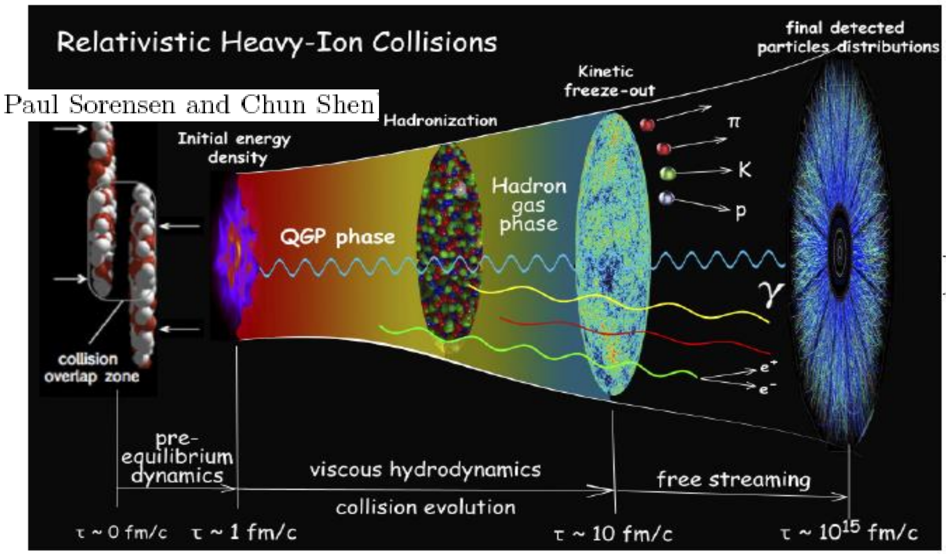
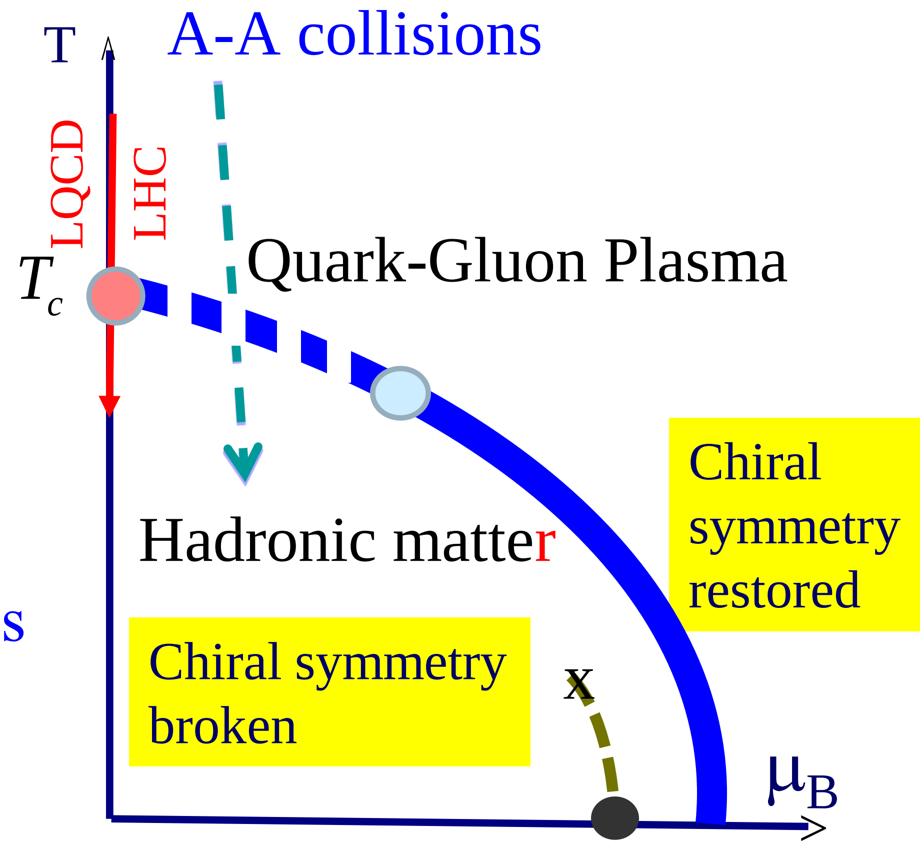
Hagedorn’s mass spectrum and QCD thermodynamics

Krzysztof Redlich, Uni Wroclaw





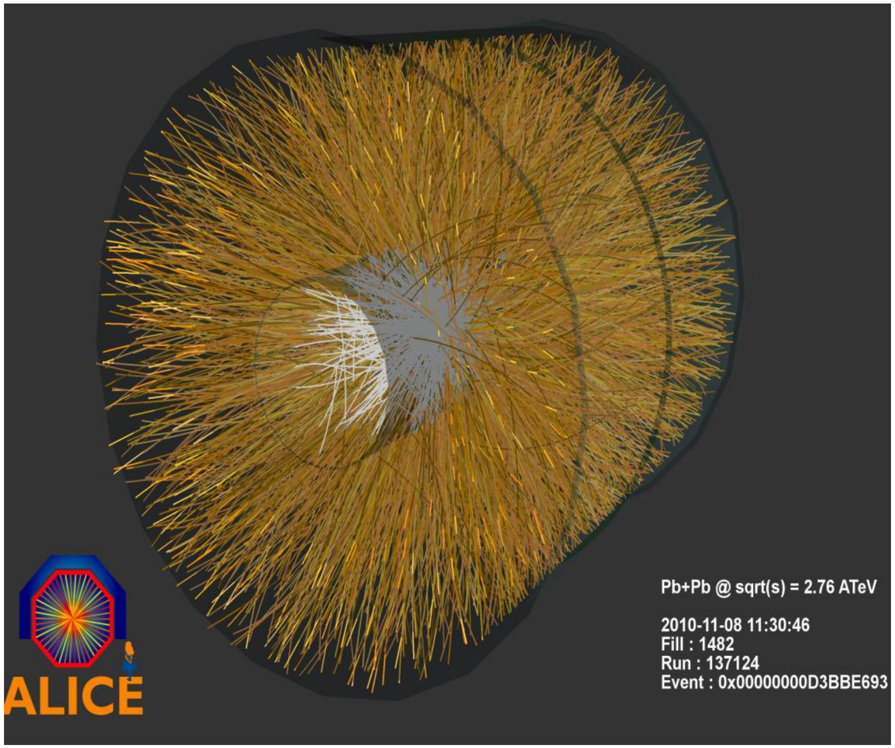
Hagedorn‘s concept of thermodynamics of strong interactions verified through

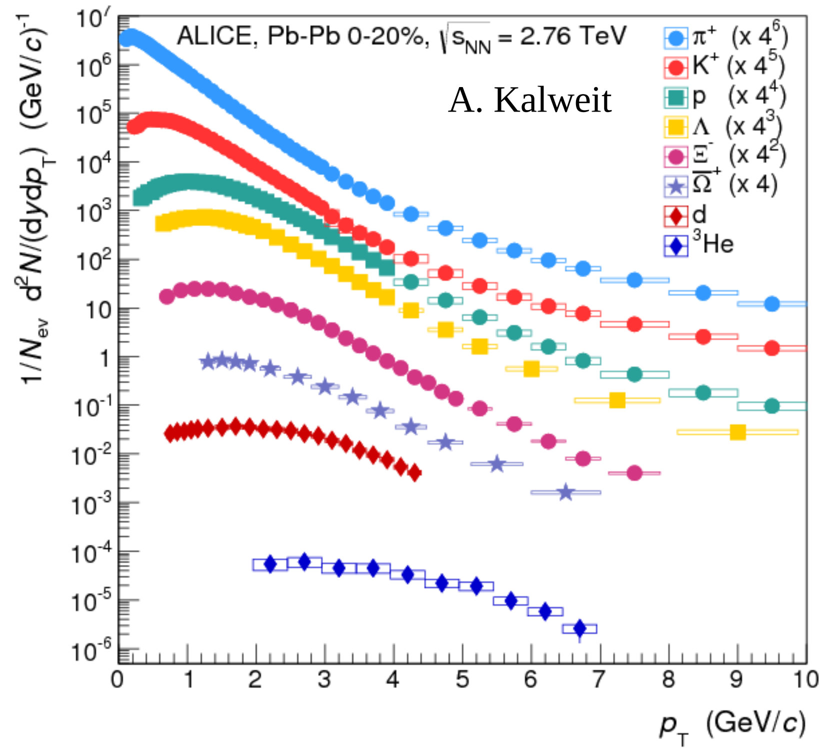
Heavy Ion Phenomenology

Lattice Gauge Theory

Excellent data of ALICE Collaboration for particle yields

ALICE Collaboration





ALICE Time Projection Chamber (TPC), Time of Flight Detector (TOF), High Momentum Particle Identification Detector (HMPID) together with the Transition Radiation Detector (TRD) and the Inner Tracking System (ITS) provide information on the flavour composition of the collision fireball, vector meson resonances, as well as charm and beauty production through the measurement of leptonic observables.

