Hamiltonian approach to QCD in Coulomb gauge: from the vacuum to finite temperatures

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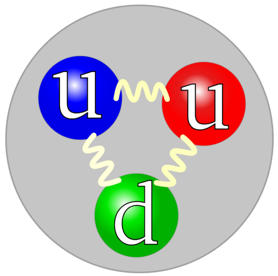
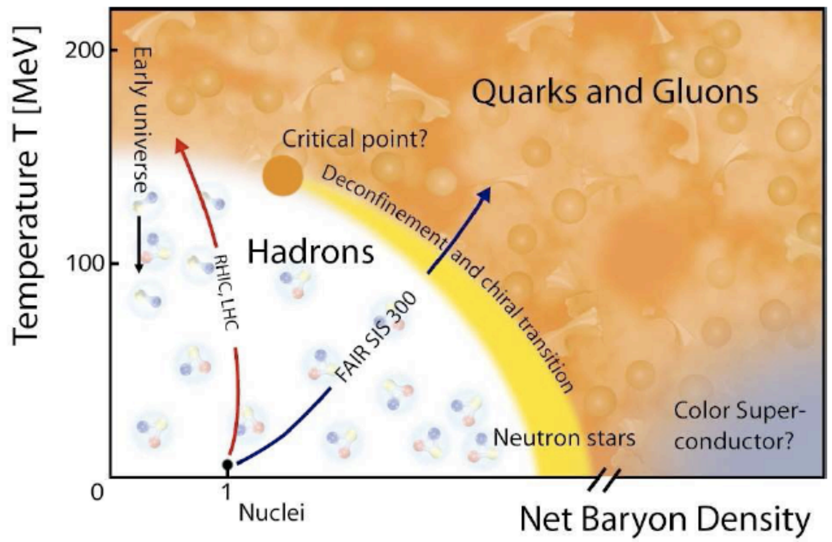
QCD

- *vacuum*
    - confinement
    - SB chiral symmetry



- *phase diagram*
    - deconfinment
    - rest. chiral symm.



- latticeMC-fail at large chemical potential
  continuum approaches required
  Hamiltonian approach

Why Hamiltonian approach?

- *QFT: functional integral approach*
  - *perturbation theory*
  - *lattice gauge theory*
- *QM: solving the Schrödinger equation is much simpler and more efficient than calculating the functional integral,* *see e.g. hydrogen atom*
- *non-perturbative continuum QFT:*
  - *Hamiltonian approach is more* efficient

Outline

- introduction:
- Hamiltonian approach to QCD in Coulomb gauge
- variational solution of the Schrödinger equation for the vacuum
- novel Hamiltonian approach to finite temperature QFT: compactification of a spatial dimension
- QCD at finite temperature
  - the Polyakov loop
- conclusions

Classical Yang-Mills theory

action

$$S = \frac{1}{4g^2} \int d^4 x (F_{\mu\nu}(x))^2$$

field strength tensor

$$F_{\mu\nu}(x) = \partial_\mu A_\nu - \partial_\nu A_\mu + \left[A_\mu , A_\nu \right]$$

Canonical Quantization of Yang-Mills theory

cartesian coordinates $A^a_\mu(x)$

momenta $\Pi^a_i(x) = \delta S / \delta \dot{A}^a_i(x) = E^a_i(x)$

$\Pi^a_0(x) = 0$ Weyl gauge : $A^a_0(x) = 0$

$$H = \tfrac{1}{2}\int d^3x(\Pi^2(x) + B^2(x))$$

quantization: $\Pi^a_k(x) = \delta / i\delta A^a_k(x)$

Taylor-Slavnov identites

Gauß law: $D\Pi\Psi = \rho_m \Psi$

residual gauge invariance $\mathrm{U}(\vec{\mathrm{x}})$: $\Psi(A^U) = \Psi(A)$

Hamiltonian approach to YMT

$$\boxed{H = \tfrac{1}{2}\int(\Pi^2 + B^2)}$$

$$\Pi = \delta / i\delta A$$

Schrödinger equation $H\Psi[A] = E\Psi[A]$

Hilbert space of gauge-invariant wave functionals

$$\langle \Phi | \ldots | \Psi \rangle = \int \mathrm{DA}\Phi^*(\mathrm{A})\ldots\Psi(\mathrm{A})$$

gauge invariant formulation: K. Johnson etal
Karabali, Nair...

more convenient: gauge fixing

Coulomb gauge $\partial \mathrm{A} = 0$

Coulomb gauge

$$\partial A = 0, \qquad A = A^{\perp}$$

curved space

$$\langle \Psi | \Phi \rangle = \int DA^{\perp} J(A^{\perp}) \Psi^{*}(A^{\perp}) \Phi(A^{\perp})$$

Faddeev-Popov

$$J(A^{\perp}) = Det(-D\partial)$$

$$\Pi = \Pi^{\perp} + \Pi^{\parallel}, \qquad \Pi^{\perp} = \delta / i\delta A^{\perp}$$

Gauß law:

$$D\Pi\Psi = \rho_m \Psi$$

resolution of
Gauß´ law

$$\Pi^{\parallel} = -\partial(-D\partial)^{-1}\rho, \qquad \rho = (-\hat{A}^{\perp}\Pi^{\perp} + \rho_m)$$

Hamiltonian approach to YMT in Coulomb gauge

$\partial A = 0$

$$\boxed{H = \tfrac{1}{2}\int (J^{-1}\Pi^{\perp} J \Pi^{\perp} + B^2) + H_C}$$

$\Pi^{\perp} = \delta / i\delta A^{\perp}$

Christ and Lee

$$J(A^{\perp}) = Det(-D\partial) \qquad D = \partial + gA$$

$$H_C = \tfrac{1}{2}\int J^{-1}\rho J(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}\rho$$

Coulomb term

color charge density: $\rho^a = -f^{abc}A^b\Pi^c + \rho^a_m$

$$\langle \Phi | ... | \Psi \rangle = \int_{\Lambda} DAJ(A)\Phi^*(A)...\Psi(A)$$

$$H\Psi[A] = E\Psi[A]$$

Variational approach

- Gaussian ansatz,

$$\Psi(A) = \exp\left[-\tfrac{1}{2}\int dx\, dy\, A(x)\omega(x,y)A(y)\right]$$

D. Schütte 1984

....

A.Szczepaniak & E. Swanson 2002

C. Feuchter & H. R. 2004

-ansatz

-FP determinant

-renormalization

- Greensite, Matevosyan,Olejnik,Quandt, Reinhardt, Szczepaniak,PRD83

Variational approach

C.Feuchter & H. R. PRD70(2004)

- trial ansatz

$$\Psi(A) = \frac{1}{\sqrt{\mathrm{Det}(-D\partial)}} \exp\left[-\tfrac{1}{2}\int dx\, dy\, A(x)\,\omega(x,y)\,A(y)\right]$$

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QM: particle in L=0 state

$$\Psi(r) = \frac{u(r)}{r} \qquad r = \sqrt{J} \qquad \int dr\, r^2 |\Psi(r)|^2 = \int dr\, |u(r)|^2$$

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gluon propagator $\quad \langle A(x)A(y)\rangle = (2\omega(x,y))^{-1}$

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gluon propagator $\langle A(x)A(y)\rangle = (2\omega(x,y))^{-1}$

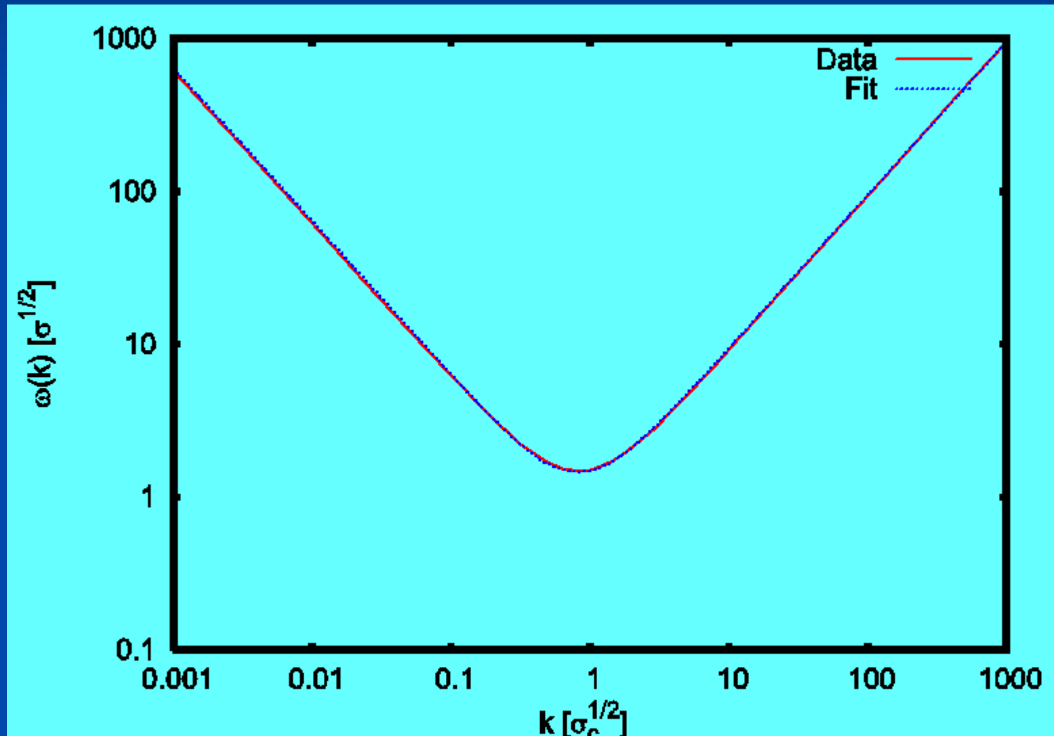
variational kernel $\omega(x,x')$ determined from

$$\boxed{\langle\Psi|H|\Psi\rangle \to \min}$$

Numerical results

gluon energy

D. Epple, H. R., W.Schleifenbaum, PRD 75 (2007)

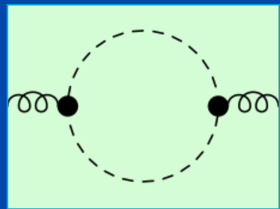


$$IR: \quad \omega(k) \sim 1/k \qquad UV: \quad \omega(k) \sim k$$

equation of motion

$$\omega^2(k) = k^2 + \chi^2(k) + \cancel{\ldots\ldots}$$

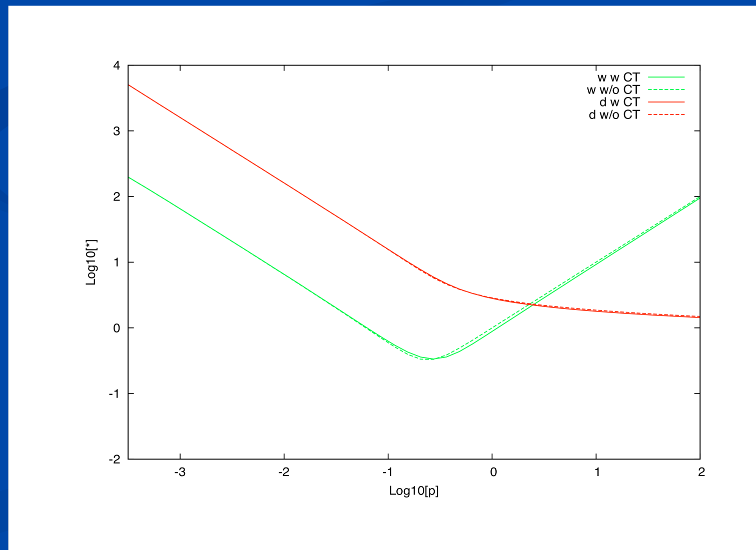
gost loop $\chi(k)$



ghost propagator

$$\left\langle (-D\partial)^{-1} \right\rangle = d\,/\,(-\Delta)$$

horizon condition $\mathrm{d}^{-1}(0) = 0$



The color dielectric function of the QCD vacuum

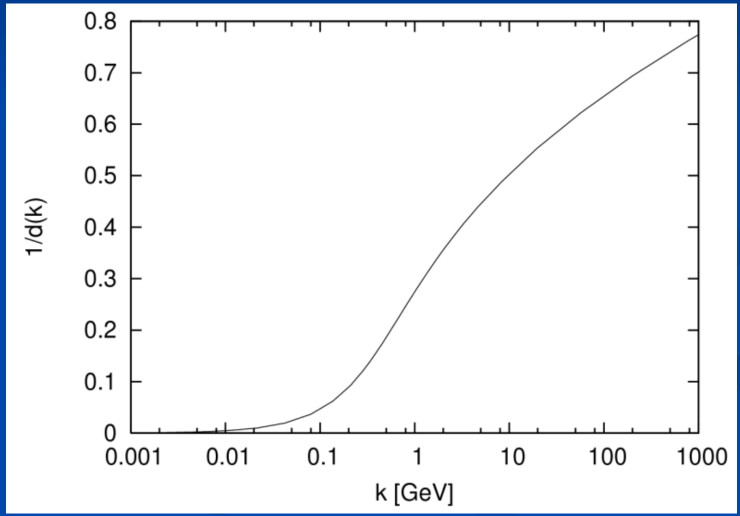
- ghost propagator

$$\left\langle (-D\partial)^{-1} \right\rangle = d\,/\,(-\Delta)$$

- dielectric „constant“

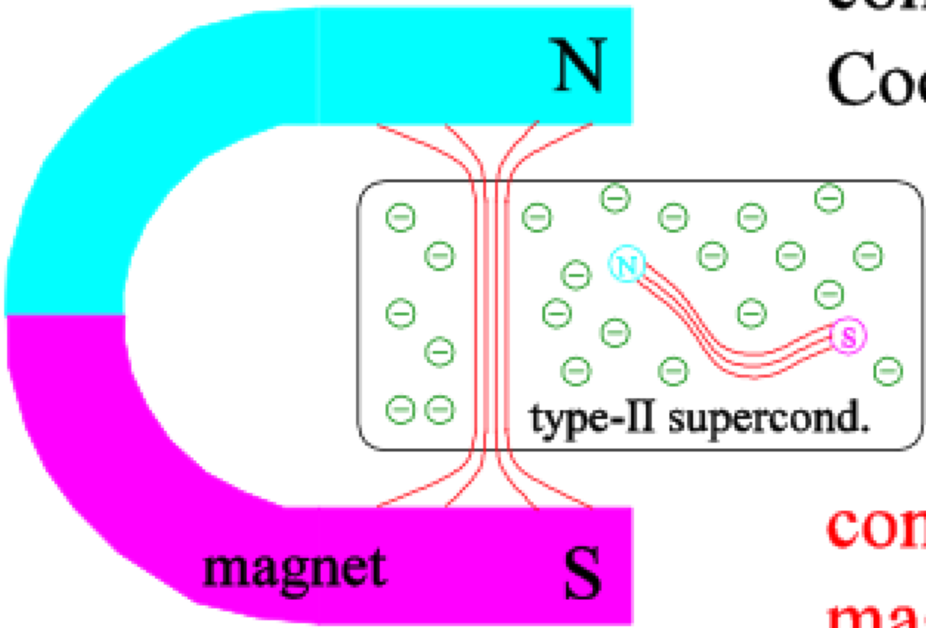
$$\varepsilon = d^{-1}$$

H.R. PRL101 (2008)



- horizon condition:
  - : $d^{-1}(k=0)=0 \qquad \varepsilon(k=0)=0$
- QCD vacuum: perfect color dia-electricum
  - dual superconductor
  - $\varepsilon(k)<1$ anti-screening

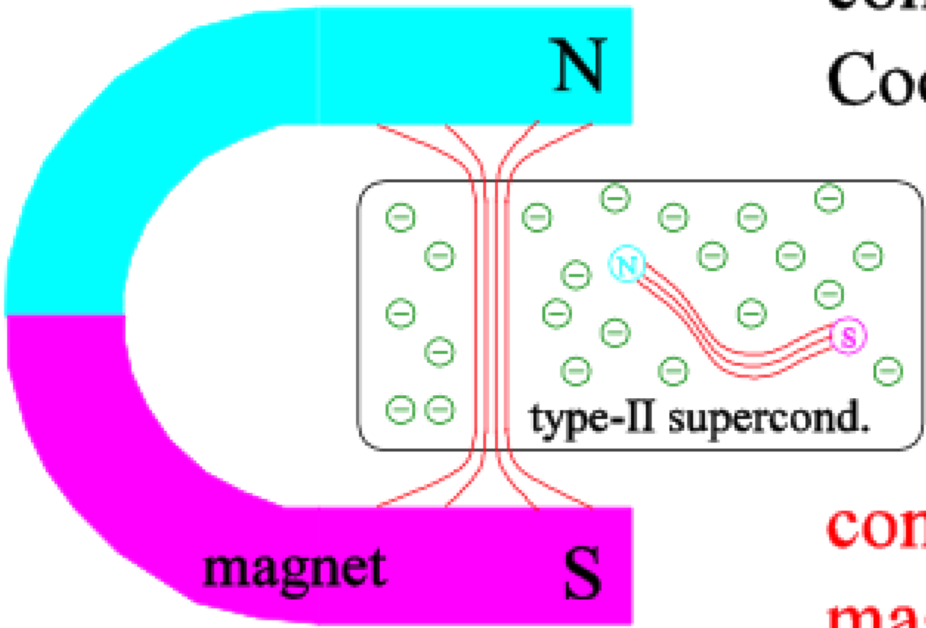
superconductor



condensate of
Cooper pairs

confinement of
magn. monopoles

superconductor



condensate of
Cooper pairs

confinement of
magn. monopoles

dual superconductor

magnetic field	$\longleftrightarrow$	*electric field*
magnetic charge		*electric charge*

