Charmonium production off nuclei: a story of surprises

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Early years of high-energy QCD: nuclear effects

Wide-spread believe in the 70s: hard processes experience no nuclear effects. Naive, but sounds natural...

\[ R_{PA} \equiv \frac{\sigma(hA)}{A\sigma(hN)} = A^\alpha \]

Thus, \(\alpha=1\) has been anticipated, and indeed was well confirmed by data.


The first surprise: NA3 experiment

K.J. Anderson et al. PRL 42(1979)944

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<th>225 GeV/c</th>
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Energy loss?

The first explanation: initial state energy loss

F. Niedermayer & B.K. 1984

\[ \frac{dE}{dz} = -\kappa \]

\( \kappa = 3 \text{ GeV/fm} \)

\[ \psi \]

\( e^{-\sigma_{\text{abs}} \rho A L} \)

\( dE/dz \) is energy independent in the string model, but also in pQCD

F. Niedermayer PRD34(1986)3494
S. Brodsky & P. Hoyer, PLB298 (1993)165

\[ x_F^\psi \Rightarrow x_F^\psi + \Delta E/E \]

\[ \sigma_{hN}(x_F) \Rightarrow \sigma_{hN}(x_F + \Delta x_F) \]

The shift in \( x_F \) causes suppression
E772/866 experiments: more surprises

Data at 200 and 800 GeV demonstrate energy independence and xF-scaling

\[ x_1 x_2 = \frac{M_{\psi_T}^2}{s} \]
\[ x_1 - x_2 = x_F \]

If suppression at large xF were related to a modification of PDFs, one would expect scaling in x2, or in

\[ p_\psi = \frac{M_{\psi}^2}{2m_N x_2} \]

Data rule out x2-scaling.

RHIC data confirm lack of x2 scaling
The data also exclude the above explanation with energy-independent energy loss. However, the nonperturbative Fock-state decomposition of the incoming hadron leads to energy loss proportional to energy. Every process measured so far exposes nuclear suppression increasing towards the kinematic bound.
Naively one should have anticipated a much stronger nuclear suppression for the radial excitation $\psi'$, which has the mean radius squared twice as big as $J/\psi$.

Two time scales controlling the nuclear effects

S. Brodsky & A. Mueller PLB206(1988)685

- Coherence time
  \[ t_c = \frac{2E_{\psi}}{4m_c^2} \]

- Formation time of the charmonium wave function
  \[ t_f = \frac{2E_{\psi}}{M_{J/\psi}(M_{\psi'} - M_{J/\psi})} \gg t_c \]

The naive picture of a charmonium propagating through the nucleus is relevant only at low energies $E_{\psi} < 10$ GeV, when $t_f$ is short, otherwise the dynamics is more involved.

A small-size $c\bar{c}$ pair propagates through the nucleus and afterwards is projected to the charmonium wave function. The latter for $\psi'$ has a node in $r$-dependence, which might cause even nuclear enhancement instead of strong suppression.

Energy dependence of $\sigma_{\text{abs}}$

$e^{-\sigma_{\text{abs}} \rho_\Lambda L}$

$J/\Psi$

¿ NA60: why does $\sigma_{\text{abs}}$ decrease with energy?
**pA: J/Ψ formation and color transparency**

A $c\bar{c}$ dipole is produced with a small separation $r_{c\bar{c}} \sim \frac{1}{m_c} \sim 0.1 \text{fm}$ and then evolves into a J/Ψ mean size $r_{J/Ψ} \sim 0.5 \text{ fm}$ during formation time $t_f = \frac{2E_{J/Ψ}}{m_{Ψ'}^2 - m_{J/Ψ}^2} = 0.1 \text{ fm} \left(\frac{E_{J/Ψ}}{1 \text{ GeV}}\right)$.