Hagedorn's Thermodynamics and Heavy Ion Collisions

Strongly interacting hadronic matter considered as

thermal medium in chemical equilibrium

- resonance production dominates the interactions in hadronic reactions

clustering of hadrons and particle antiparticle pair creations is included

- all information about interaction is hidden in the mass spectrum $\tau(m^2)\,d(m^2)$

↓

describes the number of hadrons and resonances in the mass interval $d(m^2)$

Statistical operator and mass spectrum

$$\ln(Z) = \int d^4p \frac{2V_\mu p^\mu}{(2\pi)^3} \tau(p^2) e^{-\beta_\mu p^\mu}$$

- $\tau(p^2)$ obtained from the Hagedorn's bootstrap equation

$$\varphi = 2\Phi(\varphi) - e^{\Phi(\varphi)} + 1$$

expressed by Laplace transforms

$$\Phi = \int H \tau(p^2) e^{-\beta_\mu p^\mu} d^4p$$

input function: $$\varphi = \int H[\delta(p^2 - m_\pi{}^2) + \delta(p^2 - m_N{}^2)] e^{-\beta_\mu p^\mu} d^4p$$

- Solution of SB in the limit $m \to \infty$

$$\tau(m^2) \sim m^a e^{\beta_0 m}$$

Hagedorn's statistical operator for discrete mass spectrum

- **resonance dominance (R. Hagedorn)**

$$\ln Z^{GC}(T,\vec{\mu}) = \int d^4p \frac{2V_\mu p^\mu}{(2\pi)^3} \tau(p) e^{-\beta_\mu p^\mu}$$

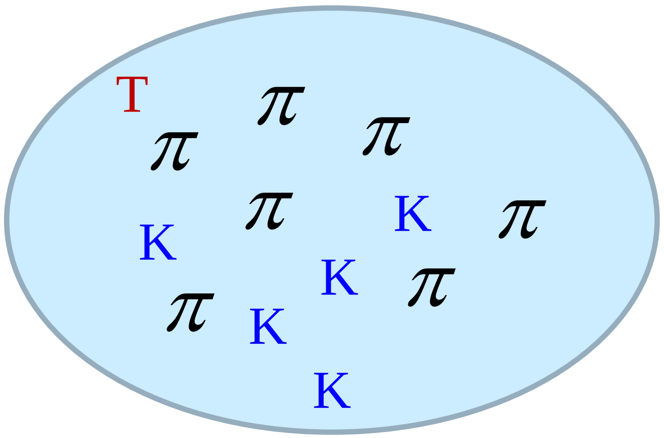
- **approximate** $\tau(m^2)$ **by experimentally known mass spectrum**

$$\ln Z(T,\vec{\mu}) \approx \frac{V}{2\pi^2} \sum_{i \in hadrons} d_i \int p^2 dp \, e^{-\beta E_i}$$

S-MATRIX APPROACH:

R. Dashen, S. K. Ma and H. J. Bernstein,
Phys. Rev. 187, 345 (1969)

W. Weinhold, et al., Phys. Lett. B 433, 236 (1998).



- Consider interacting pion and kaon gas in thermal equilibrium at temperature T
- Due to $K\pi$ scattering resonances are formed

  I =1/2, s -wave : κ(800), K0*(1430) [*JP* = 0+]

  I =1/2, p -wave : *K**(892), *K**(1410), *K**(1680) [*JP* =1−]
- In the S-matrix approach the thermodynamic pressure in the low density approximation

$$P(T) \approx P_{\pi}^{id} + P_{K}^{id} + P_{\pi K}^{\text{int}}$$

$$P_T = P^{id} / T^4 = -\int \frac{d^3\hat{p}}{(2\pi)^3} \Big\{ \ln[1 - e^{-\sqrt{\hat{p}^2+\hat{M}^2}-\hat{\mu}_S}] + \ln[1 - e^{-\sqrt{\hat{p}^2+\hat{M}^2}+\hat{\mu}_S}] \Big\}$$

S-MATRIX APPROACH INTERACTING PART

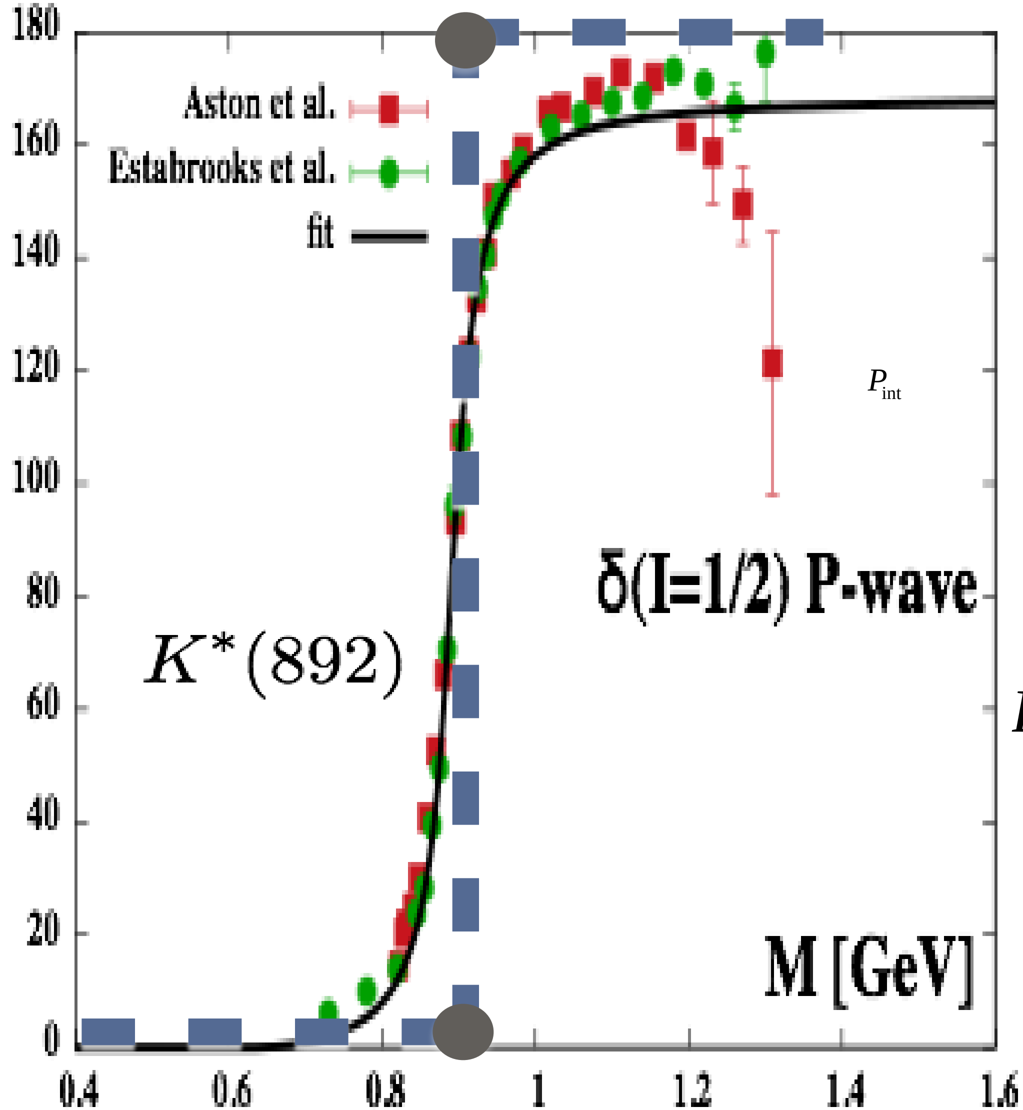
$$\hat{P}_T(M) = -2\int \frac{d^3\hat{p}}{(2\pi)^3}\left\{\ln[1 - e^{-\sqrt{\hat{p}^2+\hat{M}^2}-\hat{\mu}}]\right.$$
$$\left. + \ln[1 - e^{-\sqrt{\hat{p}^2+\hat{M}^2}+\hat{\mu}}]\right\}$$

$$\hat{P}_{int} = \int_{m_{th}}^{\infty} \frac{dM}{2\pi}\mathcal{B}(M)\hat{P}_T(M)$$

$$\mathcal{B}(M) = 2\frac{d}{dM}\delta(M)$$

Scattering phase shift

Experimental phase shift in P-wave channel



B. Friman et al, arXiv:1507.04183

For narrow resonance

$$\mathcal{B}(M) = 2\frac{d}{dM}\delta(M)$$

very well described by the Breit-Wigner form

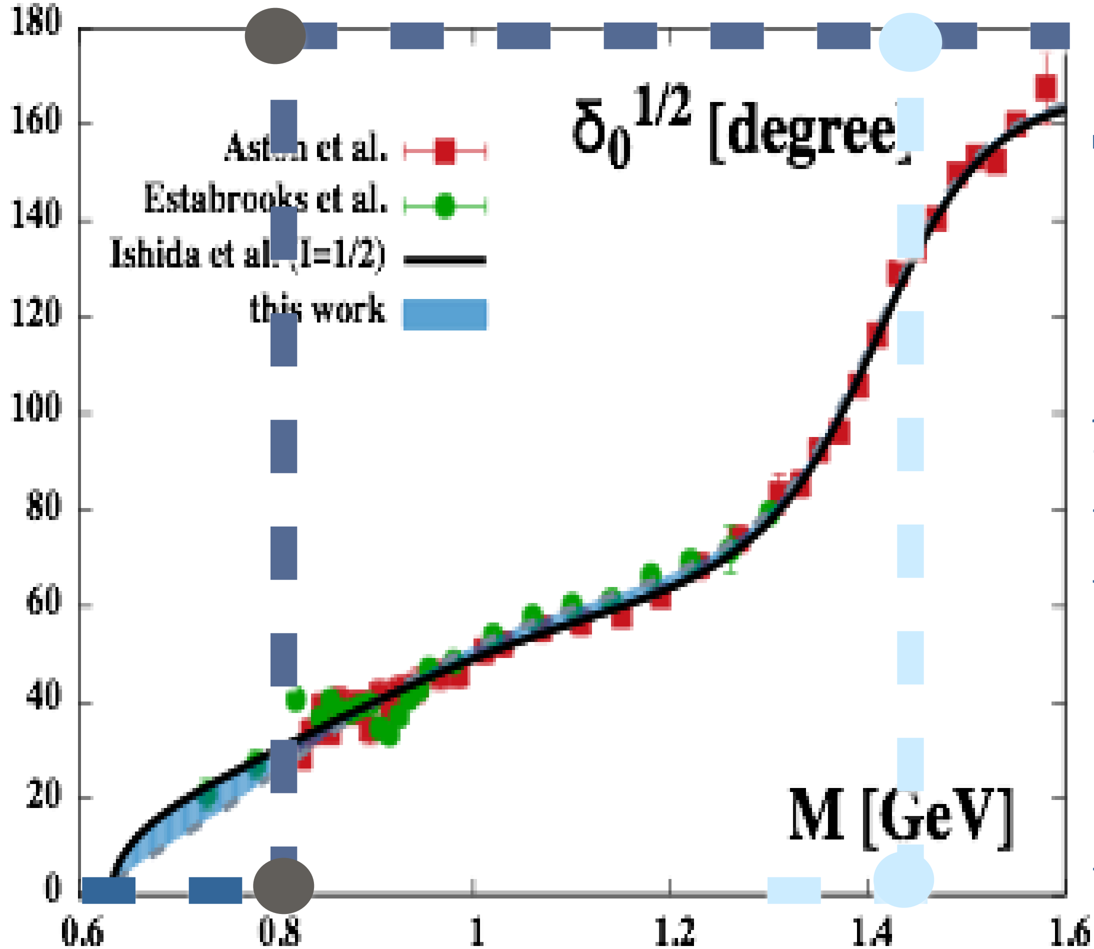
$$B(M) \approx M\frac{2M\gamma_{\rm BW}}{(M^2 - M_0^2)^2 + M^2\gamma_{\rm BW}^2}$$

for $\gamma_{BW} \to 0$

$$B(M) = \delta(M^2 - M_0^2)$$ **and**

$$P^{\rm int}_{\pi K}(T) \approx P^{id}_{K^*}(T)$$

Experimental phase shift in S channel



The kappa resonance is a special example that:

for broad resonances their contribution has to be taken with a special care, by considering the experimental or theoretically calculated phase shift