The color dielectric function of the QCD vacuum

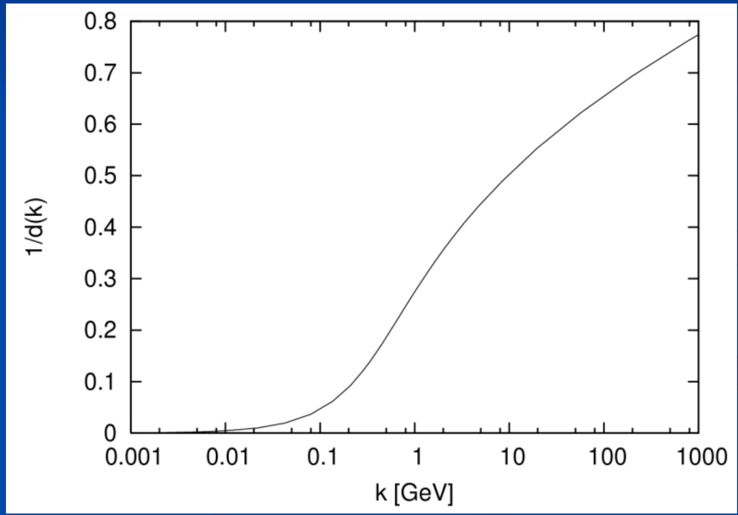
- ghost propagator

$$\left\langle (-D\partial)^{-1} \right\rangle = d / (-\Delta)$$

- dielectric „constant“

$$\varepsilon = d^{-1}$$

H.R. PRL101 (2008)



- horizon condition:
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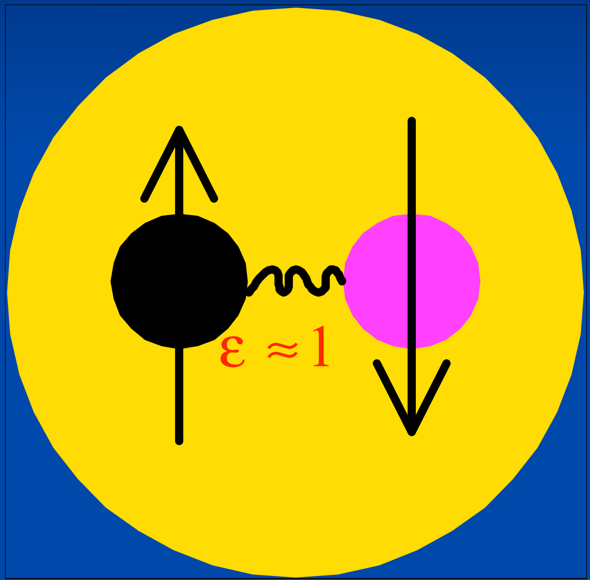
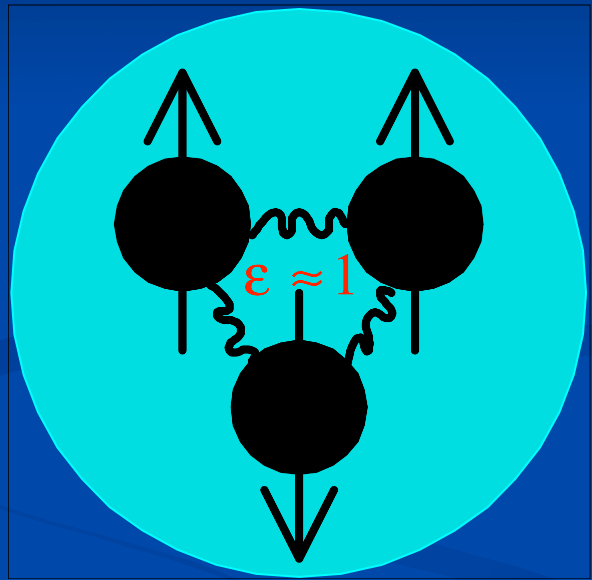
  - $\varepsilon(k)<1$ anti-screening

$$D = \varepsilon E \qquad \partial D = \rho_{free}$$



$$\varepsilon = 0$$



no free color charges in the vacuum: confinement

Static gluon propagator in D=3+1

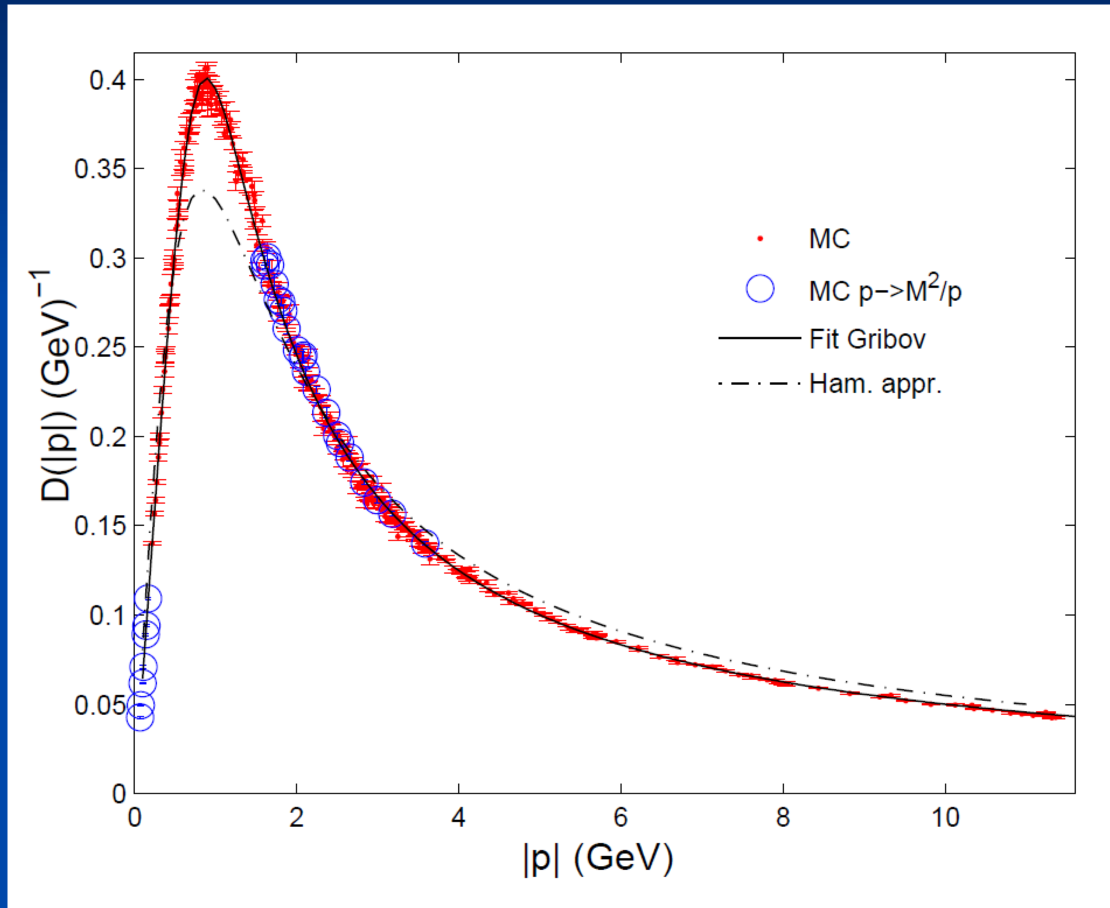
$$D(k) = (2\omega(k))^{-1}$$

Gribov's formula

$$\omega(k) = \sqrt{k^2 + \frac{M^4}{k^2}}$$

$$M = 0.88 GeV$$

missing strength in
mid momentum regime:
missing gluon loop



G. Burgio, M.Quandt , H.R., **PRL102(2009)**

Variational approach to YMT with non-Gaussian wave functional

D. Campagnari & H.R, Phys.Rev.D82(2010)

wave functional

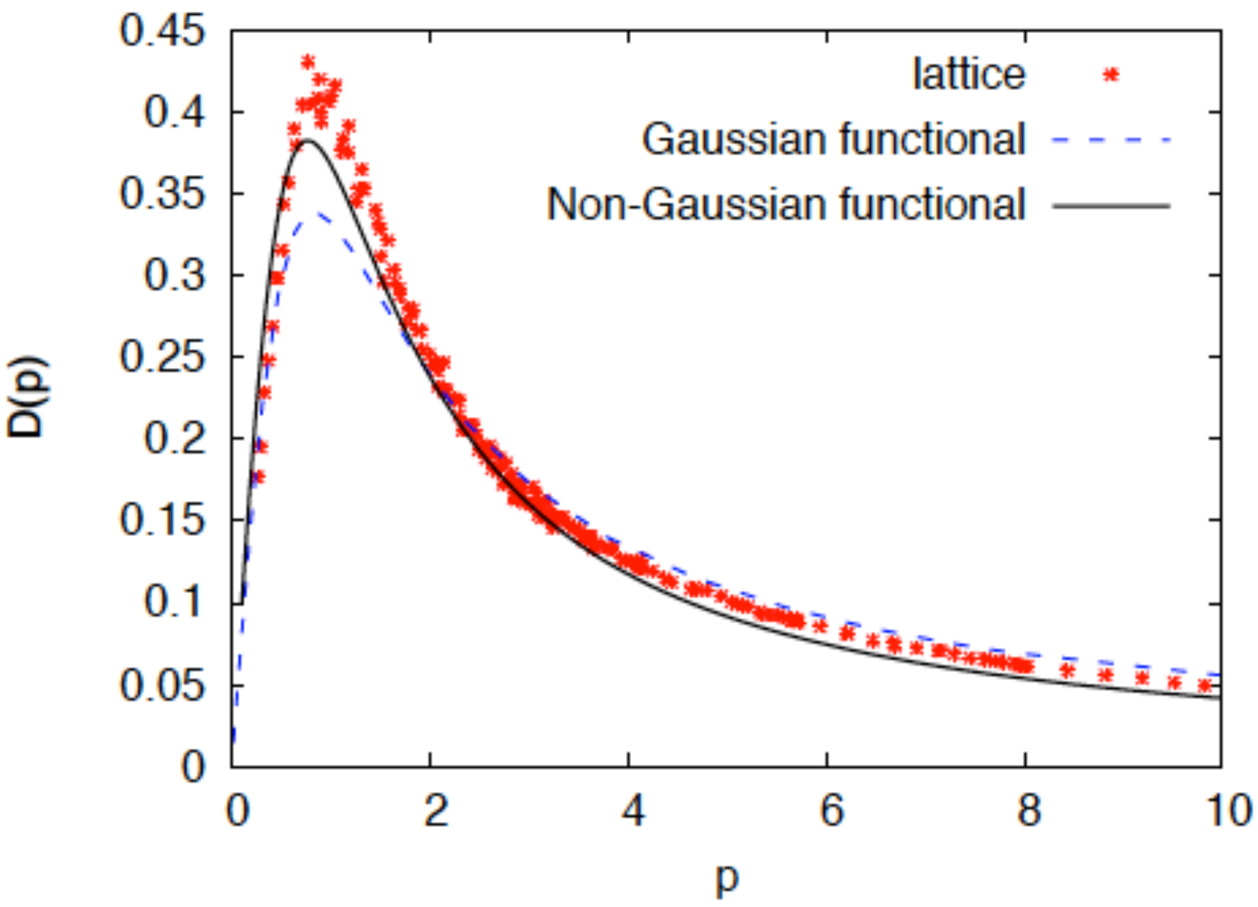
$$|\psi[A]|^2 = \exp(-S[A])$$

ansatz

$$S[A] = \int \omega A^2 + \tfrac{1}{3!}\int \gamma^{(3)} A^3 + \tfrac{1}{4!}\int \gamma^{(4)} A^4$$

exploit DSE

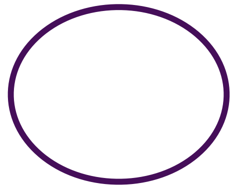
Corrections to the gluon propagator



D. Campagnari & H.R, Phys.Rev.D82(2010)

Wilson loop

$$W[A](C) = P\exp\left[i\oint_C A \right]$$



C

order parameter of confinement

$$\langle W[A](C) \rangle \sim \begin{cases} \exp(-\sigma A(C)) & \textit{confinement} \\ \exp(-\kappa P(C)) & \textit{deconfinement} \end{cases}$$

area law = linearly rising potential
between static color acharges

$\langle W[A](C) \rangle$ difficult to calculate in the continuum theory due to path ordering

approximate calulation of the Wilson loop

M. Pak & H.R., Phys. Rev 80(2009)

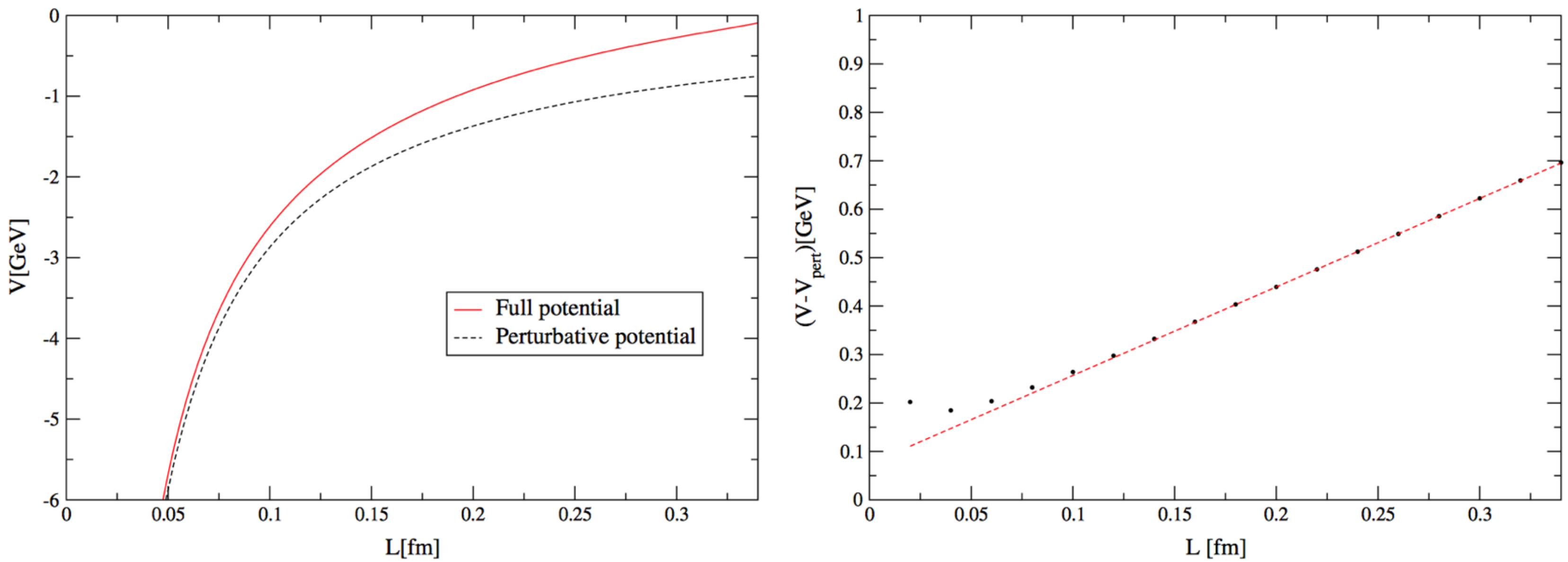


FIG. 7 (color online). Left-hand panel: The full static quark potential $V(L)$ obtained from the full propagator (32) and the perturbative potential $V_{\text{pert}}(L)$ obtained from the perturbative propagator (33). Right-hand panel: The full potential minus its perturbative part.

