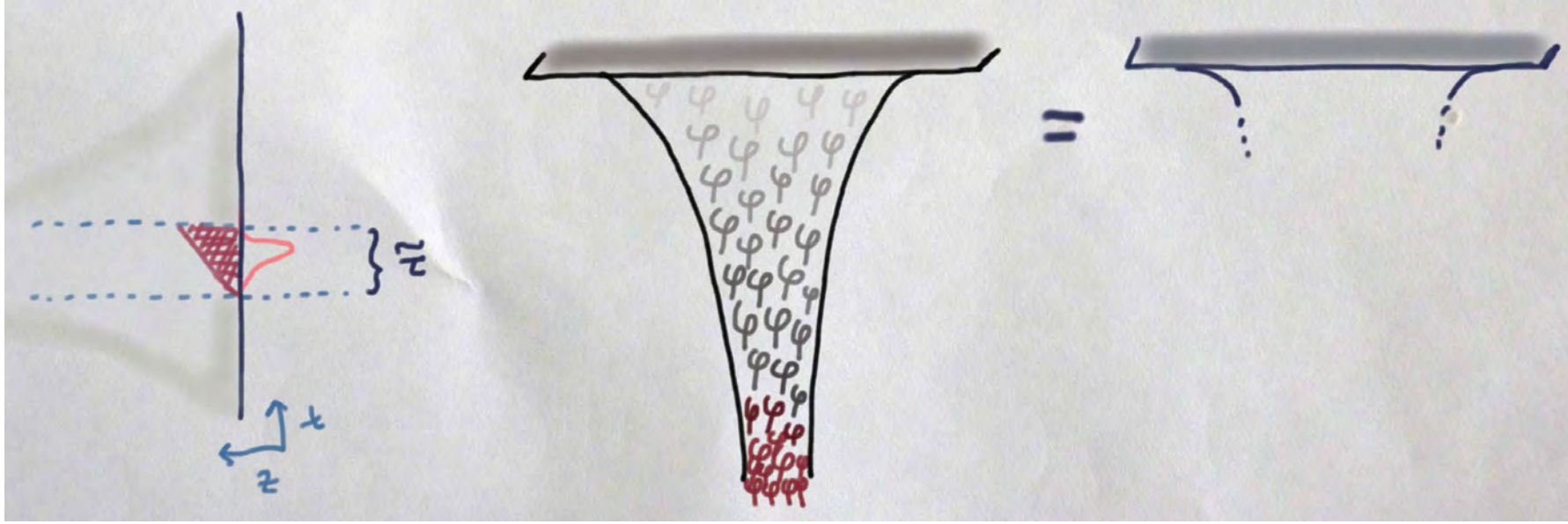
Scaling


Fixed (large) Amplitude
$\langle \hat{T}_{tt} \rangle \propto 1/\tau^2$
$\langle \hat{T}_{tt} \rangle$
$1/\tau^2$
Fixed (small) Duration
$\langle \hat{T}_{tt} \rangle \propto \delta f_0{}^2$
$\langle \hat{T}_{tt} \rangle \propto \delta f_0{}^{1.92}$
$\langle \hat{T}_{tt} \rangle$
δf_0

Fast Quenches

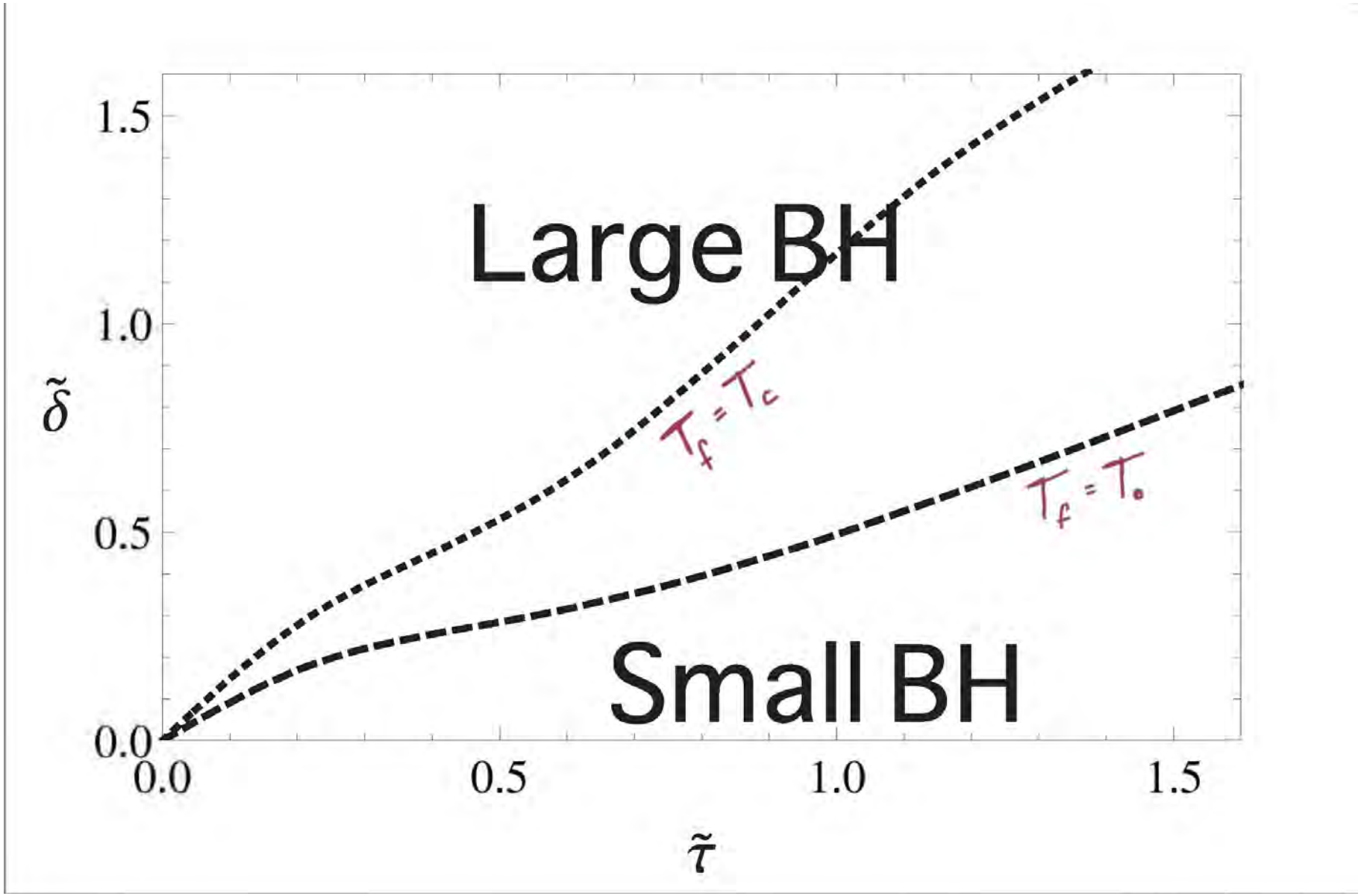
$$\langle \hat{T}_{tt} \rangle_F \sim \frac{\tilde{\delta}^2}{\tilde{\tau}^2} = \boxed{\frac{\tilde{\delta}^2}{\tilde{\tau}^{2\Delta - d}}}$$

