On the base quantities of oscimodes of valence q' qbar and 3q states

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Abstract

These notes are meant to illustrate the 'oscillatory modes of $N_{fl} = 3$ light quark flavors' , – u , d , s – , using the

$$\left(\text{SU3}_{fl} = 9 = 8 + 1 \right) \times \left(\text{SU2}_{spin} = 3 + 1 \right) \times \text{SO3} \left(\vec{L} \right)$$

broken symmetry classification for mesons , and

$$\text{SU} \left(2 N_{fl} = 6 \right) \times \text{SO3} \left(\vec{L} \right)$$ **broken symmetry classification for baryons.**

Here the orbital angular momentum operator $\vec{L}$ stands for

$\vec{L}_{12} = \vec{L}_{q'} + \vec{L}_{\overline{q}}$ **for mesons,** $\vec{L}_{123} = \vec{L}_{q_1} + \vec{L}_{q_2} + \vec{L}_{q_3}$ **for baryons.**

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$\longrightarrow$

1 - Introduction

The aim of this contribution is to present a short overview of the work in collaboration with Sonia Kabana and myself on 'Oscillatory modes of light flavored u, d, s quarks and antiquarks' , in mesons and baryons – work still in progress . I had the privilege to report on this work in a parallel session to ICNFP2013 and in the poster session to ICNFP2014. Both papers were published in the respective proceedings: ref. 1 = [1-2014] and ref. 2 = [2-2015] .

Meanwhile we were able to carry on with our collaboration in 2015, from where the present contribution results. It shall also serve the purpase to descrbe the basic elements underlying 'Oscillatory modes' . This is done in section 2 . $\longrightarrow$

2 - Timeline of the construction of oscillatory modes

The search for 'Oscillatory modes of light flavored valence quarks and antiquarks' was launched in a paper by Francis Halzen and myself in 1970 and published in ref. 3 = [3-1970] . Our collaboration with Francis Halzen had to be discontinued, whence he left CERN in fall 1971 for the position of professor for theoretical physics at the University of Madison in Madison, WI, USA . In ref. 3 = [3-1970] no construction of 'Oscillatory modes' was undertaken. Rather we investigated the shift in the pole position of the Roper resonance due to the two scattering channels

(1) $$\pi\,\mathcal{N} \longrightarrow \begin{cases} \pi\,\mathcal{N} \\ \pi\,\pi\,\mathcal{N} \end{cases}$$

$\longrightarrow$

in order to estimate deviations of Regge poles from a purely real and linear shape

$$\alpha^{\prime} M^2 (J) = J + J_0 \text{ with } \alpha^{\prime} \Delta M^2 = \Delta J = \Delta N$$

(2) $$\left(\alpha^{\prime} \right)^{-1} = 1.06 \pm 5\% \text{ GeV}^2$$

$\alpha^{\prime}$: **slope of Regge trajectories excepting the Pomeron**

$\longrightarrow$

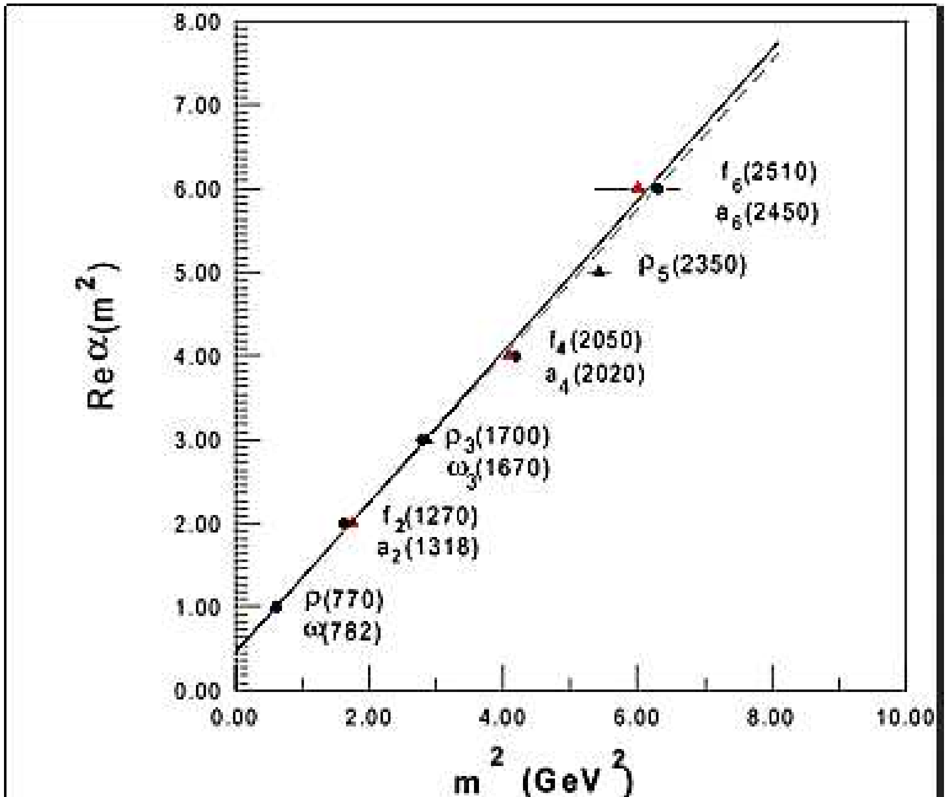

Figure 4. The Chew-Frautschi plot for some mesons and resonances.

