Appearance of Hagedorn limiting temperature in microscopic model calculations

E. Zabrodin (UiO & SINP MSU),

results obtained in collaboration with

L. Bravina, M. Gorenstein and Frankfurt ITP/FIAS group

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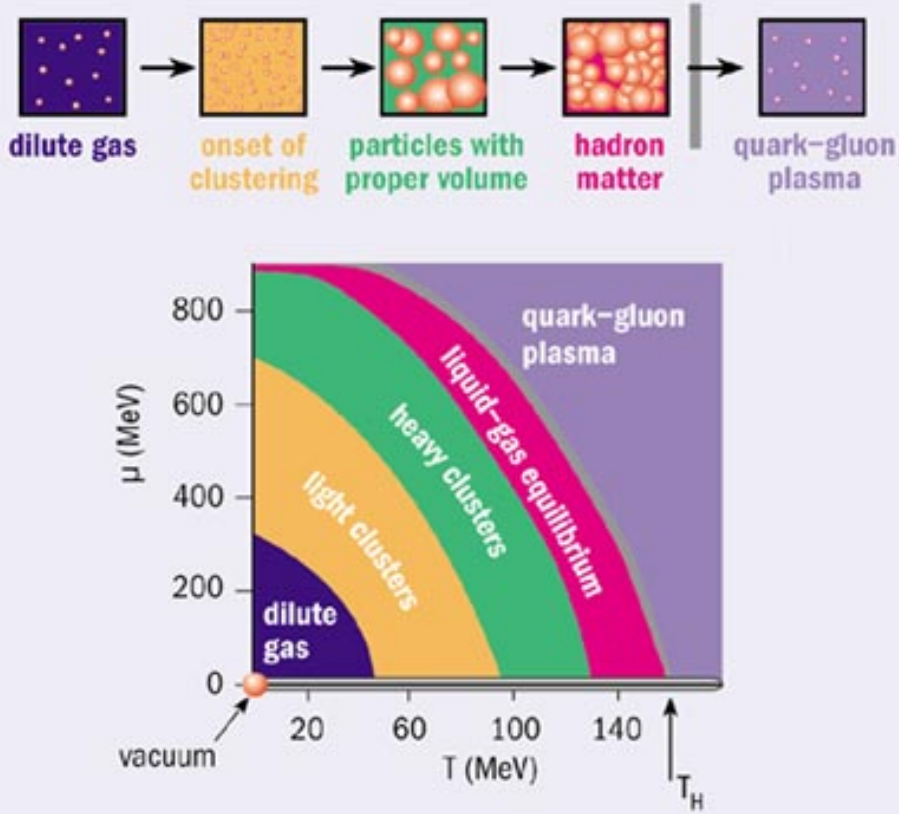
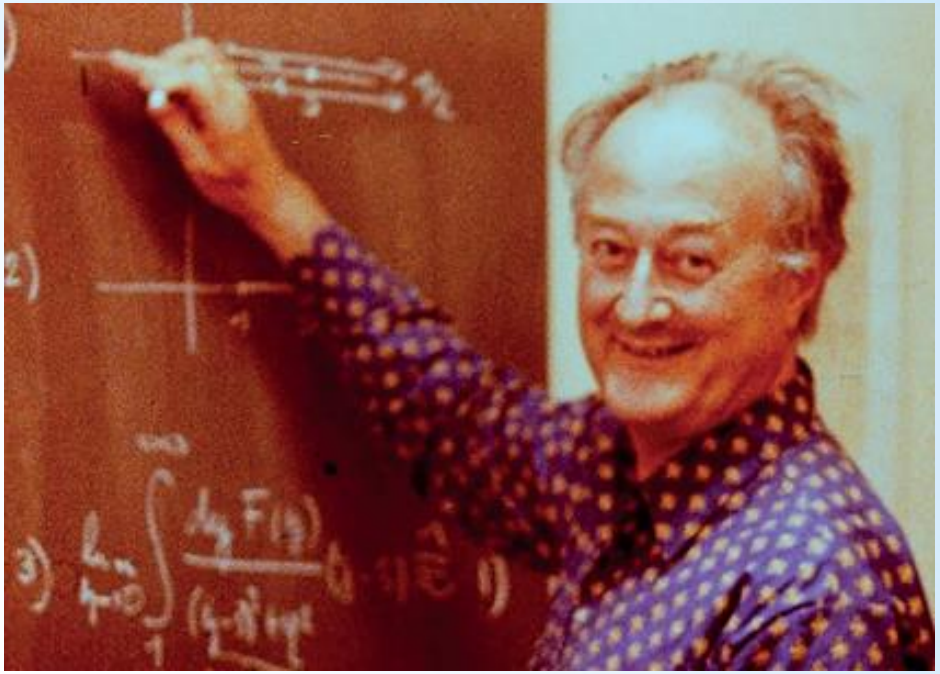
Kolymbari, Crete, Greece, 26.08.2015

Outline

I. Motivation, SBM
II. Equilibration of infinite hadron gas. Hagedorn-like limiting temperature
III. Equilibration in uRHIC's
IV. Comparison of obtained results
V. Application of Hagedorn states
VI. Conclusions

Motivation: Statistical Bootstrap Model





T.Ericson, J.Rafelski,
CERN Courier (2003)

Bootstrap Equation

$$\rho = \{\text{input particles}\} + \sum_{2}^{\infty} \int \ldots \int \{\text{products of } \delta\text{−functions and } \rho'\text{s}\}$$

R. Hagedorn, Nuovo Cim. Suppl. 3, 147 (1965)

Motivation: Statistical Bootstrap Model

See talks by Rafelski, Zakharov, Gorenstein, Redlich, Bugaev, Begun, …

$$\rho(\mathbf{m})_{\mathbf{m}\to\infty} \to \rho_0\ \mathbf{m}^{a_H}\ \exp\left(\mathbf{m}/\mathbf{T_H}\right)$$

$$a_H = -5/2$$

Density of hadronic states

$$\mathbf{P} = \int \rho(\mathbf{m})\mathbf{dm} \int \frac{\mathbf{d^3p}}{(\mathbf{2}\pi)^{\mathbf{3}}}\ \mathbf{p}\ \mathbf{f}(\mathbf{E}) + \{\ldots\}$$

Pressure

$$\mathbf{f}(\mathbf{E}) = \left(\mathbf{e}^{\mathbf{E}/\mathbf{T}} \pm \mathbf{1}\right)^{-\mathbf{1}}$$

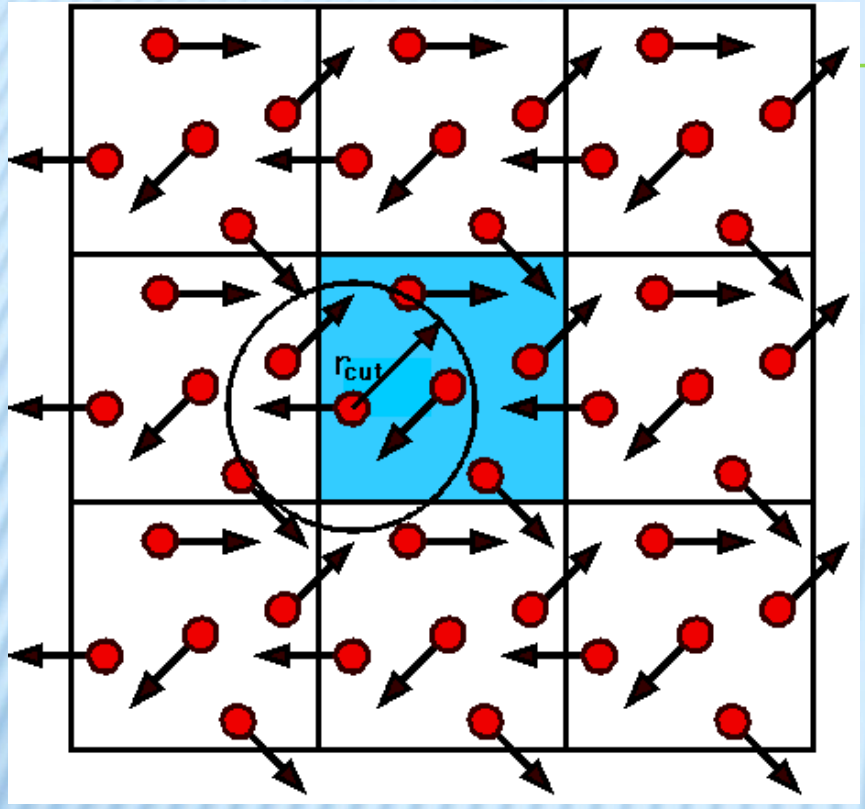
Distribution function

« *How this model is related to a real collision.- All this applies to infinitely extended hadronic matter in equilibrium.* Experiments unfortunately produce only microscopic lumps of such matter, which are never in equilibrium. The relation between the described model and the situation in a collision is, therefore, far from trivial. It seems, however, that at any given time equilibrium is nearly reached *locally*, so that the model might be applied locally and then be folded with collective motions assumed *ad hoc* or derived from special models.»

R. Hagedorn, Riv. Nuovo Cim. 6N10, 1-50 (1983)

Infinite hadron gas: a box with periodic boundary conditions

BOX WITH PERIODIC BOUNDARY CONDITIONS



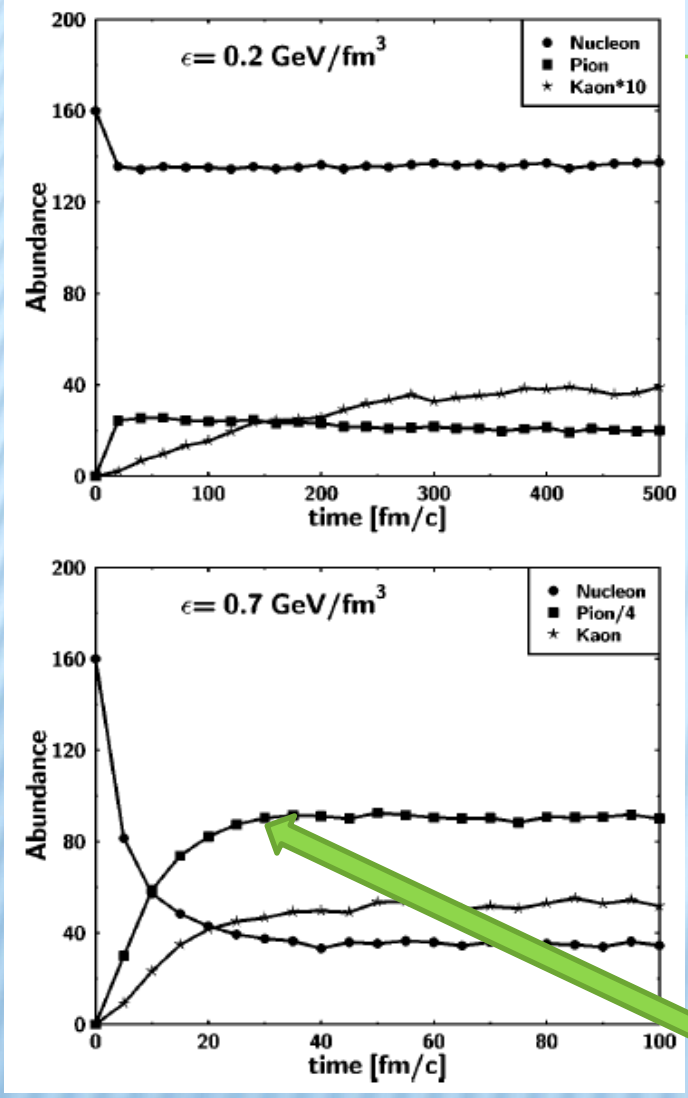
M.Belkacem et al., PRC 58, 1727 (1998)

Model employed: UrQMD
55 different baryon species
(N, Δ, hyperons and their
resonances with
$m \leq 2.25$ GeV/c^2)
32 different meson species
(including resonances with
$m \leq 2$ GeV/c^2) and their
respective antistates.
For higher mass excitations
a string mechanism is invoked.