[1307.4740]

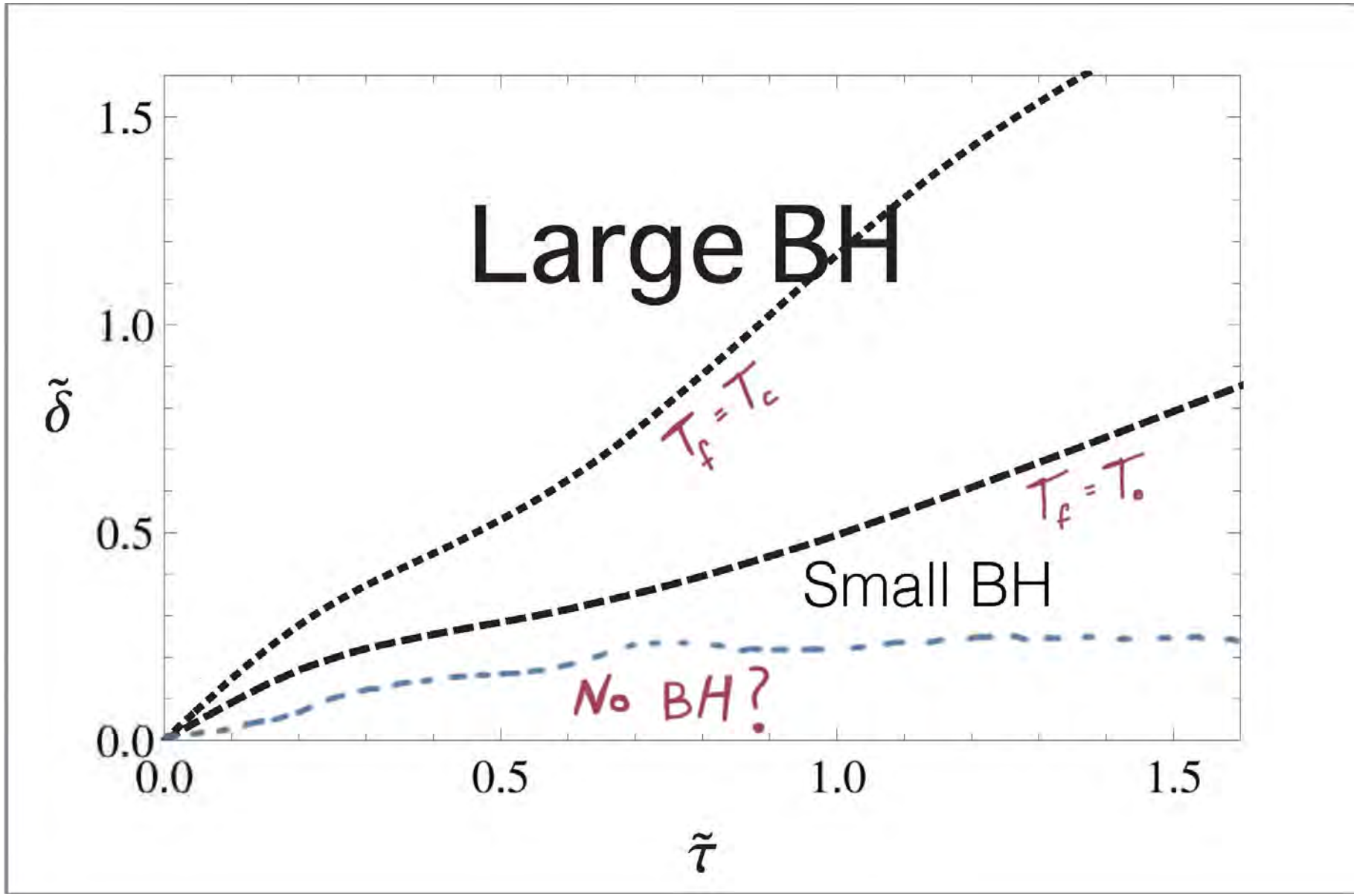


Buchel+Lehner+Myers+Niekerk, Das+Galante+Myers

The phase diagram



The full phase diagram



Conclusions and Outlook

- Rapid Transitions to the linear regime: The confinement scale plays little role for perturbations with $\frac{\langle T_{tt}\rangle}{\Lambda^4} \gtrsim 1$

- Universality and scaling in the abrupt quench limit: fast processes are only sensitive to (static) UV behavior.

- Other non-perturbative scaling regimes found, and need to be understood.

- The results are compatible (by extension) with an almost instant thermalization in QCD processes.

- The extension to the initial state being the ground state should be analyzed. A Choptuik-like phase transition is expected, but a different Choptuik exponent is expected.