Fig. (4 $\longrightarrow$) 1 : The $\varrho - f$ almost exchange Regge trajectory

$\longleftrightarrow$ from ref. 4 = [4-2005]

$\longrightarrow$

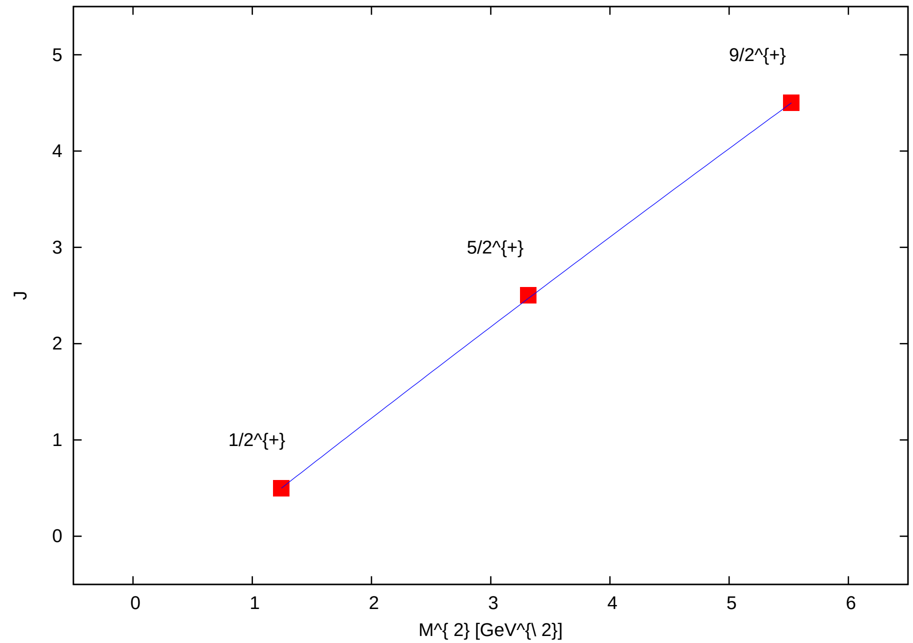

Fig. 2 : The Λ^+ Regge trajectory

$\longleftrightarrow$ **from ref. 5 = [5-2014]**

From Fig. 1 eq. 2 – for mesons – follows $\longrightarrow$

and from Fig. 2 we obtain – for baryons –

$$
\begin{array}{lcccc}
\Lambda\,,\ J^{P} & : & \frac{1}{2}^{+} & \frac{5}{2}^{+} & \frac{9}{2}^{+} \\
M_j & : & 1.115683 & 1.820 & 2.350 \\
M_j^2 & : & 1.2447485 & 3.3124 & 5.5225 \\
\frac{1}{2}\,\Delta M^2 & : & & 1.034 & 1.105
\end{array}
\tag{3}
$$

which implies

$$
\begin{aligned}
1/\alpha' &= \frac{1}{3}\left(M_2^2 - M_1^2\right) + \frac{1}{6}\left(M_3^2 - M_2^2\right) \\
&= 1.06 \pm 5\%\,\mathbf{GeV}^2
\end{aligned}
\tag{4}
$$

Within the errors the inverse slopes for mesons and baryons are identical. $\longrightarrow$

Dynamics of oscillatory modes of – u, d, s – light valence quark and antiquark flavors in mesons

The development of oscillatory modes of valence quarks and antiquarks in mesons was subject of lectures/exercises by one of us (P.M.) in 1978 . Derivations are given in condensed form in ref. 6 = [6-1978] and jointly for mesons, baryons and antibaryons in ref. 7 = [7-1980] .
I quote the textbook by Collins and Squires ref. 8 = [8-1968] for a coherent presentation of Regge theory and a recent theretical overview by Alexander Kaidalov in ref. 9 = [9-2010] on the same and related topics .

This said we turn to the synamics of oscillatory modes in mesons .

$\longrightarrow$

2.1 - The local colored fields associated with oscillatory modes – a prerequisite substrate of oscillatory modes

The initial consideration were devoted to clarify what field content – if any – would be admitted , consistent with locality and microscopic causality without revealing an associate gauge transformation associated color quantum number in the spectrum of hadronic states . In this connection the *minimum* field content versus resonance states within the well identifyable hadrons was aligned through quark and antiquark (spin $\frac{1}{2}$ - fields)

$$\left(q_{\,c\,fl} \right)_{A} (\,x\,) \qquad \leftrightarrow \qquad q\,,\,\overline{q}^{\,'} \qquad \textbf{flavored mesons}$$

$$\left(\overline{q}_{\,\dot{c}\,fl} \right)_{\dot{A}} (\,x\,) \qquad\qquad q\,,\,q^{\,'}, q^{\,''} \quad \textbf{flavored baryons}$$

$c\,,\dot{c}^{\,'} = 1, 2, 3$: **color -** ; $A\,,\,\dot{A}^{\,'} = 1, \cdots, 4$: **spinor indices**

$x = t\,,\,\vec{x}$: **space-time variables**

(5) $\longrightarrow$

We repeat eq. 5 below for clarity

$$\begin{array}{lcll} \left(q_{\,c\,fl} \right)_{A} (\,x\,) & & q\,,\,\overline{q}^{\,'} & \textbf{flavored mesons} \\ & \leftrightarrow & & \\ \left(\overline{q}_{\,\dot{c}\,fl} \right)_{\dot{A}} (\,x\,) & & q\,,\,q^{\,'},q^{\,''} & \textbf{flavored baryons} \end{array}$$

$c\,,\dot{c}^{\,'} \;=\; 1, 2, 3$: **color -** ; $A\,,\;\dot{A}^{\,'} \;=\; 1, \cdots, 4$: **spinor indices**

$x \;=\; t\;,\;\vec{x}$: **space-time variables**

(5)

In eq. 5 the labels and variables pertaining to the local quark and antiquark fields are displayed . On the phenomenological side denoted quark (antiquark) flavored mesons and baryons , besides the PDG listings of $\sim\;1970$, the assignment of resonances to Regge trajectories – excepting the Pomeron – were used , first as known in 1970 . I quote the textbook by Collins and Squires ref. 8 = [8-1968] for a coherent presentation of Regge theory . $\longrightarrow$

The first projection of investigations to be pursued in ref. 3 = [3-1970] which led to the following logical possibilities

