A Possible Evidence of Observation of Two Mixed Phases In Nuclear Collisions

Kyrill Bugaev
A. Ivanytskyi, D. Oliynychenko, V. Sagun, G.M. Zinovjev

Bogolyubov ITP, Kiev, Ukraine

V. Trubnikov
Kharkov Institute of Physics and Technology, Kharkov, Ukraine

E. Nikonov
Laboratory for Information Technologies, JINR, Dubna, Russia

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Outline

1. Motivation

2. Problems of single component Hadron Resonance Gas Models

3. Some results of multicomponent Hadron Resonance Gas Models

4. Novel and Old Irregularities at chemical freeze out

5. Shock adiabat model of A+A collisions

6. Meta-analysis of Hadronic and QGP event generators

7. Conclusions
Experiments on A+A Collisions

AGS (BNL) up to 4.9 GeV
SPS (CERN) 6.1 - 17.1 GeV
RHIC (BNL) 62, 130, 200 GeV

} Completed

Ongoing HIC experiments
LHC (CERN) > 1 TeV (high energy)
RHIC (BNL) low energy
SPS (CERN) low energy

Future HIC experiments
NICA (JINR, Dubna)
SIS$_{300}$ = FAIR (GSI)
Present Status

In 2000 CERN claimed indirect evidence for a creation of new matter.

In 2010 RHIC collaborations claimed to have created a quark-gluon plasma/liquid.

However, up to now we do not know:

1. whether deconfinement is a phase transition

2. where does the onset of deconfinement begin

In order to answer 2-nd question we need a very accurate tool to analyze data.
HRG model is a truncated Statistical Bootstrap Model with the excluded volume correction a la VdWaals for all hadrons and resonances known from Particle Data Group.

For given temperature $T$, baryonic chem. potential, strange charge chem. potential, chem. potential of isospin 3-rd projection $\Rightarrow$ thermodynamic quantities $\Rightarrow$ all charge densities, to fit data.

Chemical freeze-out - moment after which hadronic composition is fixed and only strong decays are possible. I.e. there are no inelastic reactions.
HRG: a Multi-component Model

Traditional HRG model: one hard-core radius $R=0.25-0.3$ fm

Overall description of data (mid-rapidity or $4\pi$ multiplicities) is good!

But there are problems with $K+/\pi+$ and $\Lambda/\pi-$ ratios at SPS energies!!! => Two component model was suggested
Two component models do not solve the problems!  
Hence we need more sophisticated approach.
Anomalous properties otherwise.

Normal properties, if

\[ j \]

connects (Rankine-Hugoniot-Taub (RHT) adiabat = shock adiabat)

This compressed matter move in opposite directions toward the vacuum, leaving high-density matter

This equation follows from the usual hydrodynamic conservation laws of energy, momentum and baryonic charge.

\[ \chi^2/dof = 21.8/14 \]

\[ \chi^2/dof = 79/12 \]

Too steep increase before maximum and too slow decrease after it!

Short dashed line: a desired result

Anti Lambda problem!

Simple Solution to Horn Puzzle

Use four hard-core radii: $R_{\pi}$, $R_K$ are fitting parameters; 
$R_{\text{mesons}} = 0.4$ fm, $R_{\text{baryons}} = 0.2$ fm are fixed


$p$ is pressure. $K$-th charge density of $i$-th hadron sort is $n_i^K$ ($K \in \{B, S, I3\}$)

$B$ the second virial coefficients matrix $b_{ij} = \frac{2\pi}{3} (R_i + R_j)^3$

\[
p = T \sum_{i=1}^{N} \xi_i, \quad n_i^K = Q_i^K \xi_i \left[ 1 + \frac{\xi^T B \xi}{\sum_{j=1}^{N} \xi_j} \right]^{-1}, \quad \xi = \begin{pmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_s \end{pmatrix},
\]

NO strangeness suppression is included!