B. Friman et al, arXiv:1507.04183

Hagedorn Statistical operator in HIC

- **The Hagedorn statistical sum with the PDG discrete mass spectrum**

$$\ln Z(T,\vec{\mu}) \approx \frac{VT}{2\pi^2} \sum_{i\in hadrons} d_i e^{\frac{\vec{Q}_i \vec{\mu}}{T}} \int ds\, s\, K_2\left(\frac{\sqrt{s}}{T}\right) F^{B-W}(m_i, s)$$

- **and its particle composition**

particle yield **thermal density** **BR** **thermal density of resonances**

$$< N_i > = V\,[n_i^{th}(T,\mu_B) + \sum_K \Gamma_{K\to i}\, n_i^{th-Res.}(T,\mu_B)]$$

- **Only 2-parameters needed to fix all particle yield ratios**

Thermal origin of particle yields with respect to HRG

Rolf Hagedorn => the Hadron Resonace Gas (HRG):

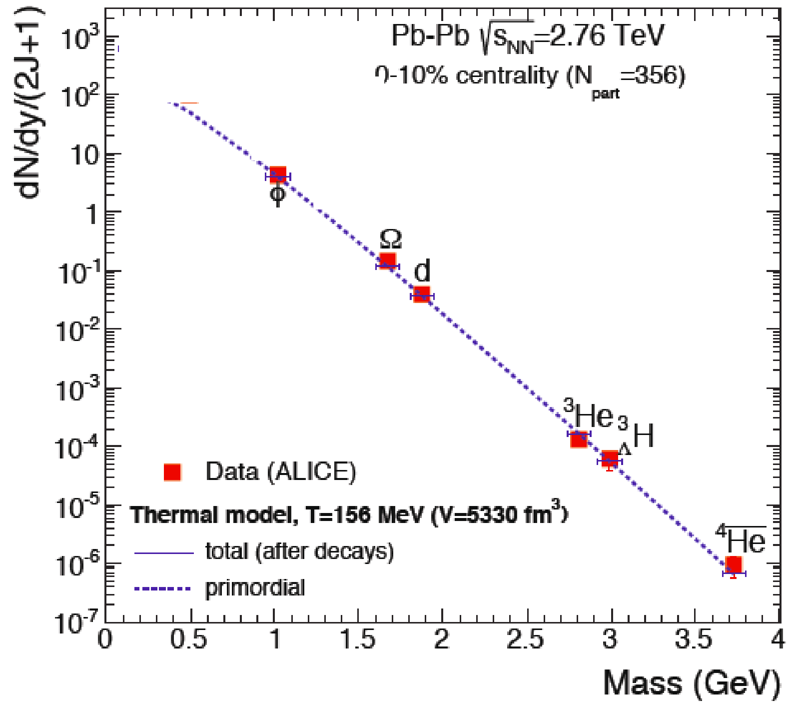
"uncorrelated" gas of hadrons and resonances

$$< N_i > = V\,[n_i^{th}(T,\vec{\mu}) + \sum_K \Gamma_{K\to i}\, n_i^{th-Res.}(T,\vec{\mu})]$$

A. Andronic, Peter Braun-Munzinger, & Johanna Stachel, et al.

Particle yields with no resonance decay contributions:

$$\frac{1}{2j+1}\frac{dN}{dy} = V(m/T)^2 K_2(m/T)$$



- Measured yields are reproduced with HRG at $T \approx 156$ MeV

Thermal origin of particle yields with respect to HRG

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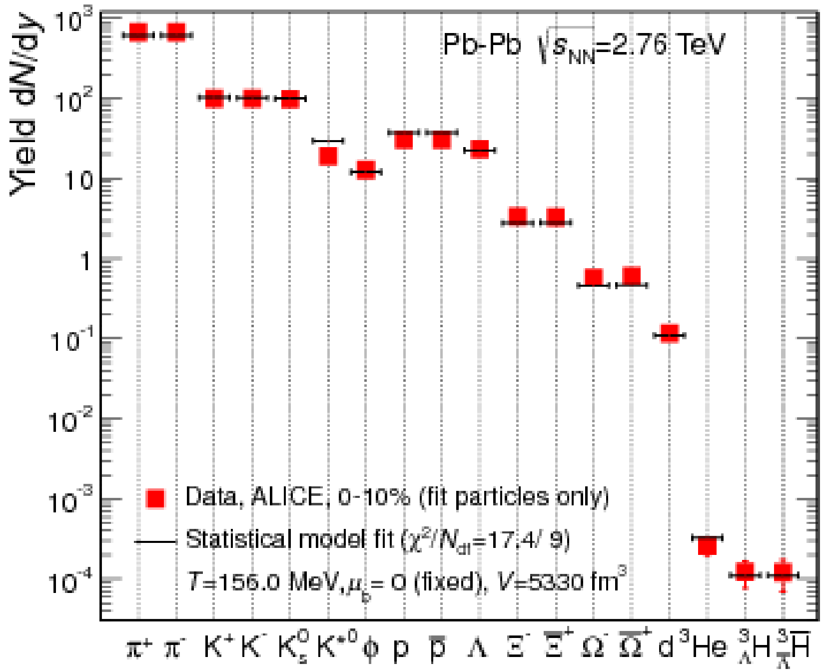
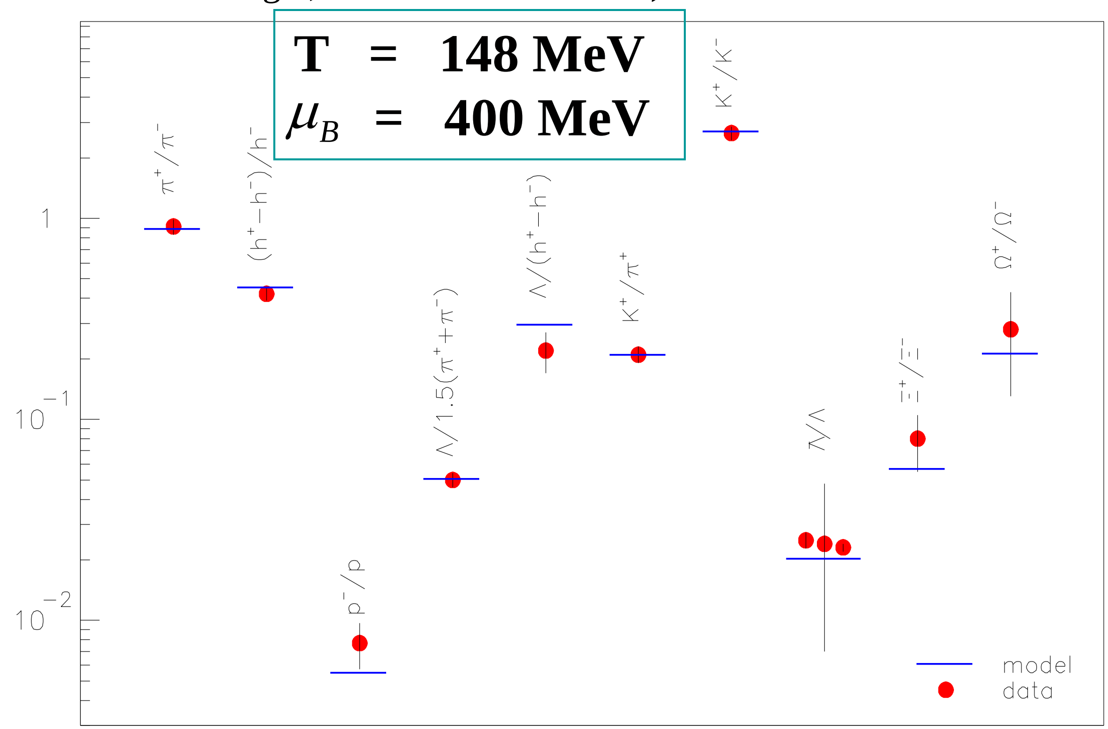
$$< N_i > = V\,[n_i^{th}(T,\vec{\mu}) + \sum_K \Gamma_{K \to i}\, n_i^{th-Res.}(T,\vec{\mu})]$$

A. Andronic, Peter Braun-Munzinger, & Johanna Stachel,



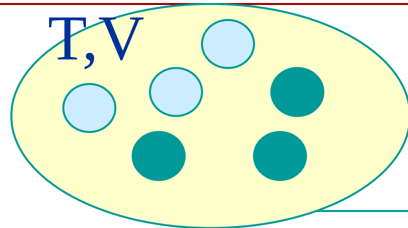


40 AGeV Pb-Pb collisions

Kinetics of abelian charges

C.M. Ko, V. Koch, Z. Lin, M. Stephanov, Xin-Nian Wang, K.R (2001)

Consider:



$$a + b \leftrightarrow q^{+} + q^{-}$$

Rate equation

$$\frac{d < N >}{dt} = \frac{G}{V} < a >< b > - \frac{L}{V} < N^2 >$$

$$< N^2 > = < N >^2 + < \delta N^2 >$$

$$\Downarrow < N >$$

:

Size of fluctuations $\Longleftrightarrow$ Equilibrium limit

$$< N > >> 1 \qquad\qquad < N > << 1$$

$$n^{GC} \approx m_{q^+}{}^2 T K_2\left(\frac{m_{q^+}}{T}\right) \qquad n^{C} \approx m_{q^+}^2 T K_2\left(\frac{m_{q^+}}{T}\right) \times V m_{q^-}{}^2 T K_2\left(\frac{m_{q^-}}{T}\right)$$