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THANK YOU

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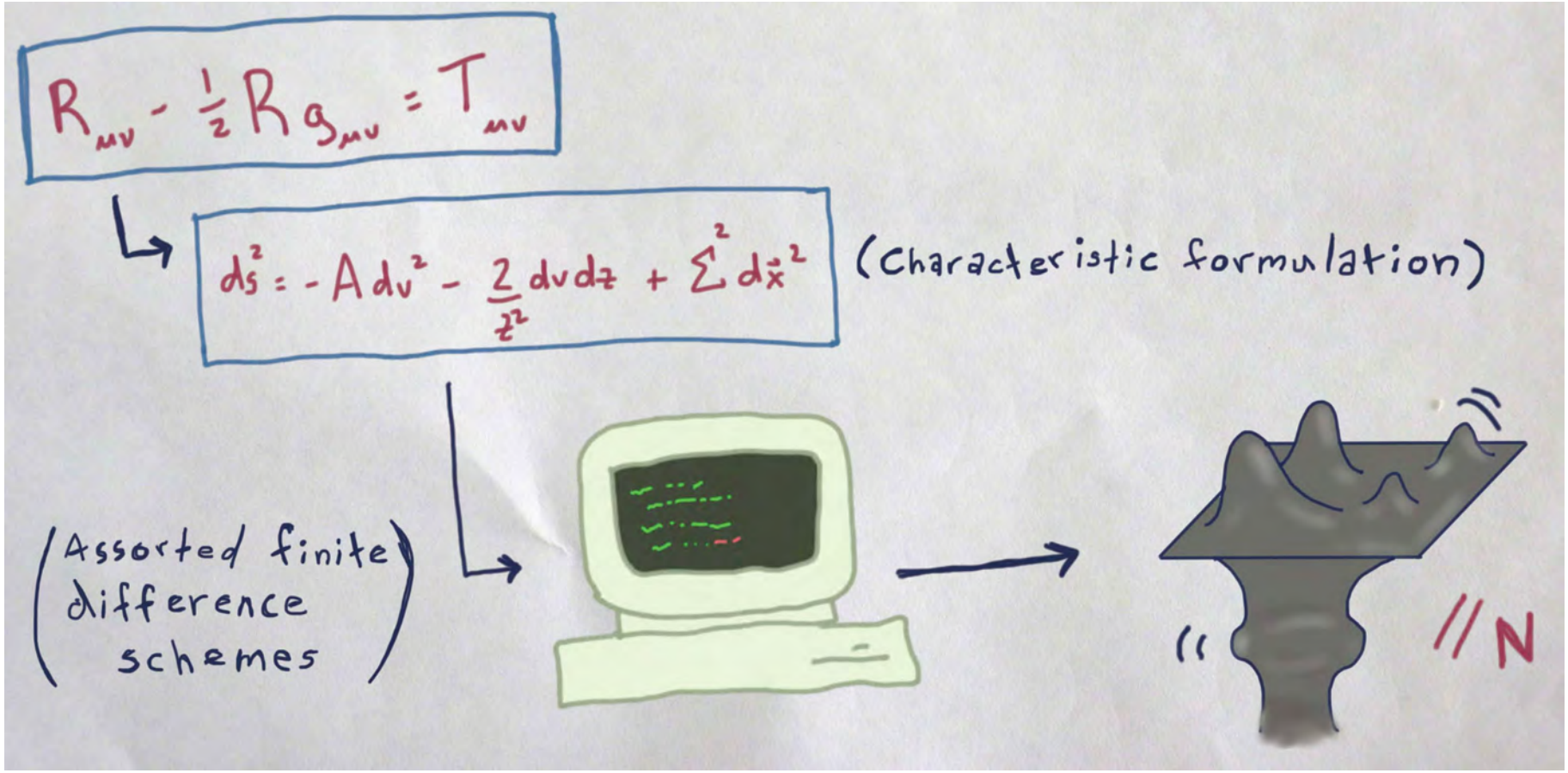
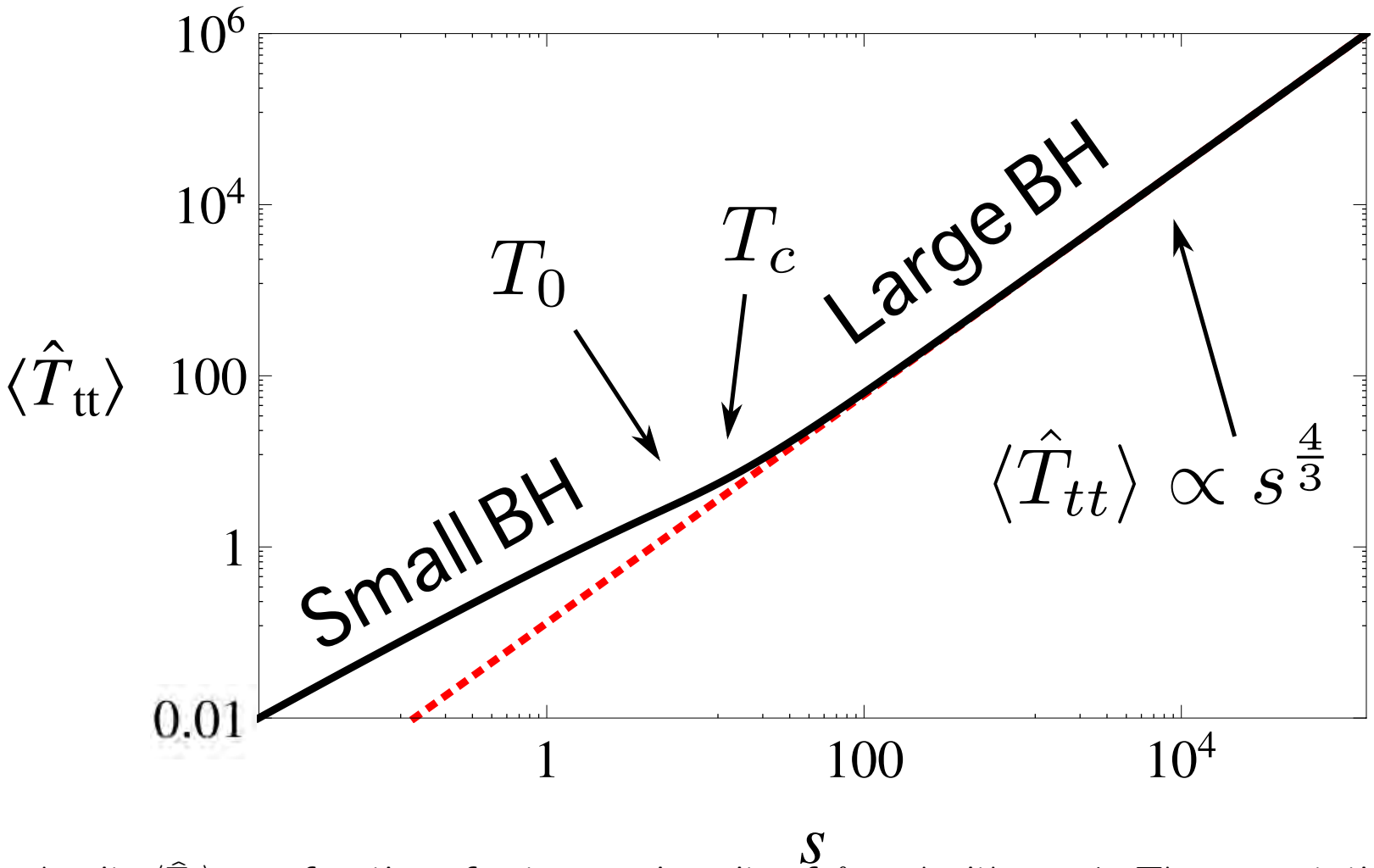
The solution procedure