the variables $\xi_i$ are the solution of the following system:

\[
\xi_i = \phi_i(T) \exp \left( \frac{\mu_i}{T} - \sum_{j=1}^{N} 2 \xi_j b_{ij} + \frac{\xi^T B \xi}{\sum_{j=1}^{N} \xi_j} \right), \quad \phi_i(T) = \frac{g_i}{(2\pi)^3} \int \exp \left( -\frac{\sqrt{k^2 + m_i^2}}{T} \right) d^3k
\]

Chemical potential of $i$-th hadron sort: $\mu_i \equiv Q_i^B \mu_B + Q_i^S \mu_S + Q_i^{I3} \mu_{I3}$

$Q_i^K$ are charges, $m_i$ is mass and $g_i$ is degeneracy of the $i$-th hadron sort.
Wide Resonances Are Important

The resonance width is taken into account in thermal densities.

In contrast to P. Braun-Munzinger & Co we found that wide resonances are VERY important in a thermal model. For instance, description of pions cannot be achieved without

$\sigma$ meson: \( m_\sigma = 484 \pm 24 \text{ MeV} \), \( \text{width } \Gamma_\sigma = 510 \pm 20 \text{ MeV} \)


\[
n_{X}^{\text{tot}} = n_{X}^{\text{thermal}} + n_{X}^{\text{decay}} = n_{X}^{\text{th}} + \sum_{Y} n_{Y}^{\text{th}} \text{Br}(Y \rightarrow X)
\]

\( \text{Br}(Y \rightarrow X) \) is decay branching of \( Y \)-th hadron into hadron \( X \)
Data and Fitting Parameters

111 independent hadronic ratios measured at AGS, SPS and RHIC energies

# of published ratios measured at mid-rapidity depends on energy =>

<table>
<thead>
<tr>
<th>$\sqrt{s_{NN}}$ (GeV)</th>
<th>$N_{rat}$ FO</th>
</tr>
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<tbody>
<tr>
<td>2.7</td>
<td>4</td>
</tr>
<tr>
<td>3.3</td>
<td>5</td>
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<td>3.8</td>
<td>5</td>
</tr>
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<td>4.3</td>
<td>5</td>
</tr>
<tr>
<td>4.9</td>
<td>8</td>
</tr>
<tr>
<td>6.3</td>
<td>9</td>
</tr>
<tr>
<td>7.6</td>
<td>10</td>
</tr>
<tr>
<td>8.8</td>
<td>11</td>
</tr>
<tr>
<td>9.2</td>
<td>5</td>
</tr>
<tr>
<td>12</td>
<td>10</td>
</tr>
<tr>
<td>17</td>
<td>13</td>
</tr>
<tr>
<td>62.4</td>
<td>5</td>
</tr>
<tr>
<td>130</td>
<td>11</td>
</tr>
<tr>
<td>200</td>
<td>10</td>
</tr>
<tr>
<td>Sum</td>
<td>111</td>
</tr>
</tbody>
</table>

# of local fit parameters cannot be larger than 4 (for all energies) or larger than 5 (for energies above 2.7 GeV)

# of local fit parameters for each collision energy = 3 (no $\gamma_s$ factor)
T, mu_B, mu_I3
Total # for 14 energies = 42

# of fit parameters with $\gamma_s$ factor is 4
Total # for 14 energies = 56

# of global fit parameters = 4
R_pi, R_K, R_mesons, R_baryons
Best global fit of all ratios gives $R_{\pi}=0.1$ fm, $R_K=0.38$ fm, $\chi^2/dof=1.16$ for fixed: $R_{\text{baryons}}=0.2$ fm, $R_{\text{mesons}}=0.4$ fm

Note that Lambda and other hyperons can be described better!

Strangeness Enhancement as Deconfinement Signal

In 1982 J. Rafelski and B. Müller predicted that enhancement of strangeness production is a signal of deconfinement. (Phys. Rev. Lett. 48(1982))

In 1991 J. Rafelski introduced strangeness fugacity $\gamma_S$ factor (Phys. Lett. 62(1991))

which quantifies strange charge chemical oversaturation ($>1$) or strange charge chemical undersaturation ($<1$)

Idea: if s-(anti)quarks are created at QGP stage, then their number should not be changed during further evolution since s-(anti)quarks number is small and since density decreases => there is no chance for their annihilation!