Pair production due to an exact conservation law,

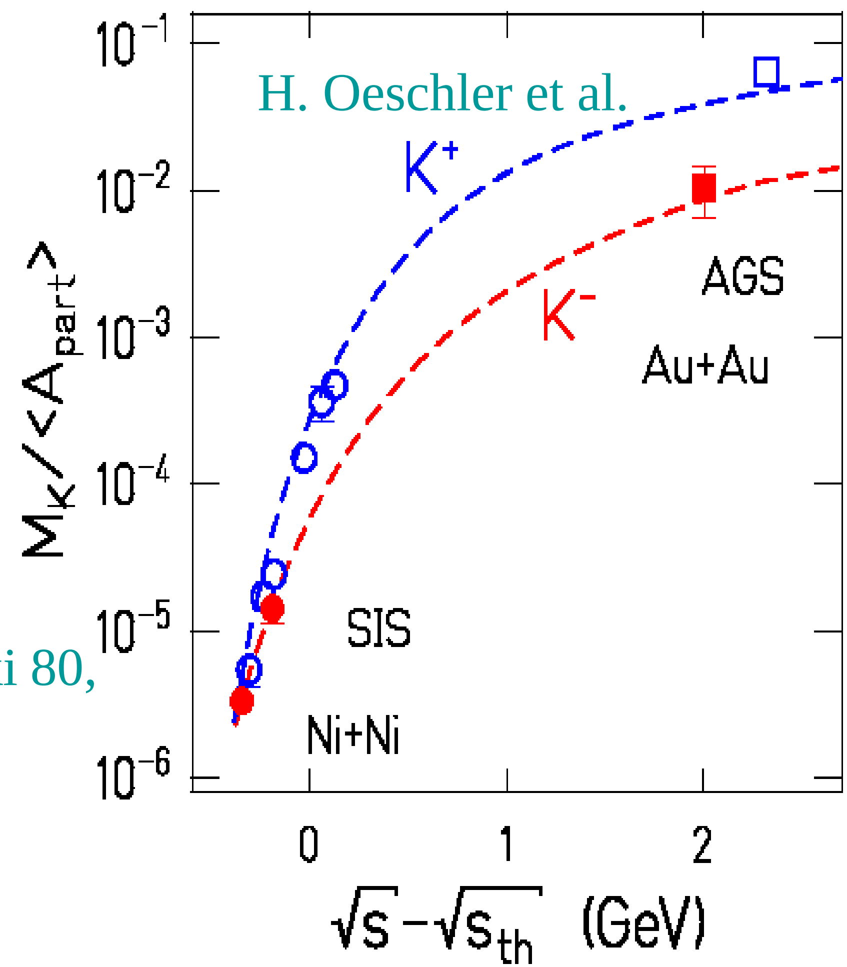
R. Hagedorn 70

Correlated strangeness production

$$N + N \rightleftarrows K^+ + Y + N \;\Rightarrow\; < K^+ >^C \sim VV_0 e^{-m_k/T} e^{-(m_Y - \mu_B)/T}$$

$$\pi + Y \rightleftarrows K^- + N$$

- If the number of produced particles carrying conserved charges is small, then it is essential to describe their thermodynamics in the canonical ensemble with respect to conservation laws

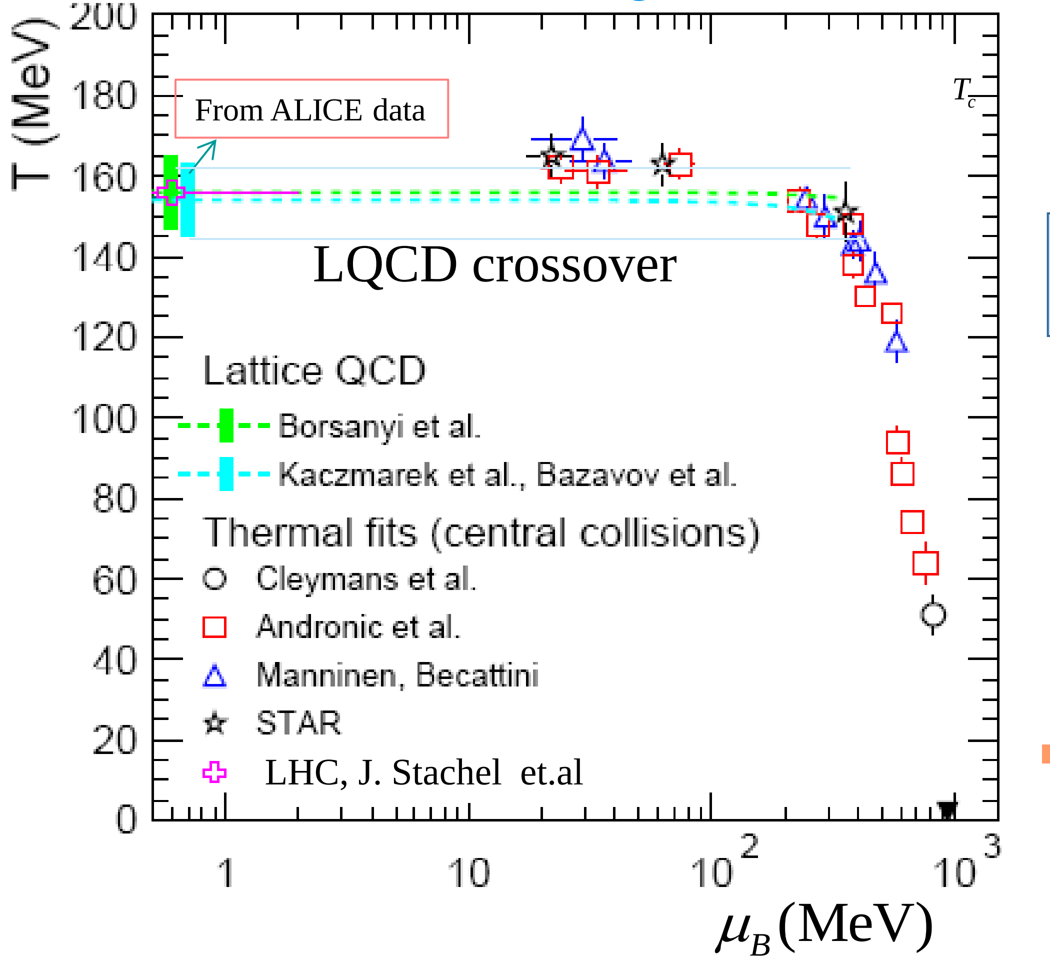


R. Hagedorn 70, E. Shuryak 72, Danos & Rafelski 80, Turko & K.R. 80, M. Gorenstein et al. 81, Rafelski & Muller 82,Hagedorn & K.R. 85, J. Cleymans et al. 91, Oeschler et al. 99, Tounsi & K.R. 2000, …..

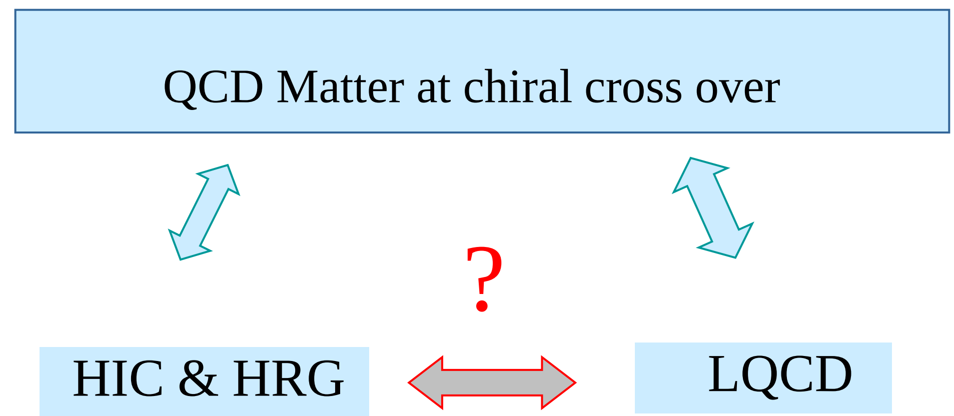
Chemical Freeze out and QCD Phase Boundary

Chemical freeze out defines a lower bound for the QCD phase boundary

A. Andronic, P. Braun-Munzinger, K.R. & J. Stachel



- The QCD phase boundary coincides with chemical freeze out conditions obtained from HIC data analyzed with the HRG model



- The HRG should describe the QCD thermodynamics in the hadronic phase

QCD at non-vanishing chemical potential $\mu_q > 0$

Bielefeld-Swansea Coll.