**Perturbative expansion**

$$r_T^2(t) = \frac{8t}{E_{c\bar{c}}} + \frac{\delta}{m_c^2}$$

The mean cross section is $L$- and $E$-dependent

$$\bar{\sigma}_{abs}(L, E_{c\bar{c}}) = \frac{1}{L} \int_0^L dl \sigma_{abs}(l) = C(E_{c\bar{c}}) \left(\frac{4L}{E_{c\bar{c}}} + \frac{\delta}{m_c^2}\right)$$

$$R_{pA} = \frac{1}{A\sigma_{abs}} \int d^2b \left[1 - e^{-\sigma_{abs}T_{A}(b)}\right]$$
**pA: Higher twist c-quark shadowing**

At higher energies $\bar{\sigma}_{\text{abs}}$ is affected by another time scale, the lifetime of a $c\bar{c}$ fluctuation

$$t_p = \frac{2E_J/\Psi}{m_c^2/\Psi} = \frac{1}{x_2 m_N} \quad \text{(5 times shorter than } t_f)$$

If $t_p \gtrsim R_A$ the initial state fluctuation $g \to \bar{q}q$ leads to shadowing corrections related to a non-zero $\bar{c}c$ separation.

Path integral technique: all possible paths of the quarks are summed up; $\sigma_{\text{abs}}(r_T, E_{\bar{c}c})$ gives the imaginary part of the light-cone potential.
Example: photoproduction of vector mesons

\[ \gamma^* \rightarrow V \quad t_c >> R_A \quad A^{1/3} \]

\[ t_c << R_A \quad A^{2/3} \]

The quantum-mechanical effect of coherence is proven theoretically and clearly seen in data.

The energy range of RHIC-LHC is well in the regime of \( t_c \gg R_A \)

Mechanisms of $J/\psi$ production in pp collisions

**Color singlet mechanism**

E. Berger & D. Jones PRD 23(1981)1521

**Collinear factorization**


**$k_T$ factorization**

F. Abe et al., PRL 79(1997)572
Mechanisms of $J/\Psi$ production in pp collisions

$k_T$ factorization

S. Baranov, A. Lipatov, N. Zotov
PR D85(2012)014034

An updated unintegrated gluon distribution
Mechanisms of $J/\psi$ production in pp collisions

Modified color singlet mechanism

NLO

\begin{align*}
\text{nLO} & \quad \text{NNLO} \\
\text{NLO} & \quad \text{NNLO}
\end{align*}

\[ J/\psi \rightarrow J/\psi \]

\[ J/\psi \rightarrow J/\psi \]

V.A. Khoze, A.D. Martin, M.G. Ryskin and W.J. Stirling


The NNLO contribution is enhanced by the factor $\ln s$, which allows to bring the cross section up to the data.
Ad hoc assumption that the characteristic time of color neutralization is as long as the formation time

\[ t \gtrsim t_f \]

Long time propagation without gluon radiation is strongly Sudakov suppressed.

The model fits the data to be explained.

“I think you should be more explicit here in step two”
Data for central Pb-Pb collisions expose a stronger $J/\psi$ suppression compared with the cold nuclear matter effects extrapolated (incorrectly) from pA to AB.

A stronger suppression was predicted in QGP due to Debye screening and melting of the bound state.

Matsui & H. Satz PLB178(1986)416

Later, however, the temperature was found to be too low to dissociate $J/\psi$

Cold nuclear matter is not cold


The radiated gluons participate in the \( \bar{c}c \) break-up, as well as in broadening

**Gluon radiation time**

\[
I^g_f = \frac{2 E_q \alpha (1 - \alpha)}{\alpha^2 m_q^2 + k^2}
\]

\[
\langle n_g \rangle = \frac{3}{\sigma_{in}(NN)} \int_{k^2_{\min}}^\infty dk^2 \int_{\alpha_{\min}}^1 d\alpha \frac{d\sigma(qN \rightarrow gX)}{d\alpha dk^2} \Theta(\Delta z - I^g_f)
\]

\[
\langle n_g \rangle = \begin{cases} 
6.9 \times 10^{-1} 
& (SPS, \sqrt{s} = 20 \text{ GeV}) \\
6.9 \times 10^{-3} 
& (RHIC, \sqrt{s} = 200 \text{ GeV})
\end{cases}
\]

The effect vanishes at the energies of RHIC and LHC.

