# Hagedorn temperature and physiccs of black holes

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# "References"

Black holes + String Theory + Entropy
is an old and intensely studied subject, e.g., book
L. Susskind "An Introduction To Black Holes, Information And The String Theory Revolution: The Holographic Universe "

This talk is mostly a review, original statements are based on 7 papers (and references therein) by
Thomas Mertens (Gent Uni → Princeton Uni),
Henri Verschelde (Gent Uni), VIZ

1305.7443, 1307.3491; 1402.2808;1408.6999; 1408.7012; 1410.8009, 1505.07798 published in JHEP

Thesis by Th. Mertens "Hagedorn String Thermodynamics in Curved Spacetimes and near Black Hole Horizons"

# Outline of the talk

- Limiting temperature,
  in hadronic world and Black Holes (BH)
- Stringy BH horizon
- Holography as a bridge between gravity and strong-interactions phenomenology
- Conclusions

# Hagedorn temperature

Ralf Hagedorn in 1965 Nuovo Cim. Suppl. 3 147 suggested density of states grow exponentially with energy at large energy:

$$\omega(E) \sim \exp\left(\beta_H E\right) \text{ , where } \beta_H \sim m_\pi^{-1}$$

Then partition function

$$z = \int_0^\infty dE\omega(E)e^{-\beta E}$$

exists only as far as $\beta > \beta_H$.

# Hagedorn temperature, cnt'd

In another language, there is a “limiting temperature"

$$\frac{1}{T} = \frac{\partial S}{\partial E}$$

and

$$T < T_H \,, \qquad T_H \equiv \frac{1}{\beta_H} = const$$

Physics: if we pump energy into the system, new higher-mass states are produced rather than the energy of already existing states is increased

# Hagedorn temparature and strings

This growth is reproduced by strings/Regge trajectories

$$E_{string} = \sigma \cdot L, \quad \sigma \equiv (2\pi\alpha^{'})^{-1}$$

where $\sigma$ is tension, $L$ is the length. For rotating string

$$M^2 = \frac{1}{\alpha^{'}} J$$

where $J$ is the angular total momentum.
The density of states is indeed exponential at high energy:

$$\omega(E) \sim \frac{\exp(\beta_H E)}{E^{1+D/2}} \, ,$$

where $D$ is the number of (non-compact) spatial directions,

# Limiting temperture vs phase transition

In reality, instead of the limiting temperature, there is deconfining phase transition, from composite hadrons to fundamental quarks and gluons

In other words, we learn that at higher temperatures, or short distances it is field theory which is fundamental, not hadronic strings

As we see next, Quantum Field Theory (Q.F.T.) becomes problematic at gravitational scale,
as is revealed by consideration of Black Holes

# Black Holes

Shwarzschild geometry

$$ds^2 = -\left(1 - \frac{2G_N M}{r}\right) dt^2 + \left(1 - \frac{2G_N M}{r}\right)^{-1} + d\mathbf{x}_\perp^2$$

$G_{00}$ component vanishes at the horizon, $r_H = 2G_N M$
Thermodynamic entropy is proportinal to the Area of BH:

$$S_{BH} = \frac{Area}{4G_N}$$

and there is Hawking radiation with temperature

$$\beta_{Hawking} = 8\pi G_N M$$

# Near-Horizon geometry

Introduce distance to the horizon
$\rho = \sqrt{8G_N M(r-2G_N M)}$ Then for $\rho \ll 4GM$

$$ds^2_{Rindler} = -\frac{\rho^2}{(4G_N M)^2}dt^2 + d\rho^2 + d\mathbf{x}^2_\perp$$

Many results apply just in this limit.
For Euclidean time $\tau$

$$ds^2_{Euclidean} = \frac{\rho^2}{(4G_N M)^2}d\tau^2 + d\rho^2 + d\mathbf{x}^2_\perp$$

which is flat space in polar coordinates, for $\tau$ periodic

$$\tau \sim \tau + \beta_{Rindler},$$

where

$$\beta_{Rindler} = 8\pi GM \equiv \beta_{Hawking} \tag{1}$$

# Blue-Shift factor

BH provides a lab to study temperatures arbitrarily high
Near horizon the Blue-shift factor

$$\chi \equiv \frac{4G_N M}{\rho} ,$$

where $\rho$ is the distance to the horizon, $M$ is the BH mass.
Hence

$$\beta_{local} = \beta_{Rindler}\chi^{-1}, \quad \beta_{local} \to 0, \quad if \quad \rho \to 0. \tag{2}$$