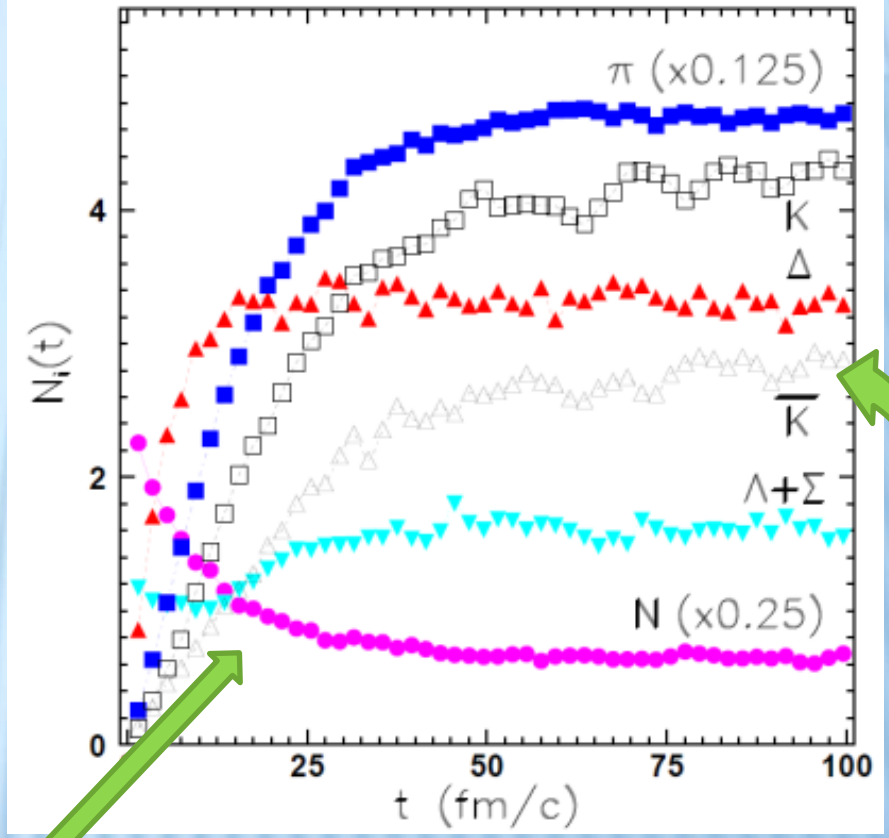
Initialization: (i) nucleons are uniformly distributed in a configuration space;
(ii) Their momenta are uniformly distributed in a sphere with random radius and then rescaled to the desired energy density.

Test for equilibrium: particle yields and energy spectra

BOX: PARTICLE ABUNDANCES



M.Belkacem et al., PRC 58, 1727 (1998)



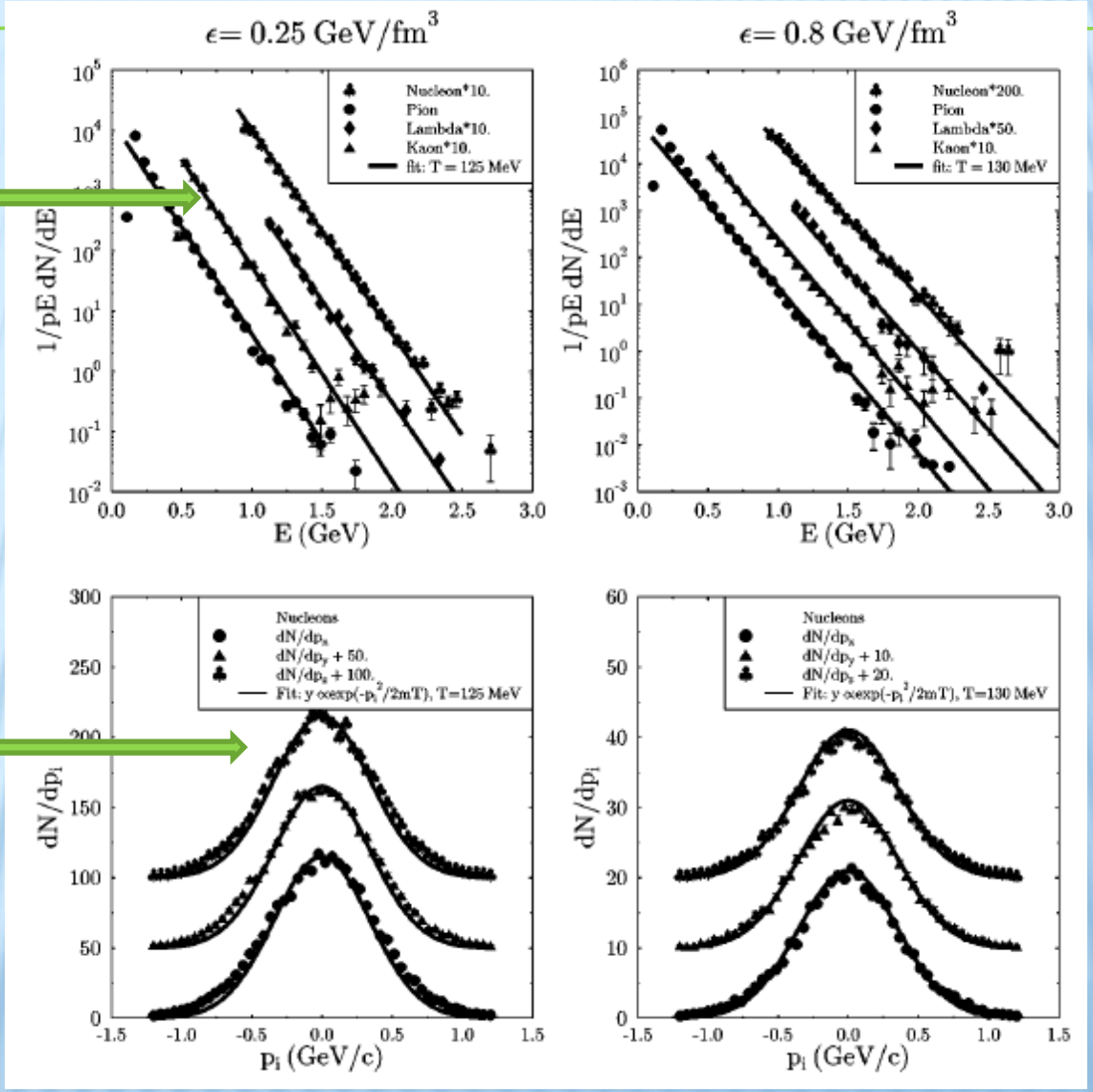
L.Bravina et al., PRC 62, 064906 (2000)

Saturation of yields after a certain time. Strange hadrons are saturated longer than others (at not very high energy densities)

BOX: ENERGY SPECTRA AND MOMENTUM DISTRIBUTIONS

Fit to Boltzmann distributions ~exp(-E/T)

Fit to Gaussian distributions ~exp(-p^2/2mT)



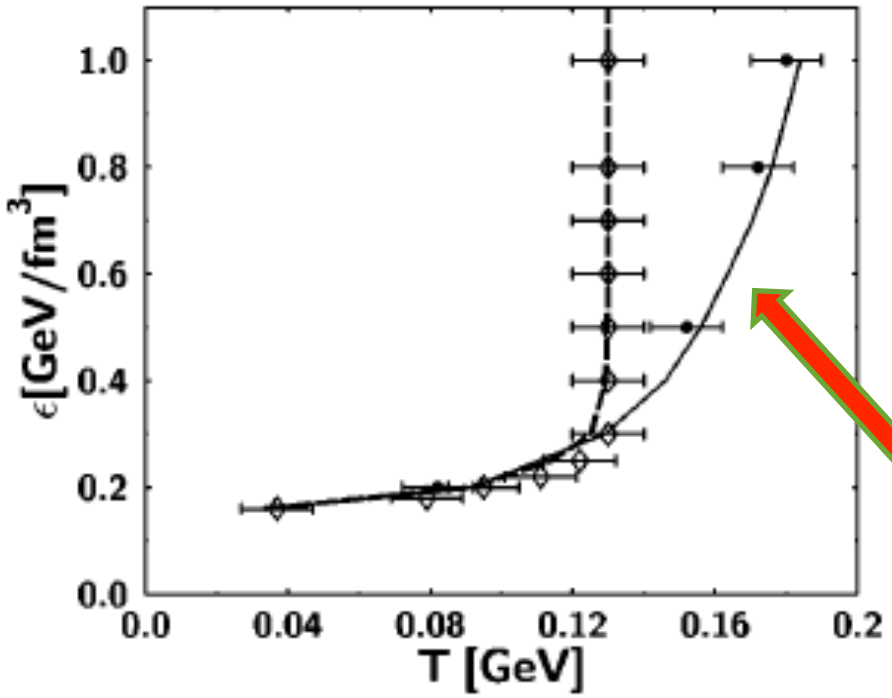
M.Belkacem et al., PRC 58, 1727 (1998)

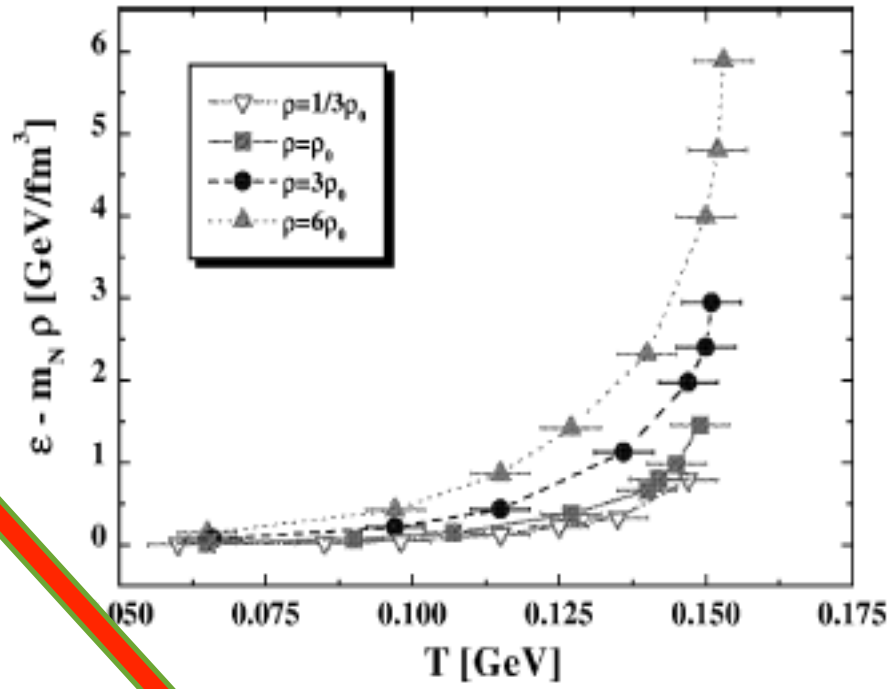
Nearly the same temperature and complete isotropy of dN/dp_i

BOX: HAGEDORN-LIKE LIMITING TEMPERATURE

M.Belkacem et al., PRC 58, 1727 (1998)

BUU+Lund





E.Bratkovskaya et al., NPA 675, 661 (2000)

UrQMD

A rapid rise of T at low ε and saturation at high energy densities. Saturation temperature depends on number of resonances in the model. W/o strings and many-N decays – no limiting T is observed.