Broadening is not additive, it is stronger in AA on the same length.
J/Ψ melting

No signal of J/Psi melting has been observed so far

The main flaws of the melting scenario

- **Prejudice:** the cold nuclear matter effect in AA can be extrapolated from pA collisions.

- **Prejudice:** once a bound level disappears, the charmonium dissociates and is terminated.

- **Prejudice:** screening of the potential is the only reason for charmonium disintegration in a dense medium.

Most of charmonia at RHIC-LHC have large $\langle p_T^2 \rangle \approx 4 - 10 \text{ GeV}^2$, so they move with relativistic velocities and the Schrödinger equation and lattice results cannot be applied.
Charmonium propagation through a medium

Path integral technique

Path integral technique

\[
\]

\[
\left[ i \frac{d}{dz} - \frac{m_c^2 - \Delta r}{E_\Psi/2} - V_{\bar{q}q}(z, r) \right] G_{\bar{q}q}(z_1, r_\perp; z, r_\perp) = 0
\]

The Green function \( G_{\bar{q}q}(z_1, r_1; l_2, r_2) \) describes propagation of the dipole.

\[
\text{Re} V_{\bar{q}q}(z, r) \text{ corresponds to the binding potential, which is known only in the rest frame of the dipole.}
\]

The imaginary part of the light-cone potential describes color-exchange interaction of the dipole with the surrounding medium, missed in previous considerations.

\[
\text{Im} V_{\bar{q}q}(z, r_\perp) = -\frac{1}{4} \dot{q}(z) r_\perp^2
\]

Transport coefficient \( \dot{q} \approx 3.6 T^3 \) is to be adjusted to data.
Survival of an unbound $c\bar{c}$

Even in the extreme case of lacking any potential between $c$ and $\bar{c}$ ($T \rightarrow \infty$), still the $J/\Psi$ can survive.

I. Potashnikova, I. Schmidt, M. Siddikov & B. K.
PRC91 (2015) 2, 024911

Path-integral description of $J/\Psi$ attenuation

$$|S(L)|^2 = \frac{m_c^2 p_\psi}{16\pi^2 L} \left( 1 + \frac{\omega}{2m_c} \right) \frac{8\pi^2}{m_c^2} \left[ \omega^2 m_c^2 + \frac{p_\psi^2}{4L^2} \left( 1 + \frac{\omega}{2m_c} \right)^2 \right]^{-1/2}$$

$$\omega = (M_{\psi'} - M_\psi) / 2$$
Lorentz boosted Schrödinger equation


The light cone fractional momentum distribution of quarks in a charmonium sharply peaks around \( x = 1/2 \). With a realistic potential

\[
\langle \lambda^2 \rangle \equiv \left\langle \left( x - \frac{1}{2} \right)^2 \right\rangle = \frac{\langle p_L^2 \rangle}{4m_c^2} = \frac{1}{4} \langle v_L^2 \rangle \approx 0.017
\]

Introducing a variable \( \zeta \) Fourier conjugate to \( \lambda \),

\[
\tilde{\Psi}_{cc}(\zeta, r_\perp) = \int_0^1 \frac{dx}{2\pi} \Psi_{cc}(x, r_\perp) e^{2im_c\zeta(x - 1/2)}
\]

and making use of smallness of \( \lambda \) and of the binding energy, we arrive at the boost-invariant Schrödinger equation for the Green function

\[
\left[ \frac{\partial}{\partial z^+} + \frac{\Delta_\perp + (\partial/\partial \zeta)^2}{p_\psi/2} - m_c^2 - U(r_\perp, \zeta) \right] G(z^+, \zeta, r_\perp; z_1^+, \zeta_1, r_1\perp) = 0
\]
Lorentz boosted binding potential

