Soft physics of heavy ion collisions

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Fluctuations and polarization

Figure 32: The CMB radiation temperature fluctuations from the 5-year WMAP data seen over the full sky. The average temperature is 2.725K, and the colors represents small temperature fluctuations. Red regions are warmer, and blue colder by about 0.0002 K.
Longer tail on the negative (low l) side! (see discussion of “Skewness” later)
In Central Heavy Ion Collisions

~ like Elliptic flow, $v_2$

~ spherical with many (16) nearly equal perturbations

$l = 2$

$l = 16$
Flow originating from initial state fluctuations is significant and dominant in central and semi-central collisions (where from global symmetry no azimuthal asymmetry could occur, all Collective $v_n = 0$)!
Fig. 2. The relative probability of finding a state of a given energy density, e, in a system of given volume, $\Omega = 10,50 \text{ fm}^3$, at a constant temperature, $T = T_c$. 

Higher order moments can be obtained from fluctuations around the critical point. Skewness and Kurtosis are calculated for the QGP → HM phase transition.

**Fig. 4:** (color online) Skewness as a function of the volume abundance of the hadronic matter (denoted as $r_h$, where 1 represents complete hadronization). The temperature scale is also indicated for clarity, the identifiers represent increments of 0.1 MeV in $T$. Results for $\Omega = 500 f_{m^3}$.

Negative skewness indicates freeze-out mainly still on the QGP side.
Global Symmetries
Symmetry axes in the global CM-frame:
- \( y \leftrightarrow -y \)
- \( x,z \leftrightarrow -x,-z \)
Azimuthal symmetry: \( \phi \)-even (\( \cos n\phi \))
Longitudinal - z-odd, (rap.-odd) for \( v_{\text{odd}} \)
Spherical or ellipsoidal flow, expansion

Theory:
\[
\frac{d^3 N}{dy dp_t d\phi} = \frac{1}{2\pi} \frac{d^2 N}{dy dp_t} \left[ 1 + 2v_1(y, p_t) \cos(\phi) + 2v_2(y, p_t) \cos(2\phi) + \cdots \right]
\]

Experiment:
\[
\frac{d^3 N}{dy dp_t d\phi} = \frac{1}{2\pi} \frac{d^2 N}{dy dp_t} \left[ 1 + 2v_1(y - y_{CM}, p_t) \cos(\phi - \Psi_{RP}) + 2v_2(y - y_{CM}, p_t) \cos(2(\phi - \Psi_{RP})) + \cdots \right]
\]

Fluctuations
Global flow and Fluctuations are simultaneously present \( \rightarrow \) \( \exists \) interference
- Azimuth - Global: even harmonics - Fluctuations: odd & even harmonics
- Longitudinal - Global: \( v_1, v_3 \) y-odd - Fluctuations: odd & even harmonics
- The separation of Global & Fluctuating flow is a must!! (not done yet)
Anisotropic Flow

Used by most experimental groups today.

[LP Csernai & H Stoecker J Phys G 41 124001]

\[
\frac{dN}{d\varphi} = \frac{\bar{N}}{2\pi} \left(1 + 2 \sum_{n=1}^{\infty} \bar{v}_n \cos(n(\varphi - \bar{\Psi}_n))\right),
\]

(1)

where \(\bar{N} \equiv \langle N \rangle\) is the mean number of selected particles per event, \(\varphi\) the azimuthal angle, and \(\bar{\Psi}_n\) the mean angle of the \(n\)-th harmonic flow plane.