Hence, we should observe chemical enhancement of strangeness with $\gamma_S > 1$

However, until 2013 the situation with strangeness was unclear:

P. Braun-Munzinger & Co found that $\gamma_S$ factor is about 1

F. Becattini & Co found that $\gamma_S$ factor is $< 1$
Systematics of Strangeness Suppression

Include $\gamma_s$ factor $\phi_i(T) \rightarrow \phi_i(T) \gamma_s^{s_i}$, into thermal density

where $s_i$ is number of strange valence quarks plus number of strange valence anti-quarks.

Thus, it is a strangeness fugacity


Typical values of $\chi^2/dof > 2$ at given energy!
Our Results on Strangeness Enhancement in 2013

High quality description of hadron multiplicities requires $T$, $\mu_B$, $\mu_{I3}$ and $\gamma_S$ factor.

\[ \chi^2/dof = 3.3/14 \]

$\chi^2/dof = 1.15$ for 111 ratios measured for c.m. energies 2.7--200 GeV


Strangeness enhancement exists where we do not expect deconfinement!

Solving problem with Kaons lead to (anti)$\Lambda$ selective suppression!
Solutions of (anti)\(\Lambda\) selective Suppression


Use these deviations from UrQMD as new suppression factor!

Our solution:

1. Introduce Hard core radius for (anti)\(\Lambda\) hyperons

2. Refit globally all hard core radii:

\[=> R_{\pi} = 0.1 \text{ fm}, \quad R_{\Lambda} = 0.1 \text{ fm}, \quad R_{b} = 0.36 \text{ fm}, \]
\[R_{K} = 0.38 \text{ fm}, \quad R_{m} = 0.4 \text{ fm}\]


Strangeness Horn and $\Lambda$ Horn in 2014

With new radii and $\gamma_s$ fit

\[
\chi^2 / 12 = 10.22 / 12 \quad \text{and} \quad \chi^2 / 8 = 6.49 / 8
\]

Total fit of 111 independent hadron ratios is the best of existing!

\[
\chi^2 / \text{dof} = 52 / 55 \simeq 0.95.
\]
Intermediate Conclusions

1. The multicomponent HRG model is a precise tool of HIC phenomenology

2. With high confidence we conclude that chemical enhancement of strangeness exists at very low energies where we do not expect deconfinement

3. Using multicomponent HRG model we can study thermodynamics at chemical freeze out
Jump of ChFO Pressure at AGS Energies

- Temperature $T_{\text{CFO}}$ as a function of collision energy $\sqrt{s}$ is rather non smooth

- Significant jump of pressure ($\simeq 6$ times) and energy density ($\simeq 5$ times)

Trace Anomaly Peaks

At chemical FO (large $\mu$)

Lattice QCD (vanishing $\mu$)

K.A. Bugaev et al., arXiv:1412.0718 [nucl-th]

Are these trace anomaly peaks related to each other?
Shock Adiabat Model for A+A Collisions

A+A central collision at $1 < \text{Elab} < 30$

From hydrodynamic point of view this is a problem of arbitrary discontinuity decay: in normal media there appeared two shocks moving outwards.


Its hydrodynamic model

Works reasonably well at these energies.
Medium with Normal and Anomalous Properties

Normal properties, if \( \Sigma \equiv \left( \frac{\partial^2 p}{\partial X^2} \right)_{s/\rho_B}^{-1} > 0 \) = convex down:

Usually pure phases (Hadron Gas, QGP) have normal properties

\[ X = \frac{\varepsilon + p}{\rho_B} \]  
- generalized specific volume

\( \varepsilon \) is energy density, \( p \) is pressure,

\( \rho_B \) is baryonic charge density

Anomalous properties otherwise.

Almost in all substances with liquid-gas phase transition the mixed phase has anomalous properties!