Statistical model of ideal hadron gas

input values

output values

$$\varepsilon^{\rm mic} = \frac{1}{V}\sum_i E_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}),$$

$$\rho_{\rm B}^{\rm mic} = \frac{1}{V}\sum_i B_i \cdot N_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}),$$

$$\rho_{\rm S}^{\rm mic} = \frac{1}{V}\sum_i S_i \cdot N_i^{\rm SM}(T,\mu_{\rm B},\mu_{\rm S}).$$

Multiplicity

$$N_i^{\rm SM} = \frac{V g_i}{2\pi^2\hbar^3}\int_0^\infty p^2 f(p,m_i)dp,$$

Energy

$$E_i^{\rm SM} = \frac{V g_i}{2\pi^2\hbar^3}\int_0^\infty p^2\sqrt{p^2+m_i^2}\, f(p,m_i)dp$$

Pressure

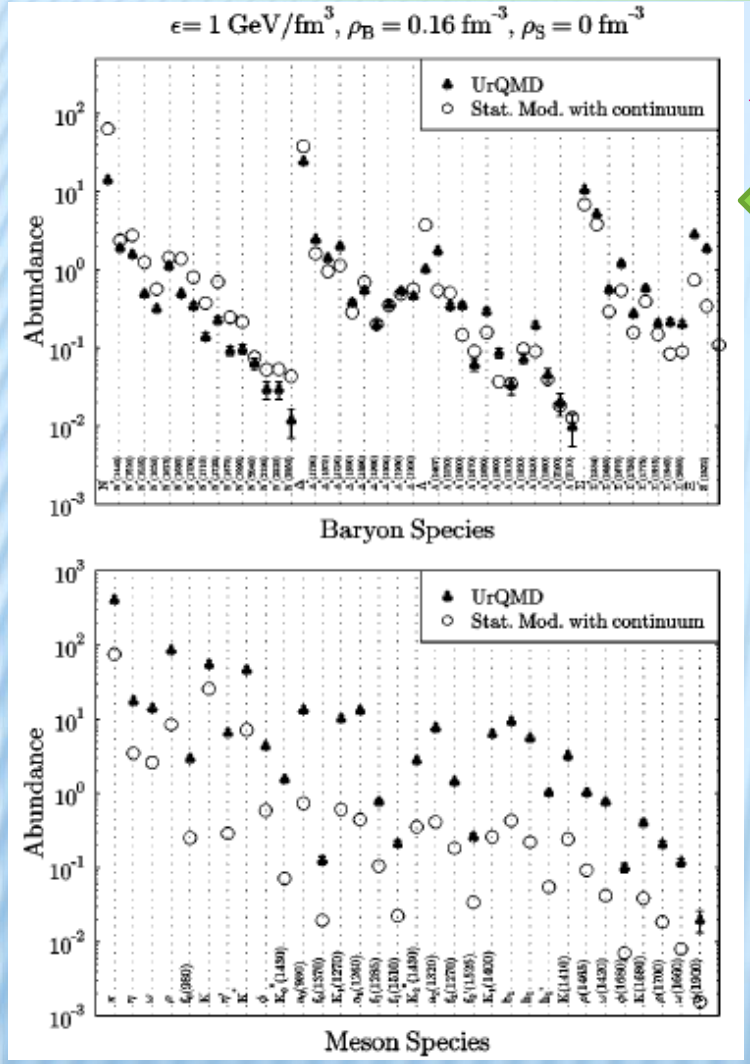
$$P^{\rm SM} = \sum_i \frac{g_i}{2\pi^2\hbar^3}\int_0^\infty p^2 \frac{p^2}{3(p^2+m_i^2)^{1/2}}\, f(p,m_i)dp$$

Entropy density

$$s^{\rm SM} = -\sum_i \frac{g_i}{2\pi^2\hbar^3}\int_0^\infty f(p,m_i)\,[\ln f(p,m_i)-1]\,p^2dp$$

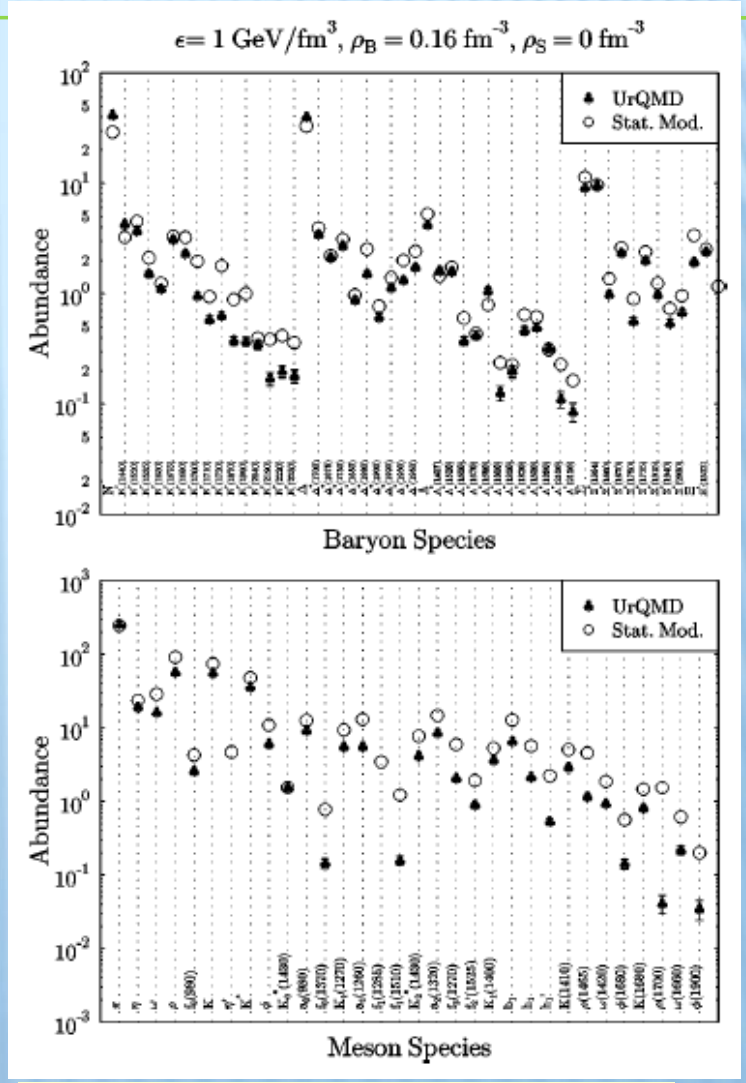
BOX: COMPARISON TO STATISTICAL MODEL

M.Belkacem et al., PRC 58, 1727 (1998)



with strings

w/o strings

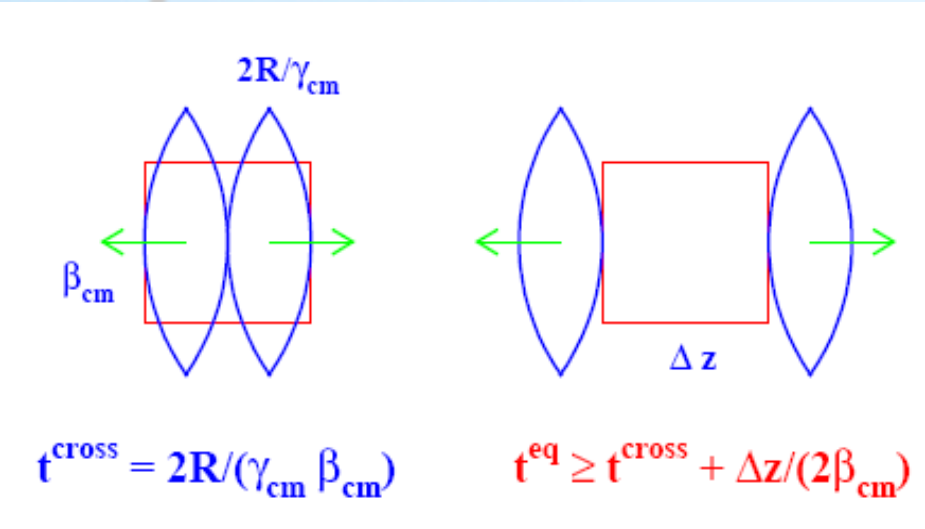


SM with the string mass spectrum

$$\rho(m) = \rho_0 \; m^{a_H} \; \exp(m/T_H)$$

Central cell: Relaxation to equilibrium

Equilibration in the Central Cell



Kinetic equilibrium:

Isotropy of velocity distributions

Isotropy of pressure

Thermal equilibrium:

Energy spectra of particles are described by Boltzmann distribution

$$\frac{dN_i}{4\pi p E dE} = \frac{V g_i}{(2\pi\hbar)^3} \exp\left(\frac{\mu_i}{T}\right) \exp\left(-\frac{E_i}{T}\right)$$

Chemical equlibrium:

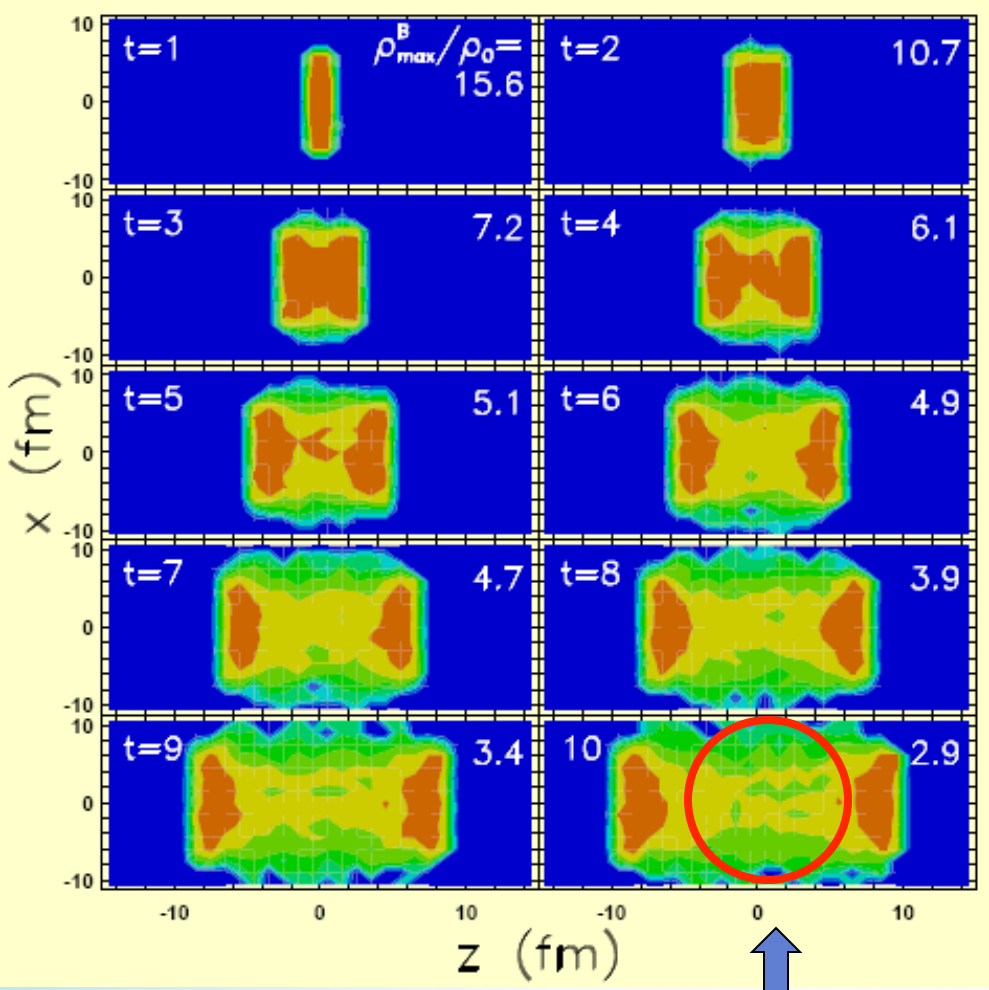
Particle yields are reproduced by SM with the same values of $(T,\ \mu_B,\ \mu_S)$:

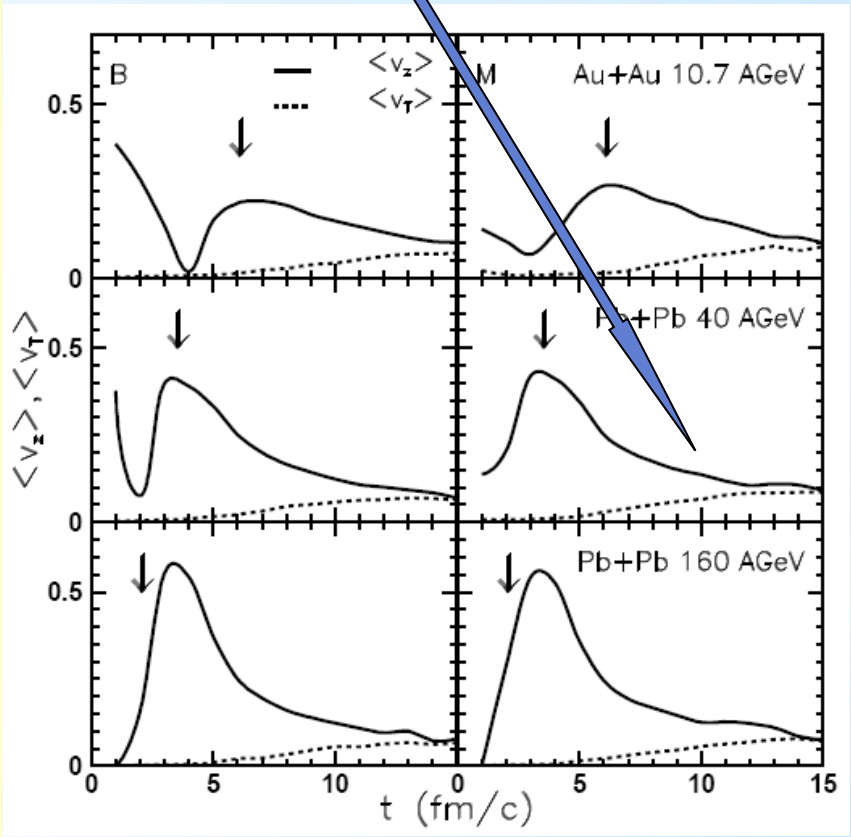
$$N_i = \frac{V g_i}{2\pi^2\hbar^3} \int_0^\infty p^2 dp \exp\left(\frac{\mu_i}{T}\right) \exp\left(-\frac{E_i}{T}\right)$$

