Appearance of Hagedorn limiting temperature in microscopic model calculations

E. Zabrodin (UiO & SINP MSU),
results obtained in collaboration with
L. Bravina, M. Gorenstein and Frankfurt ITP/FIAS group

International Conference on New Frontiers in Physics ICNFP-2015
Kolymbari, Crete, Greece, 26.08.2015
Outline

I. Motivation, SBM
II. Equilibration of infinite hadron gas. Hagedorn-like limiting temperature
III. Equilibration in uRHIC’s
IV. Comparison of obtained results
V. Application of Hagedorn states
VI. Conclusions
**Motivation: Statistical Bootstrap Model**

- **Dilute gas** → **Onset of clustering** → **Particles with proper volume** → **Hadron matter** → **Quark-gluon plasma**

![Graph showing different states of matter](image)

- **Bootstrap Equation**

\[
\rho = \{\text{input particles}\} + \sum_{2}^{\infty} \int \ldots \int \{\text{products of } \delta\text{-functions and } \rho^{\prime}\text{s}\}
\]


R. Hagedorn, Nuovo Cim. Suppl. 3, 147 (1965)
Density of hadronic states

\[ \rho(m)_{m \to \infty} \rightarrow \rho_0 \ m^a \ \exp \left( m / T_H \right) \]
\[ a_H = -5/2 \]

Pressure

\[ P = \int \rho(m) \ dm \int \frac{d^3p}{(2\pi)^3} \ p \ f(E) + \{ \ldots \} \]

Distribution function

\[ f(E) = \left( e^{E/T} \pm 1 \right)^{-1} \]

« How this model is related to a real collision. - All this applies to infinitely extended hadronic matter in equilibrium. Experiments unfortunately produce only microscopic lumps of such matter, which are never in equilibrium. The relation between the described model and the situation in a collision is, therefore, far from trivial. It seems, however, that at any given time equilibrium is nearly reached locally, so that the model might be applied locally and then be folded with collective motions assumed ad hoc or derived from special models.»

Infinite hadron gas: a box with periodic boundary conditions
Model employed: UrQMD
55 different baryon species
(N, Δ, hyperons and their
resonances with
m \leq 2.25 \text{ GeV}/c^2 )
32 different meson species
(including resonances with
m \leq 2 \text{ GeV}/c^2 ) and their
respective antistates.
For higher mass excitations
a string mechanism is invoked.

Initialization: (i) nucleons are uniformly
distributed in a configuration space;
(ii) Their momenta are uniformly distributed
in a sphere with random radius and then
rescaled to the desired energy density.

Test for equilibrium: particle yields and energy spectra
Saturation of yields after a certain time. Strange hadrons are saturated longer than others (at not very high energy densities).
BOX: ENERGY SPECTRA AND MOMENTUM DISTRIBUTIONS

Nearly the same temperature and complete isotropy of $dN/dp_i$

Fit to Boltzmann distributions $\sim \exp(-E/T)$

Fit to Gaussian distributions $\sim \exp(-p^2/2mT)$
A rapid rise of $T$ at low $\epsilon$ and saturation at high energy densities. Saturation temperature depends on number of resonances in the model. W/o strings and many-N decays – no limiting $T$ is observed.
Statistical model of ideal hadron gas

\[ \varepsilon_{\text{mic}} = \frac{1}{V} \sum_i E_i^{\text{SM}}(T, \mu_B, \mu_S), \]
\[ \rho_B^{\text{mic}} = \frac{1}{V} \sum_i B_i \cdot N_i^{\text{SM}}(T, \mu_B, \mu_S), \]
\[ \rho_S^{\text{mic}} = \frac{1}{V} \sum_i S_i \cdot N_i^{\text{SM}}(T, \mu_B, \mu_S). \]

Multiplicity  →  
Energy  →  
Pressure  →  
Entropy density  →  

\[ N_i^{\text{SM}} = \frac{V g_i}{2 \pi^2 \hbar^3} \int_0^\infty p^2 f(p, m_i) dp, \]
\[ E_i^{\text{SM}} = \frac{V g_i}{2 \pi^2 \hbar^3} \int_0^\infty p^2 \sqrt{p^2 + m_i^2} f(p, m_i) dp, \]
\[ P^{\text{SM}} = \sum_i \frac{g_i}{2 \pi^2 \hbar^3} \int_0^\infty p^2 \frac{p^2}{3(p^2 + m_i^2)^{1/2}} f(p, m_i) dp, \]
\[ s^{\text{SM}} = -\sum_i \frac{g_i}{2 \pi^2 \hbar^3} \int_0^\infty f(p, m_i) \left[ \ln f(p, m_i) - 1 \right] p^2 dp. \]
BOX: COMPARISON TO STATISTICAL MODEL

M. Belkacem et al., PRC 58, 1727 (1998)

with strings

$\rho(m) = \rho_0 \ m^{a_m} \ \exp\left(\frac{m}{T_H}\right)$
Central cell: Relaxation to equilibrium
Equilibration in the Central Cell

Kinetic equilibrium:
Isotropy of velocity distributions
Isotropy of pressure

Thermal equilibrium:
Energy spectra of particles are described by Boltzmann distribution

Chemical equilibrium:
Particle yields are reproduced by SM with the same values of $(T, \mu_B, \mu_S)$:

\[
N_i = \frac{V g_i}{2\pi^2 \hbar^3} \int_0^\infty p^2 dp \exp \left( \frac{\mu_i}{T} \right) \exp \left( -\frac{E_i}{T} \right)
\]
Pre-equilibrium Stage