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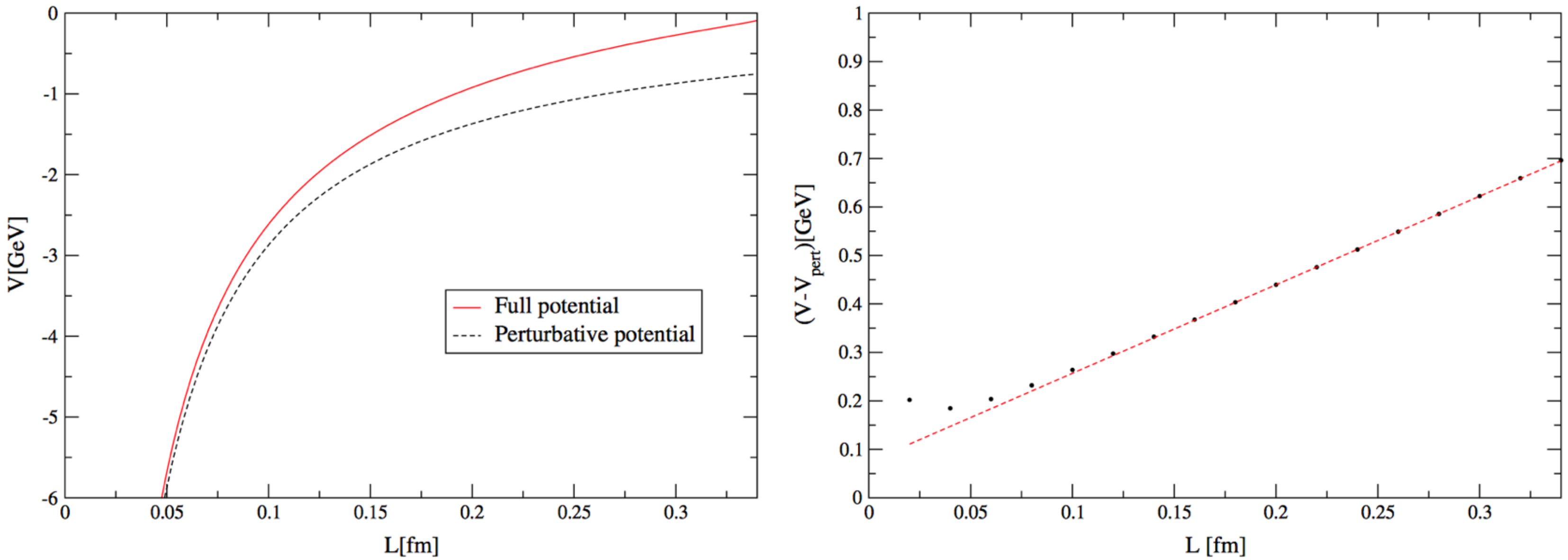

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alternative order parameters:

center of the gauge group

center Z of a group G

$$g \in G \quad z \in Z \subset G \quad \left[z, g\right] = 0$$

gauge group SU(N)

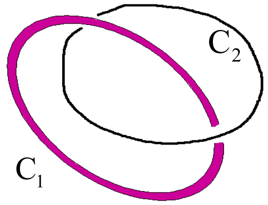
$$U = \exp\left[i\theta^a t_a\right]$$

center Z(N)

$$z_k = e^{i2\pi k/N} 1_N, \quad k = 0, 1, \ldots, N-1,$$

$$z_k = \exp\left[i\mu_k^a t_a\right], \quad \mu_k - coweights$$

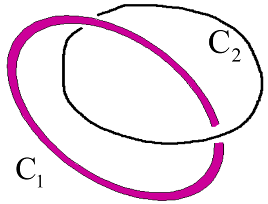
t´Hooft loop



$$\hat{V}(C_1)W(C_2) = z^{L(C_1,C_2)}W(C_2)\hat{V}(C_1)$$

z-center element
L-linking number

t´Hooft loop



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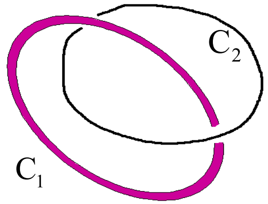
$$\langle V(C)\rangle \sim \begin{cases} \exp(-\sigma A(C)) & \textit{deconfinement} \\ \exp(-\kappa P(C)) & \textit{confinement} \end{cases}$$

t´Hooft loop



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continuum representation

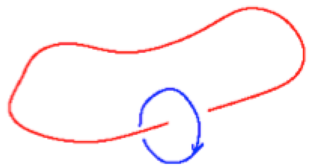
H.R. Phys. Lett. B557(2003)

$$V(C) = \exp\left[i\int_{R^3} A(C)\hat{\Pi} \right]$$

$$\Pi_k^a(x) = \delta / i\delta A_k^a(x)$$

center vortex field

$$W\left[A(C_1)\right](C_2) = z^{L(C_1,C_2)}$$



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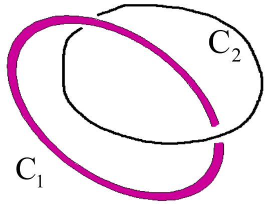

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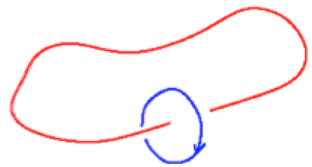
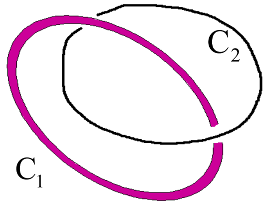


center vortex generator

$$\hat{V}(C)\Psi(A') = \Psi(A' + A(C))$$

t´Hooft loop



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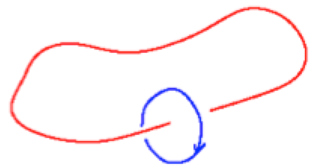
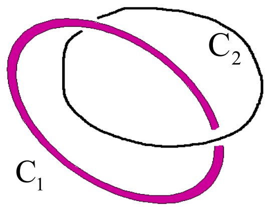


center vortex generator

$$\hat{V}(C)\Psi(A') = \Psi(A' + A(C))$$

QM: $$\exp(ia\hat{p})\Psi(x) = \Psi(x+a)$$

t´Hooft loop



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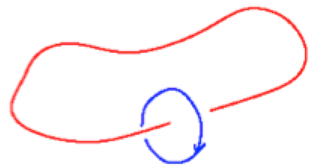

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Hamiltonian approach to YMT in Coulomb gauge: **perimeter law**

H.R. & D. Epple, Phys. Rev.D76(2007)

Hamiltonian approach to YMT in Coulomb gauge

$$\partial A = 0$$

$$\boxed{H = \tfrac{1}{2}\int (J^{-1}\Pi^{\perp} J \Pi^{\perp} + B^2) + H_C} \qquad \Pi^{\perp} = \delta / i\delta A^{\perp}$$

Christ and Lee

$$J(A^{\perp}) = Det(-D\partial) \qquad D = \partial + gA$$

$$H_C = \tfrac{1}{2}\int J^{-1}\rho J(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}\rho$$

Coulomb term

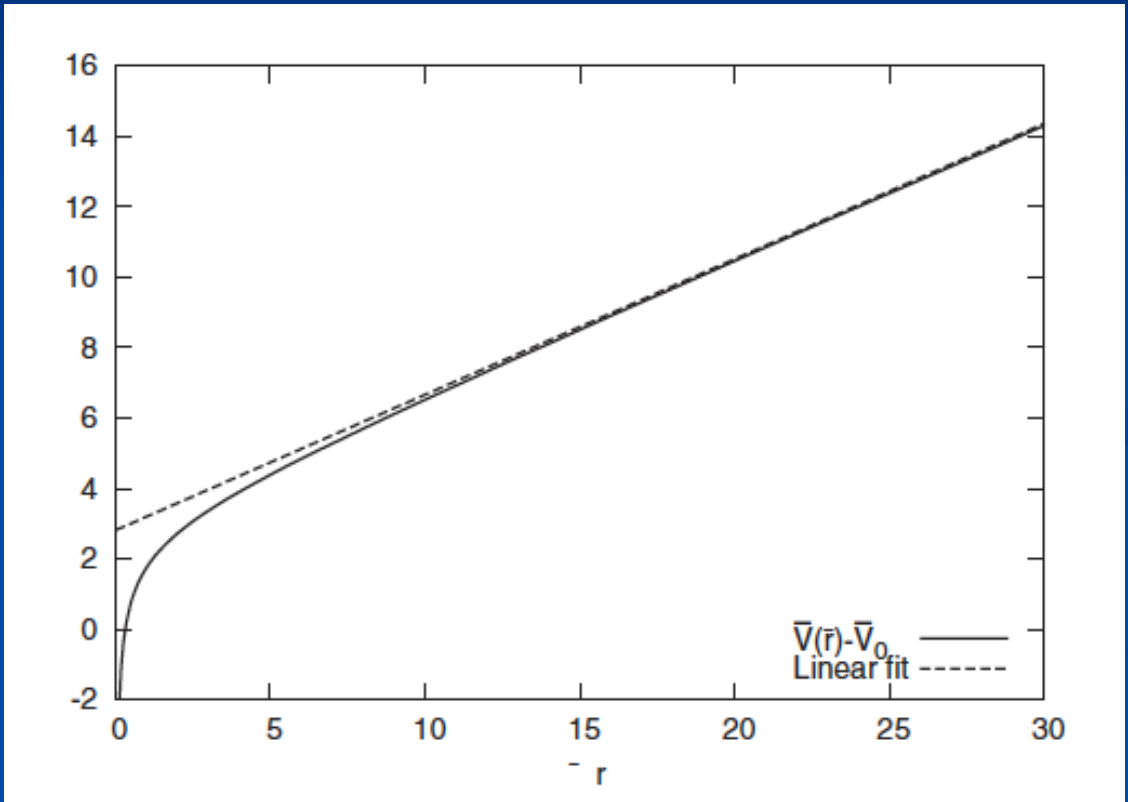
color charge density: $\rho^a = -f^{abc}A^b\Pi^c + \rho_m^a$

$$\langle\Phi|\ldots|\Psi\rangle = \int_{\Lambda} DAJ(A)\Phi^*(A)\ldots\Psi(A)$$

$$H\Psi[A] = E\Psi[A]$$

Static Coulomb potential

$$V(|x-y|) = g^2 \langle \langle x | (-D\partial)^{-1} (-\partial^2)(-D\partial)^{-1} | y \rangle \rangle$$



D. Epple, H. Reinhardt
W.Schleifenbaum,
PRD 75 (2007)

$$V(r) = \xrightarrow[r\to\infty]{} \sigma_C r, \qquad \sigma_C \geq \sigma_W \qquad \text{lattice}: \ \sigma_C = 2...3\sigma_W$$

$$V(r) = \xrightarrow[r\to 0]{} \sim 1/r$$

The QCD Hamiltonian in Coulomb gauge

$$\boxed{H_{QCD} = H_{YM} + H_C + H_q}$$

gluon part

$$H_{YM} = \tfrac{1}{2}\int (J^{-1}\Pi J \Pi + B^2) \qquad \Pi = -i\delta/\delta A \qquad J(A^{\perp}) = Det(-D\partial)$$

quark part

$$\mathrm{H_q} = \int \Psi^{\dagger}(x)[\vec{\alpha}(\vec{p} + g\vec{A}) + \beta m_0]\Psi(x) \qquad \vec{\alpha}, \beta - \textit{Dirac matrices}$$

Coulomb term

$$\mathrm{H_C} = \tfrac{1}{2}\int J^{-1}\rho(-D\partial)^{-1}(-\partial^2)(-D\partial)^{-1}J\rho$$

color charge density

$$\rho^a = -f^{abc}A^b\Pi^c + \Psi^{\dagger}(x)t^a\Psi(x)$$

quark wave functional

P. Vastag & H. R.
to be published

$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi_+^\dagger (s\beta + v\vec{\alpha}\cdot\vec{A} + w\beta\vec{\alpha}\cdot\vec{A})\Psi_-\right]|0\rangle$$

s, v, w – variational kernels $\quad \vec{\alpha}, \beta$ – Dirac matrices

$v = w = 0:$ *BCS – wave functional* — Finger & Mandula, Adler & Davis, Alkofer

$v \neq 0,\ w = 0:$ *quark - gluon - coupling* — Pak & Reinhardt, PRD88(2013)

> calculate $\langle H_{QCD}\rangle$ up to 2 loops

> variation w.r.t. **s, v, w**

$$v(p,q) = f_v[s,\omega]$$

$$w(p,q) = f_w[s,\omega]$$

$$s(p) = f_s[s,v,w;p] \qquad \textit{gap equation}$$

Renormalization

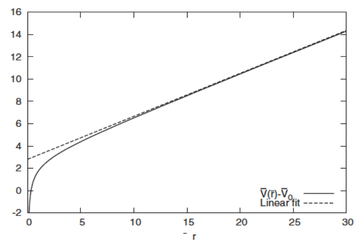
$$\langle A|\Phi\rangle_q = \exp\left[\int \Psi^\dagger (\mathrm{s}\beta + \mathrm{v}\vec{\alpha}\cdot\vec{A} + \mathrm{w}\beta\vec{\alpha}\cdot\vec{A})\Psi\right]|0\rangle$$

gap equation: $s(p) = f_s[s,v,w;p]$

strict cancelation of linear divergencies

logarithmic divergencies $\boxed{g^2 \ln(\Lambda/\mu) = \tilde{g}^2(\mu)}$

input: *non-abelian Coulomb potential*



$\Rightarrow$ *scale* $\mu = \sqrt{\sigma_C}$

lattice: $\sigma_C = 2\sigma$

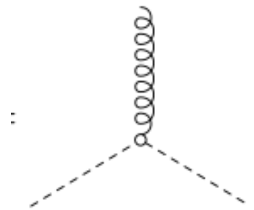
choose $\tilde{g}(\sqrt{\sigma_C})$ *to reproduce* $\langle \bar{q}q\rangle = (-235\,MeV)^3$

$\Rightarrow \tilde{g}(\sqrt{\sigma_C}) \approx 3.57$

running coupling constant

$$\alpha(p) = \frac{\tilde{g}^2(p)}{4\pi}$$

from ghost-gluon vertex

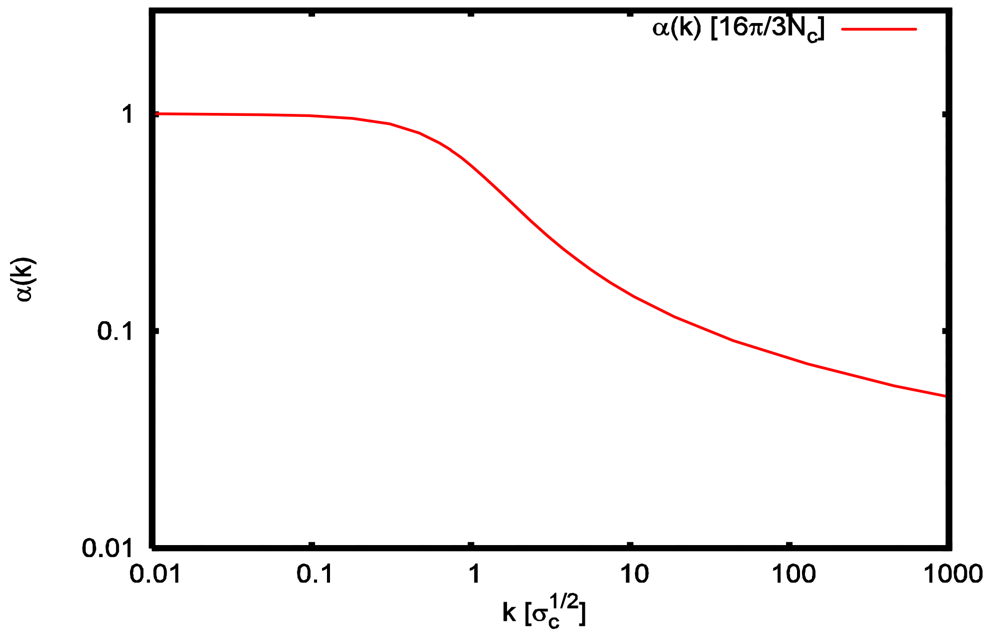


$$\tilde{g}(p \to 0) = \pi\sqrt{8/N_C} \approx 5.13$$

$$\tilde{g}(\sqrt{\sigma_C}) \approx 3.73$$

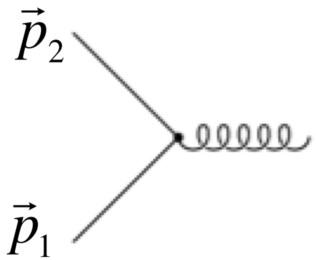
$$\langle \bar{q}q \rangle = (-235\,MeV)^3$$