Pre-equilibrium Stage

Homogeneity of baryon matter

Absence of flow





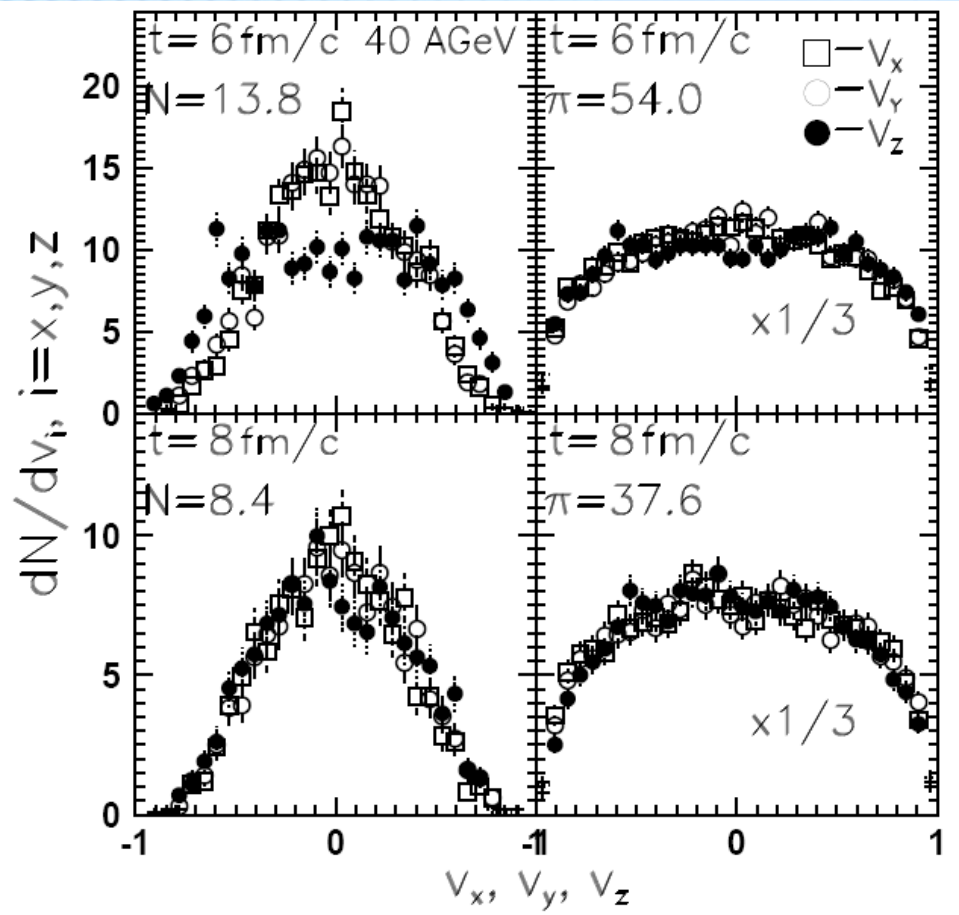
L.Bravina et al., PLB 434, 379 (1998); PRC 60, 024904 (1999)

The local equilibrium in the central zone is quite possible

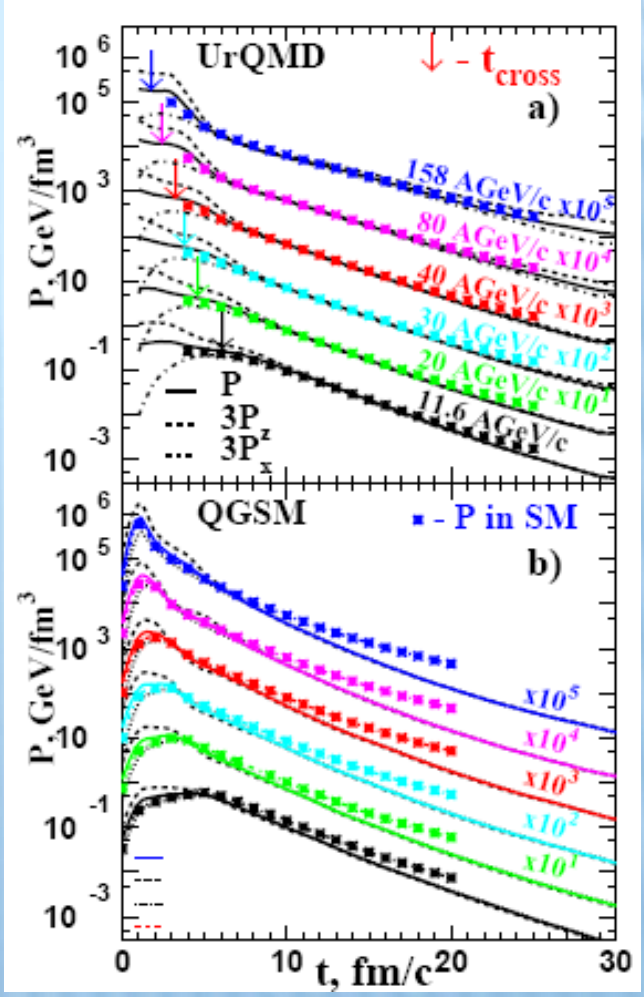
KINETIC EQUILIBRIUM

Isotropy of velocity distributions

L.Bravina et al., PRC 60, 024904 (1999)



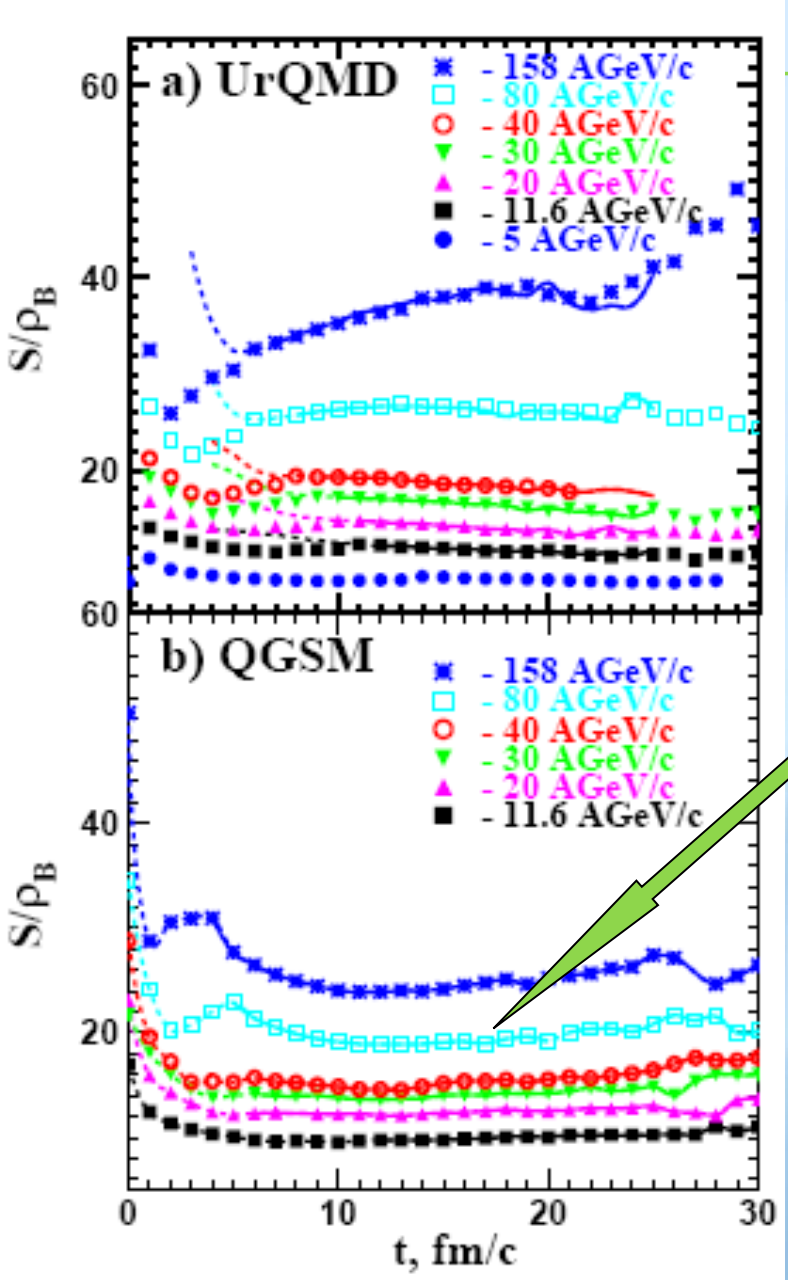
Isotropy of pressure



L.Bravina et al., PRC 78, 014907 (2008)

Velocity distributions and pressure become isotropic at all energies

ISENTROPIC EXPANSION

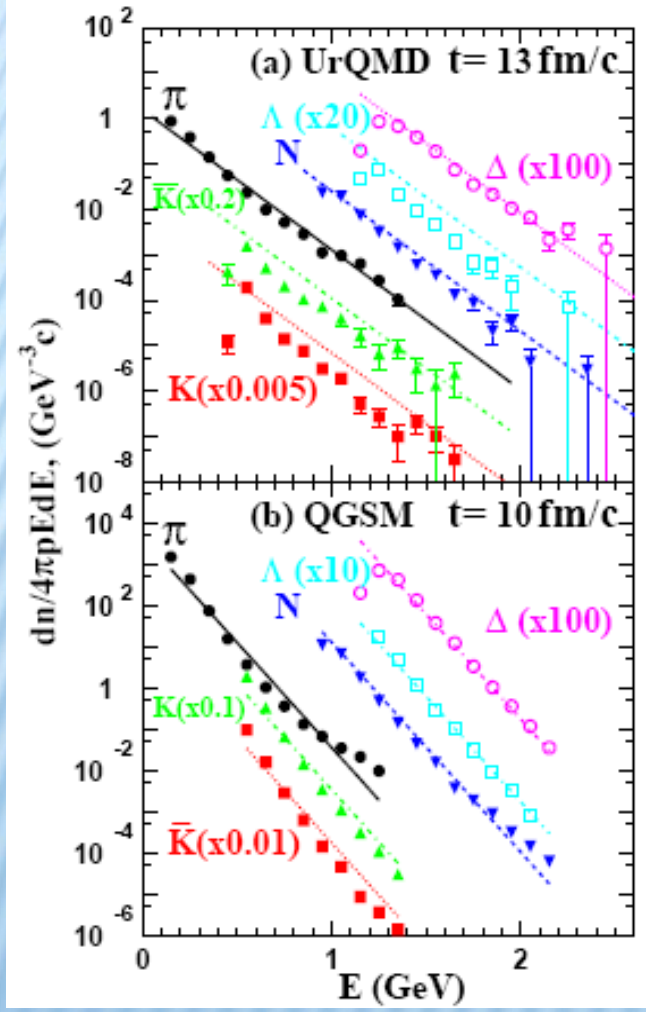


Expansion proceeds isentropically (with constant entropy per baryon). This result supports application of hydrodynamics