This is a complete ortho-normal series only if all \(\bar{\Psi}_n\)-s are given in the same reference frame with respect to some physical axis frame of the reaction, e.g. the RP

\[
\bar{v}_n(p_T, y) = \langle \langle \cos[n(\varphi - \bar{\Psi}_n)] \rangle \rangle \quad \text{or equivalently}
\]

\[
\bar{v}_n(p_T, y) = \langle \langle e^{in\varphi} e^{-in\bar{\Psi}_n} \rangle \rangle,
\]

\[
\bar{v}_n(p_T, y) = \Re \langle \langle e^{in\varphi} e^{-in\bar{\Psi}_n} \rangle \rangle
\]

where \(\langle \langle \ldots \rangle \rangle\) denotes an average in the \((p_T, y)\) bin
Anisotropic Flow

3.1. Experimental Methods

\[ \text{Re} \left\langle e^{i n (\varphi_1 - \varphi_2)} \right\rangle = \left\langle e^{i n (\varphi_1 - \Psi_n - (\varphi_2 - \Psi_n))} \right\rangle, \]
\[ = \left\langle e^{i n (\varphi_1 - \Psi_n)} e^{-i n (\varphi_2 - \Psi_n)} \right\rangle + \delta_{2,n}, \]
\[ = \langle v_n^2 + \delta_{2,n} \rangle, \]

(e.g. with 4 particle cumulant method:

\[ c_n \{ 4 \} \equiv \left\langle e^{i n (\varphi_1 + \varphi_2 - \varphi_3 - \varphi_4)} \right\rangle - 2 \left\langle e^{i n (\varphi_1 - \varphi_2)} \right\rangle^2 = \langle -v_n^4 + \delta_{4,n} \rangle. \]

Reaction plane (RP) is lost, P/T side of RP is also lost, CM is not known

Used by most experimental groups today.

[R Snellings, J Phys G 41 (2014) 24007]
[LP Csernai & H Stoecker J Phys G 41 124001]
We need an EbE reference angle (e.g. the RP). Can we find it?

**flow vector**

\[ Q_n = \sum_{i=1}^{M} e^{in\varphi_i} \rightarrow Q_1 = \sum_{i=1}^{M} e^{i\varphi_i} \rightarrow Q_1^P = \sum_{i=1}^{M} |\vec{p}_i| e^{i\varphi_i} = 0 \]

By Danielewicz and Odyniec (DO) → Separate forward & backward pt. → **c.m.**

\[ \text{DO } Q_1^P = \sum_{i=1}^{M} |\vec{p}_i| y_i e^{i\varphi_i} \neq 0 \]

\[ w Q_1^P \equiv |\vec{p}_i| \sum_{i=1}^{M} y_i e^{i\varphi_i} \neq 0 \]

\[ \tan (\bar{\Psi}_{RP}) = \frac{\text{Im } \text{DO } Q_1^P}{\text{Re } \text{DO } Q_1^P} . \]

Or one can approximate this as:

\[ \tan (\bar{\Psi}_{RP}) \approx \frac{\text{Im } Q_1^y}{\text{Re } Q_1^y} \quad \text{where } Q_1^y \equiv \sum_{i=1}^{M} y_i e^{i\varphi_i} \neq 0 \]

Weighting with \( y \) → dominates large rapidities → Use a segmented ZDC to find the **RP**!

In addition we should find the participant **c.m.** Separate out longitudinal fluctuations.
Event-shape engineering

Fluctuations

Global flow

Correlation between flow coefficients:
- Non monotonic variation
Two types of flow processes from: Fluctuations and/or Global Collective Flow

• How to split these two:
  • In theoretical models
    – Mode-by-mode hydrodynamics,
  • In experiments it is more involved
    • Average many events
    • But keeping the symmetries
Method to compensate for C.M. rapidity fluctuations

1. Determining experimentally EbE the C.M. rapidity
2. Shifting each event to its own C.M. and evaluate flow-harmonics there

Determining the C.M. rapidity:

The rapidity acceptance of a central TPC is usually constrained (e.g. for ALICE $|\eta| < \eta_{\text{lim}} = 0.8$, and so: $|\eta_{\text{C.M.}}| << \eta_{\text{lim}}$, so it is not adequate for determining the C.M. rapidity of participants.