Then shock transitions to mixed phase are unstable and more complicated flows are possible.
Generalized Shock Adiabat Model

In case of unstable shock transitions more complicated flows appear:

In each point of simple wave \( \frac{s}{\rho_B} = \text{const} \)

If during expansion entropy conserves, then unstable parts lead to entropy plateau!

Remarkably

Z model has stable RHT adiabat, which leads to quasi plateau!


FIG. 9. The entropy per baryon as a function of the bombarding energy per nucleon of the colliding nuclei for models W and Z. The points 1, 2, 3, 4 on curve W correspond to those on the generalized adiabatic as displayed in Fig. 7. The point 1 on curve Z marks the boundary to the mixed phase.
Correlated Quasi-Plateaus

Since the main part of the system entropy is defined by thermal pions => thermal pions/baryon should have a plateau!

Also the total number of pions per baryons should have a (quasi)plateau!

Entropy per baryon has wide plateaus due to large errors

Quasi-plateau in total pions per baryon ?

Thermal pions demonstrate 2 plateaus
Details on Highly Correlated Quasi-Plateaus

- Common width $M$ – number of points belonging to each plateau
- Common beginning $i_0$ – first point of each plateau
- For every $M$, $i_0$ minimization of $\chi^2$/dof yields $A \in \{s/\rho_B, \, \rho_{\pi}^{th}/\rho_B, \, \rho_{\pi}^{tot}/\rho_B\}$:

$$\frac{\chi^2}{\text{dof}} = \frac{1}{3M-3} \sum_A \sum_{i=i_0}^{i_0+M-1} \left( \frac{A - A_i}{\delta A_i} \right)^2 \Rightarrow A = \sum_{i=i_0}^{i_0+M-1} \frac{A_i}{(\delta A_i)^2} / \sum_{i=i_0}^{i_0+M-1} \frac{1}{(\delta A_i)^2}$$

<table>
<thead>
<tr>
<th>$M$</th>
<th>$i_0$</th>
<th>$s/\rho_B$</th>
<th>$\rho_{\pi}^{th}/\rho_B$</th>
<th>$\rho_{\pi}^{tot}/\rho_B$</th>
<th>$\chi^2$/dof</th>
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<td>2</td>
<td>3</td>
<td>11.12</td>
<td>0.52</td>
<td>0.85</td>
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<tr>
<td>3</td>
<td>3</td>
<td>11.31</td>
<td>0.46</td>
<td>0.89</td>
<td>0.53</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>10.55</td>
<td>0.43</td>
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</tr>
<tr>
<td>5</td>
<td>2</td>
<td>11.53</td>
<td>0.47</td>
<td>0.84</td>
<td>4.45</td>
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</table>

Low energy plateau

- High energy plateau

<table>
<thead>
<tr>
<th>$M$</th>
<th>$i_0$</th>
<th>$s/\rho_B$</th>
<th>$\rho_{\pi}^{th}/\rho_B$</th>
<th>$\rho_{\pi}^{tot}/\rho_B$</th>
<th>$\chi^2$/dof</th>
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<td>0.77</td>
<td>1.87</td>
<td>0.87</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>16.26</td>
<td>0.64</td>
<td>1.62</td>
<td>3.72</td>
</tr>
</tbody>
</table>

$S/\rho_B$, $R = 3/2(\pi^+ + \pi^-)/\rho_B$, $R = \pi_{\text{th}}/\rho_B$
Unstable Transitions to Mixed Phase

\[ X = \frac{\varepsilon + p}{\rho_B^2} \] - generalized specific volume


**GSA Model explains irregularities at CFO as a signature of mixed phase**

QGP EOS is MIT bag model with coefficients been fitted with condition \( T_c = 150 \text{ MeV} \) at vanishing baryonic density!

HadronGas EOS is simplified HRGM discussed above.
We found one-to-one correspondence between these two peaks.

Thus, sharp peak of trace anomaly at c.m. energy 4.9 GeV evidences for QGP formation.