$$s/\rho_B = \text{const} = 12(\text{AGS}),\ 20(40),\ 38(\text{SPS})$$

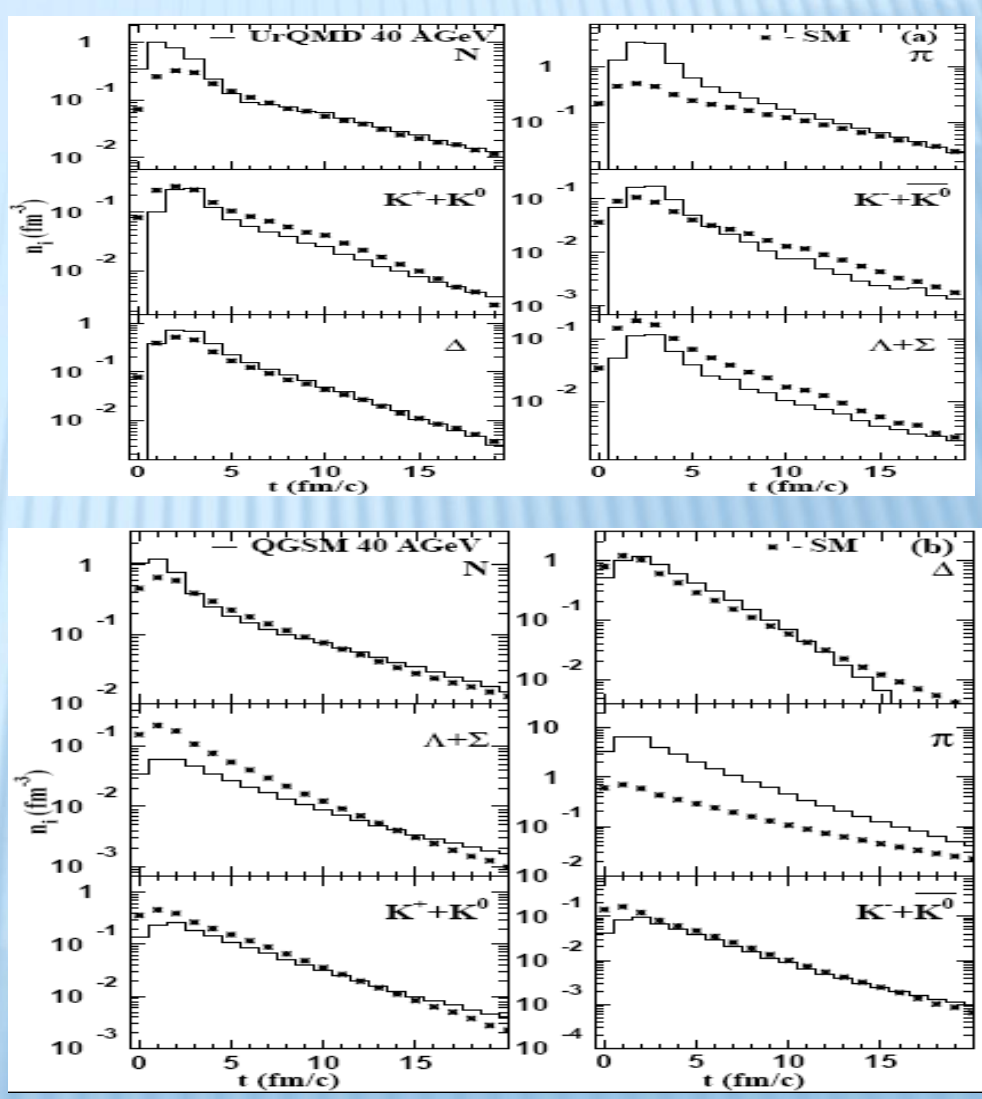
THERMAL AND CHEMICAL EQUILIBRIUM

Boltzmann fit to the energy spectra



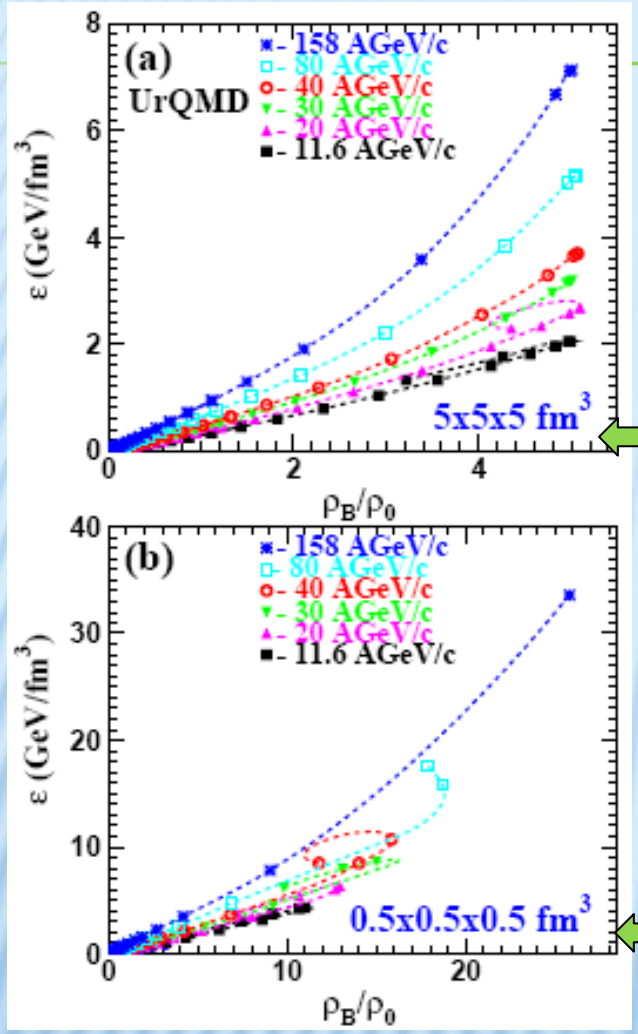
L.Bravina et al., PRC 78, 014907 (2008)

Particle yields



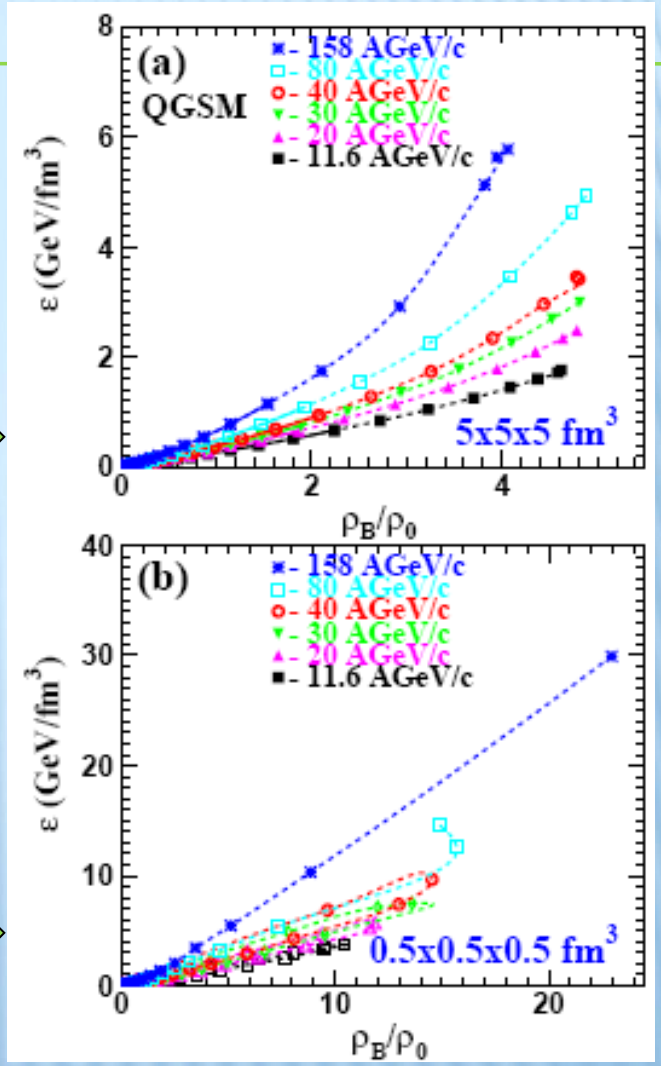
Thermal and chemical equilibrium seems to be reached

HOW DENSE CAN BE THE MEDIUM?



"Big" cell (V = 5x5x5 fm^3)

"Small" cell (V => 0)

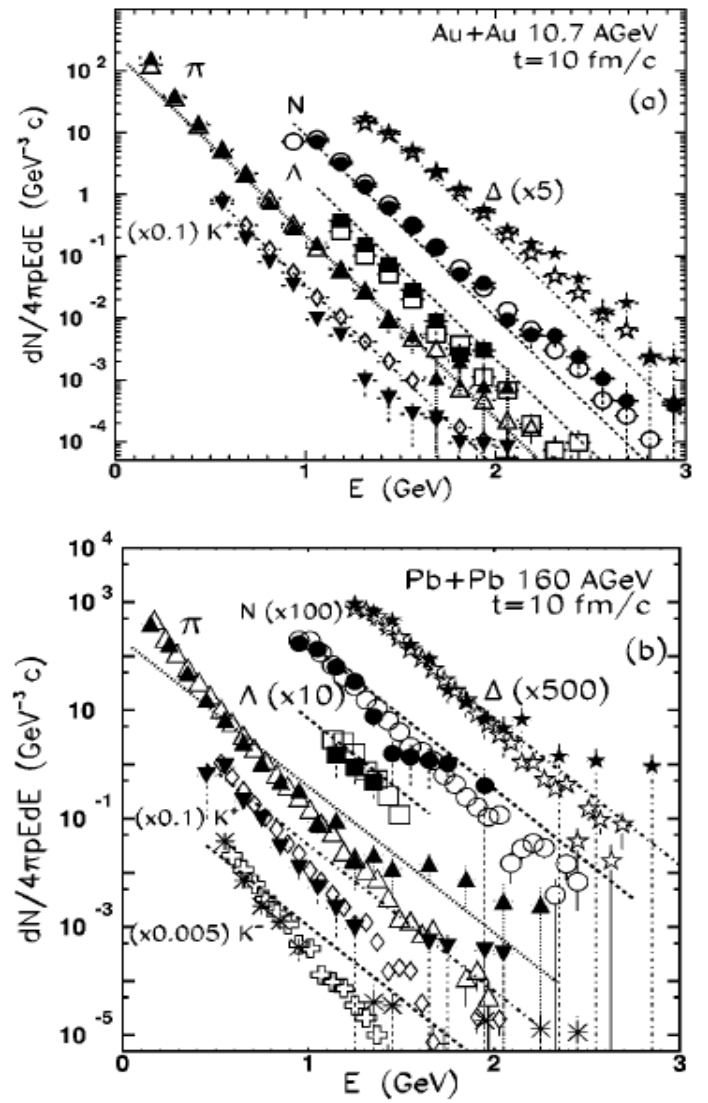


Dramatic differences at the non-equilibrium stage; after beginning of kinetic equilibrium the energy densities and the baryon densities are the same for "small" and "big" cell