$$Z(V,T,\mu) = \int DA \det M(\mu) e^{-S(V,T)}$$

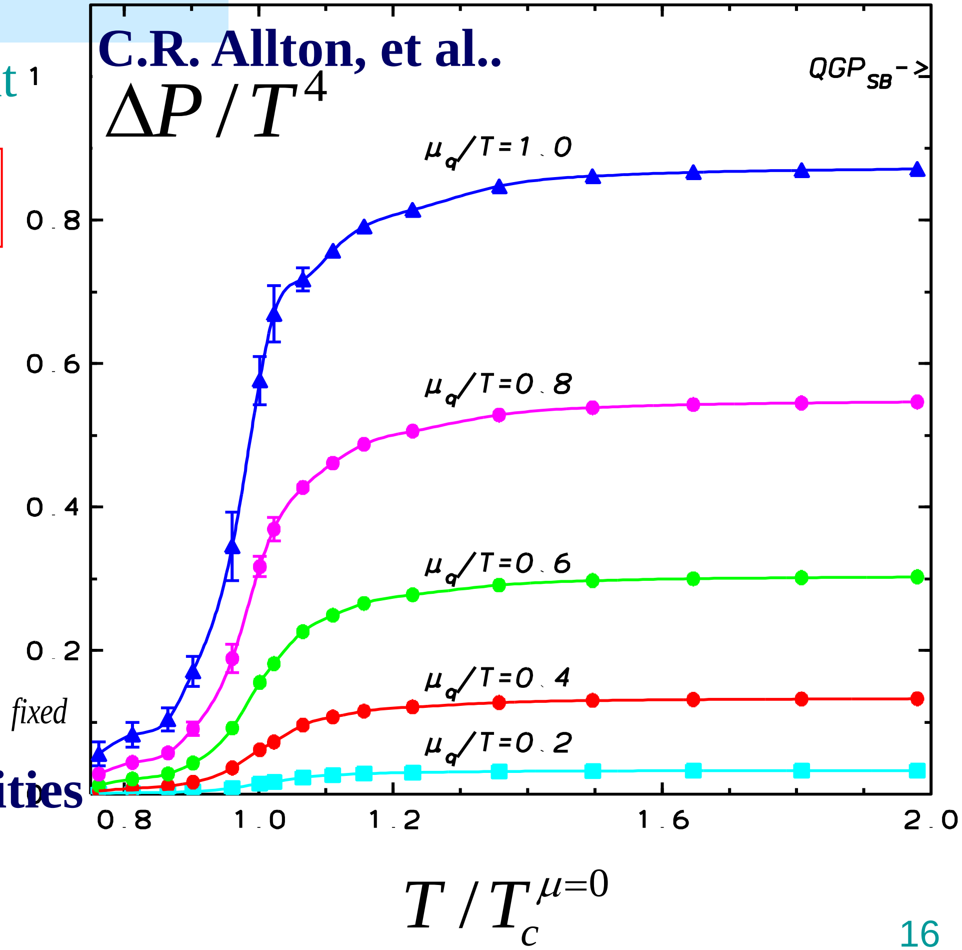
complex fermion determinant [1]

$$\Delta P = P(\mu) - P(0)$$

Taylor expansion of $P(\mu/T)$:

$$\frac{P(T,\mu)}{T^4} = \sum_{n=0}^{\infty} c_{2n}(T)\left(\frac{\mu}{T}\right)^{2n}$$

$$\frac{n_q}{T^3} = \left(\frac{\partial}{\partial(\mu/T)}\frac{P}{T^4}\right)_{T\ fixed}, \quad \frac{\chi_q}{T^2} = \left(\frac{\partial^2}{\partial(\mu/T)^2}\frac{P}{T^4}\right)_{T\ fixed}$$



From μ dependence of chiral susceptibilities

$$\frac{T_c(\mu)}{T_c(0)} \approx 1 - \alpha(m_q)\left(\frac{\mu}{T_c(0)}\right)^2$$

Taylor expansion of resonance pressure

$$P = P^M + P^B$$

baryon mass spectrum

$$\rho(m) \approx m^c e^{bm}$$

Factorization of the baryonic pressure

$$\frac{\Delta P_B}{T^4} \approx F(T)\left(\cosh\left(\frac{3\mu_q}{T}\right) - 1\right)$$

$$F(T) = \frac{1}{2\pi^2}\int dm\,\rho(m)\left(\frac{m}{T}\right)^2 K_2\left(\frac{m}{T}\right)$$

Compare with LGT results:

$$\frac{\Delta P}{T^4} \approx F(T)\left[c_2\left(\frac{\mu_q}{T}\right)^2 + c_4\left(\frac{\mu_q}{T}\right)^4 + c_6\left(\frac{\mu_q}{T}\right)^6\right]$$

$$\frac{n_q}{T^3} \approx F(T)\left[2c_2\left(\frac{\mu_q}{T}\right) + 4c_4\left(\frac{\mu_q}{T}\right)^3 + 6c_6\left(\frac{\mu_q}{T}\right)^5\right]$$

$$\frac{\chi_q}{T^2} \approx F(T)\left[2c_2 + 12c_4\left(\frac{\mu_q}{T}\right)^2 + 30c_6\left(\frac{\mu_q}{T}\right)^4\right]$$

Consequences:

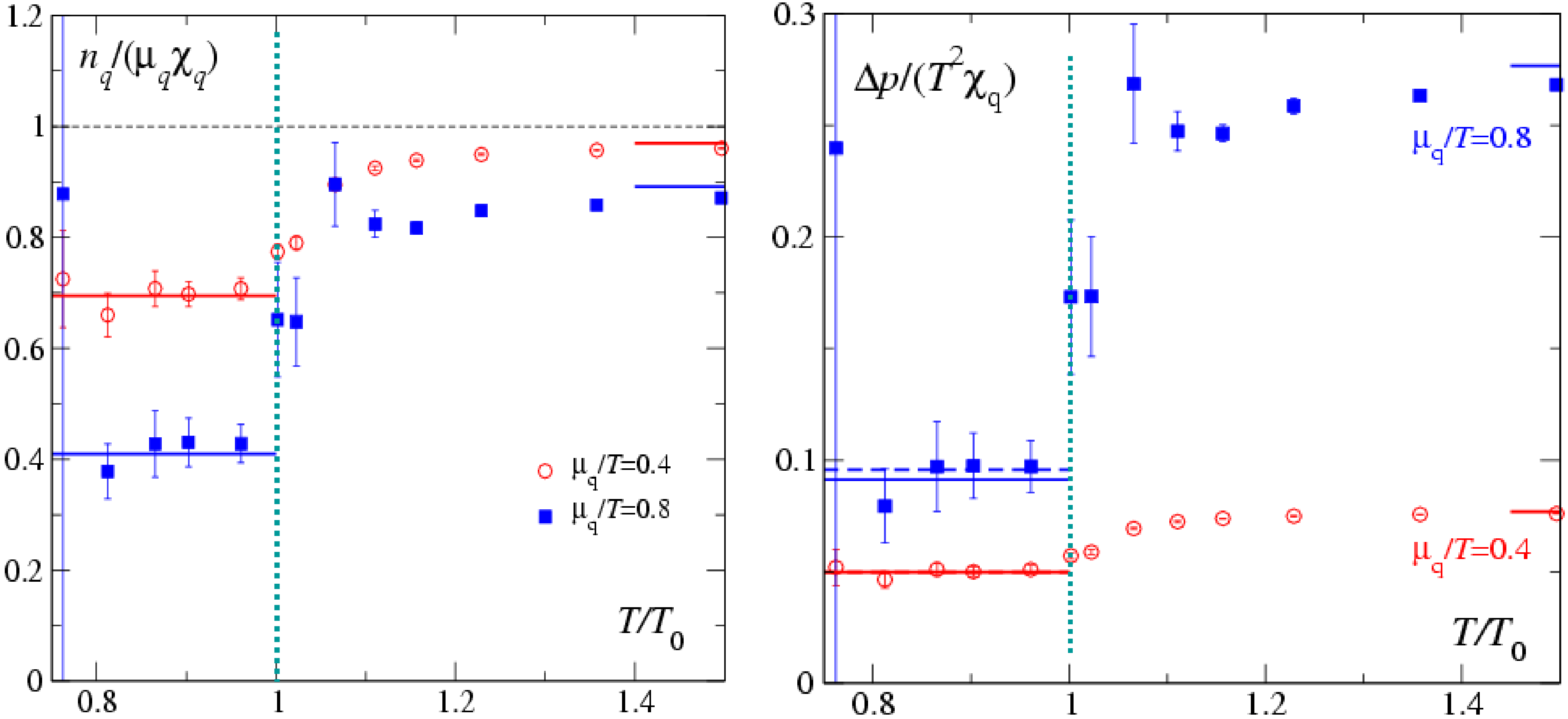
For fixed μ_q / T any ratio of these observables is T-independent

the ratio of the O(2) and O(4) coefficients:

$$\frac{c_4}{c_2} = \frac{3}{4}, \ldots, \frac{c_6}{c_4} = 0.3$$

(μ / T) -- Dependence in LGT consistent with the Hagedorn partition function

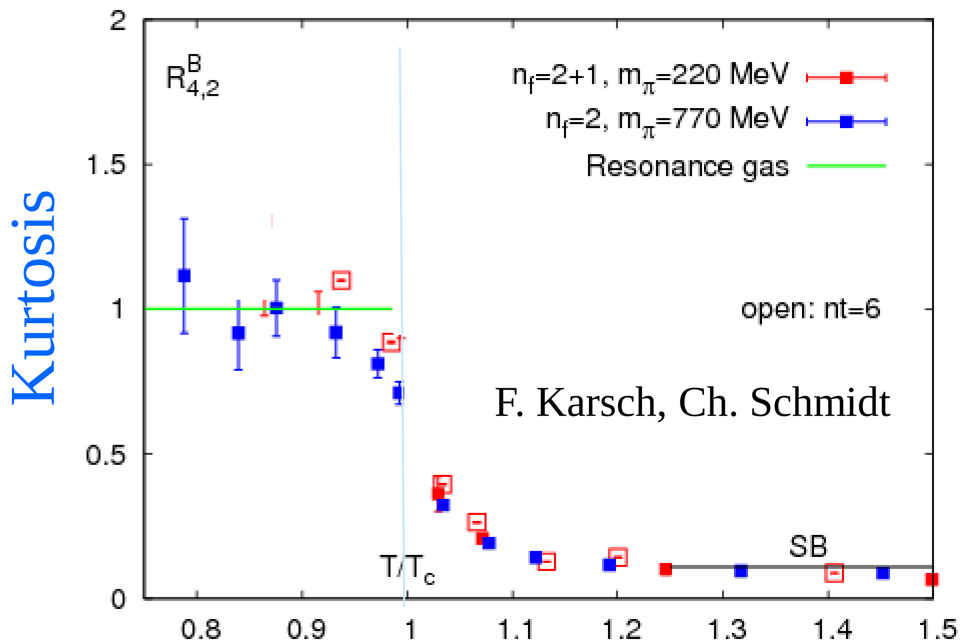
C.R. Alton et al., Phys.Rev. D71 (2005) 054508



Kurtosis as an excellent probe of deconfinement

S. Ejiri, F. Karsch & K.R.

$$R_{4,2}^{B} = \frac{1}{9}\frac{c_4}{c_2}$$



The $R_{4,2}^{B}$ measures the quark content of particles carrying baryon number

- HRG factorization of pressure:

$$P^B(T,\mu_q) = F(T)\cosh(3\mu_q/T)$$

consequently: $c_4/c_2 = 9$ in HRG

- In QGP, $SB = 6/\pi^2$
- Kurtosis=Ratio of cumulants

$$\kappa\,\sigma^2 = \frac{\chi_4^B}{\chi_2^B} \approx B^2 = \begin{pmatrix} 1 & T < T_{PC} \\ \frac{1}{9} & T > T_{PC} \end{pmatrix}$$