1) local quark and antiquark fields – as displayed in eq. 5 – do carry color and *necessarily* **appear together with an octet of gauge connection fields**

$$
\boxed{\begin{array}{l}
(\,\mathcal{W}_{\mu}\,(\,\mathcal{D}\,)\,)_{\alpha\beta}\,(\,x\,)\;=\;W_{\mu}^{\;r}\,(\,x\,)\;(\,d_{\,r}\,)_{\alpha\beta}\;\;\leftrightarrow \\
\mathcal{W}_{\mu}\,(\,\mathcal{D}\,)\;=\;-\,\mathcal{W}_{\mu}\,(\,\mathcal{D}\,)^{\,\dagger} \\[2ex]
d_{\,r}\;=\;-\,d_{\,r}^{\,\dagger}\;=\;\frac{1}{i}\,J_{\,r}\;\in\;Lie\,(\,\mathcal{D}\,) \\
\left[\,d_{\,p}\,,\;d_{\,q}\,\right]\;=\;f_{\,pqr}\,d_{\,r} \\[2ex]
r\,,\;p\,,\;q\;=\;1,\;\cdots,\;dim\,\mathcal{G}\;\;;\;\;\alpha\,,\;\beta\;=\;1,\;\cdots,\;dim\,\mathcal{D}
\end{array}} \qquad (6)
$$

$\longrightarrow$

1) (continued) : We repeat eq. 6 below for clarity

$$\boxed{\begin{array}{l} (\,\mathcal{W}_\mu\,(\,\mathcal{D}\,)\,)_{\alpha\beta}\,(\,x\,) \;=\; W_\mu^{\,r}\,(\,x\,)\,(\,d_{\,r}\,)_{\alpha\beta} \;\leftrightarrow \\ \mathcal{W}_\mu\,(\,\mathcal{D}\,) \;=\; -\,\mathcal{W}_\mu\,(\,\mathcal{D}\,)^{\dagger} \\ \\ d_{\,r} \;=\; -\,d_{\,r}^{\,\dagger} \;=\; \frac{1}{i}\,J_{\,r} \;\in\; Lie\,(\,\mathcal{D}\,) \\ [\,d_{\,p}\,,\,d_{\,q}\,] \;=\; f_{\,pqr}\,d_{\,r} \\ \\ r\,,\,p\,,\,q \;=\; 1,\,\cdots,\,dim\,\mathcal{G}\;;\;\alpha\,,\,\beta \;=\; 1,\,\cdots,\,dim\,\mathcal{D} \end{array}} \tag{6}$$

In eq. 6 $\mathcal{D}$ **denotes an irreducible unitary representation of the gauge group** $G \;=\; SU3_{\,c}$ **,** $Lie(\mathcal{D}\,)$ **the corresponding antihermitian-matrix representation of its Lie algebra. Here** $\mathcal{D}$ **is the** 3 , $(\overline{3})$ **representation for** $q\,,\,\overline{q}\,'$ **fields respectively. The hermitian gauge potentials** $W_\mu^{\,a}\,(\,x\,)$ **form the corresponding gauge connection field as shown in eq. 6 .** $\longrightarrow$

2) The main oscillatory modes : Given 1) to be valid, the first oscillatory modes to be faced are the light flavored valence $q\,\overline{q}\,'$ u, d , s mesons . The associated spectroscopy showed first of all a severe restriction concerning the local *and* **global** *exact* **gauge invariance pertaining not just to the quark-antiquark degrees of freedom , but more importantly those related to gauge potentials and their field strengths , shown in eq. 7 below . What is meant here is the exact gauge invariance , which is necessary to completely remove color and its counting from the hadron resonances in question.**
Here is a good place to quote a general collection of work concerning the construction of QCD in ref. 10 = [10-2014]

$\longrightarrow$

2) (continued)

gauge bosons : $\mathcal{L}_{B} = -\frac{1}{4\,g^{2}}\, B^{\mu\nu\, r}\, B_{\mu\nu}^{\, r}$

$$B_{\mu\nu}^{\,r} = \partial_{\mu} W_{\nu}^{\,r} - \partial_{\nu} W_{\mu}^{\,r} + f_{rst}\, W_{\mu}^{\,s}\, W_{\nu}^{\,t} \longleftarrow$$

$$\left(W_{\mu}^{\,r} \equiv -\, v_{\mu}^{\,r} \right)$$

(7) $$r, s, t = 1, \cdots, dim\,(G = SU3_{\,c}) = 8$$

Lie algebra labels, $\left[\frac{1}{2}\,\lambda^{\,r}\, ,\; \frac{1}{2}\,\lambda^{\,s} \right] = i\, f_{rst}\, \frac{1}{2}\,\lambda^{\,t}$

perturbative rescaling :

$$W_{\mu}^{\,r} = g\, W_{\mu\ pert}^{\,r}\, ,\; B_{\mu\nu}^{\,r} = g\, B_{\mu\nu\ pert}^{\,r}$$

$\longrightarrow$

3) Given 1) and 2) the absence of a sizable weak interaction channel mediated through the neutral current called for the GIM mechanism , in 1970 in ref. 11 = [11-1970] and the existence of charm as a fourth flavor somewhat more massive than three light ones .
Together with the studies extending the light quark flavors , the search was undertaken to detect the neutral current weak interaction through the neutrino or antineutrino channels . This led to the discovery of the neutral current by the Gargamelle collaboration at CERN in 1973 , ref. 12 = [12-1973] .

$\longrightarrow$

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OBSERVATION OF NEUTRINO-LIKE INTERACTIONS WITHOUT MUON OR ELECTRON IN THE GARGAMELLE NEUTRINO EXPERIMENT

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Events induced by neutral particles and producing hadrons, but no muon or electron, have been observed in the CERN neutrino experiment. These events behave as expected if they arise from neutral current induced processes. The rates relative to the corresponding charged current processes are evaluated.

We have searched for the neutral current (NC) and charged current (CC) reactions:

$$\mathrm{NC}\quad \nu_\mu/\bar{\nu}_\mu + \mathrm{N} \rightarrow \nu_\mu/\bar{\nu}_\mu + \text{hadrons}, \tag{1}$$

$$\mathrm{CC}\quad \nu_\mu/\bar{\nu}_\mu + \mathrm{N} \rightarrow \mu^-/\mu^+ + \text{hadrons} \tag{2}$$

which are distinguished respectively by the absence of any possible muon, or the presence of one, and only one, possible muon. A small contamination of $\nu_e/\bar{\nu}_e$ exists in the $\nu_\mu/\bar{\nu}_\mu$ beams giving some CC events which are easily recognised by the e^-/e^+ signature. The analysis is based on 83 000 ν pictures and 207 000 $\bar{\nu}$ pictures taken at CERN in the Gargamelle bubble chamber filled with freon of density 1.5×10^3 kg/m^3 ‡. The dimensions of this chamber are such that most

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‡ A more detailed account of the analysis of this experiment appears in a paper to be submitted to Nuclear Physics.