Overall Euclidean thermal manifold is cigar-shaped

# BH and limiting temperature

In Q.F.T. the entropy density $s \sim T^3$
and the total entropy

$$S \sim \int d\rho T^3 = \frac{Area}{\epsilon^2} , \tag{3}$$

where $\epsilon$ is small-distance or UV cut off.
Not to exceed the BH entropy, $S_{BH} = (area)/4G_N$
need limiting temperature (brick wall of 't Hooft)
Need modification of Q.F.T. at short distances
Strings are welcome back on the fundamental level

# Stretched Horizon

Stretched horizon is a surface placed close to the actual horizon, in front of it, such that $g_{00} \ll 1$
Introduced for two, actually different reasons

- as a matter of convenience, phenomenology of BH
  “Membrane paradigm"of T. Damour (1978)...
  M. Parikh and F. Wilczek (1997)
- As a matter of principle, UV cut off on validity of field theory, ('t Hooft (1993), Susskind+ (1993)...)
  as we have just discussed

# Conclusion to Introduction

Limiting temperature, modification of QFT favored by BH. (Actually, very simple and straightforward reasoning)

To continue with QFT one switches to stretched horizon

# Part II: Stringy Horizon

Consider BH formation by throwing matter focused inside

For a distant observer matter falls infiitely long

Long-string picture (of L. Susskind):
near the horizon, $\rho \sim l_s$ there is single long string. Hope:

$$S_{long\ string} = S_{BH} \qquad (?)$$

Picture motivated by the membrane paradigm, need for the limiting temperature, elements of string theory

Advantages:

- UV divergence is resolved by $l_s \neq 0$
- $S_{BH} \sim (Area)$ comes out naturally

Caution: BH physics depends on observer.

# Many questions left

Further questions left:

- What keeps the long string at $\rho \sim l_s$?
- How to get quantitatively $S = (Area)/4G_N$?
- Qualitative picture vs fundamental strings?

# Main results

(Somewhat modified) picture woks on fundamental level:

- For type II Superstrings in Rindler space ( Large BH)
- For heterotic strings in Rindler space

and does not work for bosonic strings

The structure of the Euclidean stretched horizon is explicit in terms of a zero mode of a scalar field ,
or long string in a cigar-shape background

# Main tool: thermal scalar

A complementary view on the Hagedorn transition (in flat space):

- Hagedorn divergence due to high-mass states is equivalent to Higgs-type scalar particle instability, the scalar lives in spatial dimensions only

$$m^2_{thermal\ \ scalar} = \frac{(\beta - \beta_{Hagedorn})}{2\pi(\alpha')^2}$$

To show the equivalence is an easy exercise by using the polymer, or random-walk formulation of Euclidean field theory (Atik-Witten)

# Thermal scalar as a wrapped stringy state

Mass of the thermal scalar hits zero at $\beta = \beta_{Hagedorn}$.
What happens next–not clear apriori.

It is also straightforward to demonstrate that thermal scalar correspond to the string once wrapped around compact Euclidean time. The time dependence is fixed by periodicity and we are left with 3d coordinates

To work with BH one needs to workout generalizations of thermal scalar to curved space

# Thermal scalar in curved space

$$S = \int d^{D-1}x\sqrt{G}e^{-2\phi} \quad (4)$$

$$\cdot\left(G_{ij}\nabla^i\varphi\nabla^j\varphi^* + \frac{1}{4\pi^2(\alpha')^2}(\beta^2 G_{00} - \beta^2_{Hagedorn})\varphi\varphi^*\right)$$

In Rindler space ($a$ is the Rindler accelaration)

$$\left(-\partial_\rho^2 - \frac{1}{\rho}\partial_\rho + \frac{1}{4\pi^2(\alpha')^2}(\beta^2 a^2 \rho^2 - \beta^2_{Hagedorn})\right)\varphi_n(\rho) = \lambda_n \varphi(\rho)$$

Solutions:

$$\varphi_n(\rho) = \exp\left(-\frac{a\beta\rho^2}{4\pi\alpha'}\right)L_n\left(\frac{a\beta\rho^2}{2\pi\alpha'}\right), \; \lambda_n = (a\beta(1+2n) - 2\pi)$$

Zero mode at $\beta = \beta_{Hagedorn}$, (absent in flat space).
Dominates partition function.