$$\Rightarrow \tilde{g}(\sqrt{\sigma_C}) \approx 3.57$$

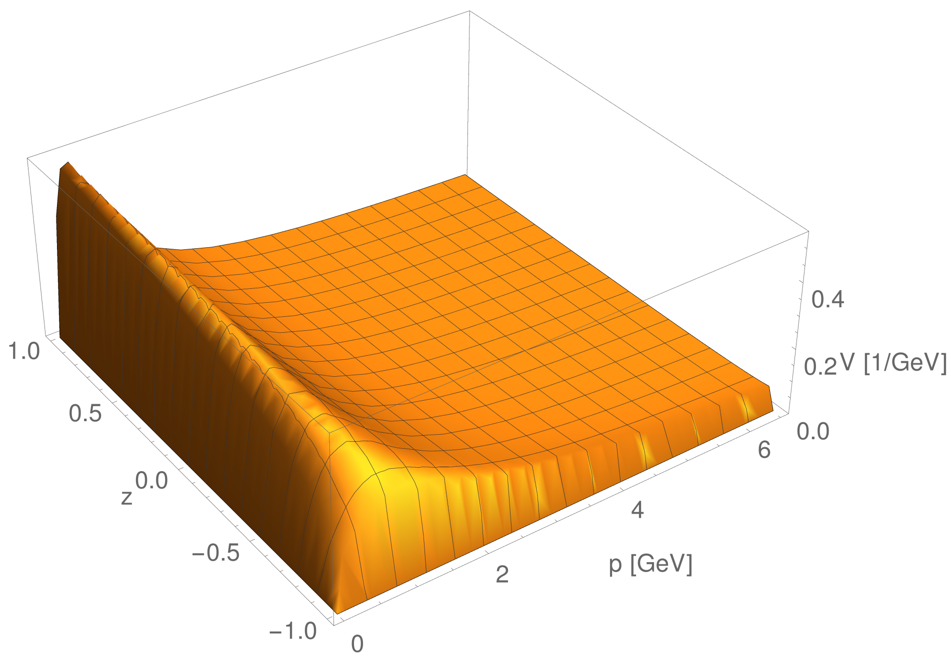


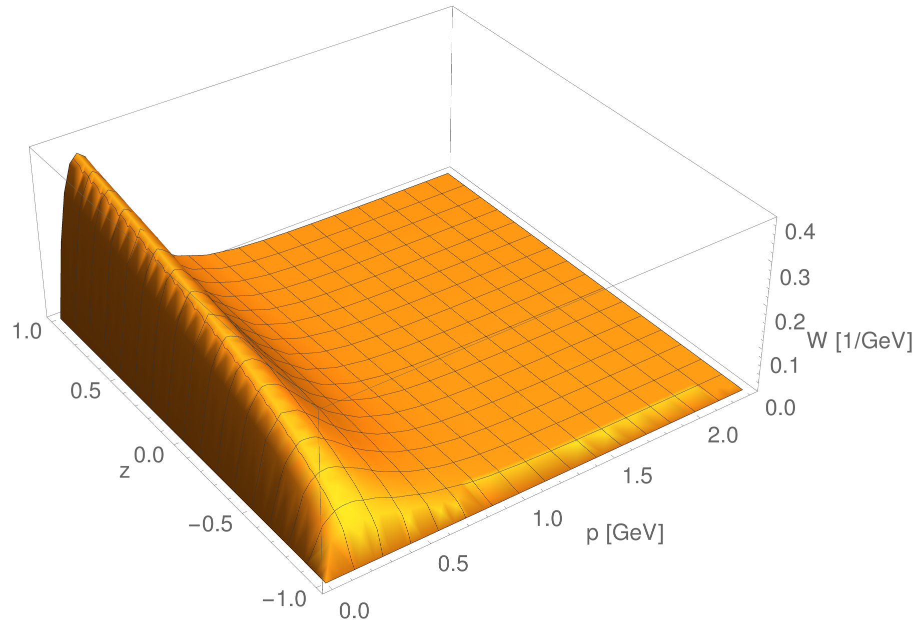
D. Epple, H. Reinhardt and W. Schleifenbaum, Phys. Rev. D75(2007)045011

vector form factors v , w

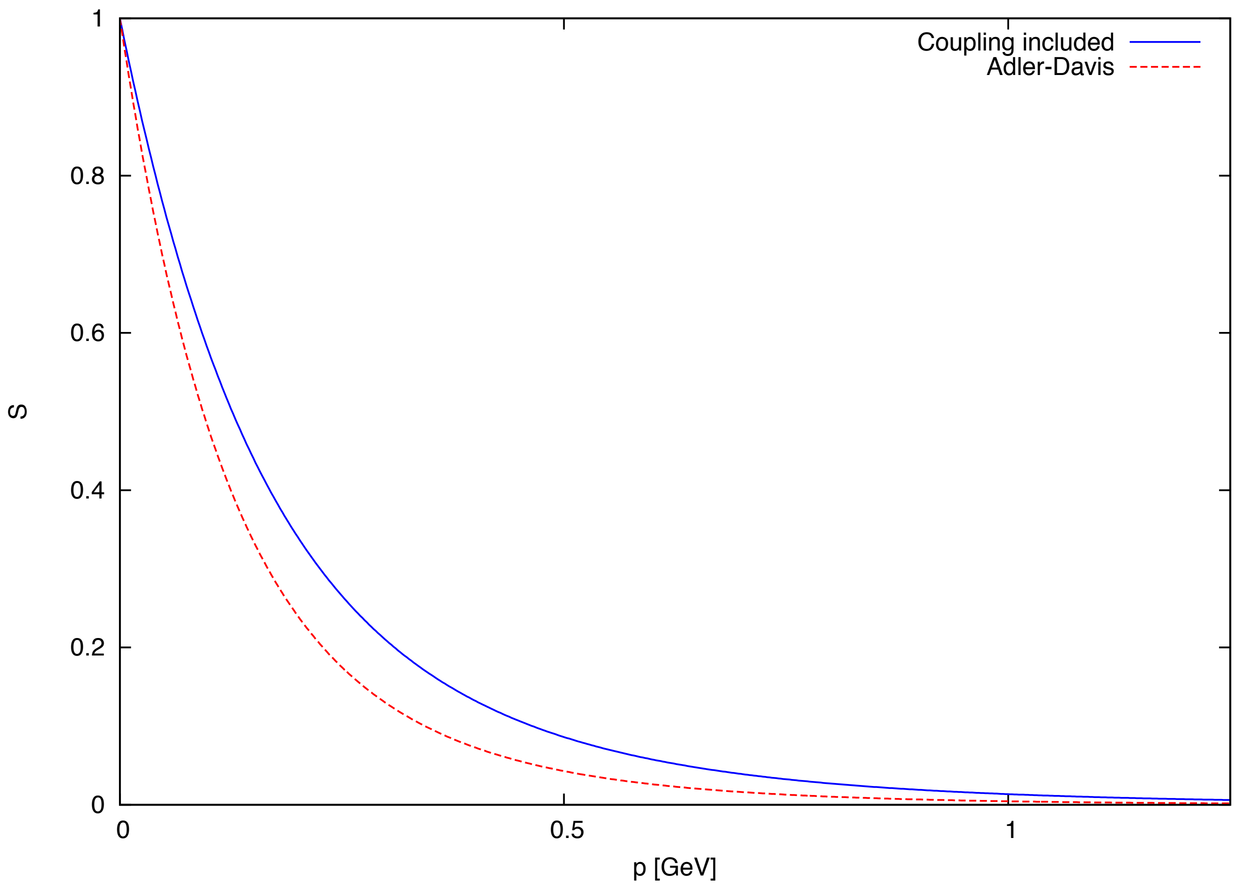


$$v, w(\vec{p}_1, \vec{p}_2): \quad p := |\vec{p}_1| = |\vec{p}_2|, \; z = \cos\sphericalangle(\vec{p}_1, \vec{p}_2)$$





scalar form factor

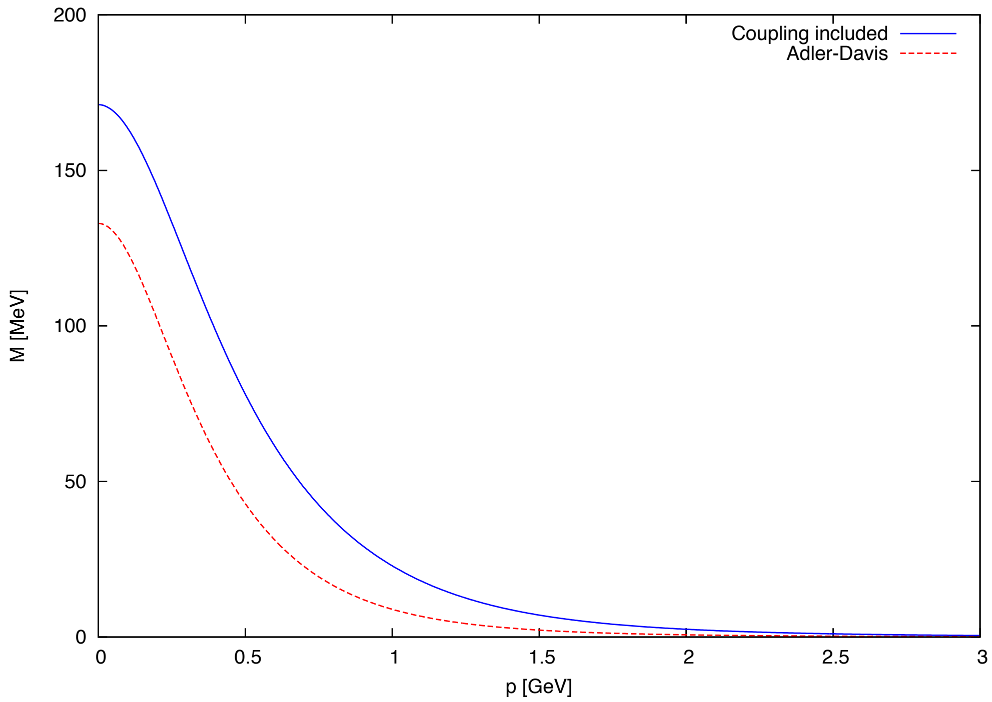


effective quark mass

$$M(p) = \frac{2ps(p)}{1 - s^2(p)}$$

$$\langle \bar{q}q \rangle_{phen} = (-235\,MeV)^3$$

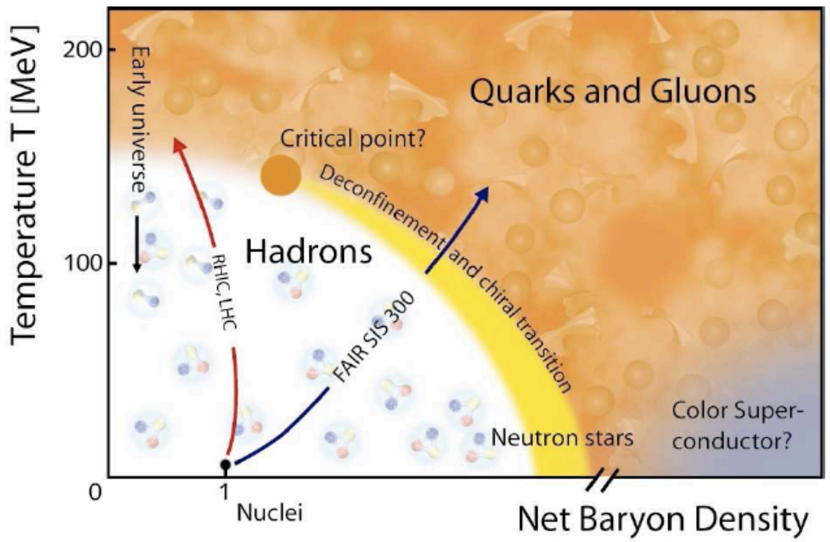
$$\langle \bar{q}q \rangle_{AD} = (-179\,MeV)^3$$



> coupling to tranversal gluons substantially increases chiral symmetry breaking

QCD at finite temperature: grand canonical ensemble

- ansatz for the density operator
- minimization of the thermodynamic potential



H.Reinhardt, D.Campagnari & A. Szczepaniak, Phys.Rev.D84(2011)
J.Heffner, H.Reinhardt & D.Campagnari, Phys.Rev.D85(2012)

Alternative Hamiltonian approach to finite temperature QFT

- *no ansatz for the density operator required*

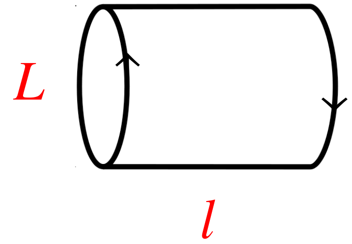
H. Reinhardt & J. Heffner,
Phys.Rev.D88(2013)
Phys.Rev.D91(2015)
and to be published

-

Finite temperature QFT

- compactification of (Euclidean) time
- bc: $A(x^0 = L/2) = \mathrm{A}(x^0 = -L/2)$ Bose fields

  $\psi(x^0 = L/2) = -\psi(x^0 = -L/2)$ Fermi fields



$S^1(L) \times \mathbb{R}^3$

- temperature $T = L^{-1}$ $\quad l \to \infty$

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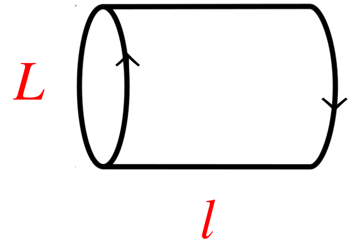
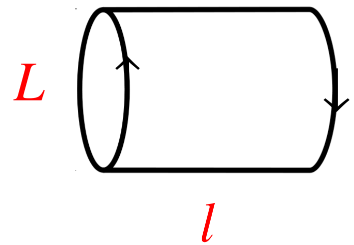


$S^1(L) \times \mathbb{R}^3$

- temperature $T = L^{-1}$ $\quad l \to \infty$
- exploit the O(4)-invariance of the Euclidean Lagrangian
  - O(4)-rotation $x^0 \to x^3 \quad \mathrm{A}^0 \to A^3 \quad \gamma^0 \to \gamma^3$
    $x^1 \to x^0 \quad \mathrm{A}^1 \to A^0 \quad \gamma^1 \to \gamma^0$
  - one compactified spatial dimension
  - bc: $A(x^3 == L/2) = \mathrm{A}(x^3 = -L/2)$ Bose fields
    $\psi(x^3 = L/2) = -\psi(x^3 = -L/2)$ Fermi fields
- spatial manifold: $\mathbb{R}^2 \times S^1(L)$



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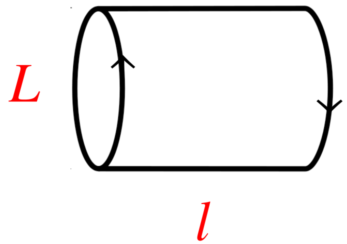

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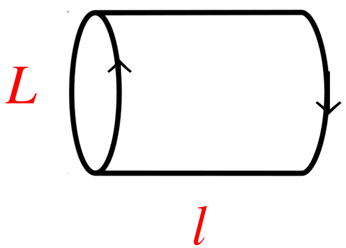


Hamiltonian approach

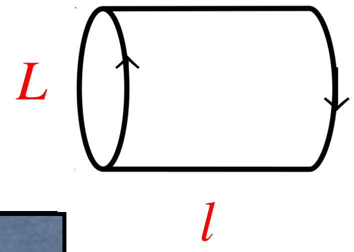
- *temperature is now encoded in one „spatial“dimension while „time“ has infinite extension independent of the temperature*

Finite temperature QFT

- partition function

$$Z(L) = \lim_{l\to\infty} Tr \exp(-lH(L)) = \lim_{l\to\infty} \sum_n \exp(-lE_n(L)) = \lim_{l\to\infty} \exp(-lE_0(L))$$

- ground state energy $\qquad E_0(L) = l^2 L e(L)$
  - on the spatial manifold: $\qquad \mathbb{R}^2 \times S^1(L)$



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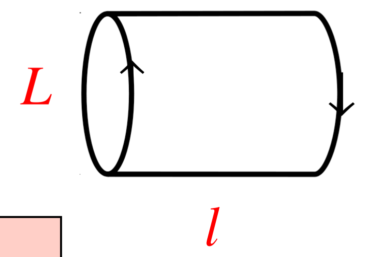
- ground state energy $E_0(L) = l^2 Le(L)$
  - on the spatial manifold: $\mathbb{R}^2 \times S^1(L)$



- pressure: $p = -e(L)$
- energy density: $\varepsilon = \partial[Le(L)]/\partial L - \mu \partial e/\partial \mu$
- Dirac fermions with finite chemical potential

$$h = \vec{\alpha}\cdot\vec{p} + \beta m \to h + i\mu\alpha^3$$

Relativistic Bose gas

- grand canonical ensemble $T = L^{-1}$

$$P = \tfrac{2}{3}\int d^3p \frac{p^2}{\omega(p)} n(p) \qquad n(p) = \frac{1}{e^{L\omega(p)} - 1} \qquad \omega(p) = \sqrt{p^2 + m^2}$$

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- proper-time regularization

$$\sqrt{A} = \frac{1}{\Gamma(-\frac{1}{2})}\lim_{\Lambda\to\infty}\int_{1/\Lambda^2}^{\infty} d\tau \exp(-\tau A)$$