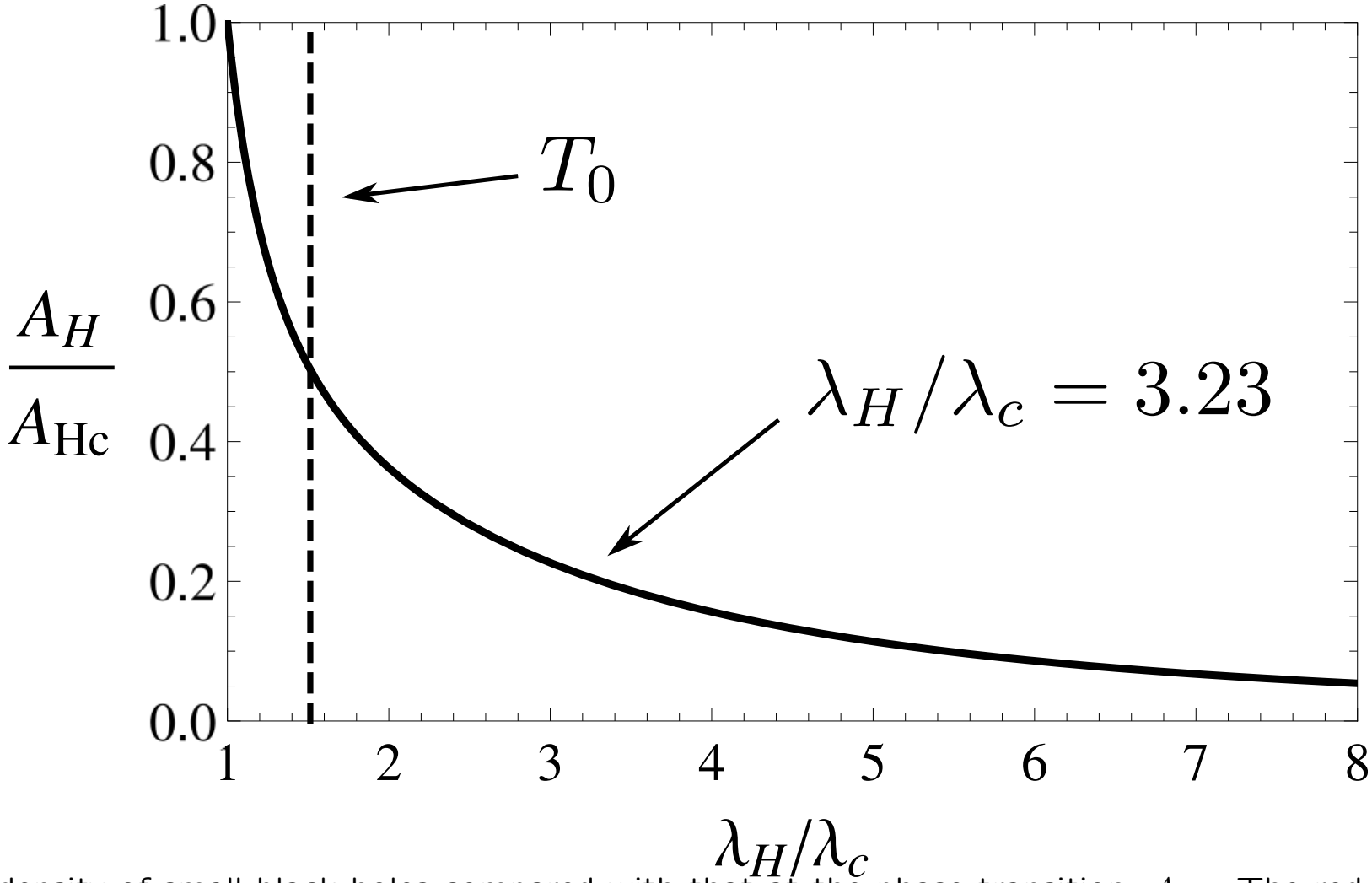


The thermodynamic functions



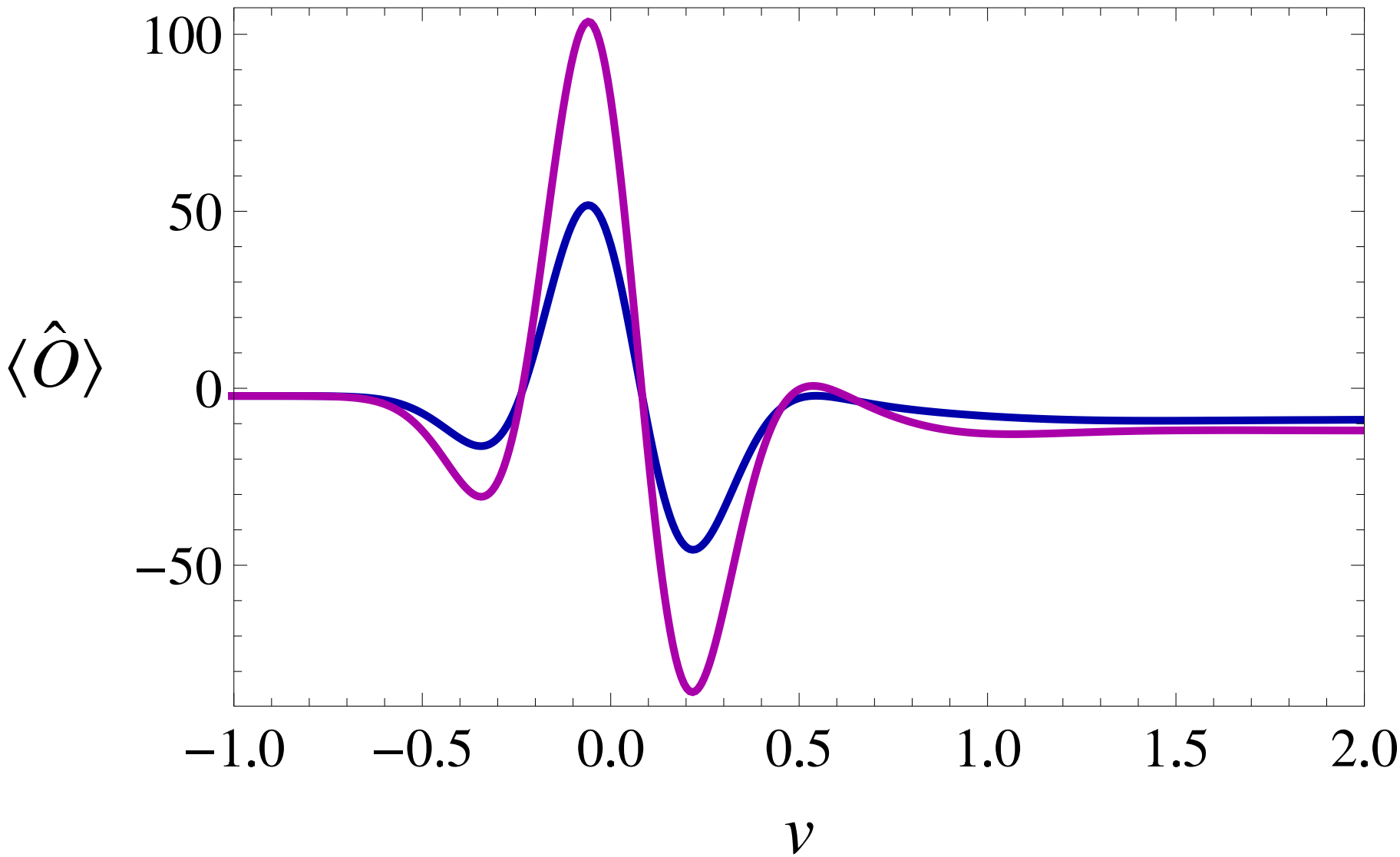
The energy density $\langle \hat{T}_{tt} \rangle$ as a function of entropy s, in units of f_0 and with $\kappa = 1$. The asymptotic behaviors are $\langle \hat{T}_{tt} \rangle \propto s^{4/3}$ and $\langle \hat{T}_{tt} \rangle \propto s\sqrt{-\ln s}$ in the limits of very large and very small black holes, and a fit for the former is plotted with a red dotted line. The green and magenta dots mark the locations of the first order phase transition at $T = T_c$ and the division between small and large black holes at $T = T_0$, respectively.