**Participant rapidity from spectators**

\[
E_B = A_B m_{B\perp} \cosh(y_B^B) = E_{\text{tot}} - E_A - E_C, \\
M_B = A_B m_{B\perp} \sinh(y_B^B) = -(M_A + M_C)
\]

\[
E_A = A_P m_N \cosh(y_0), \\
E_C = A_T m_N \cosh(-y_0),
\]

give the spectator numbers, $A_P$ and $A_T$, and

\[
M_A = A_P m_N \sinh(y_0), \\
M_C = A_T m_N \sinh(-y_0),
\]

\[
y_{E}^{CM} \approx y_B = \text{artanh} \left( \frac{-(M_A + M_C)}{E_{\text{tot}} - E_A - E_C} \right)
\]
Single neutron spectators are based on nuclear multi fragmentation studies → in experiment should be taken from data [ ALICE estimate from 1984 → ]

Results from preliminary ALICE data:

Results from preliminary ALICE data show the average and EbE fluctuations →

\[ v_1^{\text{odd}} = \sim -0.0025 \quad v_1^{\text{even}} = \sim 0 \]

ALICE PRL 2013:

\[ v_1^{\text{odd}} = \sim -0.0005 \quad v_1^{\text{even}} = \sim -0.00025 \]
Azimuthal Flow analysis with Fluctuations today

In contrast to the above formulation

\[
\frac{d^3 N}{dydp_t d\phi} = \frac{1}{2\pi} \frac{d^2 N}{dydp_t} \left[ 1 + 2v_1(y, p_t) \cos(\phi - \Psi_1^{EP}) + 2v_2(y, p_t) \cos(2(\phi - \Psi_2^{EP})) + \cdots \right],
\]

Here \( \Psi_n^{EP} \) maximizes \( v_n(y, p_t) \) in a rapidity range.

Is this a complete ortho-normal series? Yes, if the \( \Psi_n^{EP} \) values are defined ..... We can see this by using:

\[
\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta,
\]
Azimuthal Flow analysis with Fluctuations today

Is this a complete ortho-norml series? Yes, if $\Psi_n^{EP}$ values are defined ….

We can see this by $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$.

The angles $\Psi_n^{EP}$ & $\phi$ should be measured with respect to the Reaction Plane (EbE).

Separating Global Collective Flow & Fluctuations

where $y = y - y_{CM}$, and new coefficients

$c_v_n \equiv v_n\cos(n(\Psi_n^{EP}))$ and $s_v_n \equiv v_n\sin(n(\Psi_n^{EP}))$
CENTRALITY DEPENDENCE

- Odd parts multiplied with -1 for $\eta < 0$
- Mild centrality dependence for both $v_1^{\text{odd}}$ and $v_1^{\text{even}}$
- Significantly smaller magnitude of $v_1^{\text{odd}}$ compared to RHIC

$\times 0.37$
$\times 0.12$
Development of $v_1(y)$ at increasing beam energies

$v_1(y)$ observations show a central antiflow slope, $\partial v_1(y)/\partial y$, which is gradually decreasing with increasing beam energy [23]:

$$\left\{ \begin{array}{l}
-1.25\% \quad \text{for} \quad 62.4 \text{ GeV (STAR)} \\
-0.41\% \quad \text{for} \quad 200.0 \text{ GeV (STAR)} \\
-0.15\% \quad \text{for} \quad 2760.0 \text{ GeV (ALICE)}
\end{array} \right.$$ 

This can be attributed to smaller increase of $p_t$ and the pressure, and the shorter interaction time, and also to increasing rotation.

In [Cs., Magas, Stöcker, Strottman, PRC84 (2011)] we predicted this rotation, but the turnover depends on the balance between rotation, expansion and freeze out. Apparently expansion is still faster and freeze out is earlier, so the turn over to the Positive side is not reached yet.