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$$\tfrac{1}{2\pi}\sum_{k=-\infty}^{k=\infty} e^{ikx} = \sum_{n=-\infty}^{n=\infty} \delta(x - 2\pi n)$$

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$$P = -e(L) = \frac{1}{2\pi^2}\sum_{n=-\infty}^{\infty}\left(\frac{m}{n\beta}\right)^2 K_{-2}(n\beta m)$$

modified Bessel function

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  - proper-time regularization $\sqrt{A} = \frac{1}{\Gamma(-\frac{1}{2})}\lim_{\Lambda\to\infty}\int_{1/\Lambda^2}^{\infty} d\tau \exp(-\tau A)$
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modified Bessel function

- massles bosons: m=0

Stephan – Boltzmann – law

$$\boxed{P = \frac{\zeta(4)}{\pi^2}T^4 = \frac{\pi^2}{90}T^4}$$

massive bosons

$$\omega(p) = \sqrt{p^2 + m^2}$$

$$P = \tfrac{2}{3}\int d^3p\, p\,\frac{p^2}{\omega(p)}\,n(p) \qquad n(p) = \frac{1}{e^{L\omega(p)} - 1}$$

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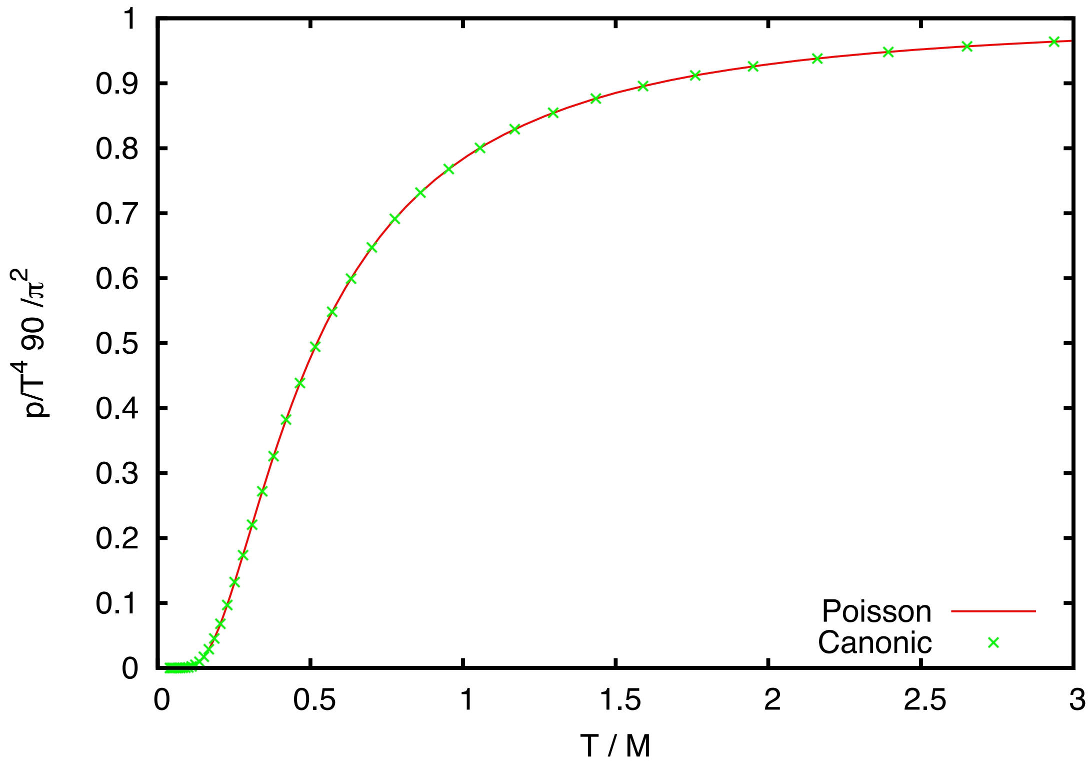
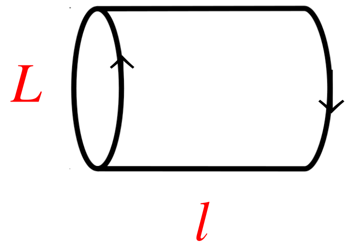


pressure of a massive relativistic Bose gas

$$e(L) = \frac{1}{2}\int d^2 p_\perp \frac{1}{L}\sum_{n=-\infty}^{\infty}\sqrt{m^2 + p_\perp^2 + \omega_n^2}$$

$$\omega_n = \frac{2\pi n}{L}$$

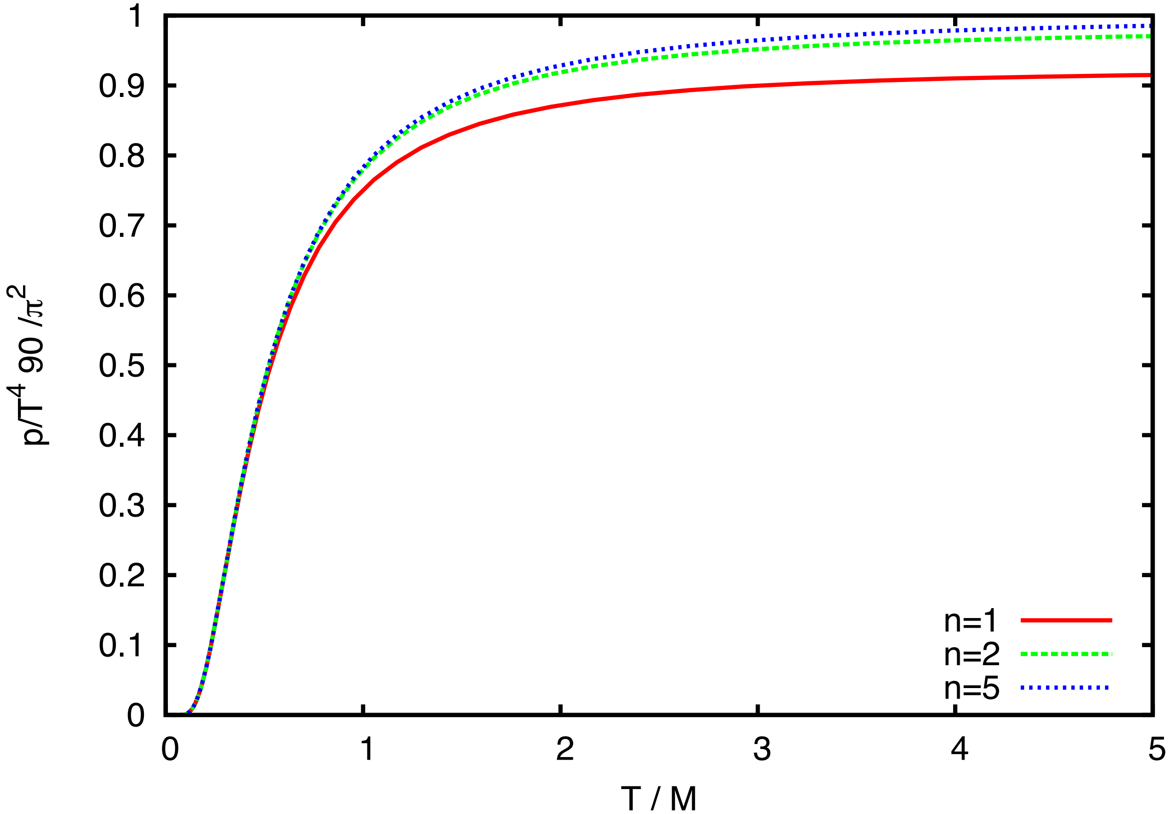


$S^1(L) \times \mathbb{R}^3$

-proper-time regularization
-Poisson resummation
-skip L-independent (div.) const.

$$P = \frac{1}{\pi^2}\sum_{n=1}^{\infty}\left(\frac{m}{n\beta}\right)^2 K_{-2}(nLm)$$

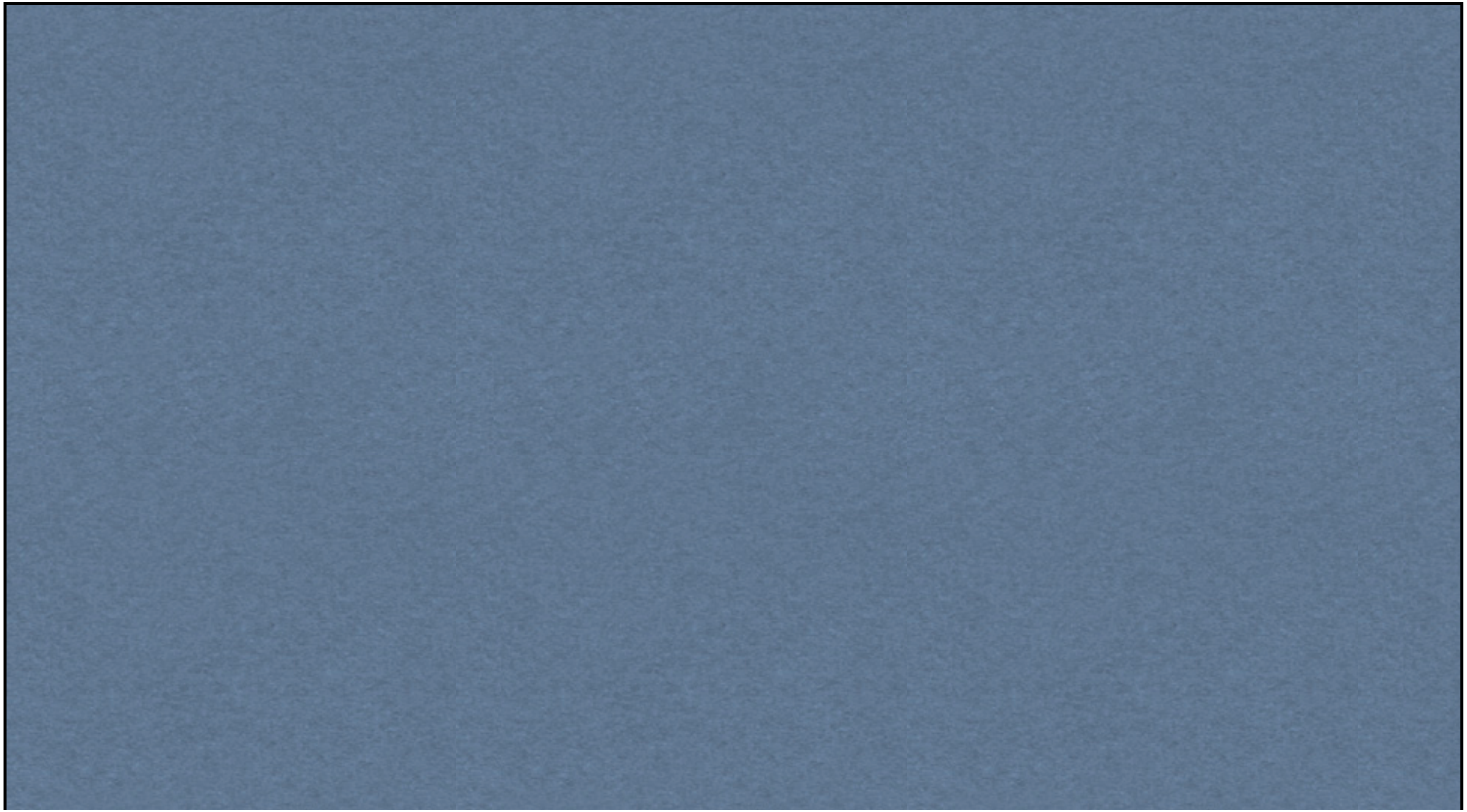
a few terms are sufficient to reproduce the result of the usual grand canonical ensemble



Relativistic Fermi gas

- grand canonical ensemble $T = L^{-1}$

$$P = \tfrac{2}{3}\int d^3p \frac{p^2}{\omega(p)}(n_+(p) + n_-(p)) \qquad n_\pm(p) = \frac{1}{e^{L(\omega(p)\mp\mu)} + 1} \qquad \omega(p) = \sqrt{p^2 + m^2}$$



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$$e(L) = -2\int d^2p_\perp \frac{1}{L}\sum_{n=-\infty}^{\infty}\sqrt{m^2 + p_\perp^2 + (\omega_n + i\mu)^2} \qquad \omega_n = \frac{2n+1}{L}\pi$$

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  - Poisson resummation

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- massles Dirac fermions: m=0
  - analytic continuationfor iμ $\longrightarrow$ x $\displaystyle\sum_{n=1}^{\infty}(-)^n\frac{\cos(nx)}{n^4} = \tfrac{1}{48}\left[-\tfrac{7}{15}\pi^4 + 2\pi^2x^2 - x^4\right]$

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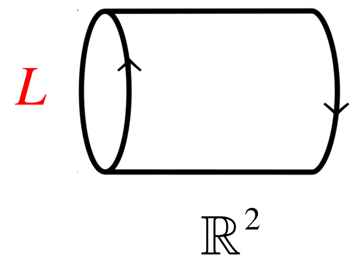
$$\boxed{P = \frac{1}{12\pi^2}\left[\frac{7}{15}\pi^4T^4+2\pi^2T^2\mu^2+\mu^4\right]}$$

QCD at finite T

- Hamiltonian approach in Coulomb gauge on the partially compactified spatial manifold $\mathbb{R}^2 \times S^1(L)$



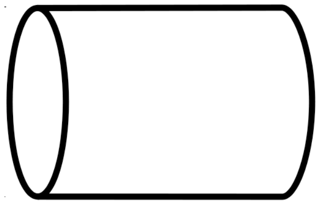
- variational solution of the Schrödinger equation for the vacuum
- finite temperature QCD is fully encoded in its vacuum
- no ansatz for density operator of the grand canonical ensemble required

YM sector:

Heffner & Reinhardt, Phys.Rev.D91(2015)

center symmetry

SU(N) gauge theory at finite T $T^{-1} = L$



periodic boundary condition $A(L) = A(0)$

allowed gauge transformations $A \to A^U$

preserving the b.c. $A^U(L) = A^U(0)$

$U(L) = zU(0) \qquad z \in Z(N)$

residual global Z(N)- symmetry which remains after gauge fixing

Polyakov loop $P[A_0](\vec{x}) = \frac{1}{d_r} tr P \exp\left[i \int_0^L dx_0 A_0(x_0, \vec{x}) \right]$ $P[A_0^U](\vec{x}) = zP[A_0](\vec{x})$

$\langle P[A_0](\vec{x}) \rangle \sim \exp[-F_\infty(\vec{x}) L]$ $F_\infty(\vec{x})$ - free energy of a static color charge

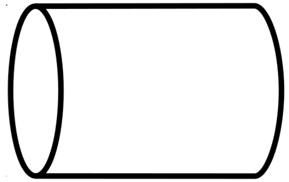
confined phase: $\langle P[A_0](\vec{x}) \rangle = 0$ *center symmetric*

deconfined phase: $\langle P[A_0](\vec{x}) \rangle \neq 0$ *center symmetry broken*

The Polyakov loop - order parameter of confinement

$$P[A_0](\vec{x}) = \tfrac{1}{d_r} tr P \exp\left[i \int_0^L dx_0 A_0(x_0, \vec{x}) \right]$$

$T^{-1} = L$



Polyakov gauge $\partial_0 A_0 = 0, \quad A_0 = diagonal$ $\qquad zP\left[A_0\right] = P\left[A_0 + \mu\right] \qquad z = e^{i\mu}$

$SU(2):$ $\boxed{P[A_0](\vec{x}) = \cos(\tfrac{1}{2} A_0(\vec{x}) L)}$ $\qquad P[A_0] - unique\ function\ of\ A_0$

alternative order parameters of confinement

$$\langle P[A_0](\vec{x}) \rangle \qquad P\left[\langle A_0 \rangle\right](\vec{x}) \qquad \langle A_0(\vec{x}) \rangle$$

- *J. Braun, H. Gies, J. M. Pawlowski, Phys. Lett. B684(2010)262*

Effective potential of the order parameter for confinement

- background field calculation $a_0 = \langle A_0(\vec{x}) \rangle -$ const, diagonal (Polyakov gauge)
- effective potential $e[a_0] \to \min \quad \Rightarrow a_0 = \bar{a}_0$
- order parameter $\langle P[A_0] \rangle \approx P[\bar{a}_0]$

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- order parameter $\quad \langle P[A_0] \rangle \approx P[\bar{a}_0]$
- 1-loop perturbation theory