The small black hole initial states

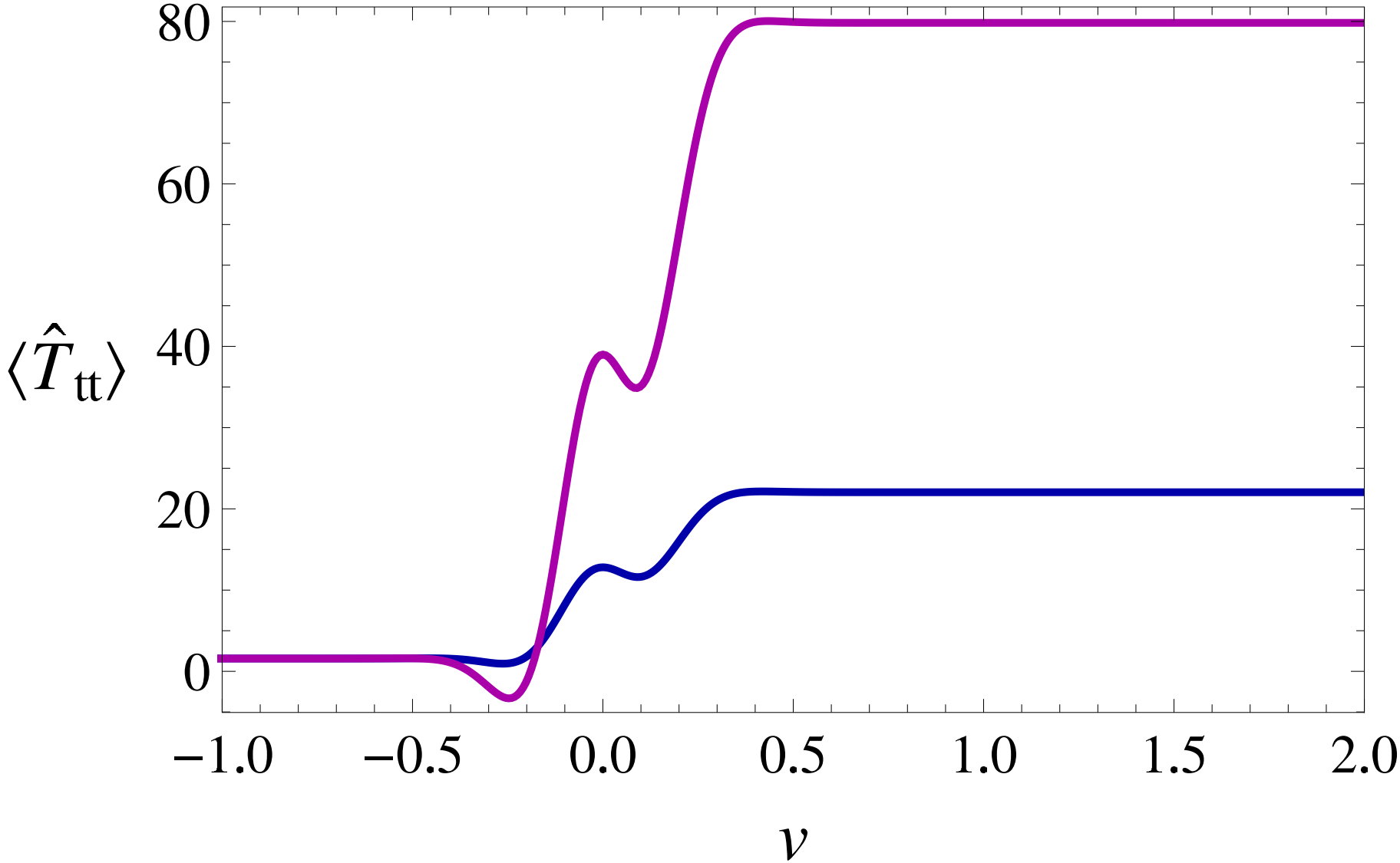


The area density of small black holes compared with that at the phase transition, A_{Hc}. The red dot marks the location in our space of solutions of the smallest black hole we perturb in this study. The dashed line indicates the location of the small and large black hole transition at $T = T_0$. As these are static black brane solutions, the apparent and event horizons coincide.