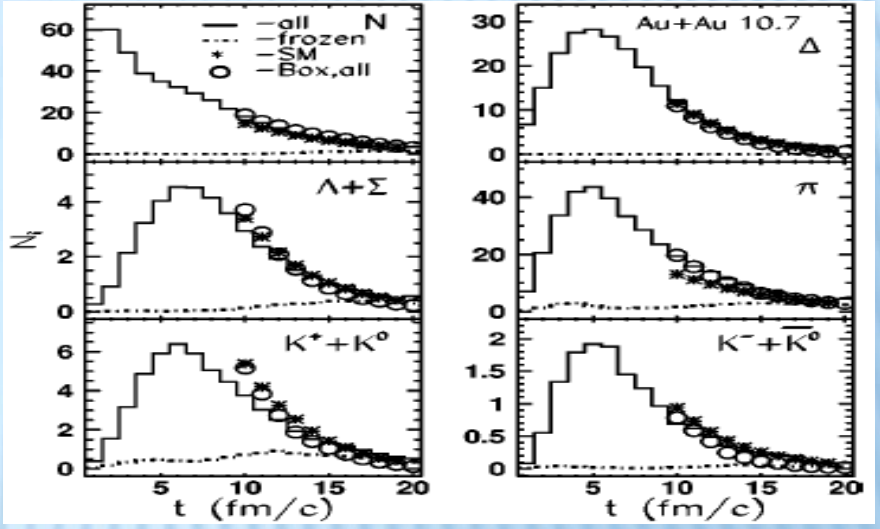
Comparison of cell and box results

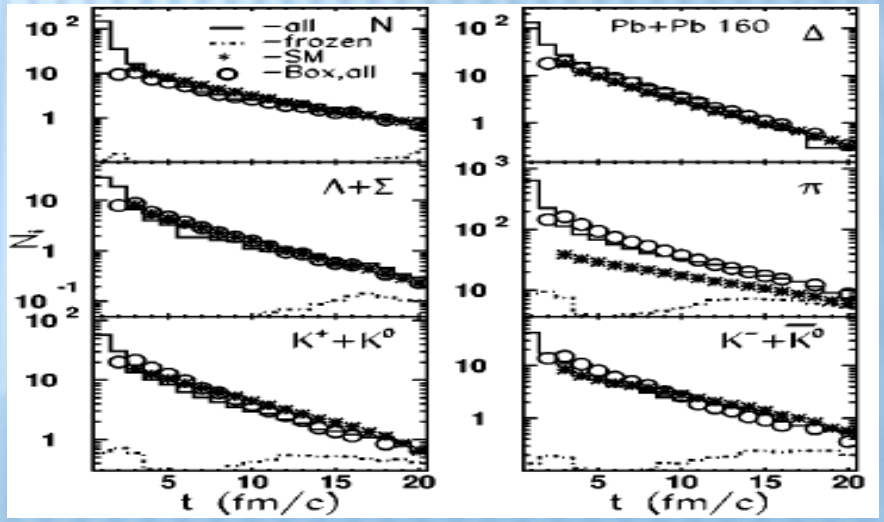
THERMAL AND CHEMICAL EQUILIBRIUM

Boltzmann fit to the energy spectra



Hadron yields



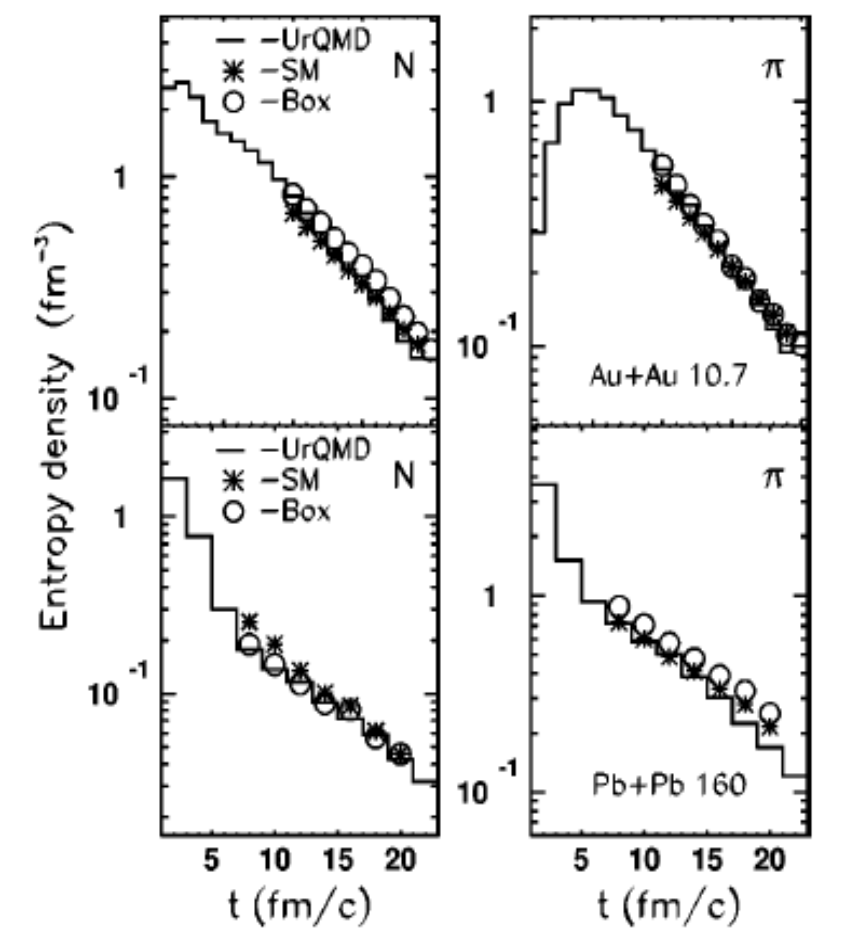


L.Bravina et al., PRC 62, 064906 (2000)

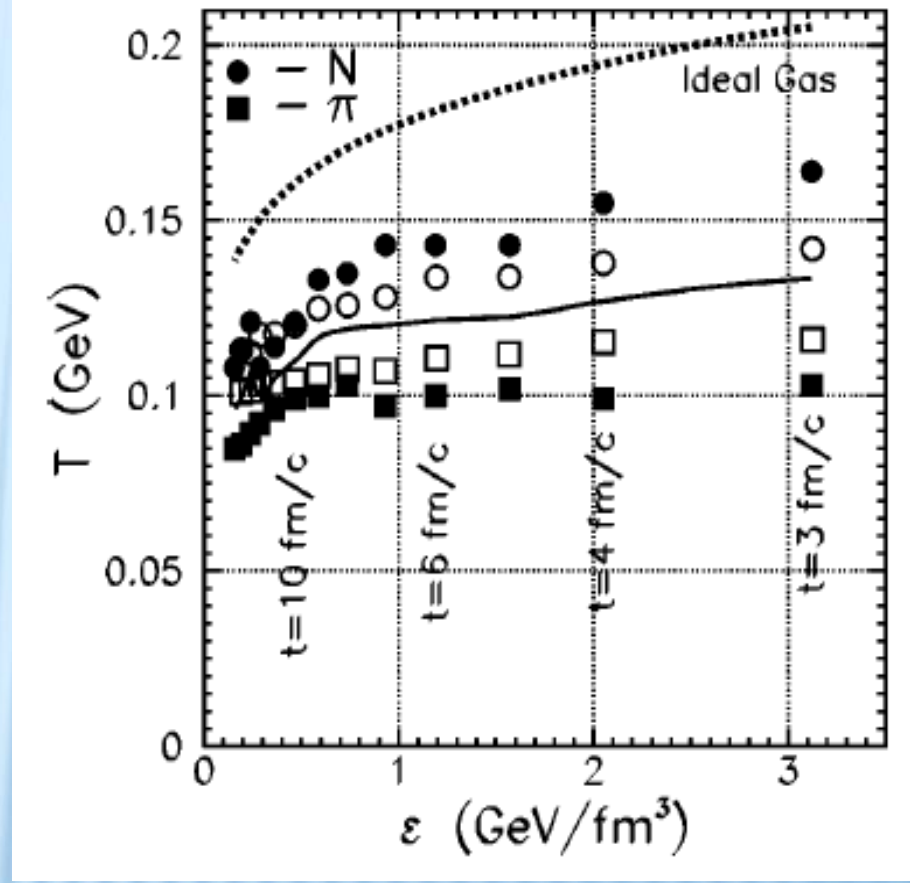
Box calculations are on the top of the cell results