# Picture emerging

Build up BH by throwing a thin shell of $\delta M$ to BH of mass $M_{initial}$. (A kind of mean-field approximation).
It ends up as a long string in a layer of thickness $\delta\rho \sim l_s$.
Density of states seen by the distant observer is

$$\omega(\delta M) \sim \frac{\exp(\beta_{Hawking}\delta M)}{\delta M}, \text{ or } \beta_{Hawking} = \beta_{Hagedorn}$$

Integrated to the Bekenstein entropy:

$$\delta S_{BH} = 8\pi G_N M \delta M \rightarrow S_{BH} = (Area)/4G_N$$

Does not fall onto BH because of the entropic (or QM) pressure, somewhat similar to Earth atmosphere.
Zero mode is exact in $\alpha'$ in Rindler case, supestrings

# Conclusions to part II

The idea that the Hagedorn temperature provides the limiting temperature for BH is supported by some first-principle calculations within string theory.

# Holography as a messanger from strings

Amusingly enough, lessons from strings on the gravitational scale might be adjusted to Yang-Mills theories (which we started from)

The means is holography: strings live in curved extra dimensions, while gauge theory lives on a flat boundary. Most famous, is the duality for $N = 4$ $SUSY$ $YM$

In case of ordinary YM, Witten constructed a model which is in the same universality class in infrared as large-$N_c$ YM. Generalized to incorporate quarks (Sakai-Sugimoto model).

From our perspective, it is crucial that the geometry in extra dimensions is the same cigar-shaped

# Compact dimensions

There are two compact dimensions:

- Euclidean time, periodic, $\tau \sim \tau + \beta_\tau(z)$ where periodocity depends on extra coordiante $z$, $z \to 0$ corresponds to Yang-Mills in UV, $z \to z_{horizon}$ correponds to YM in IR
- another coordinate $\sigma \sim \sigma + \beta_\sigma(z)$
  Wrapping around $\sigma$ counts the topological charge associated with the stringy state

From first principles, at $T = 0$ the $(\tau + z)$ space is a cylinder and $(\sigma + z)$ is cigar shaped, $\beta_\sigma(z_{horizon}) = 0$

At $T = T_{deconfinement}$ the geometries are interchanged

# Link to Yang-Mills phenomenology

For string $(action) \sim L \cdot (tension)$ and

$$\beta_\tau(z_H) = 0 \; or \; \beta_\sigma(z_H)) = 0$$

means vanishing classical action for wrapped states

At $T = 0$ this is true for instantons (topologically charged) in infrared, $z \to z_{horizon}$, True phenomenologically

At $T = T_{deconfinement}$ the $(\sigma + z)$ geometry is changed into a cylinder and instantons become surpressed; true and known

A novel feature: At $T = T_{deconfinement}$ the non-perturbative becomes 3d, instead of 4d at $T = 0$.

Dimensional reduction: phase transition at $T = T_{deconfnement}$

Specific for holography, and, probably, true on the lattice

# Phenomenological echo of the zero mode

Classically, action for defects,

$$S_{defect} = L \cdot (Tension)$$

vanishes for any wrapping number.
QM only lowest level survives as zero mode at the tip of the cigar (see discussion above)
Phenomenologically, this means that only instantons with

$$Q_{topological} = \pm 1$$

are ample even in the infrared.
Probably supported by the lattice

# Further direction: action on the stretched horizon

Originally, stretched horizon was introduced as pure fictitious, to describe observations on BH from large distance. There is fictitious liquid living on the stretched horizon. Now, duality converts properties of fictitious liquid into properties of real YN liquid in infrared.

A well-known example is prediction

$$\frac{\eta}{s} \geq \frac{1}{4\pi}$$

where $\eta$ is shear viscosity, $s$ is the entropy density

# Conclusions to part III

There is phenomenological feedback from fundamental, garvity-related strings to phenomenology of non-perturbative, or infrared physics of YM theory. But it is rather about isolated examples than about a well-developed mashinary