Fig. 3 : Titlepage of ref. 12 = [12-1973] ⟷

4) the gauge group forms a *minimal* collection including spontaneously broken subgroups
The consequence of the discovery of the neutral current weak interaction comparable in strength with the charged current one is , that beyond the exactly unbroken subgroup of $SU3_{c}$ the electroweak part – $SU2_{L} \times U1_{\mathcal{Y}}$ – form together a lowest level of combined exact and broken group

$$\begin{aligned} G_{min} &= SU3_{c} \times SU2_{L} \times U1_{\mathcal{Y}} \\ Q_{em} / |e| &= I_{3\,w} + \mathcal{Y} \end{aligned} \tag{8}$$

In eq. 8 $I_{3\,w}$ denotes the third component of weak isospin , $\mathcal{Y}$ the weak hypercharge commuting with $SU2_{L}$, Q_{em} the electric charge and $|e|$ the charge of the proton in rational units .

$\longrightarrow$

5) Quest for a unified gauge theory – of chargelike gauges – neglecting gravity

After pioneering work in this direction by by Jogesh Pati and Abdus Salam ref. 13 = [13-1973] , Howard Georgi and Sheldon Glashow ref. 14 = [14-1974] and Feza Gursey , Pierre Ramond and Pierre Sikiivie ref. 15 = [15-1976] , we initiated a systematic investigation with this goal with Harald Fritzsch by the end of 1973 giving rise to a minimal such enveloping gauge group, spin10, in ref. 16 = [16-1975] . To stay in line with the experimental discovery of charm, as predicted in ref. 11 = [11-1970] in point 3) , we recall the so called 'November revolution' in 1974 , ref. 17 = [17-1974] and ref. 18 = [18-1974] .

$\longrightarrow$

6) Reflections on the nature of interactions in the environment of gauge-invariance and gauge-breaking within unified gauge field theory(-ies)

Given the *assumed* **validity of the logical chains in points 1) - 5) I have to conclude it established , that the nature of unifyed gauge theories is singled out . It is however impossible at present, to specify the deduced subtle properties of these theories in precise mathematical terms . The restriction to gravitationless theories in uncurved 3 + 1 - dimensional space-time is of course quite restrictive .**

Recent wider discussions of these and related points can be found in ref. 19 = [19-2012] , ref. 20 = [20-2014] and ref. 21 = [21-2014] .

2.2 - Oscillatory modes of valence quarks and antiquarks $q\ ,\overline{q}^{\ \prime}\ ;\ q,q^{\ \prime}\ =$ u. d, s in mesons

The first elaboration of the mesonic oscillatory modes was not undertaken in 1970, the year of the project formulation in ref. 3 = [3-1970] . This I started during the last months of my stay at Caltech until end of February 1976 . It took two years to bring about a written (published) report – ref. 6 = [6-1978] – at the occasion of a lectture given at the 1978 Karslruhe Summer Institute . Ref. 6 could recently be updated with the help of the representatives of inspirehep.net , after a package of reprints at an unexpected location in Bern were found .

And this goes as follows : $\longrightarrow$

we report the derivations leading to the asymptotic form for the eigenvalues of the masssquare operator in the meson c.m. frame from refs. 7 = [7-1980] and 6 = [6-1978] extending eq. 9 below

$$\alpha^{\prime} M^2 \left(J \right) = J + J_0 \text{ with } \alpha^{\prime} \Delta M^2 = \Delta J = \Delta N$$

(9) $$\left(\alpha^{\prime} \right)^{-1} = 1.06 \pm 5\% \text{ GeV}^2$$

$\alpha^{\prime}$: **slope of Regge trajectories excepting the Pomeron**

In order to keep the systematics of barycentric coordinates as defined for general $N_c \equiv \mathbb{N}$ **, also appropriate for baryons discussed in ref. 7 = [7-1980] , we set for the kinematically simpler** $q\,\overline{q}^{\prime}$ **mesons (ref. 7 was achieved after obstacles had been overcome, which postponed it to 1980)** $\longrightarrow$

(10)
$$\vec{z} = \vec{z}_1 = \frac{1}{\sqrt{2}} (\vec{x}_1 - \vec{x}_2) \text{ and}$$
$$\vec{y} = \vec{x}_1 - \vec{x}_2 = \sqrt{2}\, \vec{z}$$

Next we consider the Lagrangean

$$\mathcal{L}_{q\bar{q}}{}' = - \left[\begin{array}{r} m_1 (\vec{y} ; M_1) \sqrt{1 - \vec{v}_1^{\,2}} \\ + m_2 (\vec{y} ; M_2) \sqrt{1 - \vec{v}_2^{\,2}} \end{array} \right]$$

(11)

$M_{1,2}$: **masses of** $q_1 , \bar{q}_2$ **respectively**

$\vec{v}_{1,2} = (d / dt) \vec{x}_{1,2}$; **in the meson c.m. system**

t : **overall synchronized time in the c.m. system ,**

using units such that $c = 1$

In the next step we choose the chiral limit as a valid approximation at large distances , to be specified subsequently . $\longrightarrow$