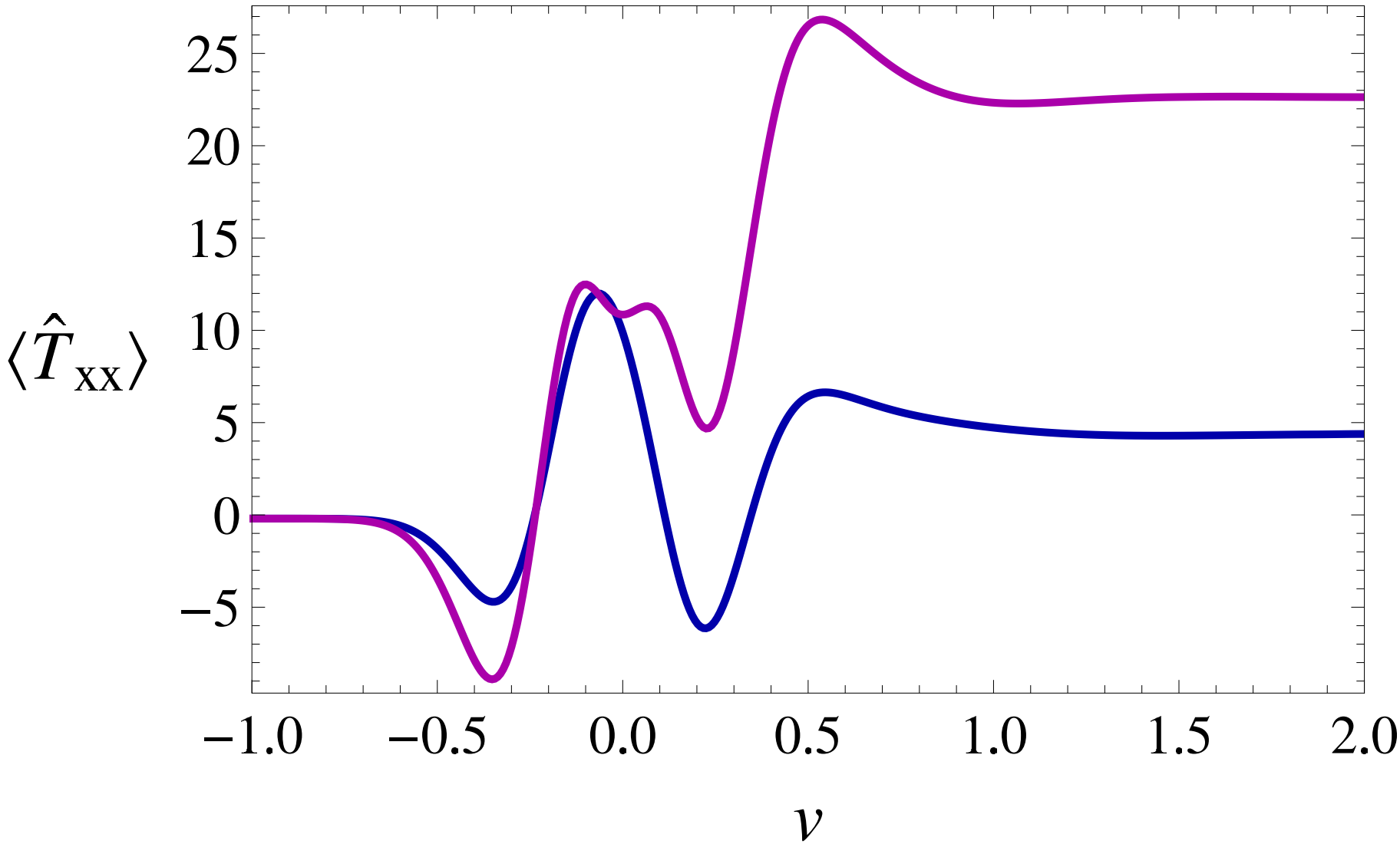
Large amplitude quench



The blue and purple lines correspond to $\tilde{\delta} = 0.5$ and 1, respectively.

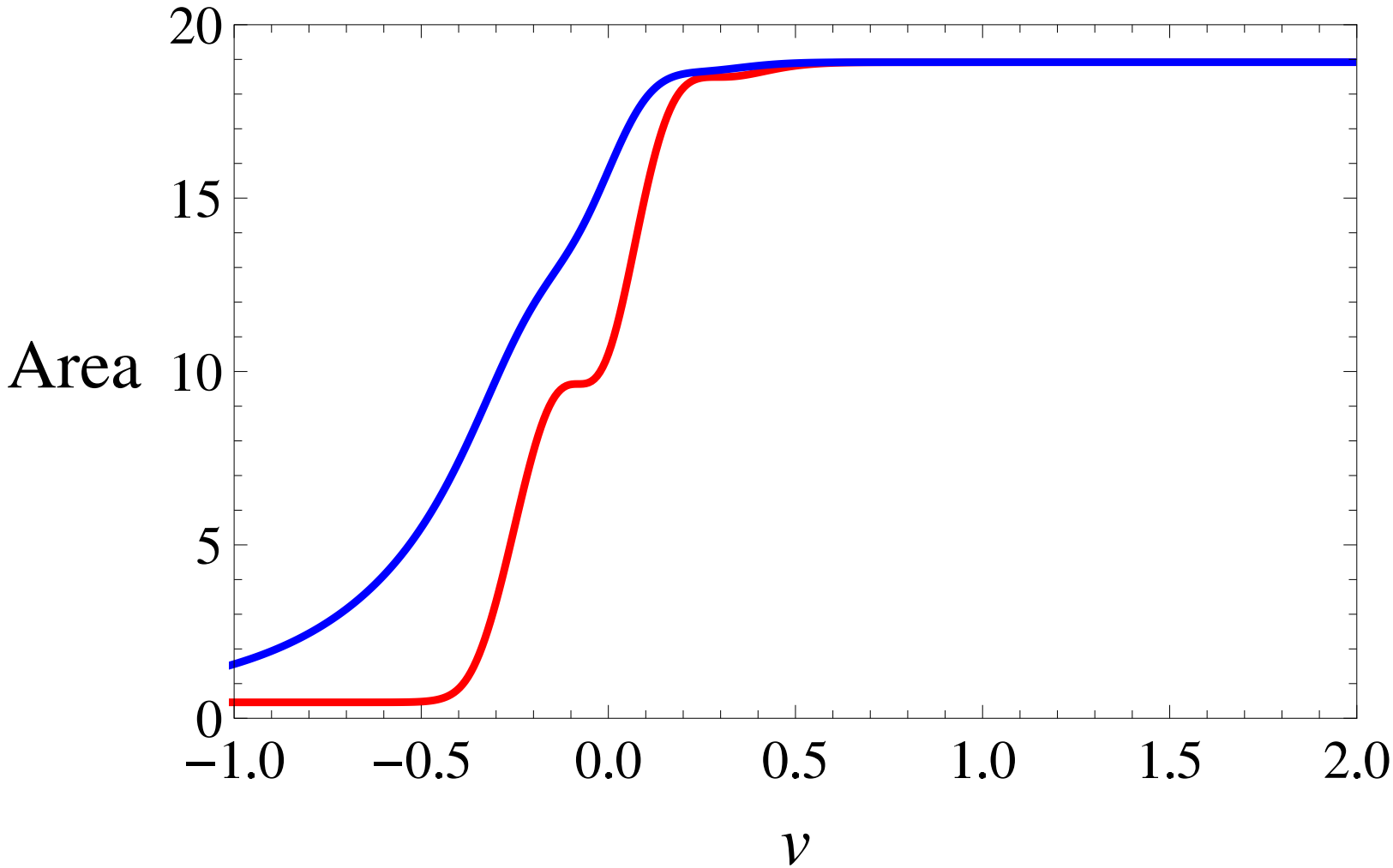


The blue and purple lines correspond to $\tilde{\delta} = 0.5$ and 1, respectively. In the case of the larger amplitude quench, it is interesting to note that the energy density appears to be driven below the ground state energy density (i.e. negative) in the first moments of the quench.



The blue and purple lines correspond to $\tilde{\delta} = 0.5$ and 1, respectively.