Debye screening of the potential for $J/\Psi$ at rest relative to the medium can be modeled,

$$V_{\bar{c}c} \left( r = \sqrt{r_\perp^2 + \zeta^2} \right) = \frac{\sigma}{\mu(T)} \left( 1 - e^{-\mu(T)r} \right) - \frac{\alpha}{r} e^{-\mu(T)r}$$

$$\mu(T) = g(T)T \sqrt{1 + \frac{N_f}{6}}, \quad g^2(T) = \frac{24\pi^2}{33 \ln(19T/\Lambda_{\overline{MS}})}$$


However, most of $J/\Psi$s are fast moving, at the LHC $\langle p_{\psi}^2 \rangle = \langle p_T^2 \rangle \approx 10 \text{ GeV}^2$

$V(r)$ is not Lorentz invariant $r$ is 3-dimensional

The procedure of Lorentz boosting of the Schrödinger equation was developed recently in E. Levin, I. Schmidt, M. Siddikov & B.K. arXiv:1501.01607, PRD2015
Results for $J/\Psi$

Survival probability

$$S_{J/\Psi}^2(b) = \left| \int \frac{d\phi}{2\pi} \int \frac{d^2s}{T_{\text{A}}(\bar{s})T_{\text{B}}(\bar{b} - \bar{s})} \right|^2$$

Calculations are done for central Pb-Pb collisions with realistic nuclear density. No ISI effects are added.

1. Net melting: $\text{Re}U \neq 0; \text{Im}U = 0$.
2. Net absorption: $\text{Re}U = 0; \text{Im}U \neq 0$.
3. Total suppression: $\text{Re}U \neq 0; \text{Im}U \neq 0$.

Azimuthal asymmetry

\[ v_2(b) = \frac{1}{S_{J/\Psi}(b)} \int_0^{2\pi} \frac{d\phi}{2\pi} \cos(2\phi) \int \frac{d^2s}{T_{AB}(b)} \frac{T_A(s)T_B(b-s)}{T_{AB}(b)} \]

\[ \times \left| \int d^2r_1 d^2r_2 d\zeta_1 d\zeta_2 \Psi_f^\dagger(\zeta_2, \tilde{r}_2) G(\infty, \zeta_2, \tilde{r}_2; l_0, \zeta_1, \tilde{r}_1) \Psi_{\text{in}}(\zeta_1, \tilde{r}_1) \right|^2 \]

\[ \int d^2r d\zeta \Psi_f^\dagger(\zeta, \tilde{r}) \Psi_{\text{in}}(\zeta, \tilde{r}) \]

\[ v_2 = \frac{1}{S_{J/\Psi}(b)} \int_0^{2\pi} \frac{d\phi}{2\pi} \cos(2\phi) \int \frac{d^2s}{T_{AB}(b)} \frac{T_A(s)T_B(b-s)}{T_{AB}(b)} \]

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\[ \int d^2r d\zeta \Psi_f^\dagger(\zeta, \tilde{r}) \Psi_{\text{in}}(\zeta, \tilde{r}) \]
Results for $\Psi'$

Projecting to the wave function of $\Psi(2S)$ one gets a stronger suppression

![Graph showing $S_{\Psi'}^2$ and $S_{\Psi'}^2/S_{\Psi}^2$ as functions of $p_T$ (GeV).]
pA at LHC: a new challenge

A perturbatively produced $c\bar{c}$, rather than $J/\Psi$, propagates through the nucleus.

The dipole transverse separation is quite small, $r^2 \sim 1/m_c^2 \sim 0.02 \text{ fm}^2$, so the dipole cross section, $\sigma(r) = C(x_2)r^2$, with $x_2 = e^{-y}M_{\bar{c}c}/\sqrt{s}$, is known, fitted to HERA DIS data. It is small, but steeply rises with energy. Correspondingly, small is the mean number of collisions

$$\langle n \rangle_A = \sigma(r)\langle T_A \rangle \approx \begin{cases} 0.1 - 0.2 & \text{(RHIC)} \\ 0.2 - 0.4 & \text{(LHC)} \end{cases}$$
As far as \( \langle n \rangle_A \ll 1 \), one can rely on the approximation of a single interaction

\[
R_{pA} = \frac{1}{A} \int d^2b \int_{-\infty}^{\infty} dz |S_{pA}(b, z)|^2,
\]

\[
S_{pA}(b, z) = \int d^2r \ W_{\bar{c}c}(r) \exp \left[ -\frac{1}{2} \sigma_3(r) T_-(b, z) - \frac{1}{2} \sigma(r) T_+(b, z) \right]
\]

\[
\sigma_3(r) = \sigma_{\bar{c}g}(r) = \frac{9}{4} \sigma(r/2) - \frac{1}{8} \sigma(r)
\]

\[
W_{\bar{c}c}(r) = \Psi_{J/\psi}^\dagger(r) \sigma(r) \Psi_g(r)
\]