Then eq. 11 simplifies through the relations

(12)
$$
\begin{aligned}
&m_1(\vec{y};\, M_1 \to 0) \to m_1(\vec{y}) \\
&m_2(\vec{y};\, M_2 \to 0) \to m_2(\vec{y}) \\
&m_1(\vec{y}) = m_2(\vec{y}) = m(\vec{y}) \\
&\vec{v}_1 = -\vec{v}_2 = \vec{v}
\end{aligned}
$$

and $\mathcal{L}_{q\bar{q}}{}'$ **in eq. 11 becomes**

(13)
$$
\begin{aligned}
&\mathcal{L}_{q\bar{q}}{}' = -\overline{m}(\vec{y})\sqrt{1-\vec{v}^{\,2}} \\
&(\overline{m} = m_1 + m_2 = 2\,m)(\vec{y}) \\
&\vec{v} = \tfrac{1}{2}\,\dot{\vec{y}}\ ;\ \dot{} = d/dt
\end{aligned}
$$

We expand the derivations following ref. 7 = [7-1980] . $\longrightarrow$

The canonical momentum relative to $\frac{1}{2}\vec{y}$ becomes

$$\vec{p} = \vec{p}_1 - \vec{p}_2 = 2\,\vec{p}_{c.m.} = \left(\mathcal{L}_{q\overline{q}}{}' \right)_{,\,\vec{v}} = \overline{m}\,\frac{\vec{v}}{\sqrt{1 - \vec{v}^{\,2}}}$$

$$\mathcal{H}_{(2)} = \vec{v} \left(\mathcal{L}_{q\overline{q}}{}' \right)_{,\,\vec{v}} - \mathcal{L}_{q\overline{q}}{}' = \frac{\overline{m}}{\sqrt{1 - \vec{v}^{\,2}}}$$

$$\vec{p}^{\,2} = \overline{m}^{\,2}\,\frac{v^2}{1 - v^2} = \mathcal{H}_{(2)}^2 - \overline{m}^{\,2}\;;\quad v^2 = \vec{v}^{\,2} \quad \longrightarrow$$

$$\vec{p}^{\,2} + \overline{m}^{\,2} = \mathcal{H}_{(2)}^2$$

(14)

$\longrightarrow$

It follows from the relations in eq. 14 that $\mathcal{H}_{(2)}$ is a constant of motion in both classical and quantum mechanical interpretations of the two body $q\,\overline{q}^{\,\prime}$ system considered . The Euler-Lagrange equations become

$$
\begin{aligned}
\dot{\vec{p}} \;&=\; \mathcal{H}_{(2)}\,\ddot{\vec{v}} = \mathcal{H}_{(2)}\,\tfrac{1}{2}\,\ddot{\vec{y}} = \left(\mathcal{L}_{q\overline{q}}{}'\right)_{,\,\frac{1}{2}\,\vec{y}} \\
&=\; -\sqrt{1-\vec{v}^{\,2}}\;2\,grad_{\,\vec{y}}\,\overline{m} \;=\; -\,\mathcal{H}_{(2)}^{-1}\,\overline{m}\,grad_{\,\vec{y}}\,\overline{m}^{\,2} \\
\longrightarrow\;\; \mathcal{H}_{(2)}\,\tfrac{1}{2}\,\ddot{\vec{y}} \;&=\; -\left(\mathcal{H}_{(2)}\right)^{-1}\,2\,\overline{m}\,grad_{\,\vec{y}}\,\overline{m} \\
&=\; -\left(\mathcal{H}_{(2)}\right)^{-1}\,grad_{\,\vec{y}}\,\overline{m}^{\,2}
\end{aligned}
\tag{15}
$$

We can see how the mass-square dynamical variable arises in the classical interpretation of the equations of motion $\longrightarrow$

The sum of the squares $S_2(N^*)$ is in the present context an *auxiliary* **function necessary for the calculation of the total number of meson states with $N \leq N^*$.**
We thus find for the number of u,d,s meson states with $N \leq N^*$

$$
\begin{aligned}
Z(N^*) &= \\
&= 18\left(S_2(N^*+1) + \tfrac{1}{2}(N^*+1)(N^*+2)\right) \\
\text{(16)} \qquad &= 18\left(\begin{array}{l} \frac{1}{6}(N^*+1)(N^*+2)(2N^*+3) \\ +\frac{1}{2}(N^*+1)(N^*+2) \end{array}\right) \\
&= 18\left((N^*+1)(N^*+2)\left(\tfrac{1}{3}(N^*+1)\right)\right) \\
&= 6\,(N^*+1)(N^*+2)(N^*+3)
\end{aligned}
$$

$\longrightarrow$

introducing the notation

(17) $$\mathcal{M}^{\,2}_{\,meson} \;=\; \mathcal{H}^{\,2}_{\,(2)}$$

Then eq. 15 takes the form

(18) $$\mathcal{M}^{\,2}_{\,meson}\,\ddot{\vec{y}} \;=\; -\,2\,grad_{\,\vec{y}}\,\overline{m}^{\,2}$$

The oscillatory modes inherit the central scale of QCD for light (u, d , s-) flavors of quark as implied by the trace anomaly – ref. 26 = [22-1975] , in the adopted chiral limit through the parameter Λ , not to be confused with $\Lambda_{\,QCD}$, valid in the perturbatively accessible region, in the Ansatz for the mass function for large values of $|\,\vec{y}\,|$, of dimension mass-square

(19) $$\overline{m}^{\,2}\,(\,\vec{y}\,) \sim \underset{|\vec{y}|\to\infty}{\longrightarrow} \sim \frac{1}{4}\,\Lambda^{\,2}\,|\,\vec{y}\,|^{\,2}\left[\,1\,+\,O\left(\frac{M_{\,q}}{\Lambda\,|\,\vec{y}\,|}\right)\,\right]$$

$\longrightarrow$

In eq. 19 M_q denote the physical quark-mass parameters Inserting the asymptotic large $|\vec{y}|$ part of the mass-square function $\overline{m}^{\,2}(\vec{y})$ into eq. 18 we obtain

$$\mathcal{M}^{\,2}_{meson}\,\ddot{\vec{y}} = - \sim \Lambda^{2}\,\vec{y} \;\leftrightarrow\; \omega_{cl} = \frac{\Lambda}{\mathcal{H}_{(2)}}$$

(20)

$$\omega^{\,2}_{cl} = \frac{\Lambda^{2}}{\mathcal{M}^{\,2}}$$

We here turn to the quantum mechanical description following from the Lagrangean given in eq. 13

$$\widehat{\vec{p}} = \frac{1}{i}\,\nabla_{\frac{1}{2}\vec{y}} = \left(\mathcal{L}_{q\overline{q}}{}'\right)_{,\vec{v}} = \left(\overline{m}\,\frac{\vec{v}}{\sqrt{1 - v^{2}}}\right)_{ordered}$$

(21)

$\longrightarrow$

In eq. 21 the suffix $ordered$ shall indicate that the operators $\overline{m}$ and $\dfrac{\vec{v}}{\sqrt{1 - v^{2}}}$, written as product, do not commute, which necessitates the ordering guaranteeing a self-adjoint operator beeing represented by the product .