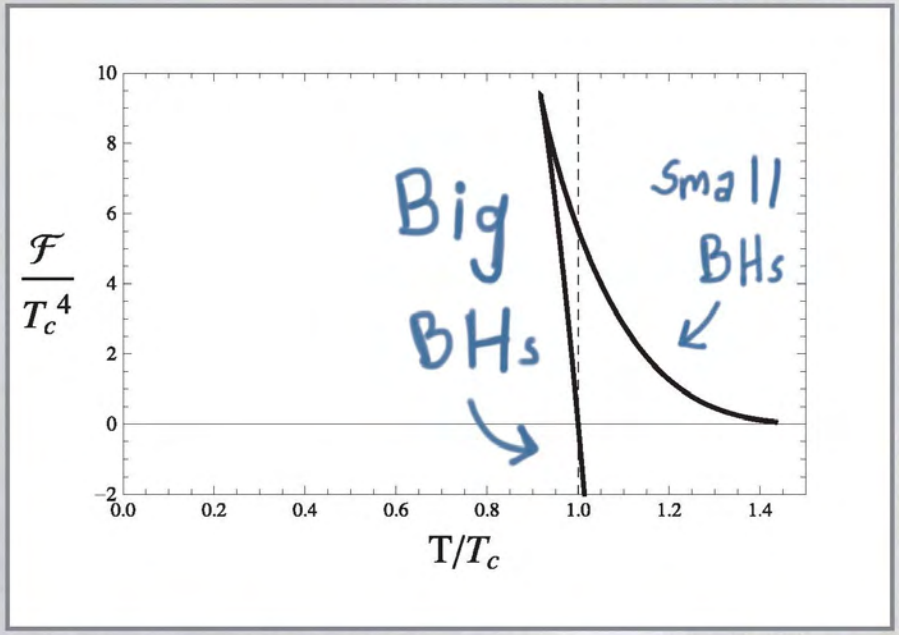
The evolution of horizons

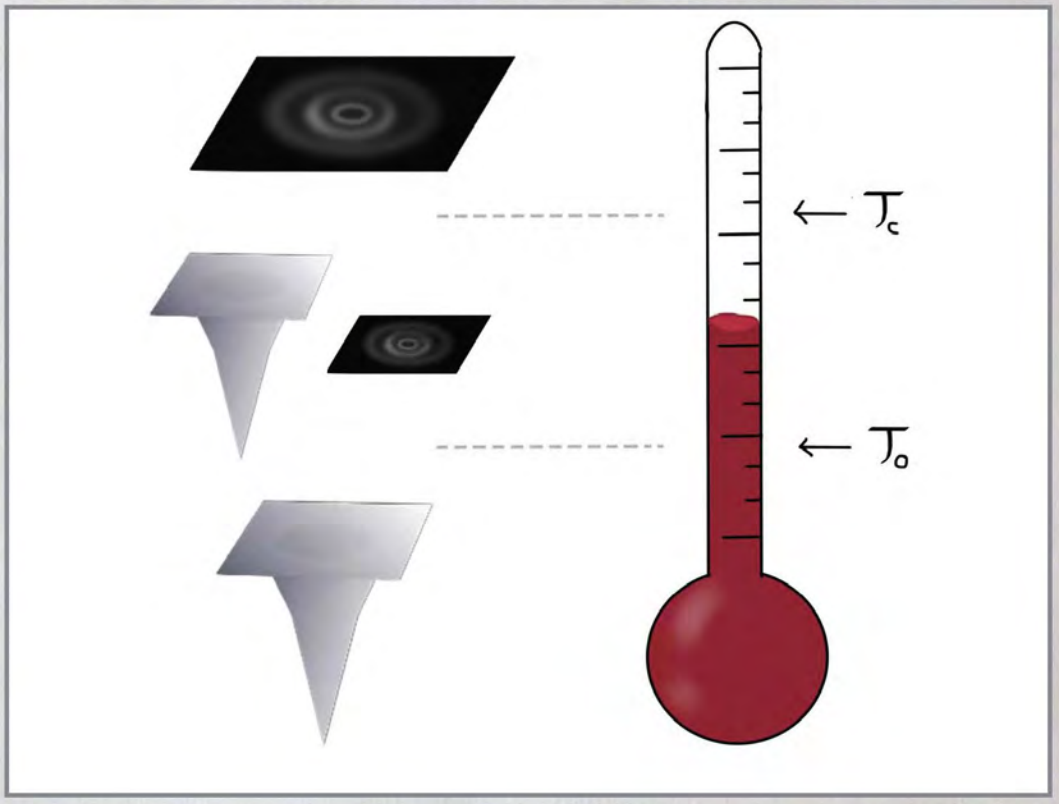


Example of the time evolution of the apparent (red) and event (blue) horizons. The event horizon coincides with the apparent horizon when the bulk solution is static, at $v \to \pm\infty$.