Interesting collective flow phenomena in low viscosity QGP →
Hot-Gluon Field $\rightarrow$ Compact IS, shear & vorticity

- [Gyulassy & Csernai, NPA460 (1986) 723]: Flux tube dominance $\rightarrow$
- Flux tube, w/ large string tension $\rightarrow$
- Longitudinal extension is limited:
- Energy & momentum conservation
- Shear flow, vorticity, rotation
- IS: 3-4 fm/c

- [Magas et al., NPA 712 (2002)167]
Detection of Global Collective Flow

We are will now discuss rotation (eventually enhanced by KHI). For these, the separation of Global flow and Fluctuating flow is important. (See ALICE v1 PRL (2013) Dec.)

- One method is polarization of emitted particles
  - This is based equilibrium between local thermal vorticity (orbital motion) and particle polarization (spin).
  - Turned out to be more sensitive at RHIC than at LHC (although L is larger at LHC)
  - At FAIR and NICA the thermal vorticity is still significant (!) so it might be measurable.

- The other method is the Differential HBT method to analyze rotation:
Viscosity vs. $T$ has a minimum at the 1st order phase transition. This might signal the phase transition if viscosity is measured. At lower energies this was done.
Kelvin-Helmholtz instability in high-energy heavy-ion collisions

L.P. Csernai, D.D. Strottman, and Cs. Anderlik
PHYSICAL REVIEW C 85, 054901 (2012)

FIG. 1: (color online) Growth of the initial stage of Kelvin-Helmholtz instability in a 1.38A + 1.38A TeV peripheral, \( b = 0.7b_{\text{max}} \), Pb+Pb collision in a relativistic CFD simulation using the PIC-method. We see the positions of the marker particles (Lagrangian markers with fixed baryon number content) in the reaction plane. The calculation cells are \( dx = dy = dz = 0.4375 \text{fm} \) and the time-step is 0.04233 \( \text{fm/c} \). The number of randomly placed marker particles in each fluid cell is 8\(^5\). The axis-labels indicate the cell numbers in the \( x \) and \( z \) (beam) direction. The initial development of a KH type instability is visible from \( t = 1.5 \) up to \( t = 7.41 \text{ fm/c} \) corresponding from 35 to 175 calculation time steps.)
FIG. 5: The classical (left) and relativistic (right) weighted vorticity calculated for all [x-z] layers at t=3.56 fm/c. The collision energy is $\sqrt{s_{NN}} = 2.76$ TeV and $b = 0.7b_{max}$, the cell size is $dx = dy = dz = 0.4375 \text{fm}$. The average vorticity in the reaction plane is 0.0538 / 0.10685 for the classical / relativistic weighted vorticity respectively.

Onset of turbulence around the Bjorken flow


- Initial state Event by Event vorticity and divergence fluctuations.
- Amplitude of random vorticity and divergence fluctuations are the same
- In dynamical development viscous corrections are negligible (→ no damping)
- Initial transverse expansion in the middle (±3fm) is neglected (→ no damping)
- High frequency, high wave number fluctuations may feed lower wave numbers

Image: Absolute value of vorticity $|\partial_1 u^2 - \partial_2 u^1|$ and divergence $|\partial_1 u^1 + \partial_2 u^2|$.
Detecting rotation: Lambda polarization

\[ \Pi(p) = \frac{\hbar \varepsilon}{8m} \frac{\int dV n_F (\nabla \times \beta)}{\int dV n_F} \]

\[ \beta^\mu(x) = \left( \frac{1}{T(x)} \right) u^\mu(x) \]

\[ \Pi_0(p) = \Pi(p) - \frac{p}{\varepsilon(\varepsilon + m)} \Pi(p) \cdot p \]

\[ [ \text{F. Becattini, L.P. Csernai, D.J. Wang, Phys. Rev. C 88, 034905 (2013)}] \]
Lambda polarization