2.1.1 - Counting oscillatory modes of valence quarks and antiquarks $q\,, \overline{q}\,'\,;\; q, q\,' \;=$ u. d, s in mesons

We first determine the number density at given main quantum number N , which amounts to calculate the power of the set of occupation numbers $n_{1}\,,\, n_{2}\,,\, n_{3}$ of the associated 3 oscillators . p is given by the number of partitions

$$p\,(N) = \{n_1, n_2, n_3|\; n_1 + n_2 + n_3 = N \;\; ; \; n_{1,2,3} = 0, 1, 2 \cdots \; N\}$$

(22)

multiplied with the multiplicity of $SU6(spin \; \times \; N_{fl}) = 36$. $\longrightarrow$

The power of the set $p(N)$ is readily written as a sum over n_3

$$p(N) = \sum_{n_3=0}^{N} p(n_1, n_2; \nu) = \sum_{\nu=0}^{N} p(n_1, n_2; \nu)$$

$$\nu = N - n_3 = 0, 1, \cdots, N \; ; \; n_{1,2} = 0.1, 2 \cdots N$$

$$p(n_1, n_2; \nu) = \{ n_1, n_2 | \; n_1 + n_2 = \nu \} = \nu + 1$$

(23)

Thus $p(N)$ defined in eq. 22 becomes

$$\begin{aligned} p(N) &= \sum_{\nu=0}^{N} (\nu + 1) = \sum_{\nu_+=1}^{N+1} \nu_+ \\ &= \tfrac{1}{2}(N+1)(N+2) \end{aligned} \qquad (24)$$

$$\nu_+ = \nu + 1$$

and $z(N) = 36\, p(N)$ is $\longrightarrow$

(25)
$$z(N) = \partial\varrho/\partial\left(\alpha' M^2\right) = 18\,(N+1)\,(N+2)$$
$$\alpha' M^2 = \alpha' \Delta M^2 + N_0 = N + N_0$$

We give here the derivation yielding the sum of squares of integers from the difference of sums of cubes

(26) $$S_2(N^*) = \sum_{\nu=0}^{N^*} \nu^2\ ,\ S_3(N^*) = \sum_{\nu=0}^{N^*} \nu^3$$

which makes use of the recursive relation

(27)
$$\begin{aligned} S_3(N^*+1) - S_3(N^*) &= \\ &= \sum_{\nu=0}^{N^*} \left[(\nu+1)^3 - \nu^3 \right] \\ &= (N^*+1)^3 \\ &= 2\,S_2(N^*) + \tfrac{3}{2}\,N^*(N^*+1) + N^* + 1 \end{aligned}$$

$\longrightarrow$

Then it follows

(28) $$S_2(N^*) = \frac{1}{6} N^*(N^* + 1)(2N^* + 1)$$

We display the function $S_2(N^*),\ N^* = 1$ **to** 5 **in table 1 below**

(29)

N^*	$S_2(N^*)$
1	1
2	5
3	14
4	30
5	55

Table 1

The sum of the squares $S_2(N^*)$ is in the present context an *auxiliary* **function necessary for the calculation of the total number of meson states with $N \le N^*$.**
We thus find for the number of u,d,s meson states with $N \le N^*$

$$Z(N^*) =$$

$$= 18\left(S_2(N^*+1) + \frac{1}{2}(N^*+1)(N^*+2)\right)$$

(30)
$$= 18\begin{pmatrix} \frac{1}{6}(N^*+1)(N^*+2)(2N^*+3) \\ +\frac{1}{2}(N^*+1)(N^*+2) \end{pmatrix}$$

$$= 18\left((N^*+1)(N^*+2)\left(\frac{1}{3}(N^*+1)\right)\right)$$

$$= 6\,(N^*+1)(N^*+2)(N^*+3)$$

$\longrightarrow$

The main results of this subsection are contained in eqs. 25 for the number density as a function of the main oscillator quantum number N: $z(N) = \partial \varrho / \partial \left(\alpha' M^2 \right)$ **and 16 for the total number of oscillatory modes below and including the limiting main quantum number** N^* **:** $Z(N^*)$ **, both equations recapitulated for u, d, s mesons** $q \overline{q}'$ **below**

(31)
$$\begin{aligned} z(N) &= \partial \varrho / \partial \left(\alpha' M^2 \right) = 18 (N+1)(N+2) \\ \alpha' M^2 &= \alpha' \Delta M^2 + N_0 = N + N_0 \end{aligned}$$

---

(32)
$$Z(N^*) = 6 (N^* + 1)(N^* + 2)(N^* + 3)$$

$\longrightarrow$

2 - Oscillatory modes of valence quarks and antiquarks $q\,,\overline{q}\,'\,;\;q, q\,'\;=$ u. d, s in mesons (continued)

We recall here the relations in eqs. 12 and 13 , repeated below

(33)

$$
\begin{aligned}
& m_{\,1}\;(\,\vec{y}\,;\;M_{\,1}\;\rightarrow\;0\,)\;\rightarrow\;m_{\,1}\;(\,\vec{y}\,) \\
& m_{\,2}\;(\,\vec{y}\,;\;M_{\,2}\;\rightarrow\;0\,)\;\rightarrow\;m_{\,2}\;(\,\vec{y}\,) \\
& m_{\,1}\;(\,\vec{y}\,)\;=\;m_{\,2}\;(\,\vec{y}\,)\;=\;m\;(\,\vec{y}\,) \\
& \vec{v}_{\,1}\;=\;-\,\vec{v}_{\,2}\;=\;\vec{v}
\end{aligned}
$$