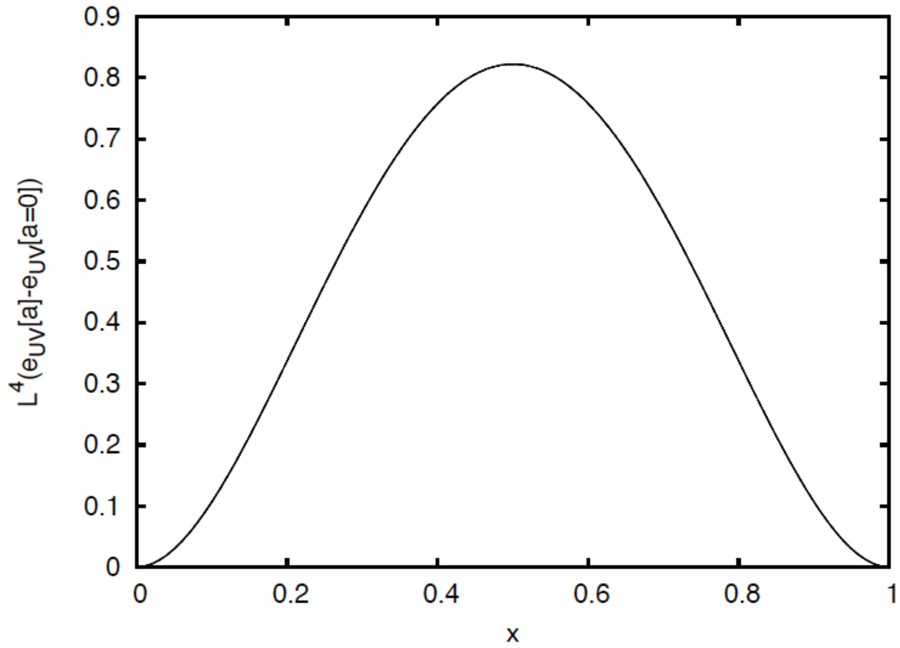
$$e_{PT}[a_0 = x 2\pi / L]$$

Gross, Pisarski, Yaffe, Rev.Mod.Pys.53(1981)

N.Weiss, Phys.Rev.D24(1981)

$$P[\bar{a}_0 = 0] = 1$$

deconfined phase



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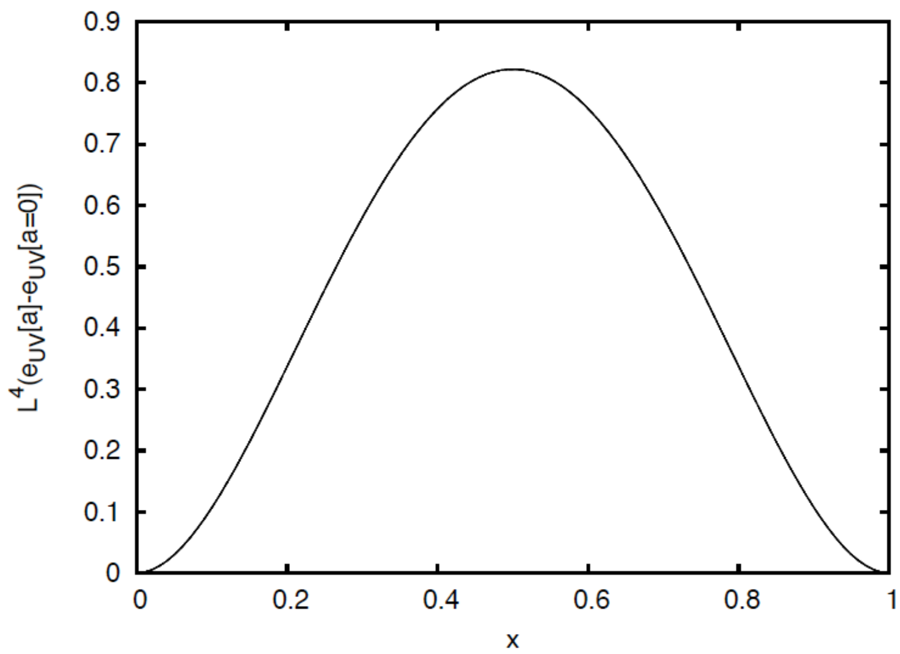


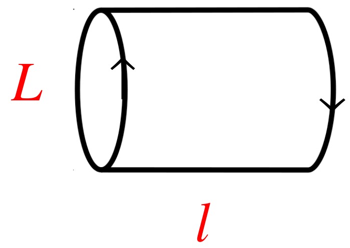
in this talk: non-perturbative evaluation of $e[a_0]$ in the Hamiltonian approach

H. Reinhardt & J. Heffner, Phys.Rev.D88(2013)

Effective potential of the order parameter for confinement

- background field calculation $a_0 = \langle A_0(\vec{x}) \rangle$ – const, diagonal (Polyakov gauge)
- effective potential $e[a_0] \to \min \qquad \Rightarrow a_0 = \bar{a}_0$
- order parameter $\langle P[A_0] \rangle \approx P[\bar{a}_0]$

- ordinary Hamiltonian approach assumes Weyl gauge $A_0 = 0$
  - O(4)-invariance
  - compactify (instead of time) one spatial axis to a circle of circumference L and interprete L^{-1} as temperature
  - Hamiltonian approach on $\mathbb{R}^2 \times S^1(L)$
  - compactify x_3 – axis $\vec{a} = a\vec{e}_3$
  - calculate the effective potential $\boxed{e[a]}$



The effective potential in the Hamiltonian approach

- effective potential $e(\vec{a})$ of a spatial background field $\vec{a}$

$$\langle H \rangle_{\vec{a}} = \min \langle H \rangle \qquad \langle \vec{A} \rangle = \vec{a}$$

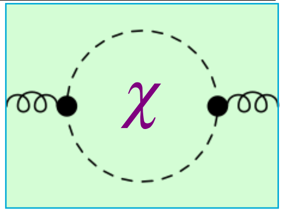
$$\langle H \rangle_{\vec{a}} = (spatial\ \ volume) \times e(\vec{a})$$

$$e(\vec{a}) - energy\ density$$

- variational calculation of $e(\vec{a})$

The gluon effective potential

- energy density $$e(a,L) = \sum_{\sigma} \tfrac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_\perp \left(\omega(p^\sigma) - \chi(p^\sigma)\right)$$



- background field $\vec{p}^{\,\sigma} = \vec{p}_\perp + (p_n - \sigma a)\vec{e}_3 \qquad p_n = 2\pi n / L \qquad \sigma - roots$
- roots

$SU(2)\!: \quad H_1 = T_3 \qquad \sigma_1 = 0,\ \pm 1 \qquad$ *positive roots*

$SU(3)\!: \quad H_1 = T_3 \qquad H_2 = T_8 \qquad \sigma = (1,0),\quad (\tfrac{1}{2}, \tfrac{1}{2}\sqrt{3}),\quad (\tfrac{1}{2}, -\tfrac{1}{2}\sqrt{3})$

- periodicity $e(a,L) = e(a + \mu_k / L, L) \qquad \exp\left(i\mu_k\right) = z_k \in Z(N)$

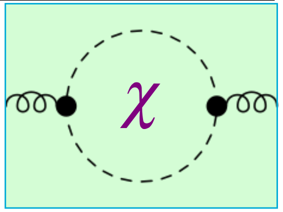
$\mu_k - coweights$

- input: $\omega(p), \chi(p)$ from the variational calculation in Coulomb gauge at T=0

C. Feuchter & H. Reinhardt,Phys. Rev.D71(2005)
D. Epple, H. Reinhardt, W. Schleifenbaum, Phys. Rev.D75(2007)

The gluon effective potential

- energy density $$e(a,L)=\sum_{\sigma}\frac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_{\perp}(\omega(p^{\sigma})-\chi(p^{\sigma}))$$

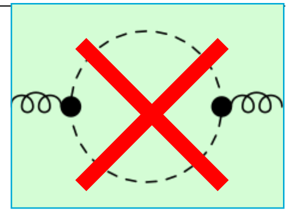


- background field $p^{\sigma}=p_{\perp}+(p_n-\sigma a)\quad p_n=2\pi n/L\quad \sigma-roots$

- periodicity $e(a,L)=e(a+\mu_k/L,L)\quad \exp(-\mu_k)=z_k\in Z(N)$

The gluon effective potential

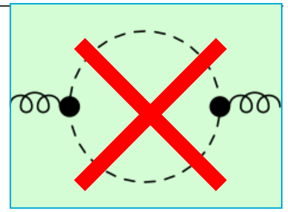
- energy density $$e(a,L) = \sum_{\sigma} \frac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_\perp (\omega(p^\sigma) - \cancel{\chi(p^\sigma)})$$



- background field $p^\sigma = p_\perp + (p_n - \sigma a)$ $\quad p_n = 2\pi n / L$ $\quad \sigma - roots$
- periodicity $e(a,L) = e(a + \mu_k / L, L)$ $\quad \exp\left(i\mu_k\right) = z_k \in Z(N)$
- neglect ghost loop $\chi(p) = 0$ $$e(a,L) = \sum_{\sigma} \frac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_\perp \omega(p^\sigma)$$
  - quasi-gluon gas

The effective potential

- energy density $$e(a,L)=\sum_{\sigma}\frac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_\perp(\omega(p^\sigma)-\cancel{\chi(p^\sigma)})$$



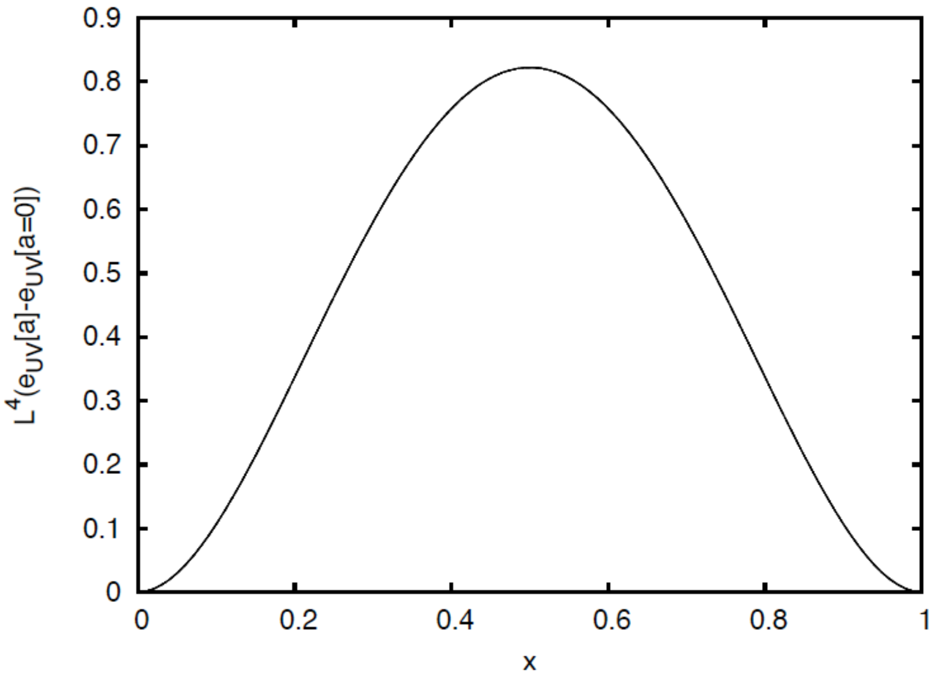
- background field $p^\sigma=p_\perp+(p_n-\sigma a)$ $\quad p_n=2\pi n/L$ $\quad \sigma-roots$
- periodicity $e(a,L)=e(a+\mu_k/L,L)$ $\quad \exp(-\mu_k)=z_k\in Z(N)$
- neglect ghost loop $\chi(p)=0$ $$e(a,L)=\sum_{\sigma}\frac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_\perp\omega(p^\sigma)$$
  - quasi-gluon gas
- limiting cases
  - UV: $\omega_{UV}(p)=p$
  - IR: $\omega_{IR}(p)=M^2/p$
  - Gribov: $\omega(p)=\sqrt{(p^2+M^4/p^2)}\approx\omega_{IR}(p)+\omega_{UV}(p)$

The gluon UV-effective potential

$$\chi(p)=0 \qquad \omega(p)=p \qquad e(a,L)=\sum_{\sigma}\tfrac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_\perp(\omega(p^\sigma)-\cancel{\chi(p^\sigma)})$$

$$e(a,L)=\frac{8}{\pi^2L^4}\sum_{n=1}^{\infty}\frac{\sin^2(naL/2)}{n^4}$$

$$=\frac{4\pi^2}{3L^4}\underbrace{\left(\frac{aL}{2\pi}\right)}_{x}^2\left[\frac{aL}{2\pi}-1\right]^2$$



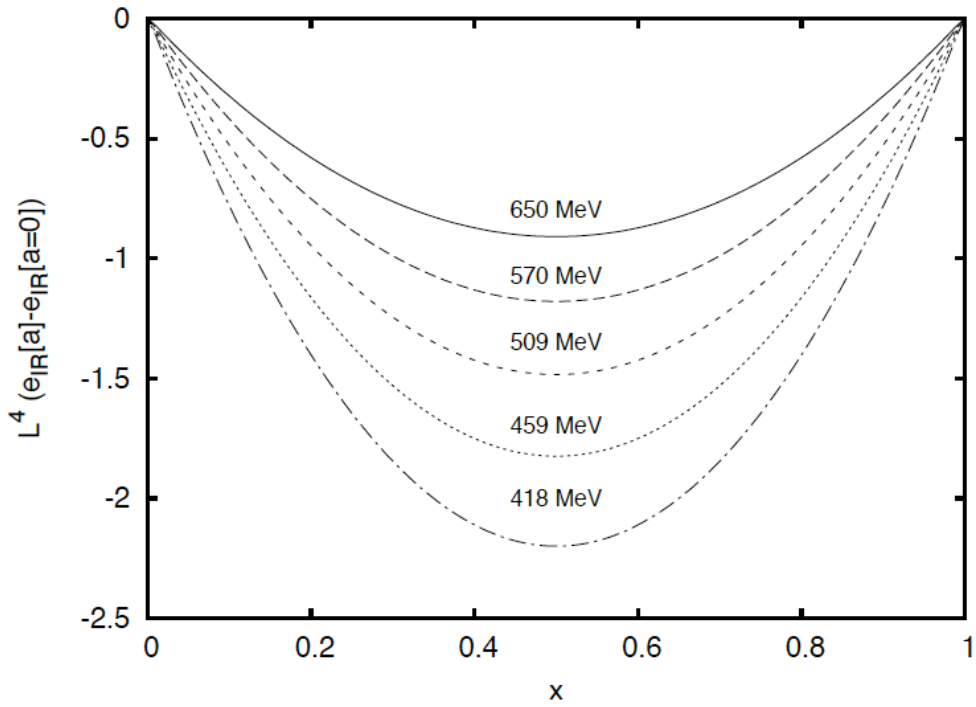
N.Weiss 1-loop PT

$$Polyakov-loop \qquad \langle P\rangle \simeq P[a_{\min}=0]=1 \qquad deconfining\quad phase$$

The gluon IR-effective potential

$$\chi(p)=0 \qquad \omega(p)=M^2/p \qquad e(a,L)=\sum_\sigma \tfrac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_\perp(\omega(p^\sigma)-\cancel{\chi(p^\sigma)})$$

$$e_{IR}(a,L)=-\frac{4M^2}{\pi^2L^2}\sum_{n=1}^{\infty}\frac{\sin^2(naL/2)}{n^2}$$
$$=\frac{2M^2}{L^2}\underbrace{\left(\frac{aL}{2\pi}\right)}_{x}\left[\frac{aL}{2\pi}-1\right]$$



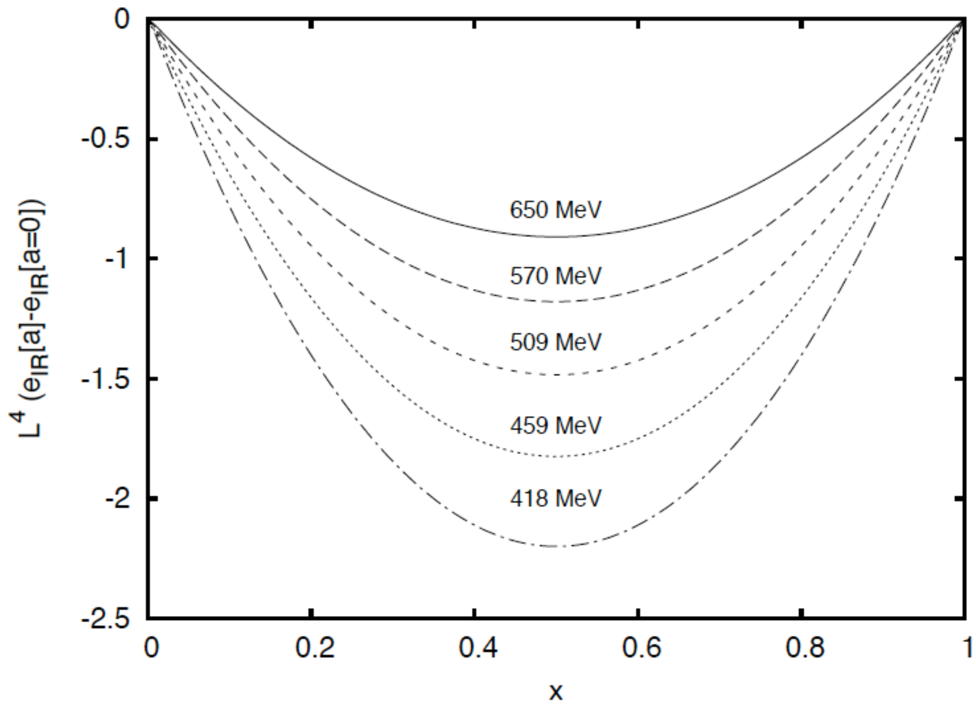
$$Polyakov-loop \qquad \langle P\rangle \simeq P[a_{\min}=\pi/L]=0 \qquad confining \quad phase$$