- The POLARIZATION of $\Lambda$ and $\bar{\Lambda}$ due to thermal equipartition with local vorticity is slightly stronger at RHIC than at LHC due to the much higher temperatures at LHC.
- Although early measurements at RHIC were negative, these were averaged over azimuth! We propose selective measurement in the reaction plane (in the +/- x direction) in the EbE c.m. frame. Statistical error is much reduced now, so significant effect is expected at $p_x \geq 3$ GeV/c.
FIG. 8: (Color online) Global polarization of $\bar{\Lambda}$–hyperons as a function of centrality. Filled circles show the results for Au+Au collisions at $\sqrt{s_{NN}}=200$ GeV (centrality region 20-70%) and open squares indicate the results for Au+Au collisions at $\sqrt{s_{NN}}=62.4$ GeV (centrality region 0-80%). Only statistical uncertainties are shown.

* Azimuth averaged ↓
* C.M. & RP$_{(P/T)}$ should be precisely determined, & only at large $p_x$ !
Exact, expanding and rotating hydro solution (E.M.)

Azimuthal HBT is Changed by Rotation (E.M.)

Azimuthally symmetric source but in phase space:

\[ C(q, K) = 1 + \exp \left( - \sum_{i,j=0,s,l} q_i q_j R_{ij}^2(K) \right) \]

We get for azimuthal HBT:

[Velle, MehrabiPari & Csernai, arXiv: 1508.01884v1 [nucl-th]]
Vorticity in E.M. & in ECHO-QGP hydro

- **EM**: YL Xie, RC Glastad, LP Csernai, arXiv: 1505.07221v1 [nucl-th]
- **ECHO-QGP hydro**: F Becattini et al., arXiv: 1501.04468v2 [nucl-th]

- Vorticity components: -y directed from rotation, [x,z] or [r,φ] directed fr. Expansion:
  \[ \text{rot} \beta \quad \text{or} \quad \partial_t \beta \]

- Different initial conditions (!): **with vs. without shear**

- In E.M. \( \text{rot} \beta = -0.132 \ldots -0.106 \), decreasing with time | HwSh.: 0.3—0.6
  \[ \partial_t \beta_r = 0.029 - 0 \times \text{y/fm}, \quad \partial_t \beta_\phi = 0.009 - 0 \times \text{r/fm}, \quad \partial_t \beta_\gamma \approx 0.0 \]

- In ECHO-QGP: (no initial shear !) mean values:
  \[ \text{rot} \beta = 0.0 \ldots -0.007 \), increasing, with viscosity

Matter is put beyond \( \pm \eta_m \), parameter to secure angular momentum without shear!

\[ \Rightarrow \]

No vorticity in the middle zone of the collision, just at high rapidities, i.e. high z

\[ \sim [x,z]-\text{component} \]
Vorticity in ECHO-QGP hydro cont.

- In the \([x,z] \sim [x,\eta]\) plane y-directed vorticity, changes \(\text{btwn: } -0.05 - 0.03\)
- Net FO vorticity is negative up to \(-0.007\) with “large” viscosity.
- No initial vorticity
- No vorticity in central domains
- Both due to lack of initial shear flow

**Figure 13:** (color online) Contour plot of \(1/\tau\)-scaled \(\eta x\) covariant component of the thermal vorticity, \(\varpi_{nx}/\tau\) over the freeze-out hypersurface for \(y = 0, \eta/s=0.1, \eta_m=2.0\).
Polarization in E.M. & in ECHO-QGP hydro

- Y-directed polarization, $\Pi_y$, is very different. In E.M. max polarization is $-11\%$, $\gg$ in ECHO-QGP it is $-0.2\%$, due to lack of initial shear flow.
Polarization in E.M. & in ECHO-QGP hydro cont.

- In the $\Pi_x$-direction the initial shear flow has no effect.

- The structure is similar, the amplitude is different and the sign is opposite. There may be different conventions (?)
Summary

• We have shown how to split Collective flow & Fluctuations
• When Collective Flow is identified: New patterns
• Small viscosity (fluctuations & instabilities)
• Kelvin-Helmholtz Instability (KHI) ~ turbulence
• These are observable in polarizations and in HBT