K.A. Bugaev et al., arXiv:1412.0718 [nucl-th]
Comparison of Hadronic and QGP event generators of HIC

Quality of Data Description = QDD

\[ \langle \chi^2/n \rangle_A^h = \frac{1}{n_d} \sum_{k=1}^{n_d} \left[ \frac{A_{k, data}^h - A_{k, model}^h}{\delta A_{k, data}^h} \right]^2 \]

Error of QDD

\[ \Delta_A \langle \chi^2/n \rangle_A^h = \left[ \sum_{k=1}^{n_d} \left( \delta A_{k, data}^h \right)^2 \right]^{1/2} \]

Meta-analysis of QDD for 6 HG models and for 4 QGP models:

1. scan of data and theoretical curves for strange hadrons
2. average QDD over observables and same kind of models
3. average QDD over hadrons and compare models

\[ \sqrt{s_{NN}} = 4.87 \text{ GeV} \]

<table>
<thead>
<tr>
<th>mT-distribution</th>
<th>rapidity distribution</th>
<th>Yields</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle \chi^2/n \rangle = 2.54 \pm 0.01, 1.07 \pm 0.002 )</td>
<td>2.75 \pm 1.66, 5.74 \pm 2.1</td>
<td>2.6 \pm 1.3 \left( \frac{dN}{dy} \right)_{y=0} &amp; 4\pi</td>
</tr>
<tr>
<td>( \Delta \text{ set 1} )</td>
<td>ARC,RQMD2.1</td>
<td>HSD &amp; UrQMD1.3(2.1)</td>
</tr>
<tr>
<td>( \langle \chi^2/n \rangle = 3.65 \pm 0.6, 2.4 \pm 0.55 )</td>
<td>4.67 \pm 1.155</td>
<td>N/A</td>
</tr>
<tr>
<td>( \Delta \text{ set 2} )</td>
<td>QuarkComb model</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Newest Signal of QGP Formation

Idea: at high energies QGP QDD must be better than HG QDD, at low energies vice versa!

Then equal QDD of two kinds of models is about mixed phase threshold

Meta-analysis gives 2 regions of intersection:
1-st mixed phase at c.m. energies 4.3-4.9 GeV
2-nd mixed phase (?) at c.m. energies 10-13.5 GeV
BOTH CAN BE CHECKED at NICA and FAIR!
Possible Interpretation

Evolution of possible «initial» states with collision energy

Appearance of 2-nd intersection at c.m. energies 10-13.5 GeV probably means that trajectory goes near critical (left) or 3critical (right) endpoint

To resolve this problem we need RHIC data at 11.5, 14.5 and 19.6 GeV

FIG. 5: Schematic pictures of possible locations of the initial states of matter formed in A+A collisions are shown on the plane of baryonic density and pressure. Each point on these trajectories (dashed curves) corresponds to a single collision energy value.

Left panel: As it is argued in the text the possible initial states correspond to the trajectories AD or BD as it follows from KTBO-plot 1 for the case of critical endpoint. The trajectory CD is located far from the mixed phase region and, hence, it cannot generate the second region in which the QDDs of HG and QGP models are equally good.

Right panel: In case of the tricritical endpoint the second region in which the QDDs of HG and QGP models are equally good may, alternatively, appear due to the second phase transition.
Conclusions

1. High quality description of the chemical FO data allowed us to find few novel irregularities at c.m. energies 4.3-4.9 GeV (pressure, entropy density jumps e.t.c.)

2. HRG model with multicomponent repulsion allowed us to find the correlated (quasi)plateaus at c.m. energies 3.8-4.9 GeV which were predicted about 25 years ago. The second set of plateaus may be a signal of another phase transition!?

3. Generalized shock adiabat model allowed us to describe entropy per baryon at chemical FO and determine the parameters of the QGP equation of state from the data.

4. The most interesting ranges of c.m. energies to probe at FAIR and NICA are 3.8-4.9 GeV and 10-13 GeV.
Thank You for Your Attention!
In 1982 J. Rafelski and B. Müller predicted that enhancement of strangeness production is a signal of deconfinement. (Phys. Rev. Lett. 48(1982)) We observe 3 regimes: at c.m. energies 4.3 GeV and 7.6 GeV slope of experimental data drastically changes!