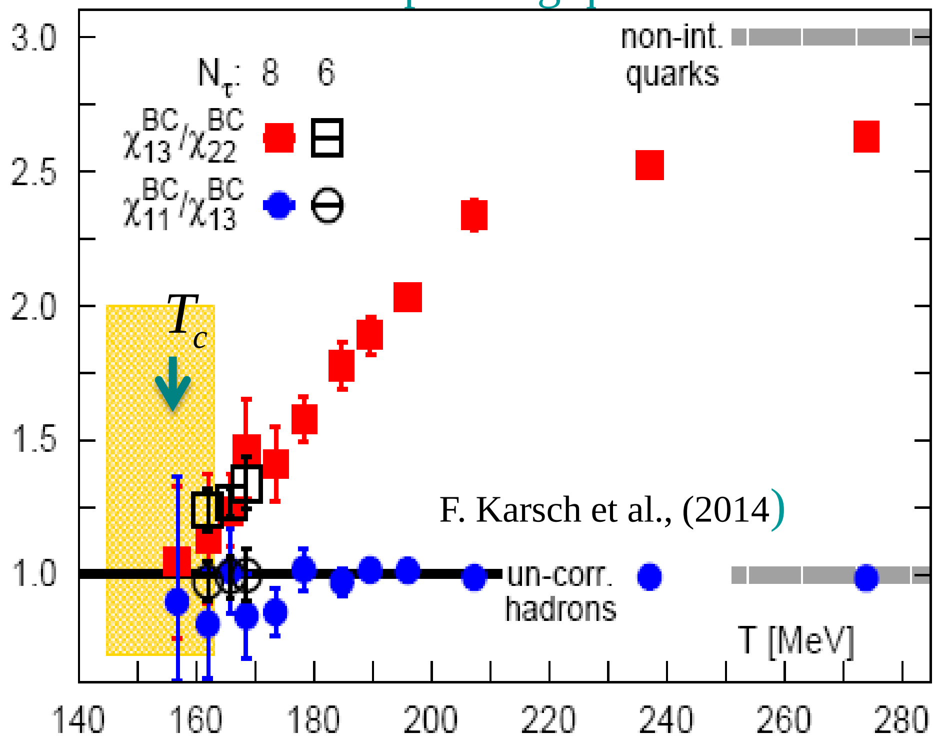
excellent probe of deconfinement

Charm deconfinement in LQCD

Ratios of cumulants

$$\chi_{n,m}^{B,C} = \frac{1}{T^4} \frac{\partial^{(n+m)} P(\mu_B, \mu_C)}{\partial \mu_B^n \partial \mu_C^m} |_{\mu=0}$$

are sensitive to the degrees of freedom that are carriers of the corresponding quantum numbers



Factorized pressure in the HRG and sQGP

$$P(T, \vec{\mu}) = F(m/T) \cdot \cosh(B\mu_B + C\mu_C)$$

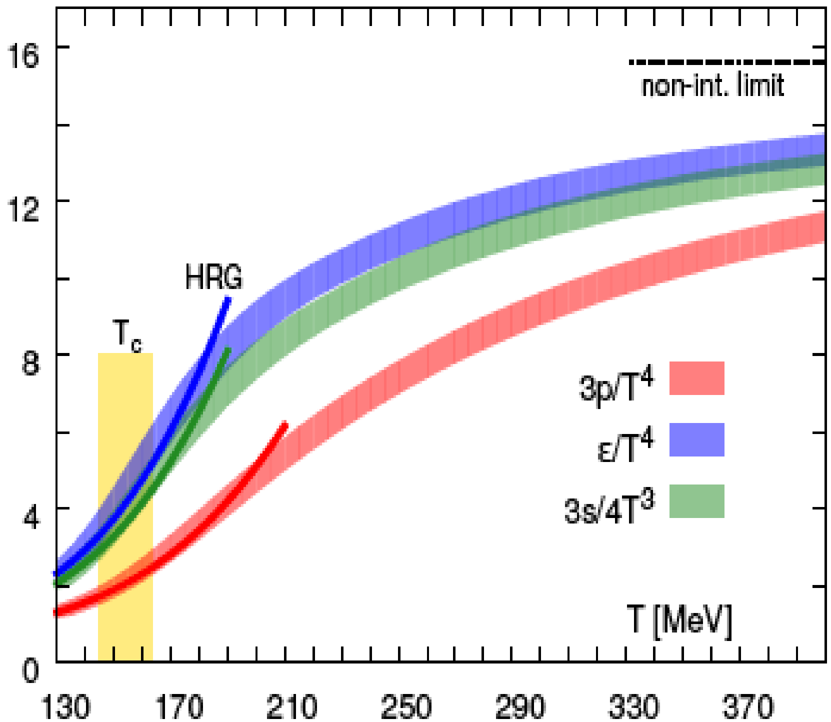
$$\frac{\chi_{1,3}^{B,C}}{\chi_{2,2}^{B,C}} \approx \frac{C}{B} = \begin{pmatrix} 1 & T < T_c \\ 3 & T \gg T_c \end{pmatrix}$$

$$\frac{\chi_{1,1}^{B,C}}{\chi_{1,3}^{B,C}} \approx \frac{1}{C^2} = \begin{pmatrix} 1 & T < T_c \\ 1 & T \gg T_c \end{pmatrix}$$

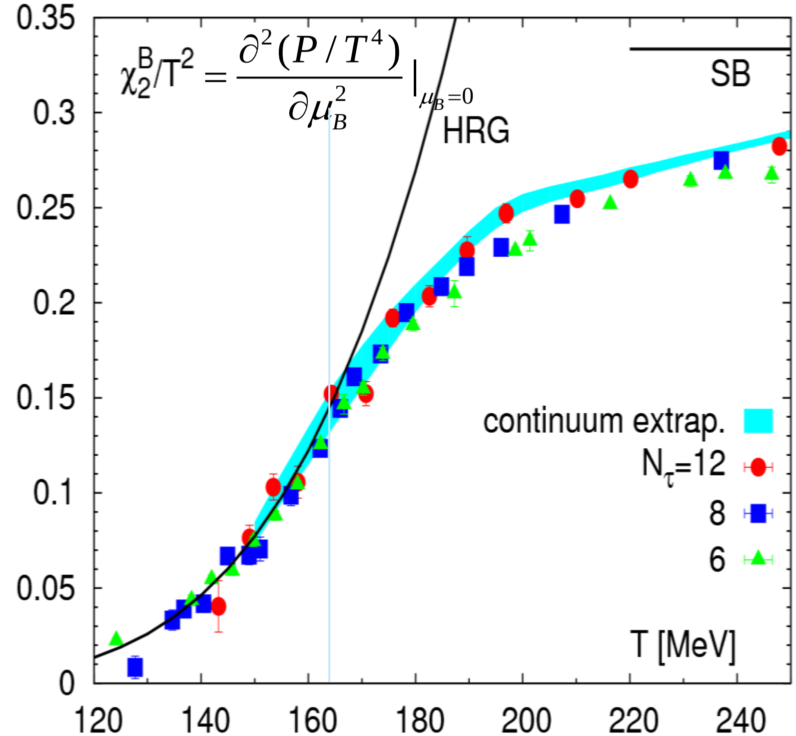
- For $T > T_c$, charmed degrees of freedom can no longer be described by hadronic states.
- The dissociation of open charm hadrons and the emergence of deconfined charm sets in just near the chiral crossover transition.