The gluon IR-effective potential

$\chi(p) = 0$ $\qquad \omega(p) = M^2 / p$ $\qquad e(a, L) = \sum_\sigma \frac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_\perp (\omega(p^\sigma) - \cancel{\chi(p^\sigma)})$

$$e_{IR}(a, L) = -\frac{4M^2}{\pi^2 L^2} \sum_{n=1}^{\infty} \frac{\sin^2(naL/2)}{n^2}$$

$$= \frac{2M^2}{L^2} \underbrace{\left(\frac{aL}{2\pi}\right)}_{x} \left[\frac{aL}{2\pi} - 1\right]$$



$$Polyakov-loop \qquad \langle P \rangle \simeq P[a_{\min} = \pi / L] = 0 \qquad confining \quad phase$$

deconfinement phase transition results from the interplay between the confining IR-potential and deconfining UV-potential

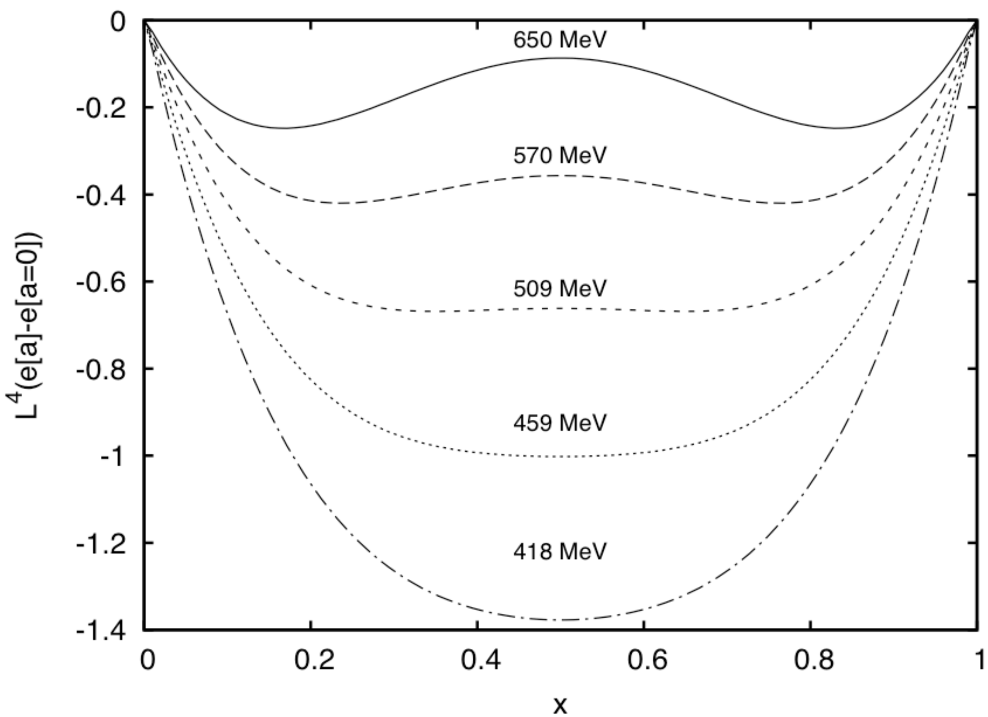
The gluon IR+UV effective potential:

$\chi(p) = 0$ $\quad\quad$ $\omega(p) = p + M^2 / p$ $\quad\quad$ $e(a, L) = e_{UV}(a, L) + e_{IR}(a, L)$

phase transition

critical temperature:

$$T_C = \sqrt{3}M / \pi$$



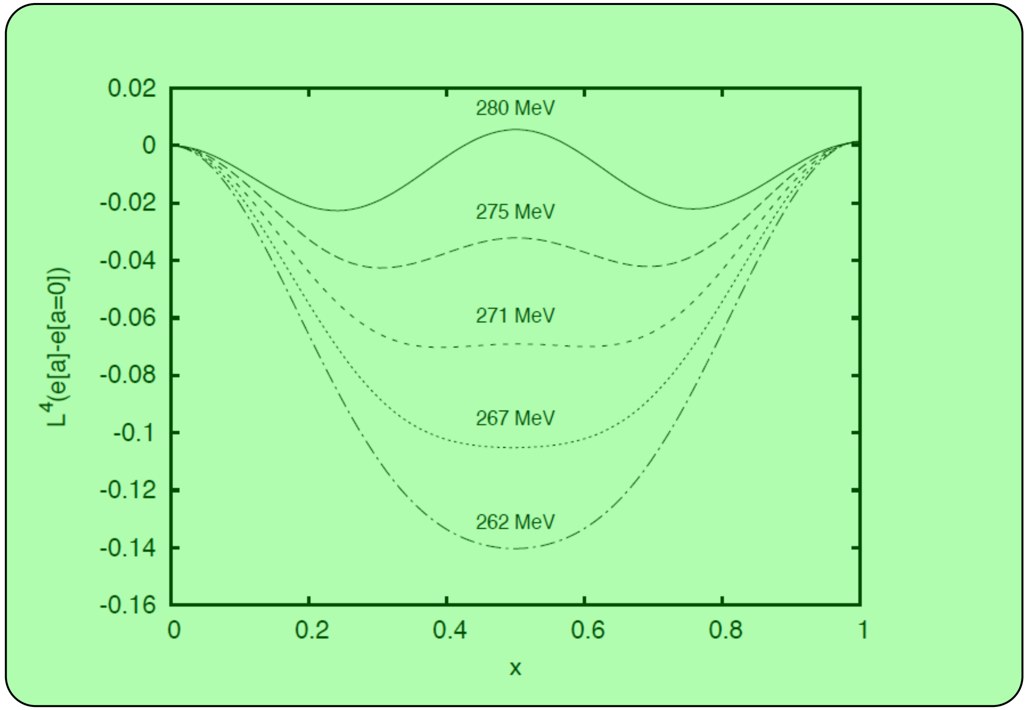
$lattice : M \simeq 880\,MeV \quad \Rightarrow \quad T_C \simeq 485\,MeV$

$\chi(p) = 0$ $\quad\quad$ $\omega(p) = \sqrt{p^2 + M^4 / p^2}$ $\quad\quad$ $T_C \simeq 432\,MeV$

The full gluon effective potential

$$e(a,L)=\sum_{\sigma}\frac{1}{L}\sum_{n=-\infty}^{n=\infty}\int d^2p_{\perp}(\omega(p^{\sigma})-\chi(p^{\sigma}))$$

variational calculation in Coulomb gauge



SU(2)

critical temperature:

$$T_C \simeq 267\,MeV$$

The effective potential for SU(3)

SU(3)-algebra consists of 3 SU(2)-subalgebras characterized by the 3 non-zero positive rooots

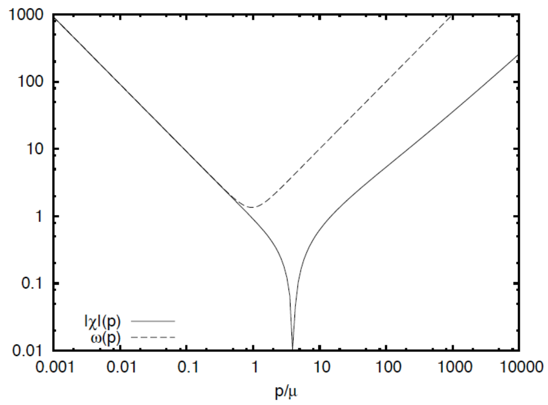
$$\boxed{\sigma = (1,0), \quad \left(\frac{1}{2}, \frac{1}{2}\sqrt{3}\right), \quad \left(\frac{1}{2}, -\frac{1}{2}\sqrt{3}\right)}$$

$$e_{SU(3)}[a] = \sum_{\sigma>0} e_{SU(2)(\sigma)}[a]$$

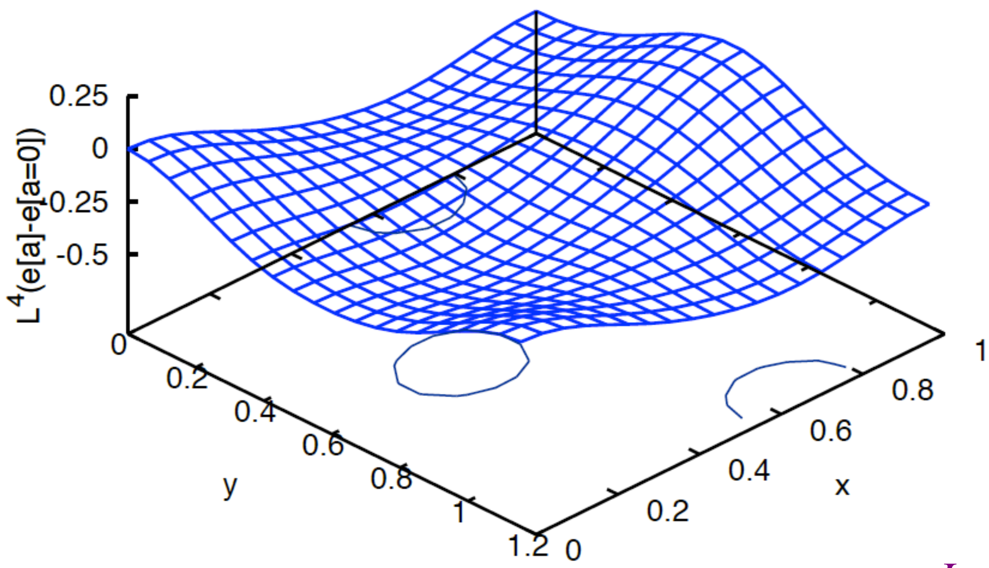
The full effective potential for SU(3)

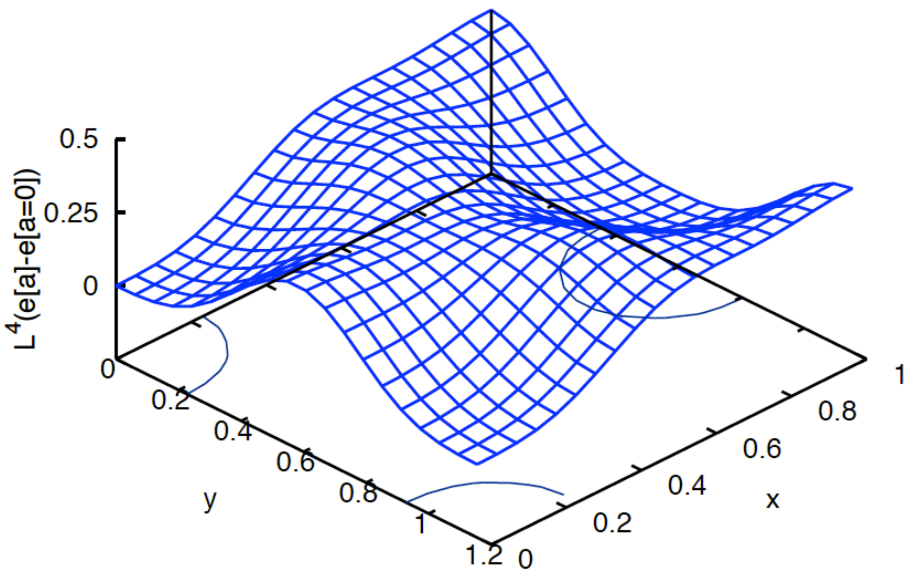
$$e(a,L) = \sum_{\sigma} \frac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_{\perp} (\omega(p^{\sigma}) - \chi(p^{\sigma}))$$

variational calculation in Coulomb gauge



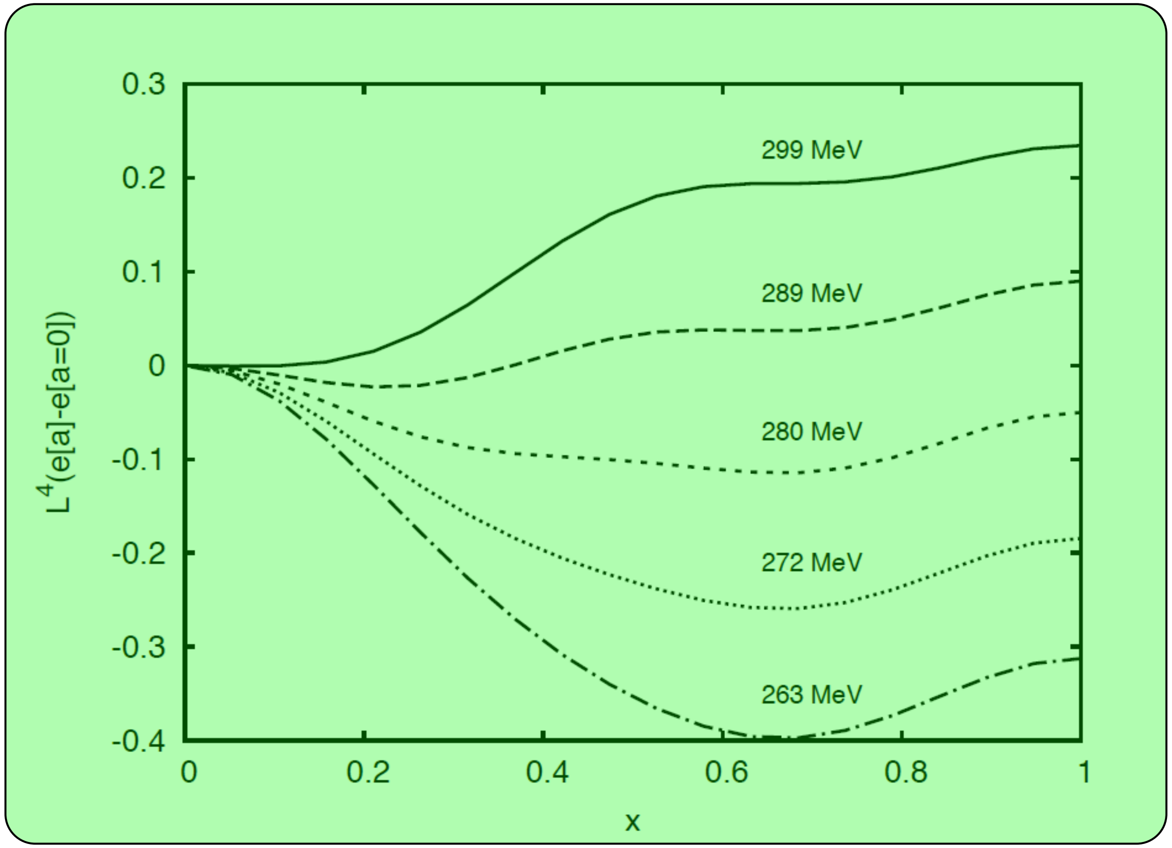
$T < T_C$ $\qquad$ $T > T_C$





$$x = \frac{a_3 L}{2\pi}, \qquad y = \frac{a_8 L}{2\pi}$$

Polyakov loop potential for SU(3)



$$x = \frac{a_3 L}{2\pi}, \qquad \mathrm{y} = \frac{a_8 L}{2\pi} = 0$$

$$input: \; SU(2) - data:$$

$$M = 880\,MeV$$

$$T_C = 283 MeV$$

critical temperature

$lattice:$ $T_C^{SU(2)} = 312\,MeV$ $T_C^{SU(3)} = 284\,MeV$

$this\ work:$ $T_C^{SU(2)} = 269\,MeV$ $T_C^{SU(3)} = 283\,MeV$

$FRG(Fister\ \&\ Pawlowski):$ $T_C^{SU(2)} = 230\,MeV$ $T_C^{SU(3)} = 275\,MeV$

lattice: *B. Lucini, M. Teper, U. Wenger, JHEP01(2004)061*

The quark effective potential

- energy density $$\boxed{e(a,L) = -N_f \sum_{\sigma} \tfrac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_\perp \varepsilon(p^\sigma)}$$
- quasi quark energy $$\varepsilon(p) = \sqrt{M^2(p) + p^2}$$
- background field $\vec{p}^{\sigma} = \vec{p}_\perp + (p_n - \sigma a + i\mu)\vec{e}_3 \qquad p_n = (2n+1)\pi / L \qquad \sigma - weights$

$$SU(2): \quad H_1 = T_3 \qquad\qquad \sigma_1 = \pm\tfrac{1}{2}$$

$$SU(3): \quad H_1 = T_3 \qquad H_2 = T_8 \qquad \sigma = (\tfrac{1}{2}, \tfrac{1}{2\sqrt{3}}), \quad (-\tfrac{1}{2}, \tfrac{1}{2\sqrt{3}}), \quad (0, -\tfrac{1}{\sqrt{3}})$$