Combining Rafelsky & Muller idea with our result that mixed phase appears at 4.3 GeV we explain this finding:

Below 4.3 GeV Lambdas appear in N+N collisions

Above 4.3 GeV and below 7.6 GeV formation of QGP produces additional s (anti)s quark pairs

Above 7.6 GeV there is saturation due to small baryonic chemical potential
Strangeness Horn and $\Lambda$ Horn in 2014

$\sqrt{S_{NN}}=4.9$ GeV

$\sqrt{S_{NN}}=8.8$ GeV
Minimum of ChFO Volume at AGS Energies

Low quality of the fit did not allow to locate this minimum earlier!

All these irregularities occur at c.m. energies 4.3-4.9 GeV!


Shock Adiabat in Normal Medium

Rankine-Hugoniot-Taub (RHT) adiabat = shock adiabat

connects $\left( X_0, p_0, \rho_{B0} \right)$ and $\left( X, p, \rho_B \right)$ states

$$\rho_B^2 X^2 - \rho_{B0}^2 X_0^2 = (p - p_0)(X + X_0)$$

by conservation laws of energy, momentum and baryonic charge

$X = \frac{\varepsilon + p}{\rho_B^2}$ - generalized specific volume

$\varepsilon$ is energy density, $p$ is pressure, $\rho_B$ is baryonic charge density

$j_B^2 = -\frac{p - p_0}{X - X_0}$ baryonic current is a straight line in $(X - p)$ plane

To solve RHT adiabat we need EOS!

Normal properties, if $\Sigma \equiv \left( \frac{\partial^2 p}{\partial X^2} \right)_{s/\rho_B} > 0$ = convex down:

pure phases have normal properties.

Anomalous properties otherwise.

Usually mixed phase is anomalous!

Shock transitions to region 1-4 are unstable and forbidden!
Details of Hadronic and QGP EOS

- Summation of hadronic spectrum \( \Rightarrow \) (anti)baryonic and mesonic contributions

\[
p = \left[ 2C_B T^A_B \text{ch} \left( \frac{\mu}{T} \right) e^{-\frac{m_B}{T}} + C_M T^A_M e^{-\frac{m_M}{T}} \right] e^{-p_{V_H} T}
\]

- Effective EoS describes (anti)baryonic and mesonic densities at CFO

\[
p_{QGP} = A_0 T^4 + A_2 T^2 \mu^2 + A_4 \mu^4 - B = A^L_0 T^4 + A^L_2 T^2 \mu^2 + A^L_4 \mu^4 - B_{eff}
\]

\[
B_{eff}(T, \mu_B) = B - (A_0 - A^L_0)T^4 - (A_2 - A^L_2)T^2 \mu^2 - A_4 - A^L_4 \mu^4
\]

Stability of Entropy per Baryon at Ch. FO

Despite the difference, both models very well describe pions and baryons!

HRGM: K.A. Bugaev et al., arXiv:1412.0718 [nucl-th]

Formally, in such a treatment two gases are separated by the wall!

HRG: a Multi-component Model

Traditional HRG model: one hard-core radius $R=0.25-0.3$ fm

Overall description of data (mid-rapidity or $4\pi$ multiplicities) is good!

Two hard-core radii: $R_{\text{pi}} = 0.62$ fm, $R_{\text{other}} = 0.8$ fm
G. D. Yen, M. Gorenstein, W. Greiner, S. N. Yang, PRC (1997) 56

Or: $R_{\text{mesons}} = 0.25$ fm, $R_{\text{baryons}} = 0.3$ fm

In both formulations the crossed second virial coefficient is forgotten!
NA49 “Signals” = Irregularities

I. There is NO a single model which can simultaneously describe these «signals»!

II. These «signals» cannot be reproduced by existing hydrodynamic and hydro-cascade models with deconfinement phase transition.

Therefore, their relation to deconfinement is unclear!

Hence, these «signals» are irregularities which require an explanation!

Furthermore, it seems that there is also something wrong with our EOS!