Excellent description of the QCD Equation of States by Hadron Resonance Gas

A. Bazavov et al. HotQCD Coll. July 2014



F. Karsch et al. HotQCD Coll.

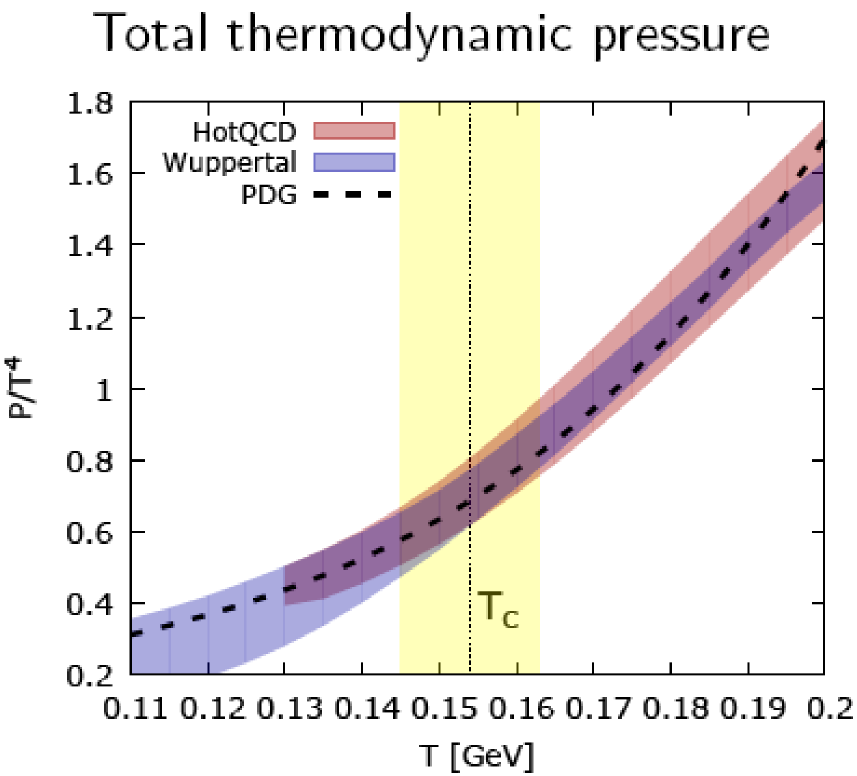


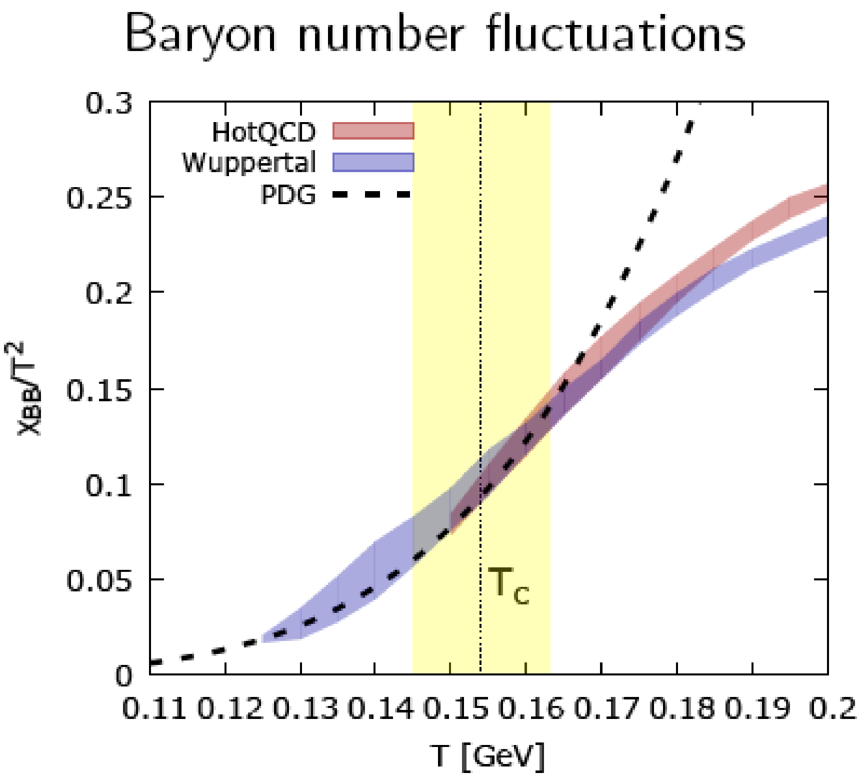
- Hagedorn Gas thermodynamic potential provides an excellent description of the QCD equation of states in the confined phase

- As well as, an excellent description of the net-baryon number fluctuations

Combined data of Hot QCD and Budapest-Wuppertal Coll.

P. M. Lo arXiv:1507.06398





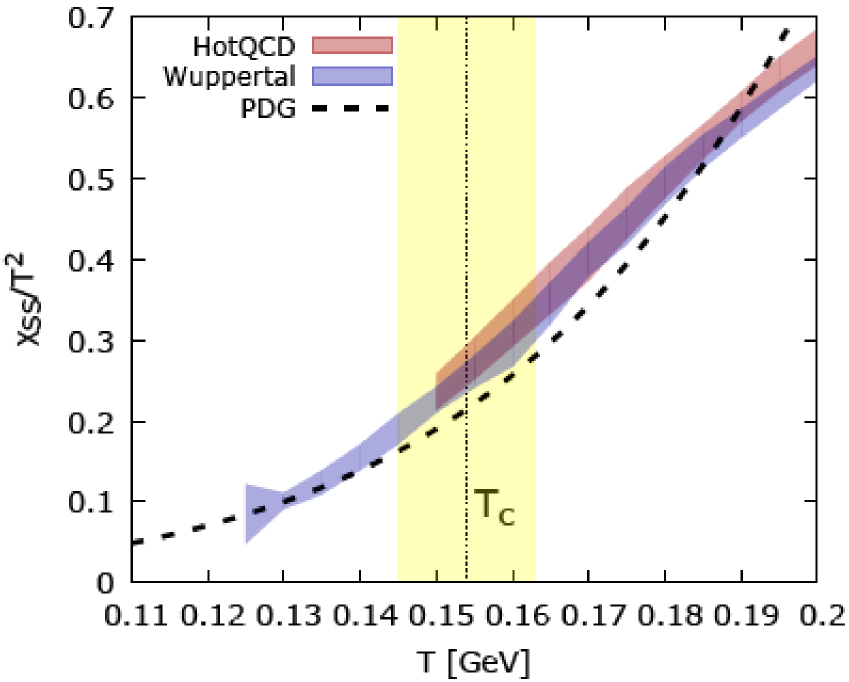
- Consistent description of the equation of state up to the chiral crossover by the HRG

Missing resonances in the strangeness sector

A. Bazavov, et al. Phys. Rev. Lett. 113 (2014)

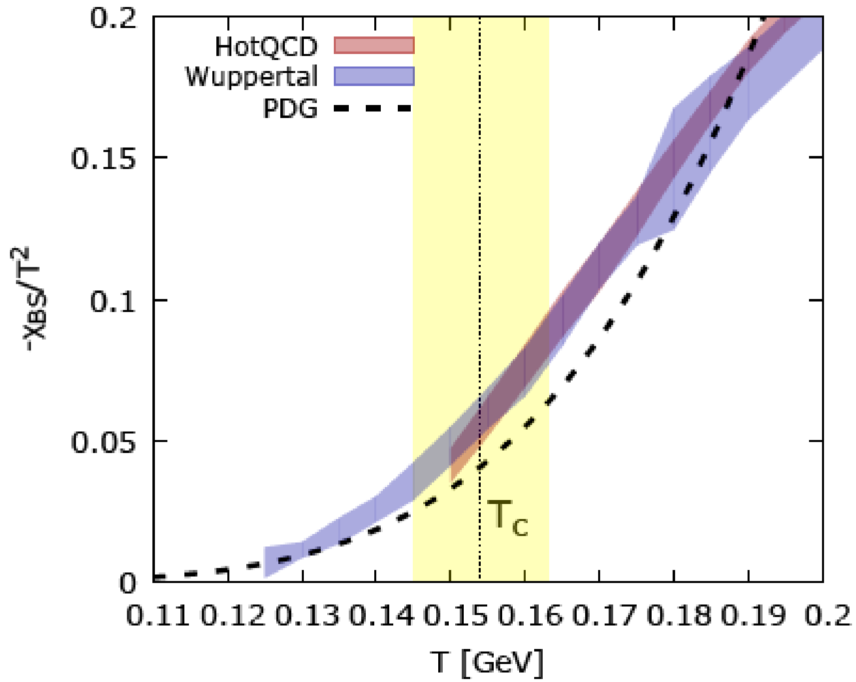
$$\chi_{SS} = \partial^2 P / \partial \mu_S^2$$

Strange mesonic sector



$$\chi_{BS} = \partial^2 P / \partial \mu_B \partial \mu_S$$

Strangeness fluctuations

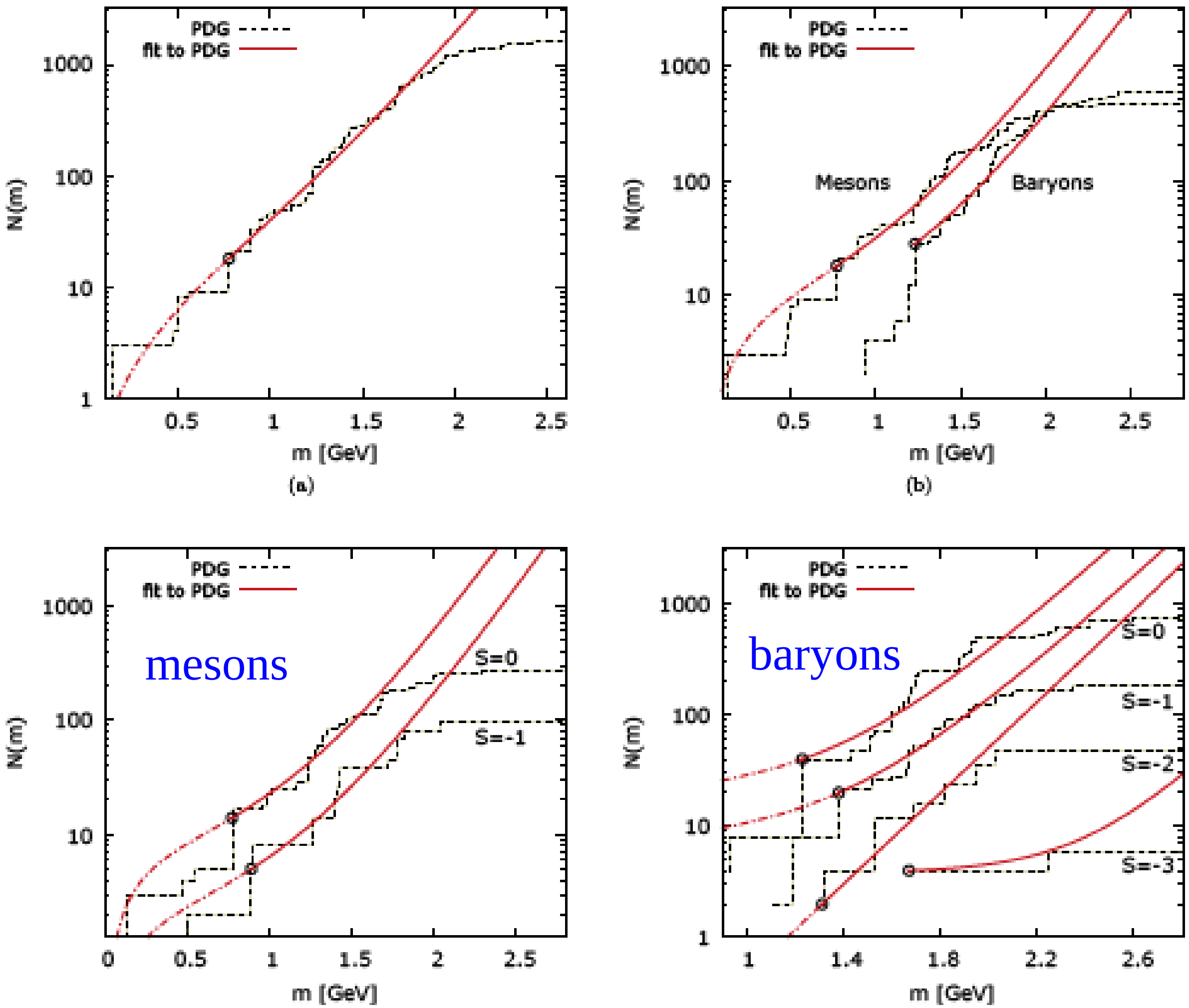


- Go beyond PDG and include resonances in the Hagedorn's continuum mass spectrum $\rho(m) \underset{m \to \infty}{\to} m^a e^{m/T_H}$