$\mathcal{L}_{\,q\overline{q}\,'}$ **in eq. 13 becomes**

(34)

$$
\begin{aligned}
& \mathcal{L}_{\,q\overline{q}\,'}\;=-\,\overline{m}\;(\,\vec{y}\,)\;\sqrt{1\,-\,\vec{v}^{\,2}} \\
& (\,\overline{m}\;=\;m_{\,1}\;+\;m_{\,2}\;=\;2\,m\,)\;(\,\vec{y}\,) \\
& \vec{v}\;=\;\frac{1}{2}\,\dot{\vec{y}}\;;\;\;{}^{\cdot}\;=\;d\,/\,dt
\end{aligned}
$$

$\longrightarrow$

Next we recall eq. 21 repeated below

$$\widehat{\vec{p}} \;=\; \frac{1}{i}\,\nabla_{\,\frac{1}{2}\,\vec{y}} \;=\; \left(\,\mathcal{L}_{\,q\overline{q}}{}'\,\right)_{,\,\vec{v}} \;=\; \left(\,\overline{m}\,\frac{\vec{v}}{\sqrt{1\;-\;v^{\,2}}}\,\right)_{ordered}$$

(35)

Eq. 14 adapted to the quantum mechanical logic becomes

$$\widehat{\vec{p}} = \widehat{\vec{p}}_1 - \widehat{\vec{p}}_2 = 2\widehat{\vec{p}}_{c.m.} = \left(\mathcal{L}_{q\overline{q}}{}'\right)_{,\,\vec{v}} = \overline{m}\,\frac{\vec{v}}{\sqrt{1-\vec{v}^2}}\Bigg|_{ordered}$$

(36)

$$\widehat{\mathcal{H}}_{\,(2)} \;=\; \vec{v}\,\left(\,\mathcal{L}_{\,q\overline{q}}{}'\,\right)_{,\,\vec{v}} \;-\; \mathcal{L}_{\,q\overline{q}}{}' \;=\; \frac{\overline{m}}{\sqrt{1\;-\;\vec{v}^{\,2}}}\Bigg|_{ordered}$$

$\longrightarrow$

$$\widehat{\vec{p}}^{\,2} = \overline{m}^{\,2} \left. \frac{v^2}{1 - v^2} \right|_{ordered} = \widehat{\mathcal{H}}^2_{(2)} - \overline{m}^{\,2} \; ; \; v^2 = \vec{v}^{\,2} \longrightarrow$$

(37)

$$\widehat{\vec{p}}^{\,2} + \overline{m}^{\,2} = \widehat{\mathcal{H}}^{\,2}_{(2)} = \widehat{\mathcal{M}}^{\,2} \; ; \; \widehat{\vec{p}}^{\,2} = -\Delta_{\frac{1}{2}\vec{y}} = -4\,\Delta_{\vec{y}}$$

$$\overline{m}^{\,2}(\vec{y}) \sim \underset{|\vec{y}| \to \infty}{\longrightarrow} \sim \tfrac{1}{4}\Lambda^2 |\vec{y}|^2 \left[1 + O\left(\frac{M_q}{\Lambda |\vec{y}|} \right) \right]$$

Eq. 37 shows the main result of this section , in particular the second order wave equation (on the second line)

(38) $$\widehat{\mathcal{M}}^{\,2} = \left[-4\,\Delta_{\vec{y}} + \tfrac{1}{4}\Lambda^2 |\vec{y}|^2 \right]$$

$\longrightarrow$

The parameter λ , of dimension mass, shall be chosen such that

(39) $4\,\lambda^{2} = \frac{1}{4}\,\lambda^{-2}\,\Lambda^{2} \;\rightarrow\; 2\,\lambda = \frac{1}{2}\,\lambda^{-1}\,\Lambda \;\rightarrow\; 4\,\lambda^{2} = \Lambda$

Substituting the last equation in eq. 39 in eq. 38 we obtain

(40) $$-4\,\Delta_{\vec{y}} + \frac{1}{4}\,\Lambda^{2}\,|\,\vec{y}\,|^{2} = \Lambda\left[-\Delta_{\vec{\zeta}} + |\,\vec{\zeta}\,|^{2}\right]$$

$\longrightarrow$

In order to exhibit the oscillator variables we substitute rescaled coordinates and derivatives relative to the spatial variable $\vec{y}$

(41) $$\vec{y} = \lambda^{-1} \vec{\zeta}\,, \;\; \nabla_{\vec{y}} = \lambda \nabla_{\vec{\zeta}}$$

The differential operator on the right hand side of eq. 42 becomes

(42) $$-4\,\Delta_{\vec{y}} + \frac{1}{4}\Lambda^2 |\,\vec{y}\,|^2 = -4\,\lambda^2 \Delta_{\vec{\zeta}} + \frac{1}{4}\lambda^{-2}\Lambda^2 |\,\vec{\zeta}\,|^2$$

$\longrightarrow$

The dimensionless rescaled spatial variables $\vec{\zeta}$ defined through eqs. 41 and 39

(43) $$\vec{\zeta} = \lambda \, \vec{y} \; ; \; \lambda = \frac{1}{2} \left(\Lambda \right)^{\frac{1}{2}}$$

and their canonically conjugate momenta

(44) $$\widehat{\vec{p}}_{\vec{\zeta}} = \frac{1}{i} \nabla_{\vec{\zeta}} = \frac{1}{i} \partial_{\vec{\zeta}}$$

in components

(45) $$\zeta^{m} \, , \, \widehat{p}_{n} = \partial_{\zeta^{n}} \; ; \; \left[\widehat{p}_{n} \, , \, \zeta^{m} \right] = \frac{1}{i} \, \delta_{n}^{\;m} \; ; m, n = 1, 2, 3$$

generate the 3 oscillator $\longrightarrow$

absorption and creation operators

$$a^{m} = \frac{1}{\sqrt{2}} \left(\zeta^{m} + i \, \widehat{p}_{m} \right) \quad a^{*m} = \frac{1}{\sqrt{2}} \left(\zeta^{m} - \partial_{\zeta^{m}} \right)$$

$$= \frac{1}{\sqrt{2}} \left(\zeta^{m} + \partial_{\zeta^{m}} \right) , \qquad = \frac{1}{\sqrt{2}} \left(\zeta^{m} - \partial_{\zeta^{m}} \right)$$

$$m = 1, 2, 3$$

(46)

obeying the commutation rules

$$\left[a^{m} , a^{*n} \right] = \delta^{mn} \, ¶ \; ; \; \left[a^{m} , a^{n} \right] = \left[a^{*m} , a^{*n} \right] = 0$$

(47)

The oscillator algebra displayed in eq. 47 is common to any (3) canonical pairs of operators , associated with a threedimensional uncurved space, as is the case here. It shows directly the U3-invariance of the commutation rules .