THERMAL AND CHEMICAL EQUILIBRIUM

Partial entropy densities



T vs ε



Note the difference between T_N and T_π

Box calculations are on the top of the cell results

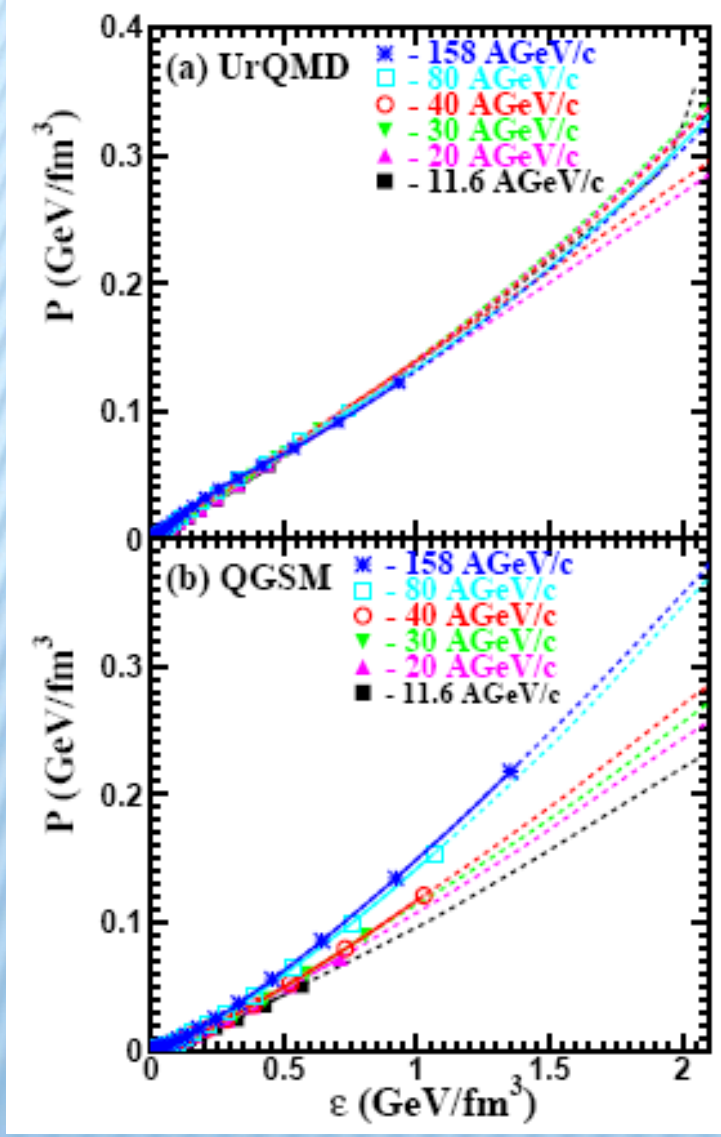
Equation of State

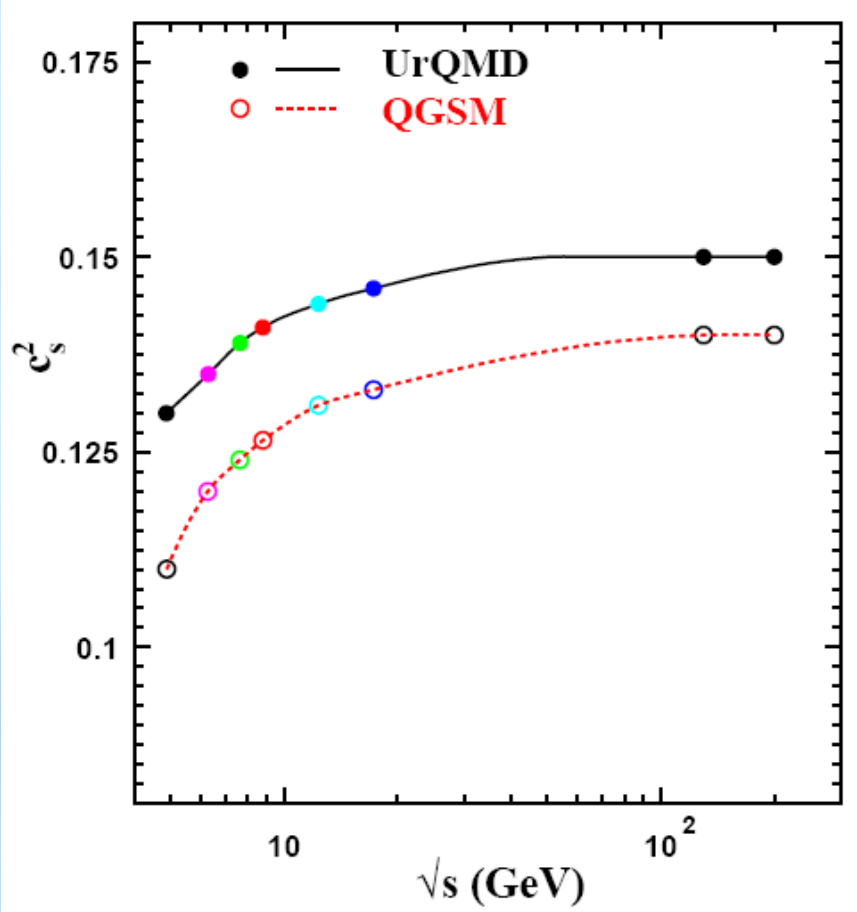
P vs. ε

EQUATION OF STATE IN THE CELL

pressure vs. energy

sound velocity





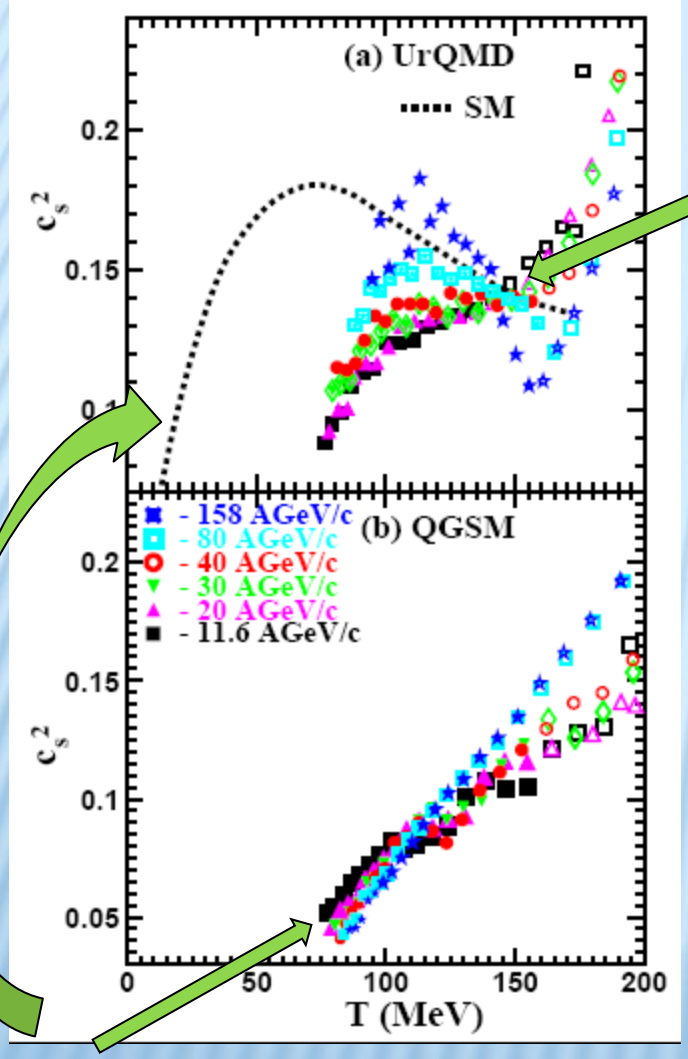
$P/\varepsilon = 0.13(\mathrm{AGS}),\ 0.14(40),\ 0.146(\mathrm{SPS}),\ 0.15(\mathrm{RHIC})$

EQUATION OF STATE:

comparison with Hagedorn model

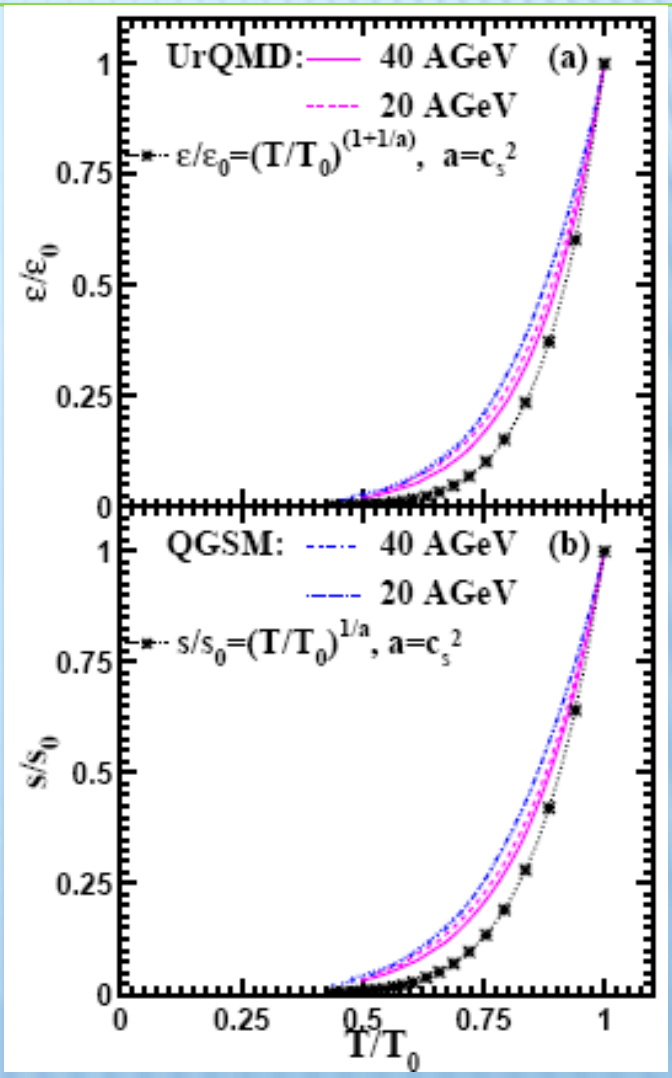
energy and entropy densities vs. T

M. Chojnacki et al., PRC 71 (2005) 044902



Heavy resonances

Big difference between models with and w/o heavy resonances



L.Bravina et al., PRC 78 (2008) 014907

Still sonic velocity drops faster than in Hagedorn model . Non-zero chemical potential ?

No difference between the models

Application of Hagedorn states

Application of Hagedorn states

C.Greiner, J.Noronha-Hostler, K.Gallmeister, …

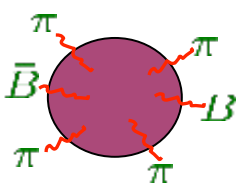
- at SPS energies chem. equil. time is 1-3 fm/c

$$\mathrm{n}_1\pi + n_2 K \leftrightarrow \bar{\bar{Y}} + p$$ (C.Greiner, Leupold, 2000)

- at RHIC energies chem. equil. time is 10 fm/c

  with same approach

- fast chem. equil. mechanism through Hagedorn states

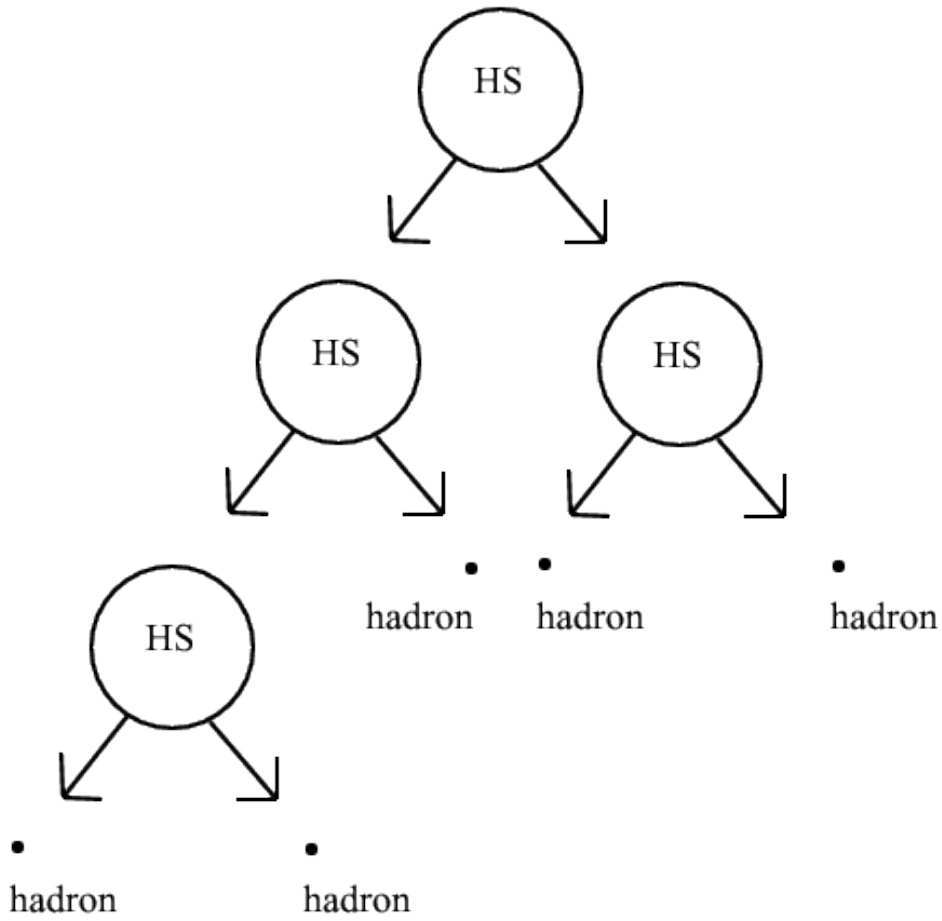


$$\left(n_1\pi + n_2 K + n_3 \bar{K} \leftrightarrow\right) HS \leftrightarrow \bar{\bar{B}} + B + X$$

- dyn. evolution through set of coupled rate equations leads to 5 fm/c for BB pairs

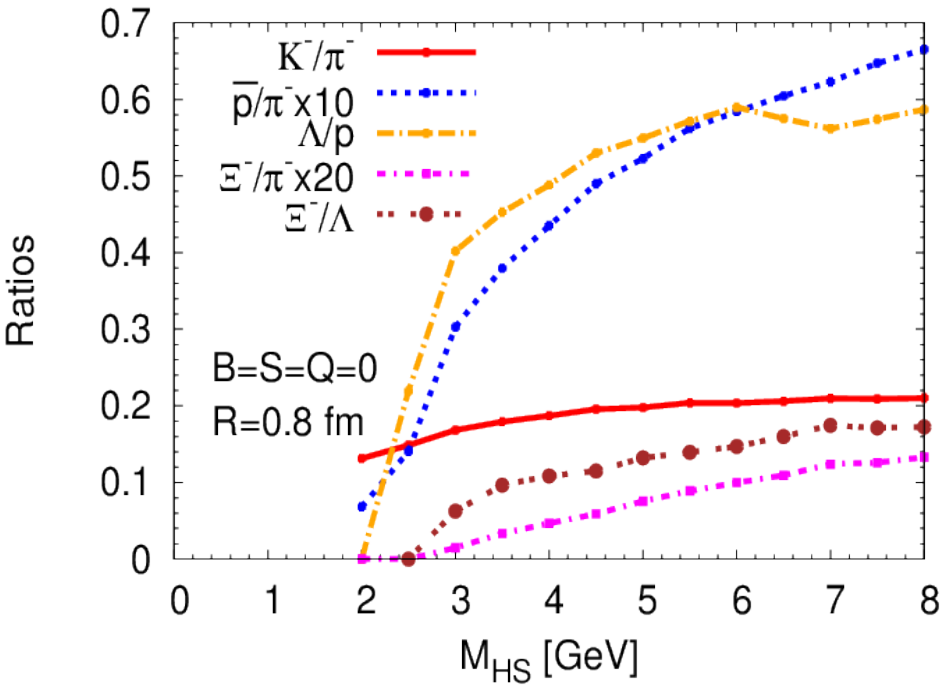
J. Noronha-Hostler et al. PRL 100 (2008)
J. Noronha-Hostler et al. J. Phys. G 37 (2010)
J. Noronha-Hostler et al. Phys. Rev. C 81 (2010)