P. M. Lo arXiv:1507.06398

Hagedorn's spectrum: parameters from the PDG data

P. M. Lo arXiv:1507.06398



- discrete mass spectrum

$$\rho(m) = \sum_i d_i \delta(m - m_i)$$

- The same information can be stored in the cumulant

$$N(m) = \sum_i d_i \theta(m - m_i)$$

such that $\rho = \partial N / \partial m$

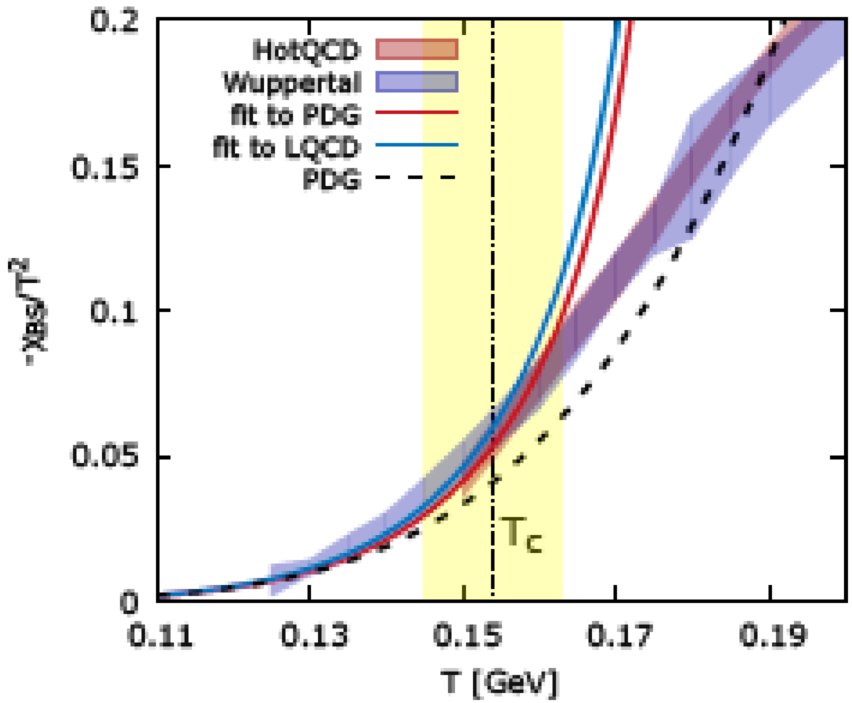
We use the following form for the mass spectrum and fit parameters to PDG

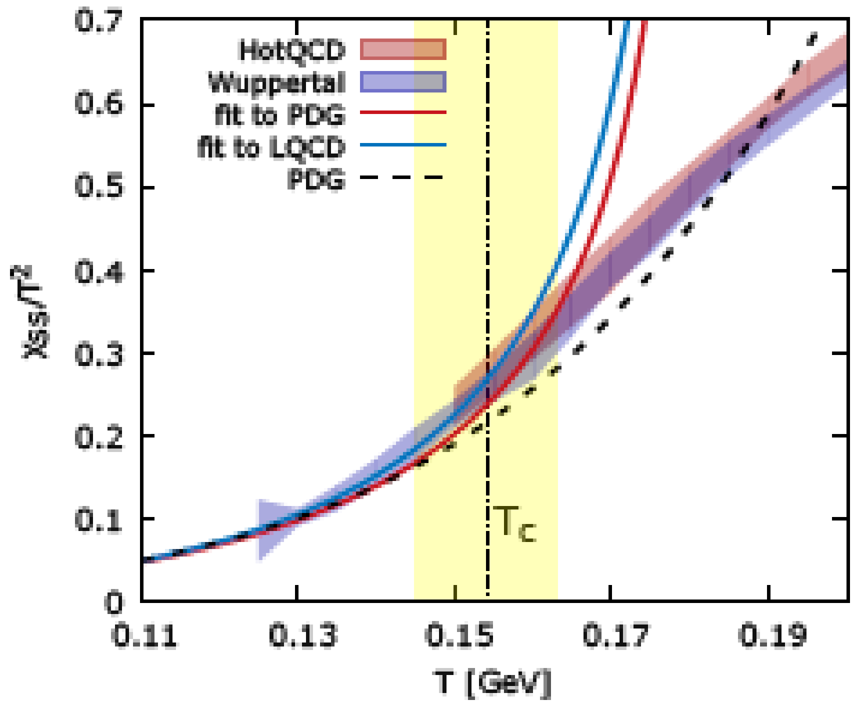
$$\rho^H(m) = \frac{A\, e^{m/T_H}}{(m^2 + m_0^2)^{5/4}},$$

$$\rho(m) = \sum_{G.S.} d_i \delta(m - m_i) + \theta(m - m_x)\rho^H(m)$$

$T_H = 0.18$ GeV common for all mesons and baryons in different sectors of quantum numbers

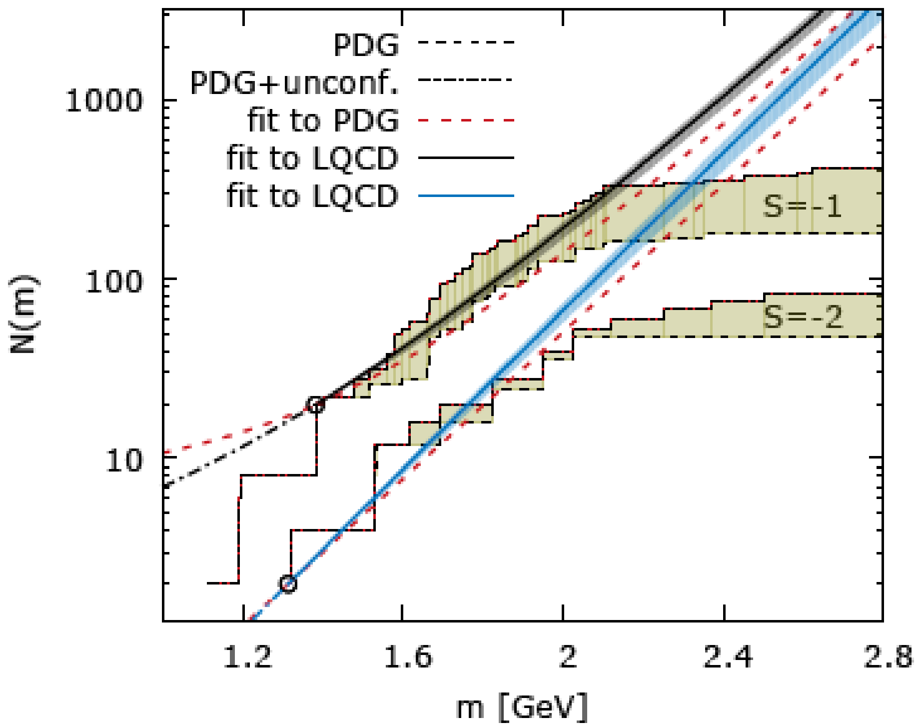
Hagedorn’s continuum mass spectrum contribution to strangeness fluctuations

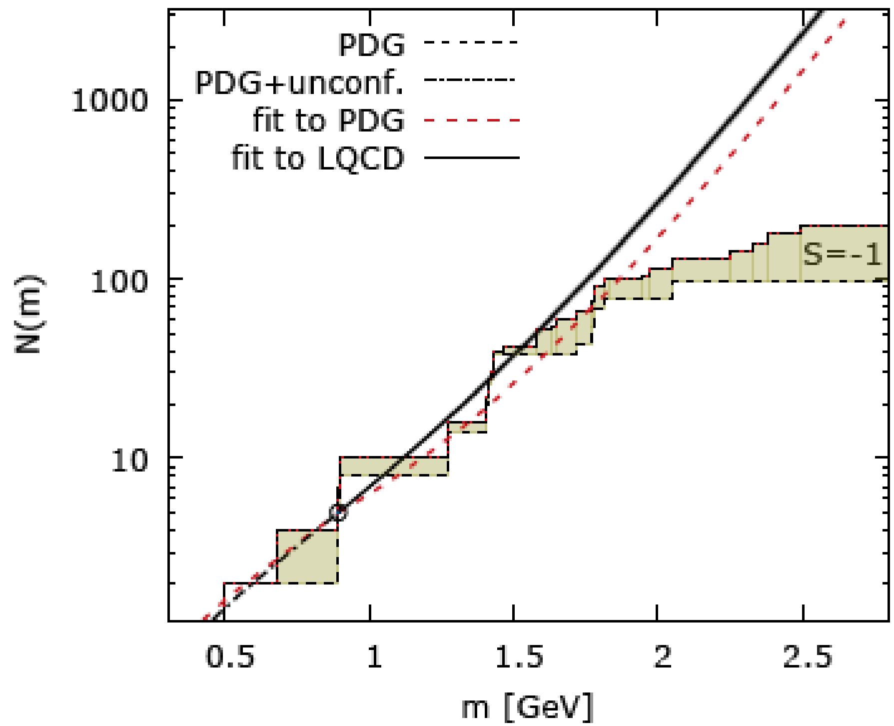




- Satisfactory description of LGT with asymptotic states from Hagedorn’s spectrum fitted to PDG
- To find optimal results: extract $\rho^H(M)$ from LGT and compare with PDG that includes expected new states

Missing resonances in the PDG:





- In the strange baryon sector the optimal mass spectrum extracted from LGT is consistent with that expected and unconfirmed states in the PDG
- In the strange meson sector one expects new resonances with the mass M< 2 GeV

Conclusions

- Hagedorn's vision and theoretical description of thermodynamics of strong interactions is excellently confirmed by HIC data and the 1-st principle LQCD calculations:
  - There is a continuous justification of the exponential hadronic mass spectrum
  - The Hagedorn's statistical operator of Hadron Resonance Gas (HRG) provides a very fair description of particle production yields in HIC from SIS up to LHC
  - The HRG is an excellent approximation of the QCD partition function in the hadronic phase,

  however, a strong deviations from HRG are expected for observables which are sensitive to critical chiral dynamics