**Simplified oscillatory potential**


**Lorentz-boosted realistic potential**


The suppression at the LHC turns out to be grossly over-predicted. This is a serious challenge, because the dipole cross section \( \sigma(\rho, x) \) steeply rises with energy (HERA data), so the nuclear matter should be more opaque at LHC than at RHIC. The LHC data are affected by a novel enhancing mechanism.
Novel mechanism: double-step $J/\psi$ production

Although the mean number of collisions of a small $c\bar{c}$ dipole is small, the single-scattering approximation might be insufficient, a double scattering correction might be important.

One can produce $J/\psi$ without gluon radiation (CSM), exchanging two gluons with different bound nucleons.

At very high energies (so far unreachable) multiple interactions become the dominant mechanism. The probability of color singlet $c\bar{c}$ production from an initial octet approaches $1/9$:

$$S(r, z) = \left[ \frac{1}{9} - \frac{1}{9} e^{-\frac{9}{8} \sigma(r) T_A(z)} \right] O_{in}(r, -\infty)$$

A.Tarasov, J.Hüfner & B.K. NPA696(2001)669

Such a correction is enhanced as $A^{1/3}$, and rises with energy, because the dipole cross section does.
Double-step photoproduction

Example: inclusive photoproduction of $J/\psi$


\[ \frac{dE}{dz} = \kappa_8 = \frac{1}{2\pi\alpha'_P} \approx 5 \text{GeV/fm} \]
Double-step production

The double-step correction, shifted down to smaller $x_1$, shows up due to the steep fall-off of the diffractive cross section.
In order to produce $1^+(J/\psi)$ in the second interaction, in the first collision, a $P$-wave antisymmetric octet state $8^-$ must be created.

The corresponding combination of dipole cross sections has the form

$$\Delta \Sigma_8(r, r') = \sigma_3(r) + \sigma_3(r') - \Sigma_8(r, r') \approx \frac{5}{8} (r_T \cdot r'_T)$$

For the denominator, $\sigma(gp \rightarrow J/\psi X)$, one can use experimental data.
The double-step correction rises with energy, because it contains the dipole cross section squared.
Bottomium production

The relative contribution of the double-scattering term for $\Upsilon$ production is $m_c^2/m_b^2$ suppressed compared with $J/\Psi$, can be neglected.

The observed energy dependence of jet quenching led to the conclusions that QGP was not created at SPS, but was at RHIC.

Why is J/ψ equally suppressed at SPS and RHIC?
More surprises come from LHC

- $J/\Psi$ is less suppressed at LHC than at RHIC!
- Suppression is independent of centrality!
Double-step (ISI/FSI) $J/\psi$ production

Initial state AA collision (cold nuclear matter stage)

Final state AA collision (hot nuclear matter stage)
The double-step term enhancing $J/\Psi$ in pA collisions rises with energy and becomes significant at the energies of LHC.

This enhancement explains the puzzling energy dependence. At higher energy the pA/pp ratio may even exceed unity.

Melting of a charmonium in QGP does not lead to its disappearance. The survival probability is still high and rises with pT.

Another source of charmonium suppression is color-exchange interaction with the medium, which breaks-up the colorless dipole.

A novel procedure for boosting the Schrödinger equation to a moving reference frame is proposed. The resulting equation is linear and does not contain any nonlocal operators.