- periodicity $e(a,L) = e(a + 2\mu_k / L, L) \qquad \exp\left(i\mu_k\right) = z_k \in Z(N)$

The quark effective potential

- energy density $$e(a,L) = -N_f \sum_\sigma \tfrac{1}{L} \sum_{n=-\infty}^{n=\infty} \int d^2 p_\perp \varepsilon(p^\sigma)$$
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$$SU(2): \quad H_1 = T_3 \qquad \sigma_1 = \pm\tfrac{1}{2}$$

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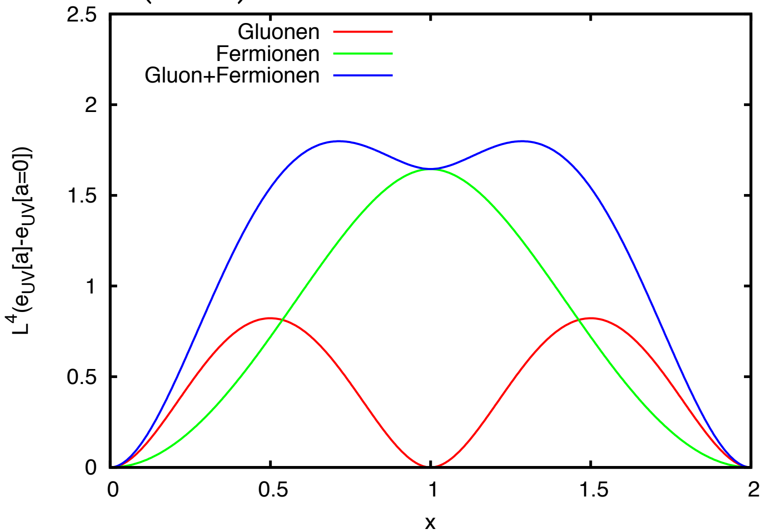
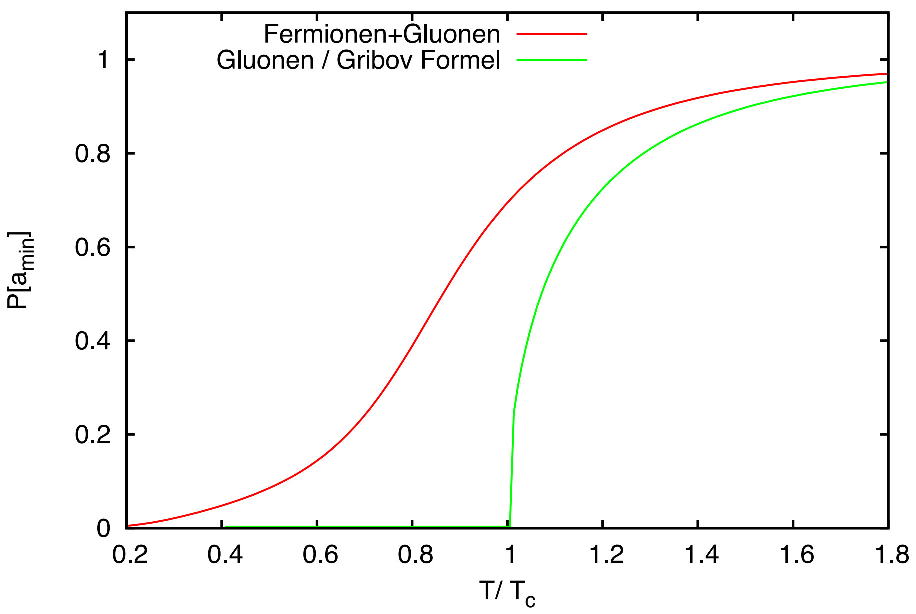
- periodicity $$e(a,L) = e(a + 2\mu_k / L, L) \qquad \exp(i\mu_k) = z_k \in Z(N)$$

UV-potential $\quad \varepsilon(p) = p$

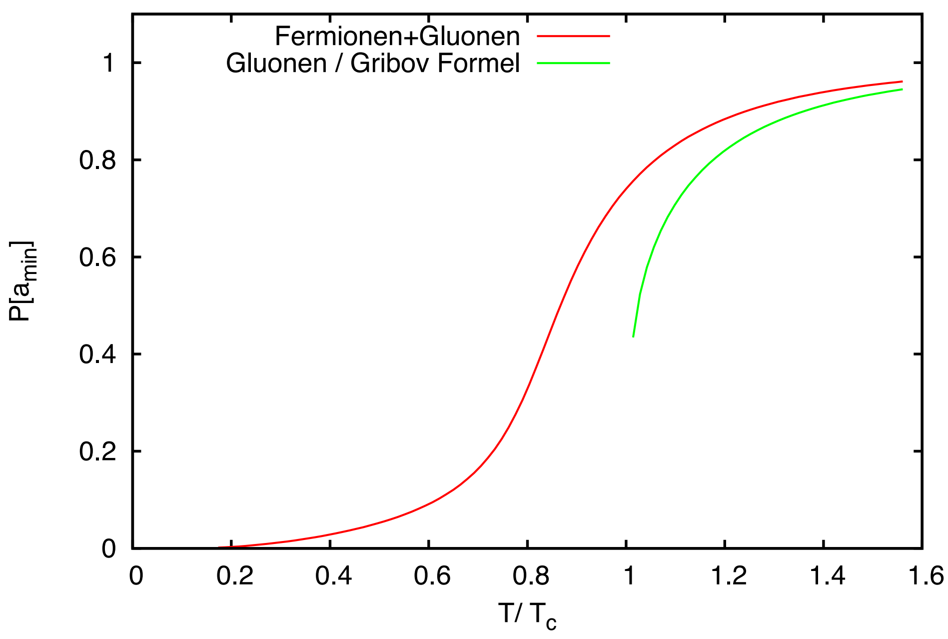
$$e(a,L) = \frac{N_f}{24\pi^2 L^4} \sum_\sigma \left[\frac{7}{15}\pi^4 + 2\pi^2 L^2 (\mu + i\sigma \cdot a)^2 + L^4 (\mu + i\sigma \cdot a)^4 \right]$$



The Polyakov loop



SU(2)



SU(3)

center symmetry

DECONFINEMENT PHASE TRANSION:

confined phase: center symmetry
deconfined phase: center symmetry broken

any observable transforming non-trivially under the center may serve as order parameter for confinement

prototype: Polyakov loop

$$P[A_0](\vec{x}) = \tfrac{1}{d_r}\, tr P \exp\left[i \int_0^L dx_0 A_0(x_0, \vec{x}) \right]$$

dual quark condensate -dressed Polyakov loop

$$\Sigma_n = \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-in\varphi} \left\langle (\bar{q}q)_\varphi \right\rangle \qquad q(L) = e^{i\varphi} q(0)$$

Σ_n *loops winding n-times around the compact time axis*

Σ_1 *dressed Polyakov loop*

Gattringer
PRL. 97(2006)

$$imaginary\ chemical\ potential: \quad \mu = i\frac{\pi - \varphi}{L}$$

$$compactified\ \ 3\text{-}axis\ potential: \quad p_3 = \Omega_n + i\mu = \frac{2\pi n + \varphi}{L}$$

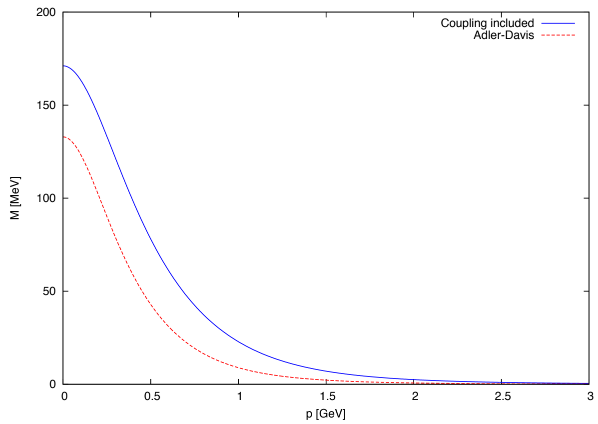
Dual quark condensate in the Hamiltonian approach in ∂A=0

$$\Sigma_n = \int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-in\varphi} \left\langle (\bar{q}q)_\varphi \right\rangle \qquad q(\beta) = e^{i\varphi} q(0)$$

after Poisson resummation

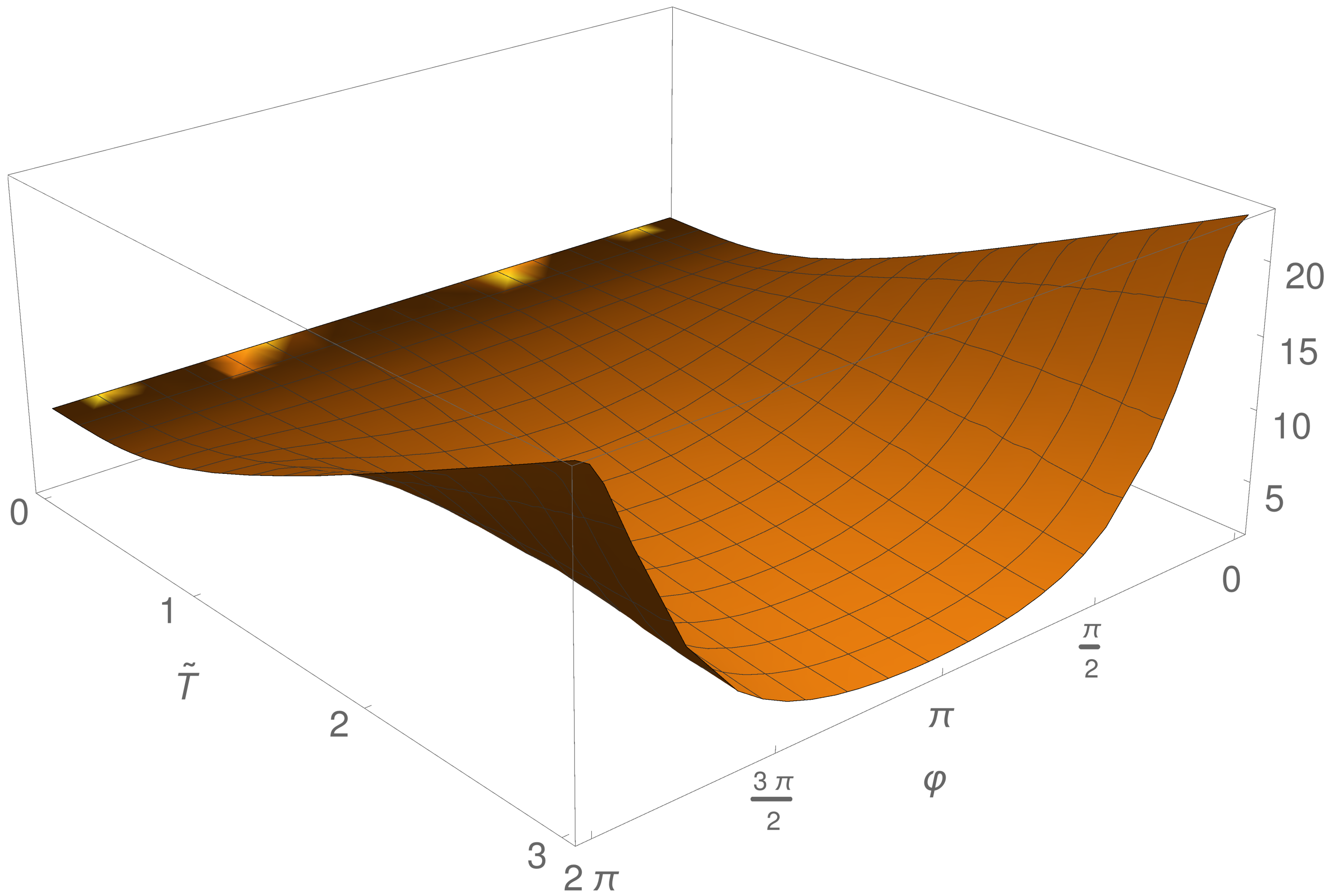
$$\Sigma_n = (-)^{n+1} \frac{2N_C}{2\pi^2} \int_0^\infty dp\, p \frac{\sin(nLp)}{nL} \frac{M(p)}{\sqrt{p^2 + M^2(p)}}$$

effective quark mass $M(p)$

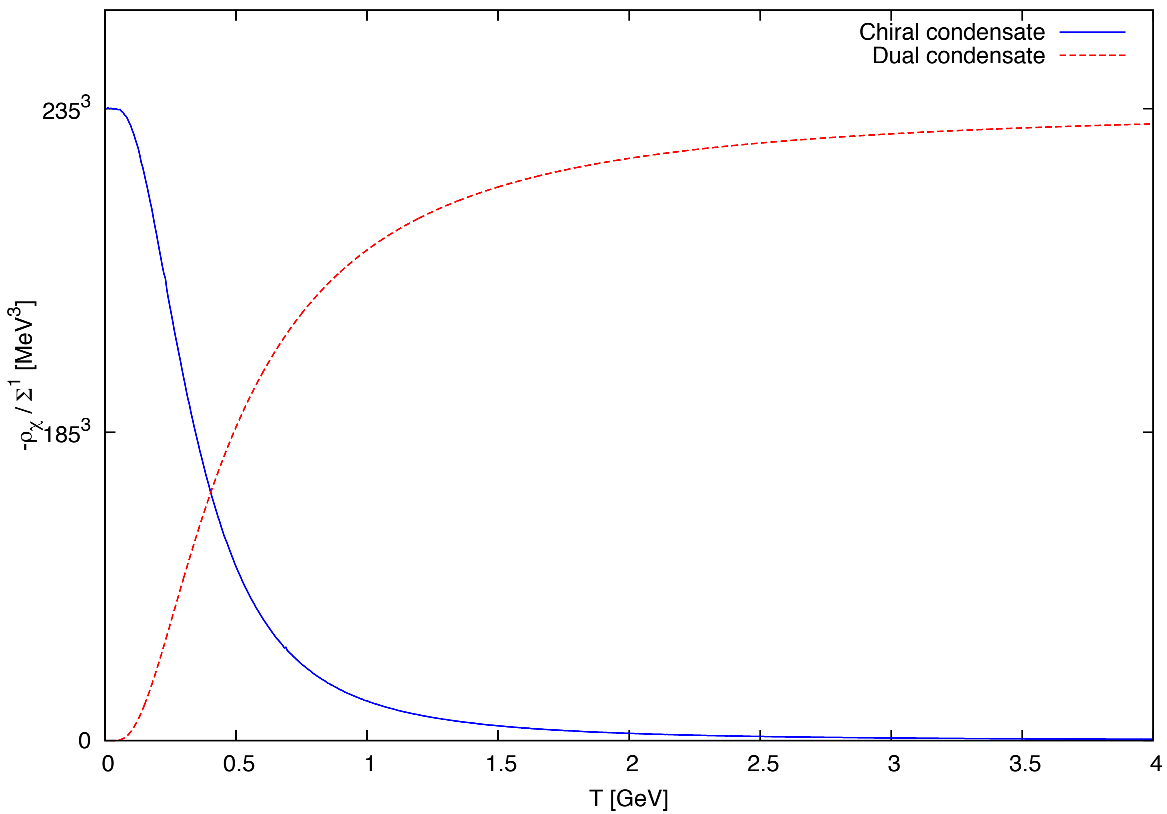


with P. Vastag & E. Ebadati

quark condensate $\left\langle (\bar{q}q)_\varphi \right\rangle$



chiral & dual condensate



$$\sigma_C = 2\sigma \qquad T_{PC} \simeq 260\,MeV$$

Conclusions

- variational approach to the Hamiltonian formulation of QCD in Coulomb gauge
- decent description of the IR-sektor of QCD
  - confinement
  - SBCS
- novel Hamiltonian approach to finite temperature QFT
  - compactification of a spatial dimension
  - the finite QFT is fully encoded in the ground state of the spatial manifold $R^2 \times S^1$
- effective potential of the Polyakov loop
  - gluonic part of the eff. potential:
    - deconfinement phase transition $T_C \simeq 275 \ldots 285\, MeV$
      - SU(2): 2.order
      - SU(3): 1.order
  - inclusion of quarks:
    - the deconfinement phase transition is turned into a crossover
- dual & chiral quark condensate
- fully self-consistent unquenched calculation have still to be done

Thanks for your attention