Black holes

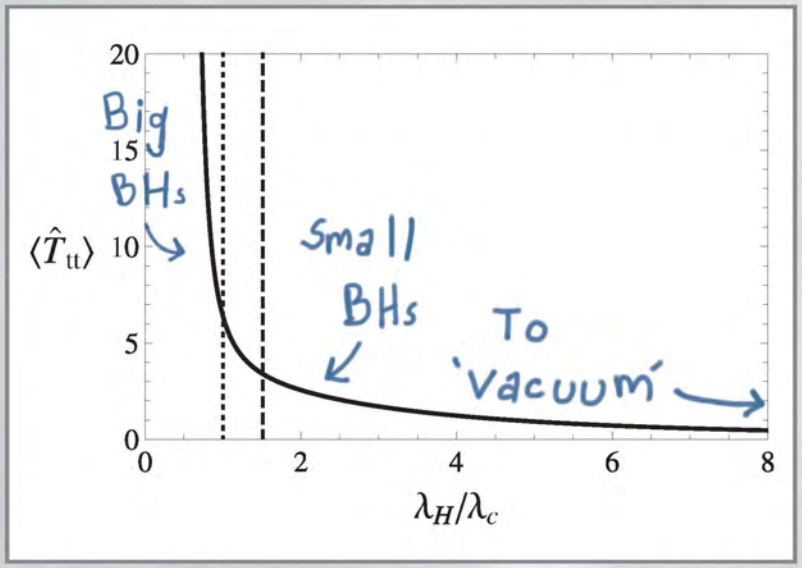
static Properties

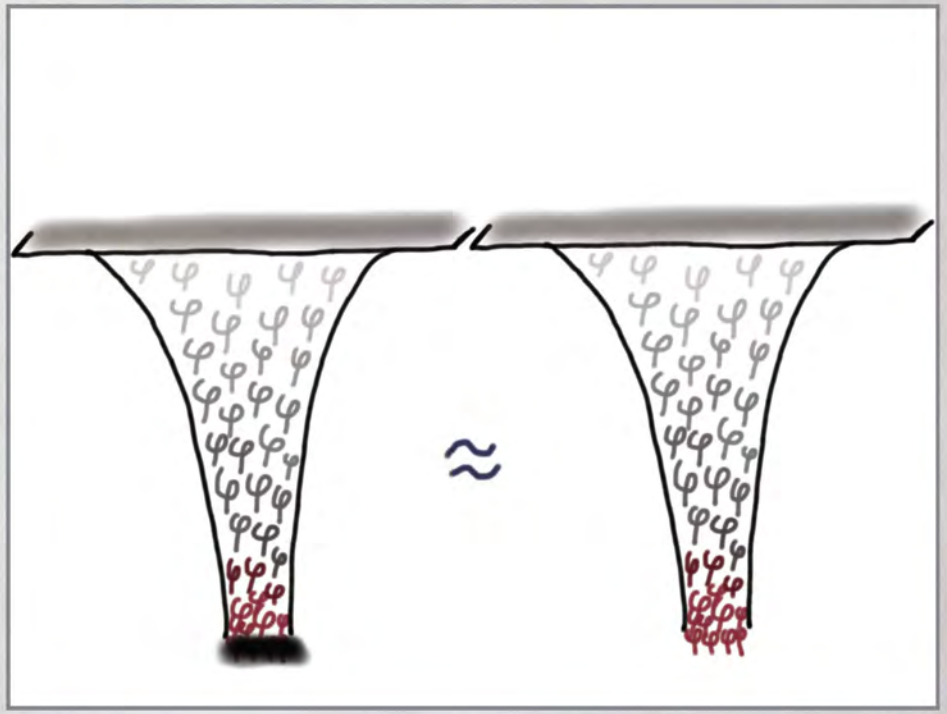




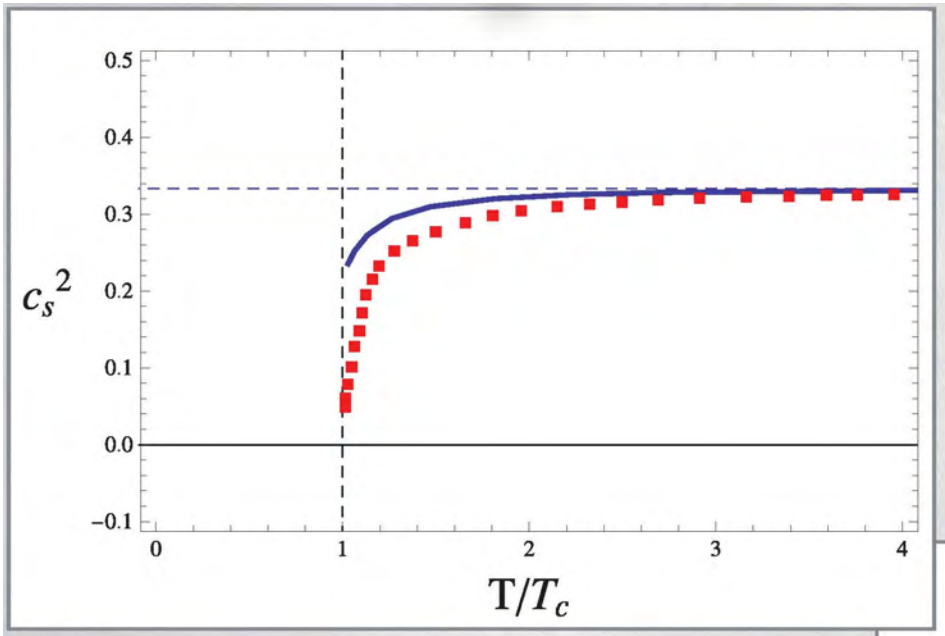
$$C_v = -T \frac{\partial^2 \mathcal{F}}{\partial T^2} \Rightarrow$$ Small BHs locally unstable

static Properties





Large Black holes



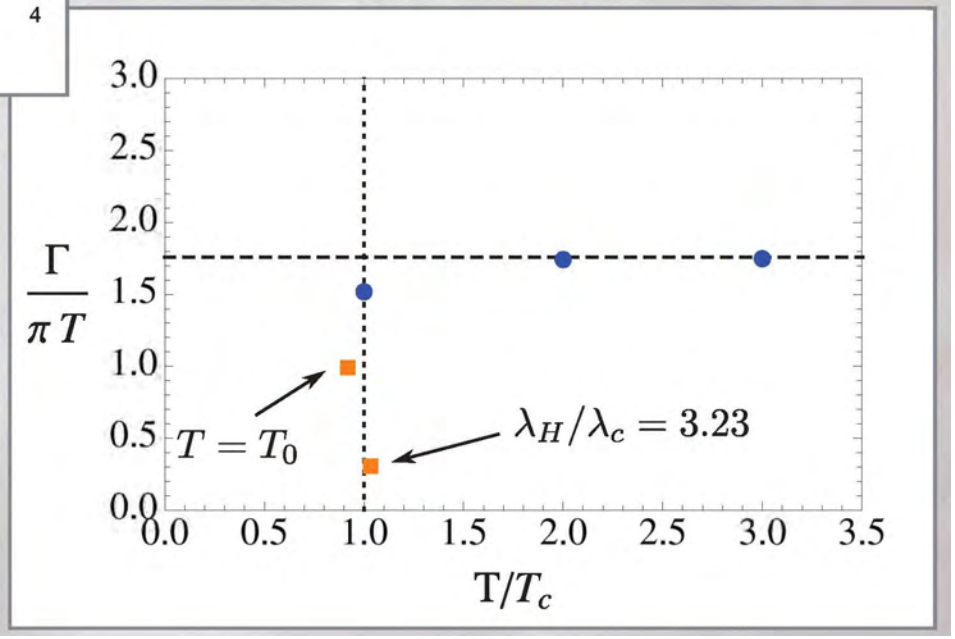
$$V(\varphi) = \frac{12\,(1+a\varphi^2)^{1/4}\cosh\frac{4}{3}\varphi - b\varphi^2}{L^2}$$

Parameters tuned towards "SU(3)-ish" glue

Lowest Lying QNM

$$\varphi(x) \sim \varphi_s(z) + \delta\varphi(z)\,e^{-i\omega t}$$

$$\omega_{QNM} = \omega^* - i\Gamma$$



Detailed plan of the presentation

- Title page 0 minutes
- Bibliography 1 minutes
- Introduction 2 minutes
- The setup for thermalization 4 minutes
- Thermalization at strong coupling 5 minutes
- Gravitational expectations 7 minutes
- Thermalization calculations 8 minutes
- The Hard Wall Model 10 minutes
- Improved Holographic QCD 15 minutes
- Black holes in Improved Holographic QCD 19 minutes
- Quench dynamics 22 minutes
- The ring-down phase 24 minutes
- The numerical data 25 minutes
- Scaling 27 minutes
- Fast Quenches 28 minutes
- The phase diagram 30 minutes
- The full phase diagram 31 minutes

- Conclusions and Outlook 32 minutes

- The solution procedure 33 minutes
- The thermodynamic functions 35 minutes
- The small black hole initial states 36 minutes
- Large amplitude Quench 39 minutes
- The evolution of horizons 41 minutes
- Black holes 44 minutes
- Large black holes 47 minutes