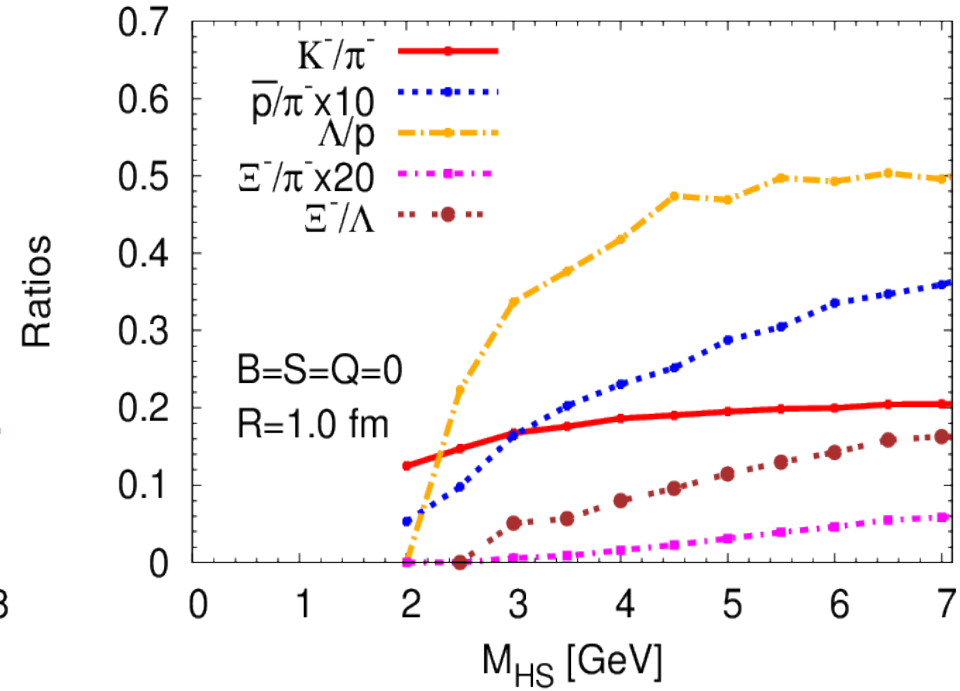
One possible Hagedorn state decay chain



Hadronic ratios from Hagedorn state cascading decay

M.Beitel, K.Gallmeister, C.Greiner, PRC 90, 045203 (2014)





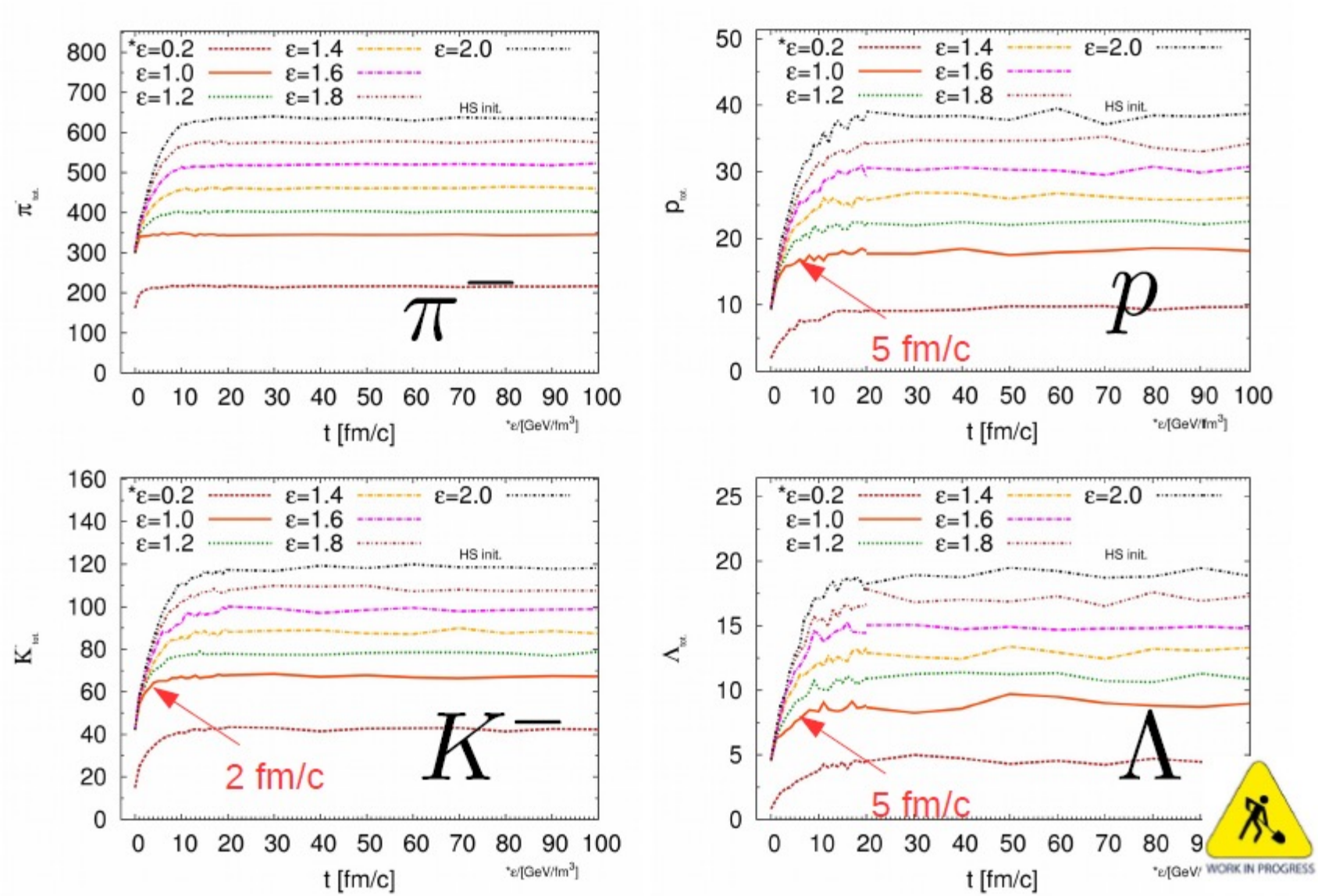
	p-p	Pb-Pb	4 GeV	8 GeV
K^-/π^-	0.123(14)	0.149(16)	0.187	0.210
$\bar{p}/\pi^-$	0.053(6)	0.045(5)	0.043	0.066
Λ/π^-	0.032(4)	0.036(5)	0.021	0.038
$\Lambda/\bar{p}$	0.608(88)	0.78(12)	0.494	0.579
Ξ^-/π^-	0.003(1)	0.0050(6)	0.0023	0.0066
$\Omega^-/\pi^- \cdot 10^{-3}$	-	0.87(17)	0.086	0.560

ALICE at LHC Ratios:

p-p @ 0.9 TeV
Pb-Pb @ 2.76 TeV

K. Aamodt et al. Eur. Phys J. C 71
B. Abelev et al. Phys Rev. C. 88
B. Abelev et al. Phys. Lett. B 728

UrQMD-Box, HS init



K. Gallmeister, talk at SQM'15 (JINR, Dubna)

Conclusions

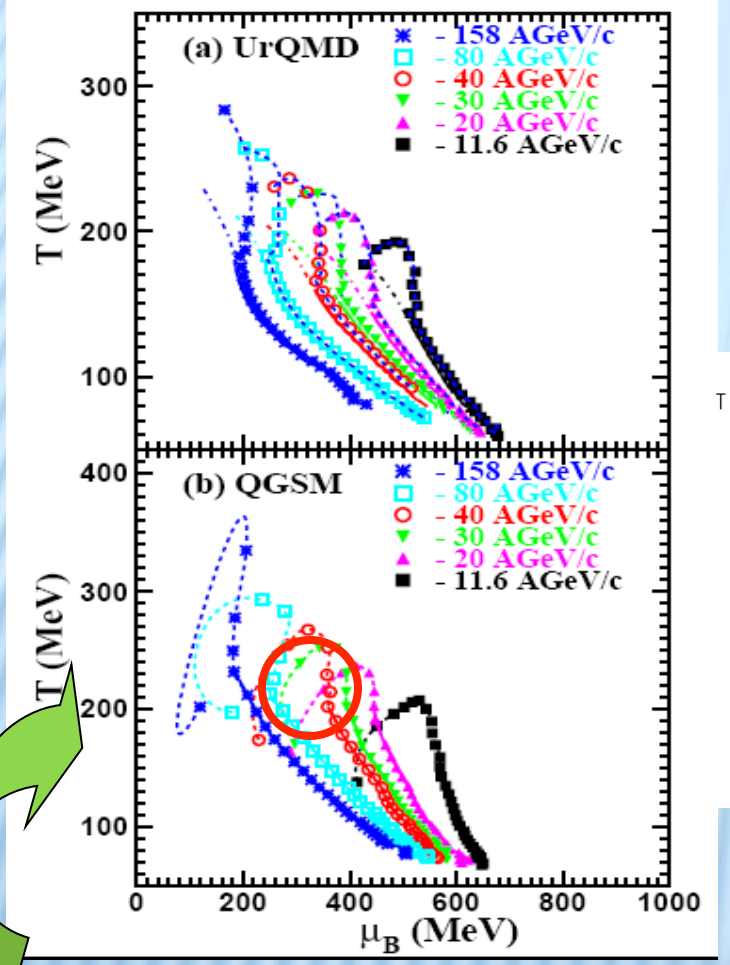
- *Infinite hadron matter: starting from random initial conditions, particle multiplicities saturate after some time*
- *The slopes of the energy spectra can be reproduced by Boltzmann fits with two temperatures T_B and T_M*
- *The EOS appears to be Hagedorn-like with a limiting temperature $T_H = 135 \pm 15$ MeV*
- *rHICs: models favor formation of equilibrated matter in the central cell for a period of 10-15 fm/c*
- *During this period the expansion of matter in the central cell proceeds isentropically with constant ratio S/B*
- *Agreement between the cell and box results; not always good between the cell/box and the SM*
- *Possible solution: Incorporation of Hagedorn states*

Back-up Slides

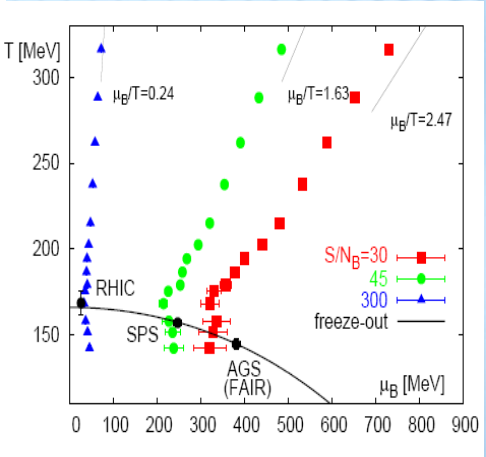
Modification of analysis (small cells)

EOS IN THE CELL: OBSERVATION OF KNEE

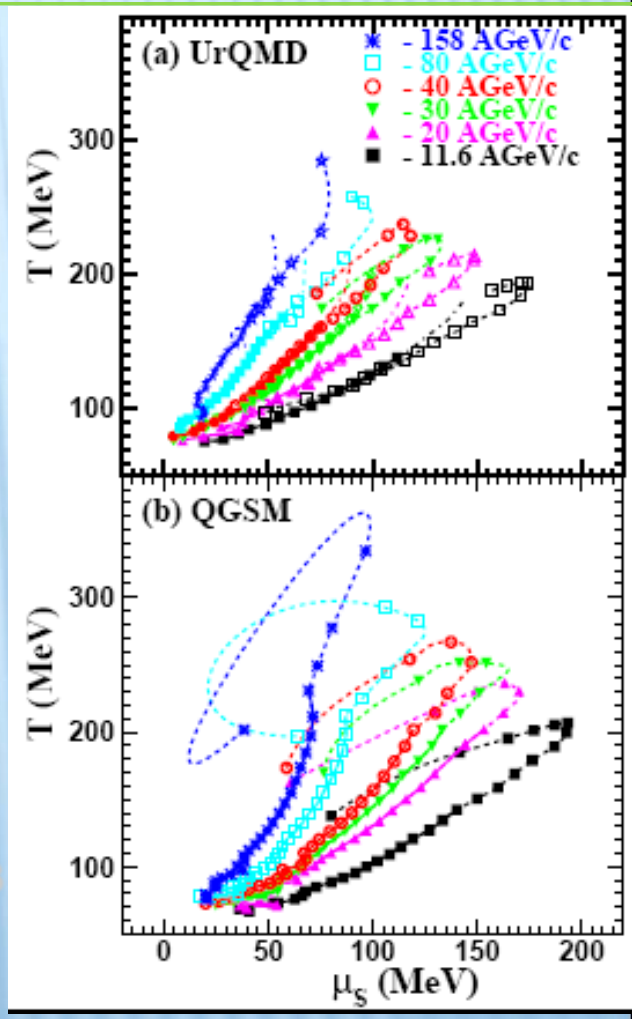
temperature vs. chemical potentials



L.Bravina et al., PRC 78 (2008) 014907



S. Ejiri et al., PRD 73 (2006) 054506



Although the "knee" is similar to that in 2-flavor lattice QCD, it is related to inelastic (chemical) freeze-out in the system