What is very special is the relation for the dynamical form of the mass square operator $\widehat{\mathcal{M}}^{\,2}$ as given in eq. 40 , which becomes

$$\begin{aligned} \widehat{\mathcal{M}}^{\,2} &= \Lambda \left[-\Delta_{\vec{\zeta}} + \vec{\zeta}^{\,2} \right] = 2\,\Lambda \left[\widehat{N} + \tfrac{3}{2}\,¶ \right] \\ \widehat{N} &= \sum_{m=1}^{3} \widehat{N}_m \ ; \ \widehat{N}_n = a^{*\,n} a^{\,n} - \tfrac{1}{2}\,¶ \end{aligned} \tag{48}$$

For an individual n we have

$$\begin{aligned} 2\,a^{*\,n}\,a^{\,n} &= \left(\zeta^{\,n} - \partial_{\zeta^{\,n}} \right) \left(\zeta^{\,n} + \partial_{\zeta^{\,n}} \right) \\ &= -\,\partial^{\,2}_{\zeta^{\,n}} + \left(\zeta^{\,n} \right)^{2} - ¶ \end{aligned} \tag{49}$$

which proves the correctness of the first relation in eq. 48 . $\longrightarrow$

Identifying $2\,\Lambda$ with the inverse slope of Regge trajectories other than the Pomeron

Since the relation in eq. 48 is only valid for large eigenvalues of the number operator $\widehat{N}$, with eigenvalues $N = n_1 + n_2 + n_3 = 0, 1, 2, \cdots$, where n_k, $k = 1, 2, 3$ denote the eigenvalues of the individual counting operators $\widehat{N}_k$, we can reparametrize within the same approximation accuracy eq. 48 in the form

$$\widehat{\mathcal{M}}^2 == 2\,\Lambda \left[\,\widehat{N} + \widehat{N}_0\right] \tag{50}$$

In eq. 50 the operator $\widehat{N}_0$ contains all effects from short distances and parametrically depends on quark masses . It does not commute with the asymptotically dominating number operators $\widehat{N}$, $\widehat{N}_k$; $k = 1, 2, 3$. $\longrightarrow$

We deal with the 'intercept'-related perturbations through the operator $\widehat{N}_0$ as characterized in eq. 50 and the text specifying its details in the sense of perturbations of *large* eigenvalues of $\widehat{N}$, the dominant operator for large eigenvalues N of the mass square operator $\widehat{\mathcal{M}}^2$, adopting the ansatz for the eigenvalues of $\widehat{\mathcal{M}}^2$

(51) $\mathcal{M}^2 \sim 2\,\Lambda\,(N + N_0) \;\; ; \;\; N = 0, 1, 2 \cdots$ **yet** $\gg 1$

We compare the structure of $\mathcal{M}^2$ in eq. 51 with the relation to this quantity along a Regge trajectory

$\longrightarrow$

taken approximately linear

$$\alpha\,(\,\mathcal{M}^{\,2}\,)\;=\;\alpha^{\,'}\,\mathcal{M}^{\,2}\;+\;\alpha_{\,0}\;\rightarrow\;\alpha^{\,'}\;:$$ **universal slope of Regge trajectories**

(52)
$$\alpha\,(\,\mathcal{M}^{\,2}\,)\;\rightarrow\;J\;\rightarrow\;N\;\;;\;\;\mathcal{M}^{\,2}\;=\;\frac{1}{\alpha^{\,'}}\;\;(\,N\;-\;\alpha_{\,0}\,)$$

Comparing the two expressions for $\mathcal{M}^{\,2}$ in eqs. 51 and 52 , we obtain the sought identification

(53)
$$2\,\Lambda\;=\;\frac{1}{\alpha^{\,'}}\;\sim\;1.06\;\pm\;0.05\;\textbf{GeV}^{\,2}$$

$\longrightarrow$

3. Summary and Outlook

In conclusion and outlook we formulate the following remarks :

1) The oscillatory modes , concentrating on the u,d,s light flavored ones forming the $q\ ,\overline{q}\ '$ mesons obey the associated harmonic oscillator differential equation in the form adapted to the inherent

$\longrightarrow$

spontaneous scale

$$\widehat{\mathcal{M}^{\,2}} = \Lambda \left[-\Delta_{\vec{\zeta}} + |\,\vec{\zeta}\,|^{\,2} \right]$$

$$\sim 2\,\Lambda \left[\sum_{n=1}^{3} \left(a^{\,*\,n}\, a^{\,n} + \tfrac{1}{2}\, ¶ \right) \right]$$

(54)

$$a^{\,m} = \frac{1}{\sqrt{2}} \left(\zeta^{\,m} + i\, \widehat{p}_{\,m} \right)$$

$$= \frac{1}{\sqrt{2}} \left(\zeta^{\,m} + \partial_{\,\zeta^{\,m}} \right)$$

$\longrightarrow$

and satisfies for large eigenvalues N of $\sum_{n=1}^{3} a^{*\,n}\, a^{\,n}$

(55) $$\alpha^{'} \mathcal{M}^{2} \sim N$$

2) The new element consists in counting these modes in mesons and baryons . We hope to be able to complete this work in progress in a careful and encompassing way .

— Thank you for your attention —

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