OUTLOOK: FSI stage in AA collisions includes initial color octet, $c\bar{c}$ produced in ISI. The contribution of double-step (ISI/FSI) production rises with energy and shows up at RHIC/LHC.
Backups

$J/\psi$ and $\psi'$: Measured/Expected

- Expected: normal nuclear absorption, from a full Glauber calculation with
  \[
  \sigma_{abs}^{J/\psi} = 4.18 \pm 0.35 \text{ mb}
  \]
  \[
  \sigma_{abs}^{\psi'} = 7.6 \pm 1.2 \text{ mb}
  \]
  
- In A-B collisions the $\psi'$ departs from the normal absorption curve “earlier” in centrality than the $J/\psi$. 
Novel mechanism: double-step $J/\psi$ production

A quark pair can be produced in $g + p \to \bar{c}c + X$ three color/space states:

(i) color singlet $(1^-)$
(ii) color octet $(8^-)$
(iii) color octet $(8^+)$

$(1^-)$ and $(8^-)$ are antisymmetric, but $(8^+)$ is symmetric relative permutations of spatial and spin variables ($\chi$ is $1$, $J/\psi$ is $1^+$).

$$\sigma(gp \to \{\bar{c}c\}_k X) = \sum_{\mu, \bar{\mu}} \frac{1}{\alpha_s} \int_0^1 d\alpha \int d^2 r \sigma^{(k)}(r, \alpha) \left| \Psi^{\mu \bar{\mu}}_g(r, \alpha) \right|^2$$

$$\sum_{\mu, \bar{\mu}} \left| \Psi^{\mu \bar{\mu}}_g(r, \alpha) \right|^2 = \frac{\alpha_s}{(2\pi)^2} \left[ m_c^2 K_0^2(m_c r) + (\alpha^2 + \bar{\alpha}^2) m_c^2 K_1^2(m_c r) \right]$$

$$\sigma^{(1^-)}(r, \alpha) = \frac{1}{8} \sigma(r); \quad \sigma^{(8^-)}(r, \alpha) = \frac{5}{16} \sigma(r)$$

$$\sigma^{(8^+)}(r, \alpha) = \frac{9}{16} \left[ 2\sigma(\alpha r) + 2\sigma(\bar{\alpha} r) - \sigma(r) \right]$$

$$\sigma^{(1^-)} + \sigma^{(8^-)} + \sigma^{(8^+)} = \frac{9}{8} \left[ \sigma(\alpha r) + \sigma(\bar{\alpha} r) \right] - \frac{1}{8} \sigma(r) \equiv \sigma_3(r, \alpha)$$

$\sigma_3$ is the 3-body, $\bar{c}c gN \to \bar{c}c X$ dipole cross section.
Evolution of the dipole is described in terms of density matrix

\[ R(r, r'|z) = S(r, r'|z) \hat{P}_1 + \frac{1}{8} O(r, r'|z) \hat{P}_8 \]

\[
\frac{d}{dz}S(r, r'|z) = \left[ -\frac{1}{2} \Sigma_1 S(r, r'|z) + \Sigma_{tr} O(r, r'|z) \right] \rho_A (b, z)
\]

\[
\frac{d}{dz}O(r, r'|z) = \left[ 8 \Sigma_{tr} S(r, r'|z) - \Sigma_8 O(r, r'|z) \right] \rho_A (b, z)
\]

\[
\Sigma_{tr} = \frac{1}{16} (b - a)
\]

\[
\Sigma_1 = c
\]

\[
\Sigma_8 = \frac{1}{16} (2b + 7a - c)
\]

\[
\hat{P}_1 = \frac{1}{3} \delta_i^j \delta^k_l
\]

\[
\hat{P}_8 = \delta_i^j \delta^k_l - \frac{1}{3} \delta_i^j \delta^k_l
\]

\[
a = 2\sigma \left( \frac{r - r'}{2} \right)
\]

\[
b = 2\sigma \left( \frac{r + r'}{2} \right)
\]

\[
c = \sigma(r) + \sigma(r')
\]

\[
\Sigma_1, \Sigma_8 \text{ are the dipole cross sections of 4-quark systems, consisted of two color singlets, or octets respectively.}
\]