Homogeneity of baryon matter

Absence of flow

The local equilibrium in the central zone is quite possible

L. Bravina et al., PLB 434, 379 (1998); PRC 60, 024904 (1999)
**KINETIC EQUILIBRIUM**

Isotropy of velocity distributions

L. Bravina et al., PRC 60, 024904 (1999)

- Velocity distributions and pressure become isotropic at all energies

Isotropy of pressure

L. Bravina et al., PRC 78, 014907 (2008)
Expansion proceeds isentropically (with constant entropy per baryon). This result supports application of hydrodynamics.

\[ s/\rho_B = \text{const} = 12(\text{AGS}), \ 20(40), \ 38(\text{SPS}) \]
Thermal and chemical equilibrium seems to be reached

L. Bravina et al., PRC 78, 014907 (2008)
HOW DENSE CAN BE THE MEDIUM?

Dramatic differences at the non-equilibrium stage; after beginning of kinetic equilibrium the energy densities and the baryon densities are the same for "small" and "big" cell.
Comparison of cell and box results
THERMAL AND CHEMICAL EQUILIBRIUM

Boltzmann fit to the energy spectra

Hadron yields

Box calculations are on the top of the cell results

L. Bravina et al., PRC 62, 064906 (2000)
THERMAL AND CHEMICAL EQUILIBRIUM

Partial entropy densities

Box calculations are on the top of the cell results

Note the difference between $T_N$ and $T_\pi$
Equation of State

$P$ vs. $\varepsilon$
EQUATION OF STATE IN THE CELL

Pressure vs. energy

Sound velocity

\[
P \propto \varepsilon
\]

\[
P/\varepsilon = 0.13(\text{AGS}), \ 0.14(40), \ 0.146(\text{SPS}), \ 0.15(\text{RHIC})
\]
**EQUATION OF STATE:** comparison with Hagedorn model

energy and entropy densities vs. T

---

Heavy resonances

Big difference between models with and w/o heavy resonances

Still sonic velocity drops faster than in Hagedorn model. Non-zero chemical potential?

No difference between the models

---

L. Bravina et al., PRC 71 (2005) 044902

M. Choinacki et al., PRC 78 (2008) 014907
Application of Hagedorn states
Application of Hagedorn states

C. Greiner, J. Noronha-Hostler, K. Gallmeister, …

- at SPS energies chem. equil. time is 1-3 fm/c

\[ n_1 \pi + n_2 K \leftrightarrow \bar{Y} + p \]  
(C. Greiner, Leupold, 2000)

- at RHIC energies chem. equil. time is 10 fm/c
  with same approach

- fast chem. equil. mechanism through Hagedorn states

\[ (n_1 \pi + n_2 K + n_3 \bar{K} \leftrightarrow \text{HS}) \leftrightarrow \bar{B} + B + X \]

- dyn. evolution through set of coupled rate equations
  leads to 5 fm/c for BB pairs

One possible Hagedorn state decay chain.
Hadronic ratios from Hagedorn state cascading decay

M. Beitel, K. Gallmeister, C. Greiner, PRC 90, 045203 (2014)

**ALICE at LHC Ratios:**

<table>
<thead>
<tr>
<th>Ratio</th>
<th>p-p (GeV=4)</th>
<th>Pb-Pb (GeV=8)</th>
<th>p-p @ 0.9 TeV</th>
<th>Pb-Pb @ 2.76 TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^-/\pi^-$</td>
<td>0.123(14)</td>
<td>0.149(16)</td>
<td>0.187</td>
<td>0.210</td>
</tr>
<tr>
<td>$\bar{p}/\pi^-$</td>
<td>0.053(6)</td>
<td>0.045(5)</td>
<td>0.043</td>
<td>0.066</td>
</tr>
<tr>
<td>$\Lambda/\pi^-$</td>
<td>0.032(4)</td>
<td>0.036(5)</td>
<td>0.021</td>
<td>0.038</td>
</tr>
<tr>
<td>$\Lambda/\bar{p}$</td>
<td>0.608(88)</td>
<td>0.78(12)</td>
<td>0.494</td>
<td>0.579</td>
</tr>
<tr>
<td>$\Xi^-/\pi^-$</td>
<td>0.003(1)</td>
<td>0.0050(6)</td>
<td>0.0023</td>
<td>0.0066</td>
</tr>
<tr>
<td>$\Omega^-/\pi^- \cdot 10^{-3}$</td>
<td>-</td>
<td>0.87(17)</td>
<td>0.086</td>
<td>0.560</td>
</tr>
</tbody>
</table>

B. Abelev et al. Phys Rev. C. 88
K. Gallmeister, talk at SQM’15 (JINR, Dubna)
Conclusions

• Infinite hadron matter: starting from random initial conditions, particle multiplicities saturate after some time
• The slopes of the energy spectra can be reproduced by Boltzmann fits with two temperatures $T_B$ and $T_M$
• The EOS appears to be Hagedorn-like with a limiting temperature $T_H = 135\pm15$ MeV
• rHICs: models favor formation of equilibrated matter in the central cell for a period of 10-15 fm/c
• During this period the expansion of matter in the central cell proceeds isentropically with constant ratio $S/B$
• Agreement between the cell and box results; not always good between the cell/box and the SM
• Possible solution: Incorporation of Hagedorn states
Back-up Slides
Modification of analysis (small cells)
Although the “knee” is similar to that in 2-flavor lattice QCD, it is related to inelastic (chemical) freeze-out in the system.